STA414: HW2 Q1 cde

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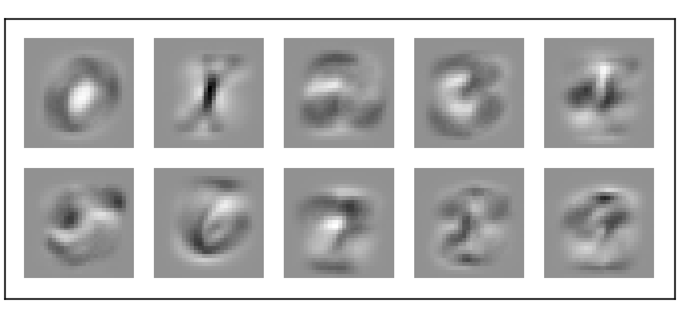
Q1(c): Report final training and test errors as well as the number of samples used in training.

Number of samples used was N = 60000.

Final training accuracy was 0.8943166666666666 = 89.43%.

Final test accuracy was 0.881 = 88.10%.

Q1(d): Figure containing each weight w_k as an image.



Q1(e): Code for Q1.

In [1]:

```python
from __future__ import absolute_import
from __future__ import print_function
from future.standard_library import install_aliases
install_aliases()
import numpy as np
from scipy.special import logsumexp
import os
import gzip
import struct
import array
import matplotlib.pyplot as plt
import matplotlib.image
from urllib.request import urlretrieve

def download(url, filename):
    if not os.path.exists('data'):
        os.makedirs('data')
    out_file = os.path.join('data', filename)
    if not os.path.isfile(out_file):
        urlretrieve(url, out_file)


def mnist():
    base_url = 'http://yann.lecun.com/exdb/mnist/'

    def parse_labels(filename):
        with gzip.open(filename, 'rb') as fh:
            magic, num_data = struct.unpack(">II", fh.read(8))
            return np.array(array.array("B", fh.read()), dtype=np.uint8)

    def parse_images(filename):
        with gzip.open(filename, 'rb') as fh:
            magic, num_data, rows, cols = struct.unpack(">IIII", fh.read(16))
            return np.array(array.array("B", fh.read()), dtype=np.uint8).reshape(num_data, rows, cols)

    for filename in ['train-images-idx3-ubyte.gz',
                     'train-labels-idx1-ubyte.gz',
                     't10k-images-idx3-ubyte.gz',
                     't10k-labels-idx1-ubyte.gz']:
        download(base_url + filename, filename)

    train_images = parse_images('data/train-images-idx3-ubyte.gz')
    train_labels = parse_labels('data/train-labels-idx1-ubyte.gz')
    test_images = parse_images('data/t10k-images-idx3-ubyte.gz')
    test_labels = parse_labels('data/t10k-labels-idx1-ubyte.gz')

    return train_images, train_labels, test_images[:1000], test_labels[:1000]


def load_mnist(N_data=None):
    partial_flatten = lambda x: np.reshape(x, (x.shape[0], np.prod(x.shape[1:])))
    one_hot = lambda x, k: np.array(x[:, None] == np.arange(k)[None, :], dtype=int)
    train_images, train_labels, test_images, test_labels = mnist()
    train_images = (partial_flatten(train_images) / 255.0 > .5).astype(float)
    test_images = (partial_flatten(test_images) / 255.0 > .5).astype(float)
    K_data = 10
    train_labels = one_hot(train_labels, K_data)
    test_labels = one_hot(test_labels, K_data)
    if N_data is not None:
        train_images = train_images[:N_data, :]
        train_labels = train_labels[:N_data, :]

    return train_images, train_labels, test_images, test_labels


def plot_images(images, ax, ims_per_row=5, padding=5, digit_dimensions=(28, 28),
                cmap=matplotlib.cm.binary, vmin=None, vmax=None):
    """Images should be a (N_images x pixels) matrix."""
    N_images = images.shape[0]
    N_rows = np.int32(np.ceil(float(N_images) / ims_per_row))
    pad_value = np.min(images.ravel())
    concat_images = np.full(((digit_dimensions[0] + padding) * N_rows + padding,
                             (digit_dimensions[1] + padding) * ims_per_row + padding), pad_value)
    for i in range(N_images):
        cur_image = np.reshape(images[i, :], digit_dimensions)
        row_ix = i // ims_per_row
        col_ix = i % ims_per_row
        row_start = padding + (padding + digit_dimensions[0]) * row_ix
        col_start = padding + (padding + digit_dimensions[1]) * col_ix
        concat_images[row_start: row_start + digit_dimensions[0],
                      col_start: col_start + digit_dimensions[1]] = cur_image
        cax = ax.matshow(concat_images, cmap=cmap, vmin=vmin, vmax=vmax)
        plt.xticks(np.array([]))
        plt.yticks(np.array([]))
    return cax


def save_images(images, filename, **kwargs):
    fig = plt.figure(1)
    fig.clf()
    ax = fig.add_subplot(111)
    plot_images(images, ax, **kwargs)
    fig.patch.set_visible(False)
    ax.patch.set_visible(False)
    plt.savefig(filename)


def train_log_regression(images, labels, learning_rate, max_iter):
    """ Used in Q1
        Inputs: train_images, train_labels, learning rate,
        and max num of iterations in gradient descent
        Returns the trained weights (w/o intercept)"""

    N_data, D_data = images.shape
    K_data = labels.shape[1]
    weights = np.zeros((D_data, K_data))

    # YOU NEED TO WRITE THIS PART

    for iter in range(max_iter):

        yi_hat = log_softmax(images, weights)
        grad = np.dot(images.T, np.exp(yi_hat) - labels)
        weights = weights - learning_rate * grad

    w0 = None  # No intercept for log-reg
    return weights, w0

def log_softmax(images, weights, w0=None):
    """ Used in Q1 and Q2
        Inputs: images, and weights
        Returns the log_softmax values."""
    if w0 is None: w0 = np.zeros(weights.shape[1])

    # YOU NEED TO WRITE THIS PART

    numerator = np.dot(images, weights) + w0
    denominator = logsumexp(numerator, axis=1)

    return numerator - denominator.reshape(-1, 1)

def cross_ent(train_labels, log_Y):
    """ Used in Q1
        Inputs: log of softmax values and training labels
        Returns the cross entropy."""

    # YOU NEED TO WRITE THIS PART
    # assume vectors

    return -np.dot(train_labels, log_Y)

def cross_ent_loss(images, labels, weights):
    """ Used in Q1
        Inputs: training images, training labels, and updated weights
        Returns: the cross entropy loss for each row of observations."""

    N = images.shape[0]
    loss = 0.0

    for i in range(N):
        xi_hat = images[i]
        yi = labels[i]

        yi_hat = log_softmax(xi_hat, weights)

        loss = loss + cross_ent(yi, yi_hat)

    return loss



def predict(log_softmax):
    """ Used in Q1 and Q2
        Inputs: matrix of log softmax values
        Returns the predictions"""

    # YOU NEED TO WRITE THIS PART


    return np.argmax(log_softmax, axis = 1)


def accuracy(log_softmax, labels):
    """ Used in Q1 and Q2
        Inputs: matrix of log softmax values and 1-of-K labels
        Returns the accuracy based on predictions from log likelihood values"""

    # YOU NEED TO WRITE THIS PART
    N = labels.shape[0]

    ytrue = np.argmax(labels, axis = 1)
    ypred = predict(log_softmax)

    correct = 0

    for i in range(N):
        if ypred[i] == ytrue[i]:
            correct = correct + 1

    return correct/len(ytrue)


def main():
    N_data = 60000 # Num of samples to be used in training
    # Set this to a small number while experimenting.
    # For log reg, finally use the entire training dataset for training (N_data=None).
    # For gda, use as many training samples as your computer can handle.

    train_images, train_labels, test_images, test_labels = load_mnist(N_data)

    # Q1: train logistic regression
    learning_rate, max_iter = .00001, 100
    weights, w0 = train_log_regression(train_images, train_labels, learning_rate, max_iter)
    save_images(weights.T, 'weights.png')

    # evaluation
    log_softmax_train = log_softmax(train_images, weights, w0)
    log_softmax_test = log_softmax(test_images, weights, w0)

    train_accuracy = accuracy(log_softmax_train, train_labels)
    test_accuracy = accuracy(log_softmax_test[:1000], test_labels[:1000])

    print("Training accuracy is ", train_accuracy)
    print("Test accuracy is ", test_accuracy)
    print(f"Number of samples used was {N_data}")



if __name__ == '__main__':
    main()
```

```
Training accuracy is  0.8943166666666666
Test accuracy is  0.881
Number of samples used was 60000
```