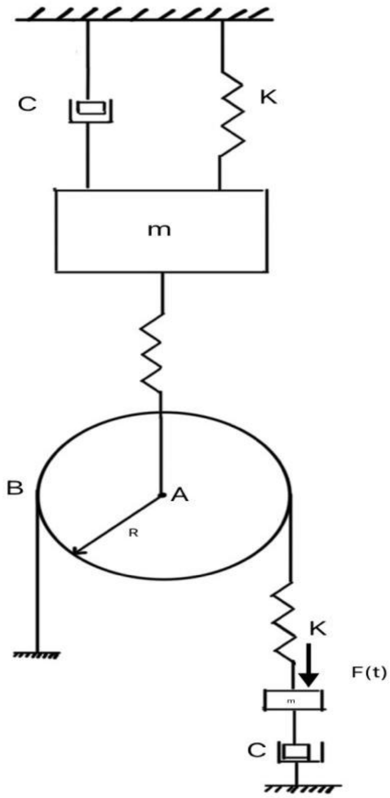

The governing equations of the system :

$$\sum F_z = m\alpha_z$$

$$[-c\dot{z} - kz - k(z-y)] = m\ddot{z}$$

$$\sum M_B = I_B\alpha$$

$$[k(z-y)R - k(2y-x)2R] = {}^3\!/_2\,(3m)R^2\,\frac{\ddot{y}}{R}$$

$$\sum F_x = m\alpha_x$$

$$[k(2y-x) + F(t) - c\dot{x}] = m\ddot{x}$$

Governing equations of the system for small oscillations around the equilibrium point：

$$m\ddot{x} + c\dot{x} + kx - 2ky = F(t)$$

$$\frac{9m}{2}\ddot{y} - 2kx + 5ky - kz = 0$$

$$m\ddot{z} + c\dot{z} - ky + 2kz = 0$$

The mass, stiffness, damping, and external excitation matrices are:

$$[M]_{3\times3} = \begin{bmatrix} m & 0 & 0 \\ 0 & \frac{9m}{2} & 0 \\ 0 & 0 & m \end{bmatrix}$$

$$[K]_{3\times3} = \begin{bmatrix} k & -2k & 0 \\ -2k & 5k & -k \\ 0 & -k & 2k \end{bmatrix}$$

$$[C]_{3\times3} = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\overrightarrow{F(t)} = \begin{bmatrix} F(t) \\ 0 \\ 0 \end{bmatrix}$$

For the state-space representation, we define the state variables x_1 to x_6 as follows:

$x_1 = x(t)$ $\quad$ $x_2 = y(t)$ $\quad$ $x_3 = z(t)$ $\quad$ $x_4 = \dot{x}(t)$ $\quad$ $x_5 = \dot{y}(t)$ $\quad$ $x_6 = \dot{z}(t)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = [M]_{3\times3}^{-1} \left\{ -[C]_{3\times3} \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} - [K]_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \cdots + \left[\overrightarrow{F(t)}_{3\times1} \right] \right\}$$

Since the system has 3 degrees of freedom, the definition of damping parameters does not have a specific meaning unless the parameters m, c, and k are considered for a single-degree-of-freedom system with the desired damping characteristic.

$$\begin{cases} Undamped\ state & \rightarrow \quad c = 0 \\ \xi = \dfrac{c}{2\sqrt{km}} = 0.25 & \rightarrow c = 0.5\sqrt{km} \\ \xi = \dfrac{c}{2\sqrt{km}} = \dfrac{\sqrt{2}}{2} & \rightarrow c = \sqrt{2km} \\ \xi = \dfrac{c}{2\sqrt{km}} = 1 & \rightarrow c = 2\sqrt{km} \end{cases}$$

$$\begin{cases} Undamped\ state \quad \rightarrow \quad c = 0 \\ \omega = 3\omega_{ni} \\ \xi = 0.25 \\ \xi = \dfrac{\sqrt{2}}{2} \end{cases}$$

Resonance occurs when the external excitation frequency is exactly equal to one of the natural frequencies of the system.

Resonant condition occurs when the natural frequencies of the system are very close to each other, and the external excitation frequency can cause synchronizing oscillations and increase the amplitude, even if the excitation frequency is not exactly equal to one of the natural frequencies.

$$\det([K] - \omega^2[M]) = \det\left(k\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & -1 \\ 0 & -1 & 2 \end{bmatrix} - m\omega^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 9/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\lambda = \frac{m\omega^2}{k} \quad \rightarrow \quad \begin{bmatrix} 1-\lambda & -2 & 0 \\ -2 & 5-\dfrac{9}{2}\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)\begin{bmatrix} 5-\dfrac{9}{2}\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 0 & 2-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)\left(\frac{9}{2}\lambda^2 - 14\lambda + 9\right) + 4(\lambda - 2) = 0$$

$$-\frac{9}{2}\lambda^3+\frac{37}{2}\lambda^2-19\lambda+1=0 \quad \rightarrow \quad \begin{cases}\lambda_1=0.0556\\ \lambda_2=1.6886\\ \lambda_3=2.3670\end{cases} \quad \rightarrow \begin{cases}\omega_1\cong\sqrt{0.0556\dfrac{k}{m}}\cong 0.2358\sqrt{\dfrac{k}{m}}\\ \omega_2\cong\sqrt{1.6886\dfrac{k}{m}}\cong 1.2994\sqrt{\dfrac{k}{m}}\\ \omega_3\cong\sqrt{2.3670\dfrac{k}{m}}\cong 1.5385\sqrt{\dfrac{k}{m}}\end{cases}$$