Sets and Parameters

- Jobs: $J = \{1,2,3,4,5\}$
- Machines: $M = \{1,2,3,4,5\}$
- Operations per job: $O = \{1,2,3\}$
- $p_{j,m}$: processing time of job j $(j \in J)$ on machine m $(m \in M)$
- $t_{m,n}$: transfer time from machine m to n $(m, n \in M)$

Decision Variables

- $x_{j,o,m} = \begin{cases} 1 & if\ operation\ o\ (o \in O)\ of\ job\ j\ (j \in J)\ is\ assigned\ to\ machine\ m\ (m \in M) \\ 0 & otherwise \end{cases}$

  $x_{j,o,m} \in \{0,1\}$

- $s_{j,o} \geq 0$: $start\ time\ of\ operation\ o\ (o \in O) of\ job\ j\ (j \in J)$
- $C_{max} = makespan\ (objective\ function\ to\ be\ minimized)$
- $y_{i,j,o,p,m} \in \{0,1\}$: $precedence\ between\ operations\ if\ they're\ on\ the\ same\ machine\ m$

  $y_{i,j,o,p,m} = \begin{cases} 1 & if\ operation\ o\ of\ job\ i\ is\ scheduled\ before\ operation\ p\ of\ job\ j\ (on\ machine\ m) \\ 0 & otherwise \end{cases}$

  $i, j \in J \quad , \quad o, p \in O \quad , \quad m \in M$

Objective

$$Minimize\ C_{max}$$

Constraints

1. **Each operation is assigned to exactly one machine:**

$$\sum_{m \in M} x_{j,o,m} = 1 \qquad \forall j, o \quad , j \in J, \ \ o \in O$$

2. **Job 5 must use machines M1 and M2:**

$$\sum_{o \in O} x_{5,o,1} \geq 1 \quad , \quad \sum_{o \in O} x_{5,o,1} \leq 2$$

$$\sum_{o \in O} x_{5,o,2} \geq 1 \quad , \quad \sum_{o \in O} x_{5,o,2} \leq 2$$

3. **Operation sequencing (avoiding overlap + considering transfer time):**

   Each electronic device (each job) cannot be on more than one machine at a time.

   For each job j, between operations o (which is done on machine m) and $o+1$ (which is done on machine n), we loop over all machine pairs (m, n), and for each combination, we write:

$$s_{j,o+1} \geq s_{j,o} + p_{j,m} + t_{m,n} - M.\left(2 - x_{j,o,m} - x_{j,o+1,n}\right), \quad \forall j \in J \;, m, n \in M$$

   If both $x_{j,o,m} = 1$ and $x_{j,o+1,n} = 1$, then the Big-M term vanishes and the constraint becomes active.

   If either machine is not used for those operations, the constraint becomes non-binding.

4. **Two operations on the same machine cannot be done at the same time:**

   If two operations (i, o) and (j, p) are assigned to the same machine m, then:

$$s_{i,o} + p_{i,m} \leq s_{j,p} + M.\left(1 - y_{i,j,o,p,m} + \left(2 - x_{i,o,m} - x_{j,p,m}\right)\right)$$

$$s_{j,p} + p_{j,m} \leq s_{i,o} + M.\left(y_{i,j,o,p,m} + \left(2 - x_{i,o,m} - x_{j,p,m}\right)\right)$$

$$i, j \in J \quad , \quad o, p \in O \quad , \quad m \in M$$

Because we don't know yet which operations are assigned to which machines (since that's decided by the optimization), we added terms to deactivate these constraints when the machine isn't assigned. So:

- If both operation (i, o) and (j, p) are assigned to machine $m \rightarrow \left(2 - x_{i,o,m} - x_{j,p,m}\right) = 0$ and constraint becomes active.
- If at least one operation is not on machine $m \rightarrow \left(2 - x_{i,o,m} - x_{j,p,m}\right) > 0$, then constraint becomes trivially true (inactive)

5. **Makespan constraint:**

   C_max must be >= end time of each job's last operation

$$C_{max} \geq s_{j,3} + \sum_{m \in M} x_{j,3,m} . p_{j,m} \qquad , \forall j \in J$$

6. $x_{j,o,m} \in \{0,1\}$
7. $y_{i,j,o,p,m} \in \{0,1\}$
8. $s_{j,o} \geq 0$
9. $C_{max} \geq 0$