Isfahan University of Technology
Game Theory
Dr. Manshaei
Homework #2
Due date: Friday, Aban 19th 1402, at 23:59

We only accept the homework **delivered via *Yekta*, before the deadline**. If you have any questions or concerns about this homework, feel free to contact Mr. Khorasanizadeh via *Telegram* (Preferred) or *Email*.

Problem 1. Let's examine the scenario of a game called Chicken where two drivers are driving towards each other at high speed. Inevitably, a collision will occur unless one of them decides to chickens out at the very last moment. If both drivers chicken out, there is no collision and they each receive a payoff of 1. If one driver swerves while the keeps driving, the non-swerving driver achieves a great success with a payoff of 2, while the swerving driver faces a great disgrace with a penalty of 1. However, if both drivers refuse to back down and neither swerves, a disastrous collision ensues, resulting in a substantial penalty denoted as M. Find the representation of this game in matrix form and determine its pure Nash equilibria.

Problem 2. Three firms will either pollute a lake in the following year or purify it. They pay 1 unit to purify, but it is free to pollute. If two or more pollute, then the water in the lake is useless, and each firm must pay 3 units to obtain the water that they need from elsewhere. If at most one firm pollutes, then the water is usable, and the firms incur no further costs. Represent this game in matrix form and determine its pure Nash Equilibria. (Hint: Write two matrices, one for the case where firm 1 purifies and one for the case where firm 1 pollutes.)

Problem 3. Consider another version of the investment game. As we discussed in class, in the investment game, every player has a choice between investing \$10 or not investing at all. In this version of the game, if more than 90% of the population invests, everyone (even the people who didn't invest) gets a \$5 payoff. However, if less than 90% of the population invests, people who had invested lose \$10. Analyze this game and talk about its pure Nash equilibrium.

Problem 4. There are n voters, of which k support candidate A and $m = n - k$ support candidate B. Each voter can either vote for his preferred candidate or abstain. Each voter gets a payoff of 2 if his preferred candidate wins, 1 if the candidates tie, and 0 if his candidate loses. If the citizen votes, he pays a cost $c \in (0, 1)$.

(a) What is the game with $m = k = 1$?

(b) Find the pure strategy Nash equilibria for $k = m$.

(c) Find the pure strategy Nash equilibria for $k < m$.

Problem 5. Omega is a robot that lives in space and amuses itself by rocketing down to Earth and giving humans difficult problems to work on. Today, Omega has chosen you as the target of its game. Omega appears in front of you with two boxes, labeled A and B, and places them on the ground in front of you. It tells you that it has placed a certain amount of money in each of the boxes. Specifically, box A is transparent and has \$1,000 in it, and box B is opaque and has either \$1,000,000 or \$0 in it (you don't know which). Omega says you have the option of choosing to open box B or both boxes A and B. But there's a twist, if Omega has predicted that you'll take both boxes A and B, then box B contains nothing and if it has predicted that you take only box B, then box B contains \$1,000,000. Write this game in matrix form and discuss the strategies that you can choose (based on your certainty of Omega's prediction) and the reasoning behind them. Then calculate payoffs for each of the strategies.

Good Luck.