Isfahan University of Technology
Game Theory
Dr. Manshaei
Homework #1
Due date: Friday, 5th Aban 1402, at 23:59

We only accept the homework **delivered via *Yekta*, before the deadline**. If you have any questions or concerns about this homework, feel free to contact Mr. MohammadReza Mirdamadiyan via *Telegram* (Preferred) or *Email*.

Problem 1. Consider the following Extended Prisoner's Dilemma game:

	confess	don't confess	run away
confess	-5,-5	0,-6	-5,-10
don't confess	-6,0	-1,-1	0,-10
run away	-10,-6	-10,0	-10,-10

Find the result by iteratively eliminating all the strictly dominated strategies. State the rationality/knowledge assumptions corresponding to each elimination.

Problem 2. Consider a 2-player game in which each player announces an integer between 0 and 100. Let a_1 be the announcement of Player 1 and a_2 be the announcement of Player 2. The payoffs are determined as follows:

- If $a_1 + a_2 \leq 100$: Player 1 receives a_1 and Player 2 receives a_2.
- If $a_1 + a_2 > 100$ and $a_1 < a_2$: Player 1 receives a_1 and Player 2 receives $100 - a_1$.
- If $a_1 + a_2 > 100$ and $a_1 > a_2$: Player 1 receives $100 - a_2$ and Player 2 receives a_2.
- If $a_1 + a_2 > 100$ and $a_1 = a_2$: Both players receive 50.

Solve this game with iterative deletion of dominated strategies.

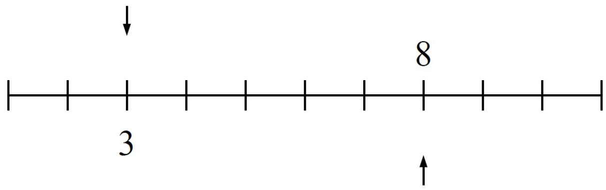
Problem 3. The members of a group of people are affected by a policy, modeled as a number. Each person i has a favorite policy, denoted x_i^*. She prefers the policy y to the policy z if and only if y is closer to x_i^* than is z. The number n of people is odd. The following mechanism is used to choose a policy: each person names a policy, and the policy chosen is the *median* of those named. (That is, the policies named are put in order, and the one in the middle is chosen. If, for example, there are five people, and they name the policies -3, 0.2, 1, 4, and 9, then the policy 1 is chosen.)

(a) Show that for each player i, the action of naming her favorite policy x_i^* weakly dominates all her other actions.

(b) Consider the variant of the mechanism described above in which the policy chosen is the *mean*, rather than the *median*, of the policies named by the players. Does a player's action of naming her favorite policy weakly dominate all her other actions?

Problem 4. Assume that the players are two vendors who simultaneously choose a location. Then the customers choose the closest vendor. The profit for each vendor equals the number of customers it attracted.

To be more specific, we assume that the vendors choose a location from the set $\{1, \ldots, n\}$ of natural numbers, viewed as points on a real line, and that at each location there lives exactly one customer. For example, for $n = 11$ we have 11 locations, and if the players choose respectively the locations 3 and 8, we have $p_1(3,8) = 5$ and $p_2(3,8) = 6$.



If a customer has an equal distance from both vendors, we assume that he is 'shared' between the vendors. For example, if the vendors choose the locations 3 and 7 respectively, they share the customer living in location 5. Hence, they end up with fractional payoffs $p_1(3,7) = 4.5$ and $p_2(3,7) = 6.5$.

(a) Write the payoff function $p_i(s_i, s_{-i})$ for $n \in \mathbb{N}$.

(b) Find the outcome of this game assuming $n = 2k - 1$.

(c) Is the outcome unique for all $n \in \mathbb{N}$?

Problem 5. *Secret Sharing* in threshold cryptography and its failure in presence of rational parties has been discussed in the class. Do a little research about the reason: Write the exact scenario and payoff functions, and explain why it does not survive iterated deletion of weakly dominated strategies. No need to mention any kind of cryptographic formulation.

Good Luck.