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1 Survival analysis for customer churn

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1.1 Problem statement

In this project I am building a survival model to predict customer churn for an internet & phone service company. The idea is to predict churn before it happens and try to prevent it by targeting at-risk customers for marketing campaigns offering a discount or a new package. Survival analysis can create an estimate of the risk of attrition over our time and that is exactly what we are interested in in this project.

The dataset includes information about:

- Services that each customer has signed up for – phone, multiple lines, internet, online security, online backup, device protection, tech support, and streaming TV and movies
- Customer account information – how long they have been a customer, contract, payment method, paperless billing, monthly charges, and total charges
- Demographic info about customers – gender, senior citizen or not, and if they have partners and dependents
- Customers who left within the last month – the column is called Churn

The dataset is available here (https://www.kaggle.com/blastchar/telco-customer-churn).

1.2 Why use survival analysis instead of linear or logistic regression?

- With linear regression (dependent variable = time of churn) we cannot use the data from the users who have not churned already (censored data). However, survival analysis uses that data.

- With logistic regression we either assume tenure does not have an effect on the churn probability (not a feature) or we assume after reaching some tenure, tenure will no longer have an effect on churn and customers who have been around long enough never tend to leave (sigmoid curve). However, survival analysis does not assume a particular shape for risk as a function of time.

1.3 Summary of results

In the first step, I have provided Kaplan Meier curves which allow for comparing different cohorts of customers in terms of survival probablity. Next, I built a Cox proportional hazard model for cutomers with internet and phone services (4835 customers). I have excluded a few features (monthly and total charges) from the model due to high multicollinearity with service types. My model achieved a concordance score of 0.95 and showed that 13 features were significant (alpha = 0.05).

For the next step, I plan to use regularization and cross validation to build a model with Lasso regularization for automatic feature selection. I will also have to work on checking the assumptions of the model.

2 Exploring the data

2.1 Import modules

In [1]:

```
import datetime as dt
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.dates as mdates
import matplotlib.pyplot as plt

from lifelines import CoxPHFitter
from lifelines import KaplanMeierFitter
from lifelines.plotting import plot_lifetimes

from patsy import dmatrices
from statsmodels.stats.outliers_influence import variance_inflation_factor

pd.set_option('display.max_columns', None)
sns.set_context("poster", font_scale=1.2)
```

2.2 Helper functions

In [2]:

```python
def print_full(x):
    ''' display the dataframe fully'''
    pd.set_option('display.max_rows', len(x))
    pd.set_option('display.max_columns', None)
    pd.set_option('display.width', 2000)
    pd.set_option('display.float_format', '{:20,.2f}'.format)
    pd.set_option('display.max_colwidth', None)
    print(x)
    pd.reset_option('display.max_rows')
    pd.reset_option('display.max_columns')
    pd.reset_option('display.width')
    pd.reset_option('display.float_format')
    pd.reset_option('display.max_colwidth')

def col_name_func(df):
    ''' format the column labels'''
    df.columns = [column.replace("(", "") for column in df.columns]
    df.columns = [column.replace(")", "") for column in df.columns]
    df.columns = [column.replace(" ", "_") for column in df.columns]
#     df.columns = [column.replace("/", "_") for column in df.columns]
#     df.columns = [column.lower() for column in df.columns]
```

2.3 Import data

In [3]:

```python
df = pd.read_csv("Telco-Customer-Churn.csv")
df.head()
```

Out[3]:

	customerID	gender	SeniorCitizen	Partner	Dependents	tenure	PhoneService	MultipleLines	InternetSe
0	7590-VHVEG	Female	0	Yes	No	1	No	No phone service	
1	5575-GNVDE	Male	0	No	No	34	Yes	No	
2	3668-QPYBK	Male	0	No	No	2	Yes	No	
3	7795-CFOCW	Male	0	No	No	45	No	No phone service	
4	9237-HQITU	Female	0	No	No	2	Yes	No	Fiber

In [4]:

```
## check for data types and nulls: TotalCharges has be converted to float
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 7043 entries, 0 to 7042
Data columns (total 21 columns):
 #   Column            Non-Null Count  Dtype  
---  ------            --------------  -----  
 0   customerID        7043 non-null   object 
 1   gender            7043 non-null   object 
 2   SeniorCitizen     7043 non-null   int64  
 3   Partner           7043 non-null   object 
 4   Dependents        7043 non-null   object 
 5   tenure            7043 non-null   int64  
 6   PhoneService      7043 non-null   object 
 7   MultipleLines     7043 non-null   object 
 8   InternetService   7043 non-null   object 
 9   OnlineSecurity    7043 non-null   object 
 10  OnlineBackup      7043 non-null   object 
 11  DeviceProtection  7043 non-null   object 
 12  TechSupport       7043 non-null   object 
 13  StreamingTV       7043 non-null   object 
 14  StreamingMovies   7043 non-null   object 
 15  Contract          7043 non-null   object 
 16  PaperlessBilling  7043 non-null   object 
 17  PaymentMethod     7043 non-null   object 
 18  MonthlyCharges    7043 non-null   float64
 19  TotalCharges      7043 non-null   object 
 20  Churn             7043 non-null   object 
dtypes: float64(1), int64(2), object(18)
memory usage: 1.1+ MB
```

In [5]:

```
## Convert TotalCharges to numeric
df['TotalCharges']=pd.to_numeric(df['TotalCharges'],errors='coerce')

## Replace Yes and No in the Churn column to 1 and 0. 1 for the event and 0 for the cens
df['Churn']=df['Churn'].apply(lambda x: 1 if x == 'Yes' else 0 )
```

In [6]:

```
## after converting the column TotalCharges to numeric
df.info()  ## Column TotalCharges has missing values
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 7043 entries, 0 to 7042
Data columns (total 21 columns):
 #   Column            Non-Null Count  Dtype  
---  ------            --------------  -----  
 0   customerID        7043 non-null   object 
 1   gender            7043 non-null   object 
 2   SeniorCitizen     7043 non-null   int64  
 3   Partner           7043 non-null   object 
 4   Dependents        7043 non-null   object 
 5   tenure            7043 non-null   int64  
 6   PhoneService      7043 non-null   object 
 7   MultipleLines     7043 non-null   object 
 8   InternetService   7043 non-null   object 
 9   OnlineSecurity    7043 non-null   object 
 10  OnlineBackup      7043 non-null   object 
 11  DeviceProtection  7043 non-null   object 
 12  TechSupport       7043 non-null   object 
 13  StreamingTV       7043 non-null   object 
 14  StreamingMovies   7043 non-null   object 
 15  Contract          7043 non-null   object 
 16  PaperlessBilling  7043 non-null   object 
 17  PaymentMethod     7043 non-null   object 
 18  MonthlyCharges    7043 non-null   float64
 19  TotalCharges      7032 non-null   float64
 20  Churn             7043 non-null   int64  
dtypes: float64(2), int64(3), object(16)
memory usage: 1.1+ MB
```

In [7]:

```
## Impute the null value with the median value
df.TotalCharges.fillna(value=df['TotalCharges'].median(),inplace=True)
```

In [8]:

```
## Create a list of categorical columns
cat_cols = [i  for i in df.columns if df[i].dtype==object]

## customerID has been removed because it is unique for all the rows.
cat_cols.remove('customerID')
```

In [9]:

```python
## lets have a look at the categories and their distribution in all the categorical colu
for i in cat_cols:
    print('Column Name: ',i)
    print(df[i].value_counts())
    print('-----------------------------')
```

```
Column Name:  gender
Male      3555
Female    3488
Name: gender, dtype: int64
-----------------------------
Column Name:  Partner
No     3641
Yes    3402
Name: Partner, dtype: int64
-----------------------------
Column Name:  Dependents
No     4933
Yes    2110
Name: Dependents, dtype: int64
-----------------------------
Column Name:  PhoneService
Yes    6361
No      682
Name: PhoneService, dtype: int64
-----------------------------
Column Name:  MultipleLines
No                  3390
Yes                 2971
No phone service     682
Name: MultipleLines, dtype: int64
-----------------------------
Column Name:  InternetService
Fiber optic    3096
DSL            2421
No             1526
Name: InternetService, dtype: int64
-----------------------------
Column Name:  OnlineSecurity
No                     3498
Yes                    2019
No internet service    1526
Name: OnlineSecurity, dtype: int64
-----------------------------
Column Name:  OnlineBackup
No                     3088
Yes                    2429
No internet service    1526
Name: OnlineBackup, dtype: int64
-----------------------------
Column Name:  DeviceProtection
No                     3095
Yes                    2422
No internet service    1526
Name: DeviceProtection, dtype: int64
-----------------------------
Column Name:  TechSupport
No                     3473
Yes                    2044
No internet service    1526
Name: TechSupport, dtype: int64
-----------------------------
Column Name:  StreamingTV
```

```
No                        2810
Yes                       2707
No internet service       1526
Name: StreamingTV, dtype: int64
-----------------------------
Column Name:  StreamingMovies
No                        2785
Yes                       2732
No internet service       1526
Name: StreamingMovies, dtype: int64
-----------------------------
Column Name:  Contract
Month-to-month    3875
Two year          1695
One year          1473
Name: Contract, dtype: int64
-----------------------------
Column Name:  PaperlessBilling
Yes    4171
No     2872
Name: PaperlessBilling, dtype: int64
-----------------------------
Column Name:  PaymentMethod
Electronic check             2365
Mailed check                 1612
Bank transfer (automatic)    1544
Credit card (automatic)      1522
Name: PaymentMethod, dtype: int64
-----------------------------
```

In [10]:

```
## Drop the customerID
df_r= df.drop(columns=['customerID'])
## Focus on the customers with Phone and Internet Services to avoid multicollinearity
## the other segments can be studied separately
df_r = df_r[(df_r['InternetService'] != 'No') & (df_r['PhoneService'] != 'No')]
df_r.head()
```

Out[10]:

	gender	SeniorCitizen	Partner	Dependents	tenure	PhoneService	MultipleLines	InternetService	Online
1	Male	0	No	No	34	Yes	No	DSL	
2	Male	0	No	No	2	Yes	No	DSL	
4	Female	0	No	No	2	Yes	No	Fiber optic	
5	Female	0	No	No	8	Yes	Yes	Fiber optic	
6	Male	0	No	Yes	22	Yes	Yes	Fiber optic	

In [11]:

```
## How many customers?
len(df_r)
```

Out[11]: 4835

3 Kaplan Meier curve

Survival function or probability $S(t)$ is the probability that a subject survives longer than a certain time t which means $S(t) = Probability(T > t)$.

$$\hat{S}(t) = \prod_{t_i < t} \frac{n_i - d_i}{n_i}$$

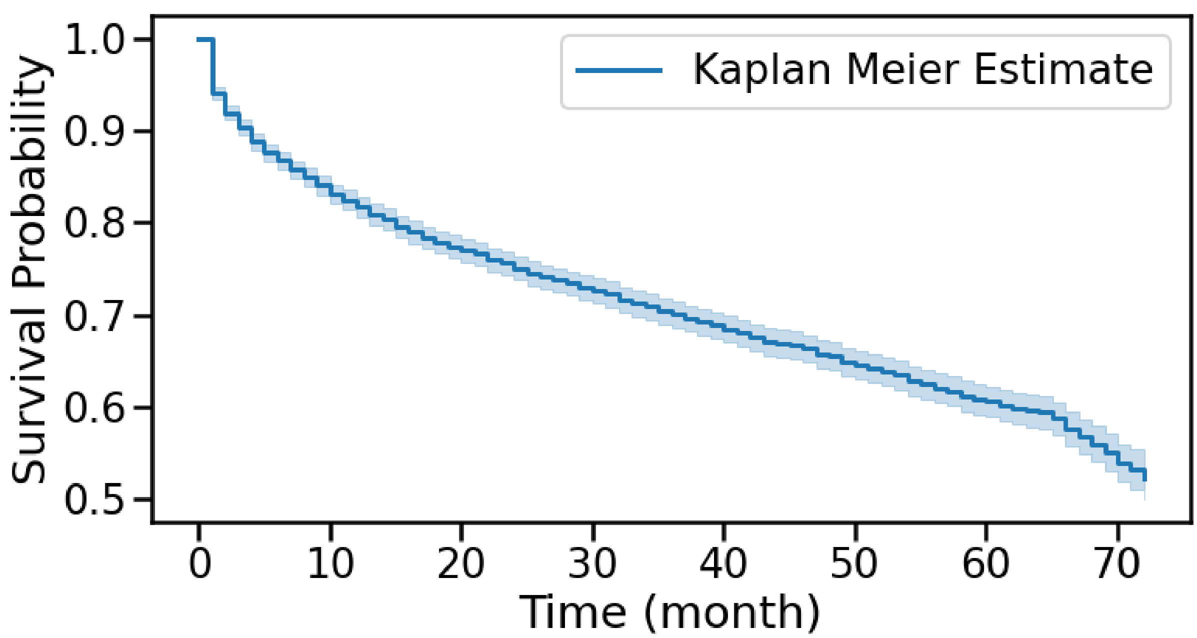
where d_i are the number of death events at time t and n_i is the number of subjects at risk of death just prior to time t.

First I will plot an overall curve, without breaking it into groups of covariates or cohorts. Then I will plot a few plots breaking data into cohorts.

3.1 An overall Kaplan Meier curve

In [12]:

```
durations = df_r.tenure
event_observed = df_r.Churn

## create a kmf object
kmf = KaplanMeierFitter()

## Fit the data to the model
kmf.fit(durations, event_observed, label='Kaplan Meier Estimate')

## Create an estimate
fig, ax = plt.subplots(1, 1, figsize=(12,6))
kmf.plot()
plt.xlabel('Time (month)')
plt.ylabel('Survival Probability');
```

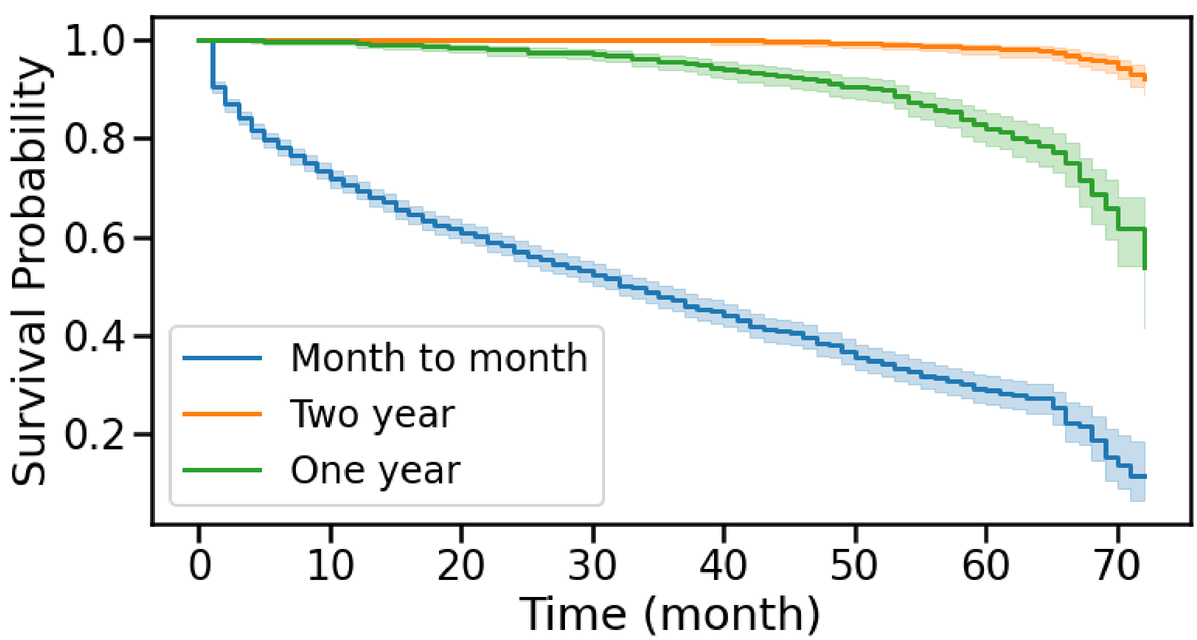


3.2 Kaplan Meier curves for cohorts

3.2.1 Cohort for contract type (2 yrs vs. 1 yr vs. month-to-month)

In [13]:

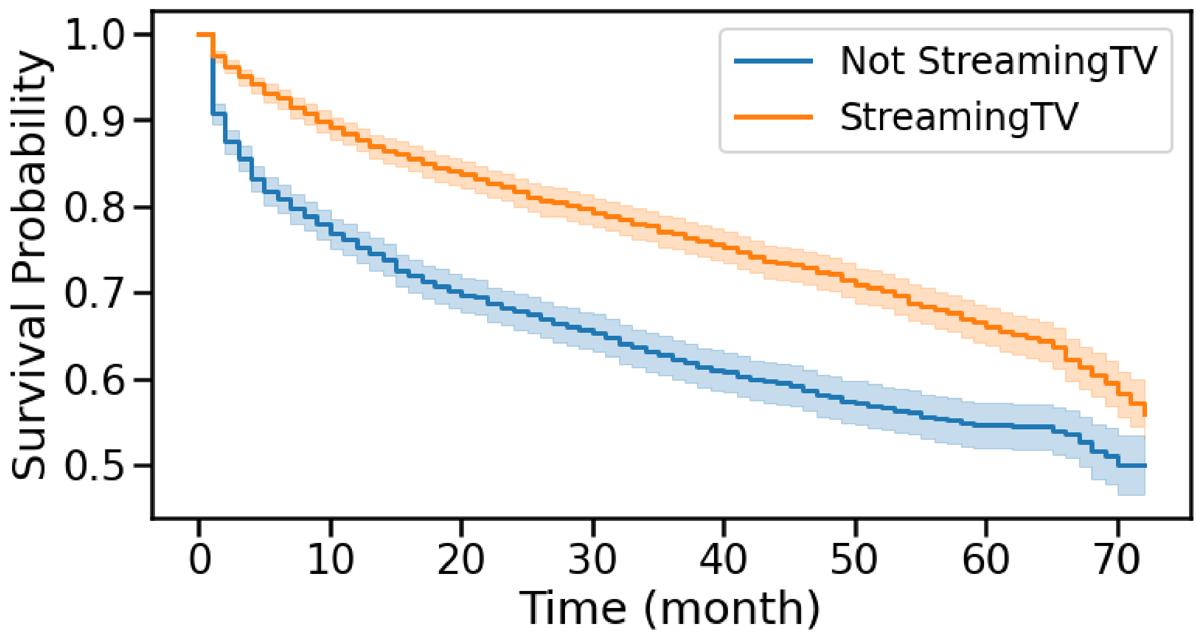
```
## create a kmf object
kmf = KaplanMeierFitter()

## time to event
T = df_r['tenure']
## event occurred or censored
E = df_r['Churn']

## Create the cohorts from the 'Contract' column with indexes
## Cohort 1
ix1 = (df_r['Contract'] == 'Month-to-month')
## Cohort 2
ix2 = (df_r['Contract'] == 'Two year')
## Cohort 3
ix3 = (df_r['Contract'] == 'One year')

## plot
fig, ax = plt.subplots(1, 1, figsize=(12,6))

## fit the cohort 1 data
kmf.fit(T[ix1], E[ix1], label='Month to month')
kmf.plot(ax=ax)

## fit the cohort 2 data
kmf.fit(T[ix2], E[ix2], label='Two year')
kmf.plot(ax=ax)

## fit the cohort 3 data
kmf.fit(T[ix3], E[ix3], label='One year')
kmf.plot(ax=ax)

plt.xlabel('Time (month)')
plt.ylabel('Survival Probability')
plt.legend(fontsize = 24);
```



3.2.2 Cohort for Streaming TV vs. not Streaming TV

In [14]:

```python
## create a kmf object
kmf = KaplanMeierFitter()

# indexes for cohorts
ix11 = (df_r['StreamingTV']  == 'No')
ix22 = (df_r['StreamingTV']  == 'Yes')

## plot
fig, ax = plt.subplots(1, 1, figsize=(12,6))

## fit the model for 1st cohort
kmf.fit(T[ix11], E[ix11], label='Not StreamingTV')
kmf.plot(ax=ax)

## fit the model for 2nd cohort
kmf.fit(T[ix22], E[ix22], label='StreamingTV')
kmf.plot(ax=ax)

plt.xlabel('Time (month)')
plt.ylabel('Survival Probability')
plt.legend(fontsize = 24);
```



4 Cox Proportional Hazard Model (Survival Regression)

With Cox Regression we can identify relationship between survival probability and predictors. The hazard function (conditional failure rate) is the basis of Cox regression model and gives the instantaneous potential per unit time for the event to occur, given that the individual has survived up to time t.

Cox regression is a semi parametric model which makes no assumption about shape of hazard function. However, it assumes the hazard ratio (or relative risk) of two groups remains about the same over time.

$$\underbrace{h(t|x)}_{\text{hazard}} = \overbrace{b_0(t)}^{\text{baseline hazard}} \underbrace{\exp\overbrace{\left(\sum_{i=1}^{n} b_i(x_i - \overline{x_i})\right)}^{\text{log-partial hazard}}}_{\text{partial hazard}}$$

4.1 Data processing

In [15]:

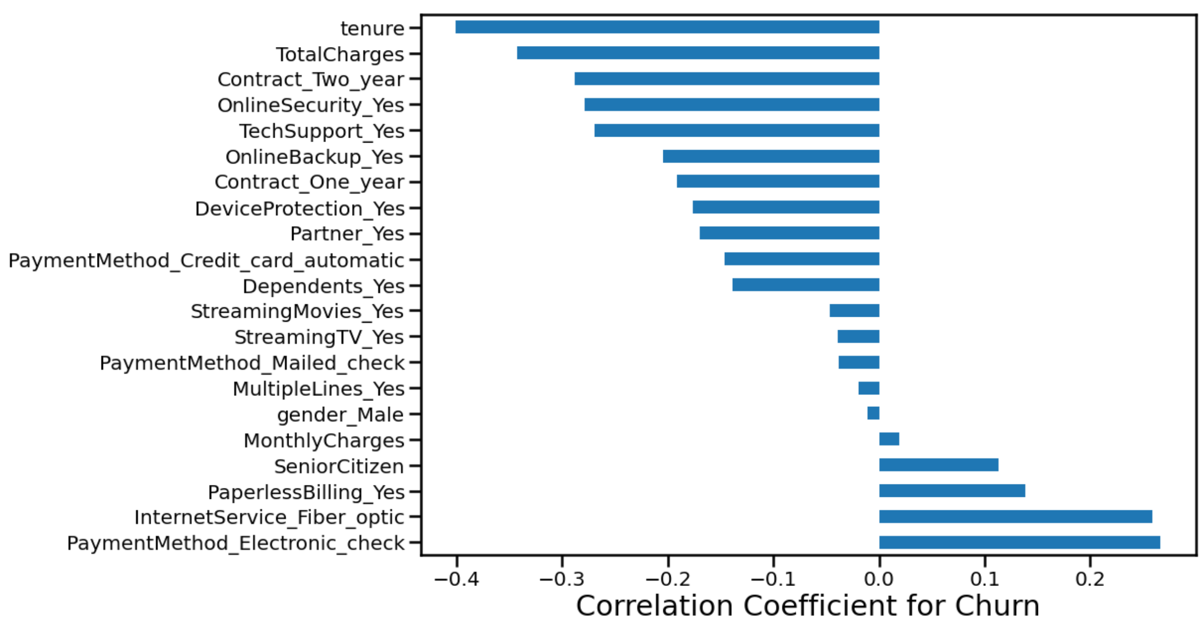
```python
## Create dummy variables
df_dummy = pd.get_dummies(df_r, drop_first=True)

## Clear the column labels
col_name_func(df_dummy)
df_dummy.head()
```

Out[15]:

	SeniorCitizen	tenure	MonthlyCharges	TotalCharges	Churn	gender_Male	Partner_Yes	Dependents_Yes
1	0	34	56.95	1889.50	0	1	0	0
2	0	2	53.85	108.15	1	1	0	0
4	0	2	70.70	151.65	1	0	0	0
5	0	8	99.65	820.50	1	0	0	0
6	0	22	89.10	1949.40	0	1	0	1

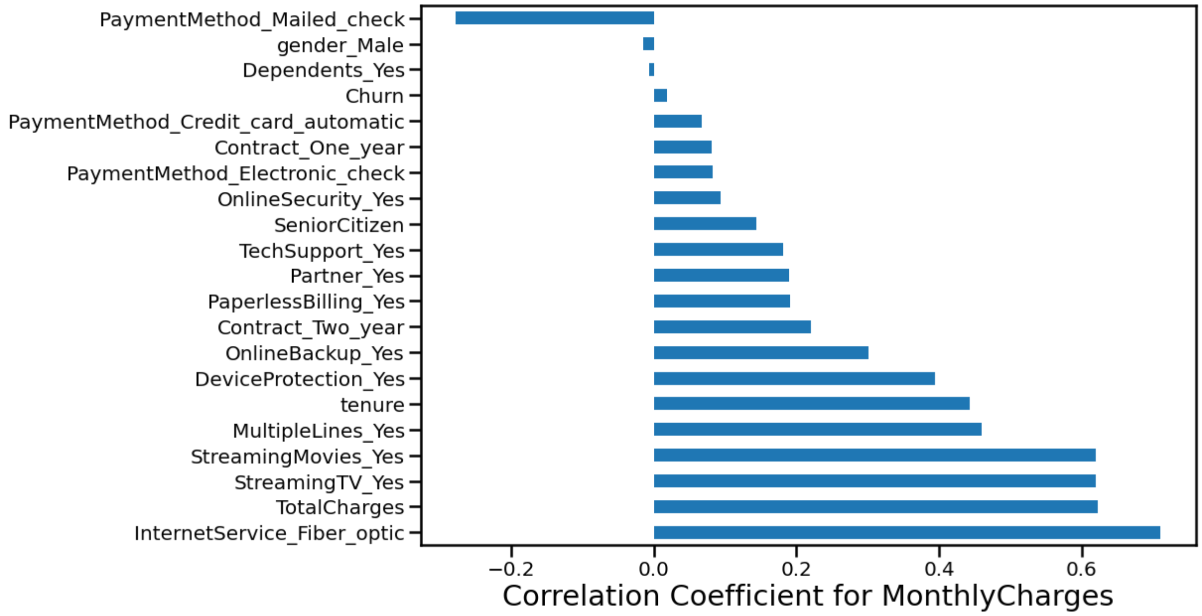
In [16]:

```python
## Investigate the correlation with churn
## Total and Monthly charges are definitely correlated with the service types
plt.figure(figsize=(14,10))
df_dummy.corr()['Churn'].drop('Churn').sort_values(ascending = False).plot(kind='barh',
plt.xlabel('Correlation Coefficient for Churn');
```



In [17]:

```
## Total and Monthly charges are definitely correlated with the service types so includ
plt.figure(figsize=(14,10))
df_dummy.corr()['MonthlyCharges'].drop('MonthlyCharges').sort_values(ascending = False)
plt.xlabel('Correlation Coefficient for MonthlyCharges');
```



In [18]:

```python
## VIF without TotalCharges = tenure * MonthlyCharges

features = ''
features = "+".join(df_dummy.drop(columns=['Churn', 'TotalCharges']).columns)

# find design matrix for the linear regression model using 'churn' as response variable
yy, XX = dmatrices("Churn ~" + features, data=df_dummy, return_type="dataframe")

# calculate VIF for each explanatory variable
vif = pd.DataFrame()
vif["variable"] = XX.columns
vif["VIF"] = [variance_inflation_factor(XX.values, i) for i in range(XX.shape[1])]

vif
```

Out[18]:

	variable	VIF
0	Intercept	1561.310621
1	SeniorCitizen	1.124230
2	tenure	3.093472
3	MonthlyCharges	255.578934
4	gender_Male	1.003903
5	Partner_Yes	1.420219
6	Dependents_Yes	1.314397
7	MultipleLines_Yes	6.017541
8	InternetService_Fiber_optic	110.908972
9	OnlineSecurity_Yes	5.760891
10	OnlineBackup_Yes	5.933602
11	DeviceProtection_Yes	6.085306
12	TechSupport_Yes	5.807752
13	StreamingTV_Yes	20.338033
14	StreamingMovies_Yes	20.350799
15	Contract_One_year	1.666367
16	Contract_Two_year	2.529730
17	PaperlessBilling_Yes	1.092097
18	PaymentMethod_Credit_card_automatic	1.552153
19	PaymentMethod_Electronic_check	1.927865
20	PaymentMethod_Mailed_check	1.574037

In [19]:

```
# VIF after dropping Monthly Charges: good
features = ''
features = "+".join(df_dummy.drop(columns=['Churn', 'TotalCharges', 'MonthlyCharges']).c
yy, XX = dmatrices("Churn ~" + features, data=df_dummy, return_type="dataframe")

# calculate VIF for each explanatory variable
vif = pd.DataFrame()
vif["variable"] = XX.columns
vif["VIF"] = [variance_inflation_factor(XX.values, i) for i in range(XX.shape[1])]

vif
```

Out[19]:

	variable	VIF
0	Intercept	14.253231
1	SeniorCitizen	1.124222
2	tenure	3.093427
3	gender_Male	1.003799
4	Partner_Yes	1.419650
5	Dependents_Yes	1.314387
6	MultipleLines_Yes	1.258475
7	InternetService_Fiber_optic	1.393007
8	OnlineSecurity_Yes	1.309785
9	OnlineBackup_Yes	1.239779
10	DeviceProtection_Yes	1.302679
11	TechSupport_Yes	1.366549
12	StreamingTV_Yes	1.361021
13	StreamingMovies_Yes	1.362355
14	Contract_One_year	1.666347
15	Contract_Two_year	2.529446
16	PaperlessBilling_Yes	1.091861
17	PaymentMethod_Credit_card_automatic	1.552135
18	PaymentMethod_Electronic_check	1.927752
19	PaymentMethod_Mailed_check	1.574036

In [20]:

```
## Drop 'MonthlyCharges' and 'TotalCharges' columns
df_dummy.drop(columns =['MonthlyCharges', 'TotalCharges'], inplace=True)
```

4.2 Building the model

In [21]:

```python
## Instantiate the class to create a Cox Proportional Hazards model object
cph = CoxPHFitter()
cph.fit(
    df_dummy,
    duration_col="tenure",
    event_col="Churn",
#     strata=[
#         "MultipleLines_Yes",
#         "OnlineSecurity_Yes",
#         "TechSupport_Yes",
#         "StreamingTV_Yes",
#         "StreamingMovies_Yes",
#         "Contract_One_year",
#         "Contract_Two_year",
#         "PaymentMethod_Mailed_check",
#     ],
)
cph.print_summary()
# dir(cph)
```

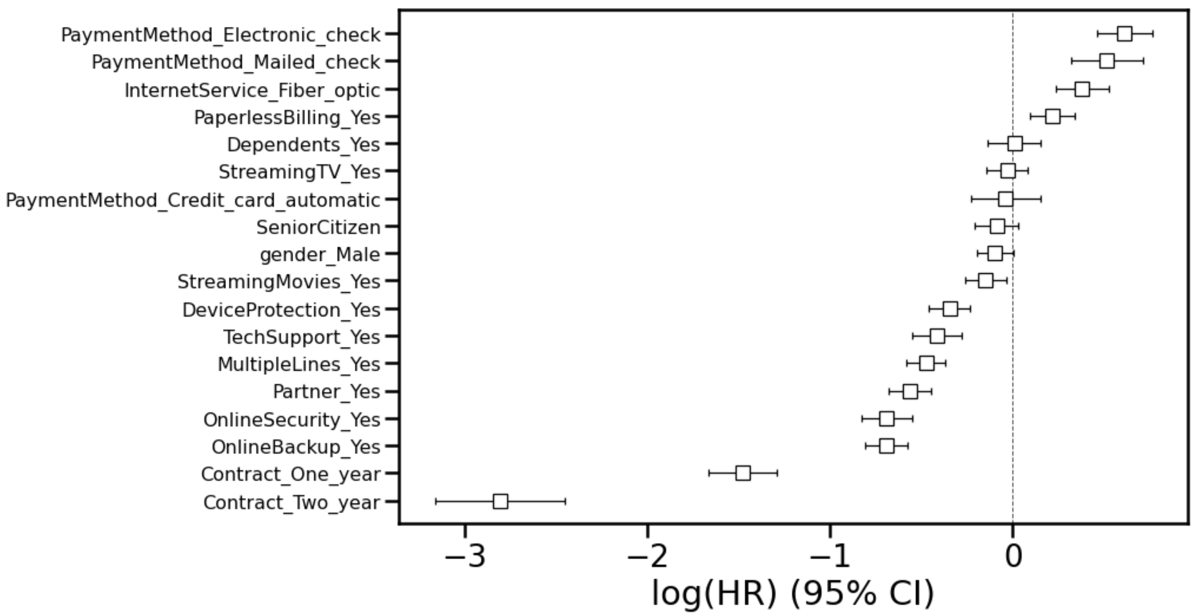
model	lifelines.CoxPHFitter
duration col	'tenure'
event col	'Churn'
baseline estimation	breslow
number of observations	4835
number of events observed	1586
partial log-likelihood	-11337.71
time fit was run	2020-11-20 09:49:09 UTC

	coef	exp(coef)	se(coef)	coef lower 95%	coef upper 95%	exp(coef) lower 95%	exp(coef) upper 95%	
SeniorCitizen	-0.09	0.92	0.06	-0.21	0.03	0.81	1.03	-
gender_Male	-0.10	0.91	0.05	-0.20	0.00	0.82	1.00	-
Partner_Yes	-0.56	0.57	0.06	-0.68	-0.45	0.51	0.64	-
Dependents_Yes	0.01	1.01	0.07	-0.14	0.16	0.87	1.17	
MultipleLines_Yes	-0.47	0.62	0.05	-0.58	-0.37	0.56	0.69	-
InternetService_Fiber_optic	0.38	1.46	0.07	0.24	0.53	1.27	1.69	
OnlineSecurity_Yes	-0.69	0.50	0.07	-0.83	-0.55	0.44	0.58	-
OnlineBackup_Yes	-0.69	0.50	0.06	-0.80	-0.58	0.45	0.56	-1
DeviceProtection_Yes	-0.34	0.71	0.06	-0.46	-0.23	0.63	0.79	-
TechSupport_Yes	-0.41	0.66	0.07	-0.55	-0.28	0.58	0.76	-
StreamingTV_Yes	-0.03	0.97	0.06	-0.14	0.08	0.87	1.09	-
StreamingMovies_Yes	-0.15	0.86	0.06	-0.26	-0.03	0.77	0.97	-
Contract_One_year	-1.48	0.23	0.10	-1.66	-1.29	0.19	0.28	-1
Contract_Two_year	-2.81	0.06	0.18	-3.16	-2.45	0.04	0.09	-1
PaperlessBilling_Yes	0.22	1.24	0.06	0.10	0.34	1.10	1.41	
PaymentMethod_Credit_card_automatic	-0.04	0.96	0.10	-0.23	0.15	0.79	1.16	-
PaymentMethod_Electronic_check	0.61	1.85	0.08	0.46	0.76	1.59	2.15	

	coef	exp(coef)	se(coef)	coef lower 95%	coef upper 95%	exp(coef) lower 95%	exp(coef) upper 95%
PaymentMethod_Mailed_check	0.52	1.67	0.10	0.32	0.71	1.37	2.04

Concordance	0.85
Partial AIC	22711.41
log-likelihood ratio test	2665.01 on 18 df
-log2(p) of ll-ratio test	inf

In [22]:
```
## Plot feature coefficients
fig, ax = plt.subplots(1, 1, figsize=(12,8))
cph.plot(ax=ax)
plt.yticks(fontsize=16);
```



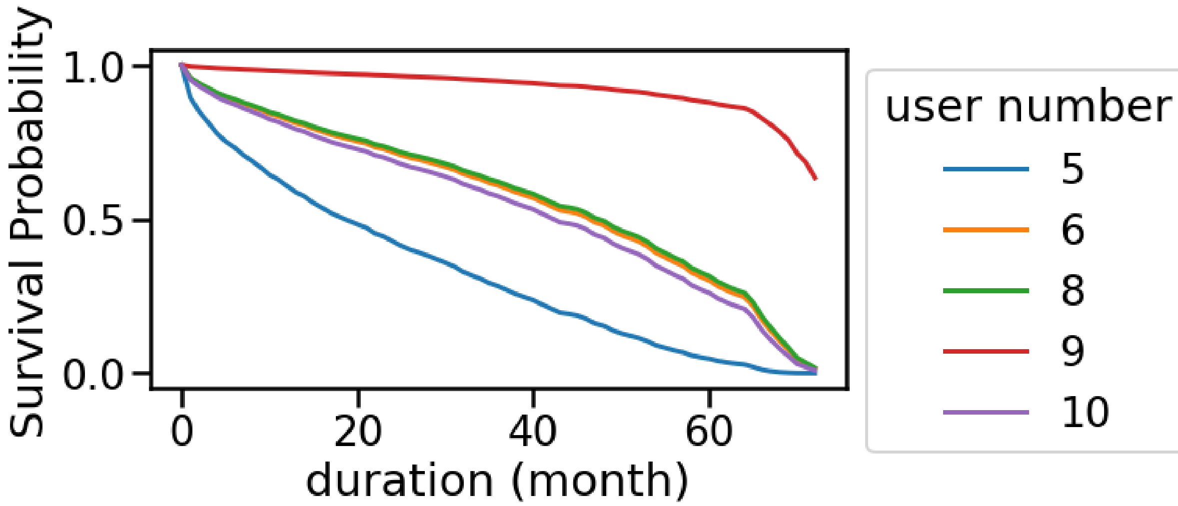
In [23]:
```
## Lets see the survival curves at the customer level. I have selected 6 customers (rows
rows = df_dummy.drop(columns='Churn')[3:8]
rows
```

Out[23]:

	SeniorCitizen	tenure	gender_Male	Partner_Yes	Dependents_Yes	MultipleLines_Yes	InternetService_Fit
5	0	8	0	0	0	1	
6	0	22	1	0	1	1	
8	0	28	0	1	0	1	
9	0	62	1	0	1	0	
10	0	13	1	1	1	0	

In [24]:

```
## Lets predict the survival curve for the selected customers.
fig, ax = plt.subplots(1, 1, figsize=(8,4))
cph.predict_survival_function(rows).plot(ax=ax)
plt.xlabel('duration (month)')
plt.ylabel('Survival Probability')
plt.legend(title = 'user number',bbox_to_anchor=(1, 1));
```



In [25]:

```
## Assumptions were not met for all the features, future work!
cph.check_assumptions(df_dummy)
```

```
The ``p_value_threshold`` is set at 0.01. Even under the null hypothesis of no viol
ations, some
covariates will be below the threshold by chance. This is compounded when there are
many covariates.
Similarly, when there are lots of observations, even minor deviances from the propo
rtional hazard
assumption will be flagged.

With that in mind, it's best to use a combination of statistical tests and visual t
ests to determine
the most serious violations. Produce visual plots using ``check_assumptions(..., sh
ow_plots=True)``
and looking for non-constant lines. See link [A] below for a full example.
```

In [ ]: