Isfahan University of Technology
Game Theory
Dr. Manshaei
Homework #6
Due date: Friday, Dey 22^{nd} 1402, at 23:59

We only accept the homework **delivered via *Yekta*, before the deadline**. If you have any questions or concerns about this homework, feel free to contact Danial Khorasanizadeh via *Telegram* (Preferred) or *Email*.

Problem 1. Alex is deciding whether or not to make a loan to Brian who is poor and has a bad credit history. Simultaneous to Alex making this decision, Brian must decide whether or not to buy himself clothes. If he buys them, he will be unable to repay the loan. If he does not buy clothes, he will repay the loan. If Alex refuses to give Brian a loan, then Brian will have to go to a loan shark. The payoffs in this game are as follows: if Alex refuses to make a loan to Brian and Brian buys clothes then both Alex and Brian get 0. If Alex refuses to make a loan to Brian and Brian does not buy clothes then Alex gets 0 and Brian gets -1. If Alex makes a loan to Brian and Brian buys clothes then Alex gets -2 and Brian gets 7. If Alex makes a loan to Brian and Brian does not buy clothes, then Alex gets a payoff of 3 and Brian gets a payoff of 5.

(a) Suppose this game is played just once. Find the Nash equilibria of the game.

Now suppose that the game is repeated. Suppose that (for all players) a dollar tomorrow is worth 2/3 of a dollar today. In addition, suppose that, after each period (and regardless of what happened in the period), Brian has a 1/2 chance of escaping poverty. Assume that, if Brian escapes poverty then he will not need a loan from either Alex or a loan shark: if effect, Brian will exit the game. Assume that, if Brian escapes poverty, he will never return. Thus, after each period, there is only 1/2 chance of the game continuing. Given this, the effective discount factor for the game between Alex and Brian is $\frac{1}{2}\frac{2}{3} = \frac{1}{3}$. Consider the following strategy profile. In period one, Alex makes Brian a loan. Thereafter, Alex continues to make Brian loans (if he is still poor) and as long as Brian has always got a loan and repaid it in the past. But if Brian ever does not repay (or does not get a loan) then Alex never makes a loan to Brian again. In period one, Brian does not buy clothes (and hence repays the loan if he gets one). Thereafter (as long as he is still poor), Brian does not buy clothes (and hence repays the loan if he gets one) as long as he has always got a loan and repaid it in the past. But if Brian ever does not repay (or does not get a loan) then he will return to buying clothes and hence never repay a loan again.

(b) Is this strategy profile an SPNE of the repeated game?

(c) Suppose that the government introduces regulation of loan sharks. As a consequence, Brian's payoff in each period in which he still needs a loan but does not get it from Alex is 1 if he does not buy clothes and 2 if he buys clothes. Explain whether or not this policy is likely to be good for Brian.

(d) Suppose that the government abandons its loan-shark policy and replaces it with a job scheme that increases the probability after each period of Brian escaping poverty to 2/3 (i.e., 1/3 chance of returning to the loan game). Explain the likely consequences of this policy for the business relationship between Alex and Brian.

Problem 2. Find all Nash equilibria and determine which of the symmetric equilibria are evolutionarily stable in the following games:

$$\begin{bmatrix} (4,4) & (2,5) \\ (5,2) & (3,3) \end{bmatrix} \begin{bmatrix} (4,4) & (3,2) \\ (2,3) & (5,5) \end{bmatrix}$$

Problem 3. Consider the following game:

	A	B	C
A	(0,0)	(6,2)	(-1,-1)
B	(2,6)	(0,0)	(3,9)
C	(-1,-1)	(9,3)	(0,0)

Find two mixed Nash equilibria, one supported on {A, B}, the other supported on {B, C}. Show that they are both ESS, but the {A, B} equilibrium is not stable when invaded by an arbitrarily small population composed of half B's and half C's.

Problem 4. In the escalation, two countries are on the brink of war. Player 1 begins by accepting the status quo or issuing a threat. If he accepts the status quo, the game ends. If he threatens, player 2 decides whether to concede or escalate the conflict. The game ends if she concedes. If she escalates, player 1 chooses whether to launch war or back down. Either way, the game ends. If player 1 accepts the status quo, each player earns 0. If player 2 concedes, player 1 makes a slight gain. Meanwhile, player 2 receives a slight loss and suffers a diminished reputation from the concession. Thus, player 1 earns 1 for this outcome, while player 2 earns -2. If player 2 escalates and player 1 backs down, the situation is reversed, and player 1 earns -2 while player 2 earns 1. Finally, if player 1 ultimately declares war, both sides suffer losses but save their reputations, giving both a payoff of -1. Write the game in extensive form then find the Nash equilibria of the game using backward induction.

Problem 5. (Bonus Question) Fish being sold at the market is fresh with probability $\frac{2}{3}$ and old otherwise, and the customer knows this. The seller knows whether the particular fish on sale now is fresh or old. The customer asks the fish-seller whether the fish is fresh, the seller answers, and then the customer decides to buy the fish or to leave without buying it. The price asked for the fish is \$12. It is worth \$15 to the customer if fresh and nothing if it is old. Thus, if the customer buys a fresh fish, her gain is \$3. The seller bought the fish for \$6, and if it remains unsold, then he can sell it to another seller for the same \$6 if it is fresh, and he has to throw it out if it is old. On the other hand, if the fish is old, the seller claims it to be fresh, and the customer buys it, then the seller loses \$R in reputation. Draw the game tree for this Bayesian game and determine the normal-form representation of the game. Then find the nash equilibria of the game in two cases: $R < 12$ and $R \geq 6$.

Good Luck.