Isfahan University of Technology

Game Theory Dr. Manshaei Homework #5

Due date: Tuesday, Azar 28th 1402, at 23:59



We only accept the homework **delivered via** *Yekta*, **before the deadline**. If you have any questions or concerns about this homework, feel free to contact Mr. Ravaee via *Telegram* (Preferred) or *Email*.

Problem 1. Three oligopolists operate in a market with inverse demand function given by P(Q) = a - Q where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has constant marginal cost of production, c, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 > 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3 . Find the subgame perfect outcome.

Problem 2. Consider a two player game in which player 1 can choose A or B. The game ends if he chooses A while it continues to player 2 if he chooses B. Player 2 can then choose C or D, with the game ending after C and continuing again with player 1 after D. Player 1 then can choose E or E, and the game ends after each of these choices.

- (a) Model this as an extensive form game tree. Is it a game of perfect or imperfect information?
- (b) How many terminal nodes does the game have? How many information sets?
- (c) How many pure strategies does each player have?
- (d) Imagine that the payoffs following choice A by player 1 are (2,0), following C by player 2 are (3,1), following E by player 1 are (0,0) and following E by player 1 are (1,2). What are the Nash equilibria of this game? Does one strike you as more "appealing" than the other? If so, explain why.

Problem 3. Consider the following normal form game where Player 1 chooses the row (either T or B), Player 2 chooses the column (either r or l), Player 3 chooses the table (either R or L)

		Player 3					
		L			R		
		Player 2			Player 2		
		- 1	r		- 1	r	
Player 1	Т	1, 1, 1	0, 0, 0		0, 0, 0	0, 0, 0	
	В	0, 0, 0	0, 0, 0		0, 0, 0	4, 4, 4	

- (a) Find all Nash equilibria in pure strategies
- (b) Assume that player 1 moves first, then player 2 and finally player 3; every player, before to play, observes the choices of the predecessors.
 - Represent the game using the extensive form
 - Find all subgame perfect Nash equilibria
- (c) Assume that player 3 is not able to see the choice of player 2
 - Represent the game using the extensive form

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• Find all subgame perfect Nash equilibria

Problem 4. Tariffs and imperfect international competition: There are two identical countries denoted by i = 1, 2. One homogeneous good is produced in each country by a firm, firm i in country i. A share h_i of this product is sold in the home market and a share e_i is exported in the other country. Governments choose tariffs, i.e., a tax on the import. Government of country i chooses tariff t_i . In country i the inverse demand function is $P_i(Q_i) = a - Q_i$, where $Q_i = h_i + e_j$. The firm's payoff (profits) is $\pi = [a - h_i - e_j]h_i + [a - h_j - e_i]e_i - c[h_i + e_i] - t_je_i$ where c > 0 is the marginal cost. The government's payoff is $W_i = 0.5Q_i^2 + \pi + t_ie_j$.

Timing: Governments simultaneously choose tariffs (t_1,t_2) ; Firms observe (t_1,t_2) and simultaneously choose quantities (h_1, e_1) (h_2, e_2) .

Find the backward induction outcome and describe the subgame perfect Nash equilibrium

Good Luck.