تکلیف سری سوم رباتیک

مهسا امینی ۹۸۱۷۸۲۳

سوال سه

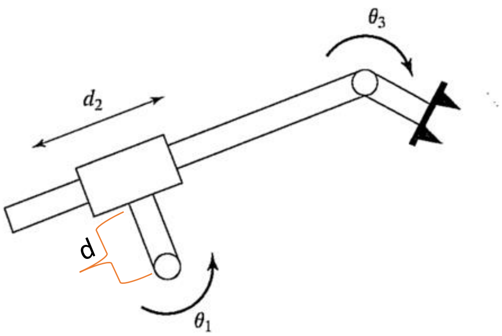

FIGURE 3.36: RPR planar robot (Exercise 3.16).

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_3 & -c\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

میخواهیم سرعت سرپنجه را نسبت به قاب صفر بیان کنیم.

در ابتدا یک محور دورانی داریم.

$$ {}^{i+1}w_{i+1} = {}^{i+1}_{i}R\, {}^{i}w_i + \dot{\theta}_{i+1}\, {}^{i+1}Z_{i+1} $$

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R\ ({}^{i}v_i + {}^{i}w_i \times {}^{i}P_{i+1})$$

$$^{1}w_1 = {}^{1}_{0}R\,{}^{0}w_0 + \dot{\theta}_1\,{}^{1}Z_1$$

$$^{0}w_0 = 0$$

$$^{1}w_1 = \dot{\theta}_1\,{}^{1}Z_1$$

$$^{1}w_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$^{1}v_1 = {}^{1}_{0}R({}^{0}v_0 + {}^{0}w_0 \times {}^{0}P_1)$$

$$^{0}v_0 = 0$$

$$^{0}w_0$$

$$^{1}v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

مفصل دوم یک مفصل منشوری است.

$$^{i+1}w_{i+1} = {}^{i+1}_{i}R\,{}^{i}w_i$$

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R\ ({}^{i}v_i + {}^{i}w_i \times {}^{i}P_{i+1}) + \dot{d}_{i+1}\,{}^{i+1}Z_{i+1}$$

$$^{2}w_2 = {}^{2}_{1}R\,{}^{1}w_1$$

$${}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}_2^1R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} $$

$$ {}_1^2R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} $$

$$ {}^2w_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} $$

$$ {}^2v_2 = {}_1^2R\ ({}^1v_1 + {}^1w_1 \times {}^1P_2) + \dot{d}_2\, {}^2Z_2 $$

$$ {}^2v_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} d \\ -d_2 \\ 0 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 0 \\ \dot{d}_2 \end{bmatrix} $$

$$ {}^2v_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} d_2\dot{\theta}_1 \\ d\dot{\theta}_1 \\ \dot{d}_2 \end{bmatrix} = \begin{bmatrix} d_2\dot{\theta}_1 \\ \dot{d}_2 \\ -d\dot{\theta}_1 \end{bmatrix} $$

مفصل سوم یک مفصل دورانی است.

$$ {}^3w_3 = {}_2^3R\, {}^2w_2 + \dot{\theta}_3\, {}^3Z_3 $$

$$ {}_3^2T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_3 & -c\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_3^2R = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ 0 & 0 & 1 \\ -s\theta_3 & -c\theta_3 & 0 \end{bmatrix} $$

$$ {}_2^3R = \begin{bmatrix} c\theta_3 & 0 & -s\theta_3 \\ -s\theta_3 & 0 & -c\theta_3 \\ 0 & 1 & 0 \end{bmatrix} $$

$$ {}^3w_3 = \begin{bmatrix} c\theta_3 & 0 & -s\theta_3 \\ -s\theta_3 & 0 & -c\theta_3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_3 \end{bmatrix} $$

$$^{3}v_3 = {}^{3}_{2}R\ ({}^{2}v_2 + {}^{2}w_2 \times {}^{2}P_3)$$

$$^{3}v_3 = \begin{bmatrix} c\theta_3 & 0 & -s\theta_3 \\ -s\theta_3 & 0 & -c\theta_3 \\ 0 & 1 & 0 \end{bmatrix} \left(\begin{bmatrix} d_2\dot{\theta}_1 \\ \dot{d}_2 \\ -d\dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} c\theta_3 d_2\dot{\theta}_1 + s\theta_3 d\dot{\theta}_1 \\ -s\theta_3\dot{d}_2 + c\theta_3 d\dot{\theta}_1 \\ \dot{d}_2 \end{bmatrix}$$

$$^{0}v_3 = {}^{0}_{3}R\, {}^{3}v_3$$

$${}^{0}_{3}R = {}^{0}_{1}R\, {}^{1}_{2}R\, {}^{2}_{3}R$$

$${}^{0}_{1}R = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}_{2}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^{2}_{3}R = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ 0 & 0 & 1 \\ -s\theta_3 & -c\theta_3 & 0 \end{bmatrix}$$

$${}^{0}_{3}R = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ 0 & 0 & 1 \\ -s\theta_3 & -c\theta_3 & 0 \end{bmatrix}$$

$${}^{0}_{3}R = \begin{bmatrix} c\theta_{13} & -s\theta_{13} & 0 \\ s\theta_{13} & c\theta_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{0}v_3 = \begin{bmatrix} c\theta_{13} & -s\theta_{13} & 0 \\ s\theta_{13} & c\theta_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 d_2\dot{\theta}_1 + s\theta_3 d\dot{\theta}_1 \\ -s\theta_3\dot{d}_2 + c\theta_3 d\dot{\theta}_1 \\ \dot{d}_2 \end{bmatrix} = \begin{bmatrix} c\theta_{13}(c\theta_3 d_2\dot{\theta}_1 + s\theta_3 d\dot{\theta}_1) - s\theta_{13}(-s\theta_3\dot{d}_2 + c\theta_3 d\dot{\theta}_1) \\ s\theta_{13}(c\theta_3 d_2\dot{\theta}_1 + s\theta_3 d\dot{\theta}_1) + c\theta_{13}(-s\theta_3\dot{d}_2 + c\theta_3 d\dot{\theta}_1) \\ \dot{d}_2 \end{bmatrix}$$

$$^{0}v = {}^{0}J(\Theta)\dot{\Theta}$$

$$^{0}v = \begin{bmatrix} ^{0}v \\ ^{0}\omega \end{bmatrix}$$

$$ {}^{3}v_3 = \begin{bmatrix} c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1 \\ -s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1 \\ \dot{d}_2 \end{bmatrix} $$

$$ {}^{3}J(\Theta) = \begin{bmatrix} \dfrac{\partial(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1)}{\partial \dot{\theta}_1} & \dfrac{\partial(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1)}{\partial \dot{\theta}_3} \\ \dfrac{\partial(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1)}{\partial \dot{\theta}_1} & \dfrac{\partial(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1)}{\partial \dot{\theta}_3} \end{bmatrix} = \begin{bmatrix} c\theta_3 d_2 + s\theta_3 d & 0 \\ c\theta_3 d & 0 \end{bmatrix} $$

$$ {}^{0}v_3 = \begin{bmatrix} c\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) - s\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1) \\ s\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) + c\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1) \\ \dot{d}_2 \end{bmatrix} $$

$$ {}^{0}J(\Theta) = \begin{bmatrix} \dfrac{\partial(c\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) - s\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_1} & \dfrac{\partial(c\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) - s\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_3} \\ \dfrac{\partial(s\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) + c\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_1} & \dfrac{\partial(s\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) + c\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_3} \end{bmatrix} $$

$$ {}^{0}v_3 = \begin{bmatrix} c\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) - s\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1) \\ s\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) + c\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1) \\ \dot{d}_2 \end{bmatrix} $$

$$ {}^{0}J(\Theta) = \begin{bmatrix} \dfrac{\partial(c\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) - s\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_1} & \dfrac{\partial(c\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) - s\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_3} \\ \dfrac{\partial(s\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) + c\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_1} & \dfrac{\partial(s\theta_{13}(c\theta_3 d_2 \dot{\theta}_1 + s\theta_3 d \dot{\theta}_1) + c\theta_{13}(-s\theta_3 \dot{d}_2 + c\theta_3 d \dot{\theta}_1))}{\partial \dot{\theta}_3} \end{bmatrix} $$

$$ {}^{0}J(\Theta) = \begin{bmatrix} c\theta_{13}(c\theta_3 d_2 + s\theta_3 d) - s\theta_{13} c\theta_3 d & 0 \\ s\theta_{13}(c\theta_3 d_2 + s\theta_3 d) + c\theta_{13} c\theta_3 d & 0 \end{bmatrix} $$