Problem Set #7

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1 Specification of the environment

1.1 Population of agents

I will analysis individuals dynamic choices of consumption and investment.

1.2 Preferences

I assumed the utility function is natural logarithm: $u(c) = \ln(c)$

1.3 Technology

I assume there is no production and households starts with an given i_0. And each period Households will receive returns from their precedent investments.

1.4 Information

In every period, He will choose how much to consume and Invest . for investment, individual will gain interest.that is, there is only one way for saving in this economy which is investing i_{t+1} The interest rate r_t is known in period t but for period t+1 it will be effected by shock ε_t. That is, when household decides to consume, ε_t is unknown for him. ε_t distribution is independently and identically normally distributed. $\varepsilon_t \sim i.i.d.N(0, \sigma)$

1.5 Household Maximization Problem

$$\begin{gathered} \max_{\{c_t, i_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t ln\left(c_t\right) \\ s.t. \\ c_t + i_t = R_t i_{t-1} \\ R_t = \rho R_{t-1} + \varepsilon_t \end{gathered} \tag{1}$$

1.5.1 Variables and Parameters

1.State Variable: i_{t-1} investment at t-1, ε_t exogenous shock in period t
2.Control Variable: c_t consumption at time t,i_t investment at time t
3.i_0 is endowment which is given and $i_0 > 0$
4.Parameters:
4.1 $0 < \beta < 1$
4.2 R_t is a rate of return on individual's investment which is given in first period.but it will be affected by the shock (ε_t) in the future
4.3 $c_0 > 0$
4.4 $\rho \in (-1, 1)$
4.5 $\varepsilon_t \sim i.i.d.N(0, \sigma)$

1.6 Bellman Equation

$$V(i,R)=\mathrm{Max}_{i'}\ln\left(R'i-i'\right)+\beta E_{\varepsilon'\varepsilon}V\left(i',R'\right) \tag{2}$$

1.6.1 First Order Necessary Conditions and Variables

First order condition:

$$\frac{\partial v(i,R)}{\partial i'}=\frac{-1}{R'i-i'}+\beta E\left(v'_{i'}\left(i',R'\right)\mid i,R\right)=0 \tag{3}$$

Applying the Benveniste-Scheinkman and envelope theorem:

$$\frac{\partial v(i,R)}{\partial i}=\frac{R'}{R'i-i'} \tag{4}$$

Updating One period :

$$v'_{i'}\left(\left(i',R'\right)\mid i,R\right)=\frac{\partial v\left(i',R'\right)}{\partial i'}=\frac{R''}{R''i'-i''} \tag{5}$$

From 3 and 5 :

$$\frac{1}{R'i-i'}=\beta E\left(\frac{R''}{R''i'-i''}\right) \tag{6}$$

From 7 Euler equation is:

$$\frac{1}{c_t}=\beta E\left(\frac{R_{t+1}}{c_{t+1}}\right) \tag{7}$$