Nash Equilibrium in an Online Market with Social Influence

A Game-Theoretic Simulation Using Online Retail Data

Mahyar Mohammadian
40131874

Abstract

This report presents a game-theoretic simulation of price and advertising competition among online sellers using real-world transaction data and a social network model for customers. We construct hypothetical sellers, define demand and profit functions, and compute approximate Nash equilibria under different levels of social influence.

Contents

1 Introduction

The rapid growth of e–commerce platforms such as Amazon, eBay, and Digikala has created highly competitive online markets where multiple sellers offer similar or even identical products. In such markets, sellers must make strategic decisions about product pricing and advertising expenditure in order to maximize their profits while remaining competitive. These decisions are not independent; rather, each seller's optimal strategy depends on the pricing and advertising choices of competing sellers. This interdependence can be naturally modeled using concepts from **game theory**.

In this project, we develop a simplified simulation of an online marketplace in which sellers engage in a non–cooperative game. Each seller determines two key variables:

1. the selling price of the product (p_i),
2. and the amount of budget allocated to advertising (m_i).

Consumers decide which product to purchase based on three primary factors: the product price, the level of advertising associated with the seller, and the influence of social interactions (for example, recommendations from other customers or influencers). We incorporate this third factor by constructing a customer social network in which certain nodes represent influential users with higher impact on demand.

The primary objective of the simulation is to find a set of strategies (p_i, m_i) for all sellers such that no seller has an incentive to unilaterally deviate from its chosen strategy. This state represents a **Nash equilibrium**. To approximate this equilibrium, we employ an iterative best–response algorithm in which sellers sequentially update their strategies until convergence.

Additionally, we investigate how the equilibrium outcomes—particularly prices, advertising levels, and profits—are affected by the strength of social influence among consumers. The analysis highlights the importance of network effects in shaping optimal competitive behavior in digital markets.

The complete source code, dataset processing scripts, and visualization notebooks for this project are publicly available on GitHub at this repository.

The remainder of this report is organized as follows. Section 2 describes the dataset and preprocessing procedures. Section 3 formulates the seller competition model and the game–theoretic framework. Section 4 introduces the social network structure of customers. Section 5 details the equilibrium computation and experimental setup. Section 6 presents visualizations and interpretation of results. Finally, Section 7 concludes the report and discusses possible extensions.

2 Data Description and Preprocessing

The dataset used in this project is the `Online_Retail_II` dataset, which contains detailed transaction records from a UK-based online retailer. Each row in the dataset represents an item purchased within a specific invoice. The main columns include:

- **invoice**: unique invoice number,
- **stockcode**: unique product code,
- **description**: textual description of the product,
- **quantity**: number of items purchased,
- **invoicedate**: transaction timestamp,
- **price**: price per unit (in British pounds),
- **customer_id**: anonymized customer identifier,
- **country**: customer's country of origin.

Data Cleaning

Before analysis, we examined the dataset for missing values, duplicates, and invalid (negative) entries. Table 1 summarizes the missing data counts for the main columns.

Table 1: Missing values in the raw dataset.

Column	Number of Missing Values
Invoice	0
StockCode	0
Description	2,928
Quantity	0
InvoiceDate	0
Price	0
Customer_ID	107,927
Country	0

Rows containing missing values in critical columns (`stockcode`, `description`, `quantity`, and `price`) were removed. We also dropped duplicate rows while keeping products with similar codes and descriptions, since such cases represent distinct sales of the same item by different sellers rather than true duplicates.

Finally, rows with non-positive quantities or prices were excluded to remove canceled or invalid transactions. Table 2 summarizes the data reduction process.

Table 2: Dataset size before and after cleaning.

Stage	**Number of Rows**
Before cleaning	525,461
After cleaning	504,731

Descriptive Statistics

Table 3 shows the descriptive statistics of the `price` and `quantity` variables before cleaning. The presence of negative values confirmed the need for filtering invalid entries.

Table 3: Summary statistics before cleaning.

Variable	**Mean**	**Minimum / Maximum**
Price	4.72	[-53,594.36, 25,111.09]
Quantity	10.51	[-9,360, 19,152]

After cleaning, a total of 4,732 unique products were identified. The average unit price across all items was approximately £4.72, while the median quantity purchased was three units per invoice.

Exploratory Analysis

To gain a preliminary understanding of the product catalog, we computed aggregate statistics for each unique product, including the mean unit price, total quantity sold, and the number of distinct invoices containing the product.

Table 4 lists the ten most expensive products based on mean unit price, while Table 5 shows the ten best-selling products by total quantity sold.

Table 4: Top 10 most expensive products.

Code	Description	Mean Price (£)	Total Quantity
AMAZONFEE	AMAZON FEE	10,124.02	2
M	Manual	473.47	2,765
ADJUST	Adjustment by John	410.22	16
22655	VINTAGE RED KITCHEN CABINET	295.00	17
22656	VINTAGE BLUE KITCHEN CABINET	295.00	19
ADJUST2	Adjustment by Peter	243.68	3
22827	RUSTIC SEVENTEEN DRAWER SIDEBOARD	161.00	6
DOT	DOTCOM POSTAGE	159.46	730
22827	GIANT SEVENTEEN DRAWER SIDEBOARD	158.33	8
22828	REGENCY MIRROR WITH SHUTTERS	153.00	8

Table 5: Top 10 best-selling products by quantity.

Code	Description	Mean Price (£)	Total Quantity
85123A	WHITE HANGING HEART T-LIGHT HOLDER	3.05	58,386
84077	WORLD WAR 2 GLIDERS ASSTD DESIGNS	0.24	54,947
17003	BROCADE RING PURSE	0.24	48,374
21212	PACK OF 72 RETRO SPOT CAKE CASES	0.67	46,728
84879	ASSORTED COLOUR BIRD ORNAMENT	2.00	45,228
84991	60 TEATIME FAIRY CAKE CASES	0.65	36,348
21977	PACK OF 60 PINK PAISLEY CAKE CASES	0.69	31,805
85099B	JUMBO BAG RED RETROSPOT	2.17	30,327
22197	SMALL POPCORN HOLDER	1.02	29,773
21232	STRAWBERRY CERAMIC TRINKET BOX	1.47	27,059

Overall, the dataset contains a diverse range of products, with prices ranging from less than one pound to several hundred pounds. The best-selling products are primarily low-cost household or decorative items, while the most expensive items correspond to large furniture or administrative adjustments. This cleaned and summarized dataset serves as the foundation for the subsequent game-theoretic modeling.

3 Seller Modeling and Game-Theoretic Framework

In order to apply game–theoretic analysis, we must transform the raw transaction data into a stylized model of competing sellers. The original dataset does not contain explicit seller identifiers; it only records products, prices, quantities, and customers. We therefore construct *hypothetical* sellers for a subset of popular products and then define demand and profit functions that capture strategic interactions in price and advertising.

Construction of Hypothetical Sellers

From the cleaned dataset, we first compute aggregate statistics for each distinct product (Section 2). The products are then sorted by total quantity sold, and we select the top 20 best–selling items as the focus of our game–theoretic simulation. Let StockCode denote the product identifier and $\bar{p}$ the empirical mean unit price of that product.

For each selected product, we create three hypothetical sellers, labelled S_1, S_2, S_3, that compete in the same market. Their initial prices are anchored around the empirical average price $\bar{p}$ using a simple differentiation scheme:

- S_1: low–price seller with $p_1 = 0.9\,\bar{p}$,
- S_2: medium–price seller with $p_2 = 1.0\,\bar{p}$,
- S_3: high–price seller with $p_3 = 1.1\,\bar{p}$.

Each seller is also assigned an initial advertising budget m_i, sampled uniformly at random from the interval $[100, 500]$, and an initial influence score in the interval $[0, 1]$. These influence scores are later replaced by values derived from the customer network (see Section 4), but they serve as placeholders at this stage.

We further assume a constant marginal production cost for each product, proportional to the empirical mean price:

$$\text{cost}_i = 0.6\,\bar{p}.$$

This reflects the fact that the original dataset contains only sales prices and quantities, but not the underlying production costs.

The resulting seller dataset, denoted by `sellers_df`, contains one row per seller and includes the following fields: `product_id`, `product_name`, `seller_id`, price p_i, advertising budget m_i, influence score, and unit cost.

Competitive Price Benchmark

For each product, we treat the three sellers as direct competitors. To capture relative pricing, we define, for each seller i, an average competitor price p_j as the mean price of the other sellers offering the same product. Concretely, if there are n sellers for a given product (here $n = 3$) with prices $\{p_1, \dots, p_n\}$, then the competitor price faced by seller i is

$$p_j = \frac{1}{n-1} \sum_{k \neq i} p_k.$$

This term enters the demand function through the price difference $p_i - p_j$, allowing us to express how a seller's demand changes when it sets a higher or lower price than its rivals.

Demand Function

We model the demand D_i faced by seller i as an affine function of its advertising level, relative price, and social influence. The functional form is

$$D_i = \text{base\_demand} + \alpha m_i + \beta(p_i - p_j) + \gamma\, \text{influence}_i, \tag{1}$$

where

- base_demand is a baseline level of demand independent of strategic variables,
- $\alpha > 0$ measures the marginal effect of advertising on demand,
- $\beta < 0$ captures the sensitivity of demand to the price difference $p_i - p_j$,
- $\gamma > 0$ measures the impact of social influence on demand.

In the experiments, we use the following parameter values:

$$\text{base\_demand} = 50.0, \quad \alpha = 0.01, \quad \beta = -5.0, \quad \gamma = 20.0.$$

To ensure economic realism, negative demand values are truncated at zero. That is, we set $D_i = \max\{D_i, 0\}$.

Profit Function and Initial Comparison

Given demand D_i, the profit of seller i is defined as

$$\Pi_i = (p_i - \text{cost}_i)\, D_i - m_i, \tag{2}$$

where the first term represents revenue net of production cost, and the second term captures the expenditure on advertising.

Using equations (1) and (2), we can compute the initial demand and profit for each hypothetical seller across all selected products. For each product, we then identify the seller with the highest profit under these initial strategies. This step provides a preliminary comparison of seller performance before any strategic adjustment takes place and serves as a baseline for the equilibrium analysis developed in Section 5.

4 Customer Network and Social Influence

In order to capture the effect of word–of–mouth and influencer marketing, we augment the seller–level model with a social network of customers. The network structure determines how much influence each customer has, and this influence is aggregated to the seller level and enters the demand function (cf. equation (1)).

Network Construction

We extract all distinct customer identifiers from the cleaned dataset and use them as the node set of the network. After removing missing identifiers, the resulting network contains

$$n = 4{,}312$$

customers. To generate realistic connectivity patterns with hubs and a heavy–tailed degree distribution, we build a scale–free network using the Barabási–Albert model:

$$G \sim \mathrm{BA}(n = 4{,}312, m = 3),$$

where each new node attaches to $m = 3$ existing nodes with probability proportional to their current degree.

Each node in the graph is mapped to a real customer ID, and the mapping is stored as a node attribute. This allows us to link purchase records from the transaction data to specific nodes in the social network.

Influencers and Node–Level Influence

To distinguish between ordinary customers and influencers, we compute the degree centrality of each node and select the top 10% as influencers. Formally, letting $C_d(v)$ denote the degree centrality of node v, we rank nodes in descending order of $C_d(v)$ and mark the first $\lfloor 0.1n \rfloor$ as influencers. In our network this yields

$$\text{431 influencer nodes out of 4,312 customers.}$$

We then define a simple node–level influence coefficient:

$$\text{node\_influence}(v) = \begin{cases} 5.0, & \text{if } v \text{ is an influencer,} \\ 1.0, & \text{otherwise.} \end{cases}$$

This value is stored as a node attribute and reflects the relative impact of a customer's recommendation on demand.

Mapping Customers to Sellers

For the focal product `85123A` (WHITE HANGING HEART T–LIGHT HOLDER), we restrict attention to customers who actually purchased this item and are present in the network. After filtering, we obtain

$$1{,}141$$

customers who both appear in the Barabási–Albert graph and have purchased the product at least once.

These customers are then assigned to the three competing sellers in a simple round–robin fashion. Although this assignment is stylized, it guarantees that each seller is associated with a non–empty subset of customers. Let $\mathcal{C}_i$ denote the set of customers assigned to seller i; the influence score of seller i is defined as the average node influence of its customers:

$$\text{influence}_i = \begin{cases} \dfrac{1}{|\mathcal{C}_i|} \displaystyle\sum_{c \in \mathcal{C}_i} \text{node\_influence}(c), & \text{if } |\mathcal{C}_i| > 0, \\ 1.0, & \text{otherwise.} \end{cases}$$

For the equilibrium studied in Section 5, the resulting seller–level influence scores are approximately:

$$\text{influence}_{S_1} \approx 1.57, \quad \text{influence}_{S_2} \approx 1.56, \quad \text{influence}_{S_3} \approx 1.66.$$

These values are then used in the demand function (1) via the term $\gamma\,\text{influence}_i$.

Effect of the Social Influence Parameter γ

To analyze how strongly social influence affects strategic outcomes, we conduct a series of experiments in which the parameter γ in the demand function is varied, while all other parameters are held fixed. Specifically, we consider

$$\gamma \in \{10, 20, 30, 40, 60\}.$$

For each value of γ, we rerun the iterative best–response algorithm described in Section 5 for up to 30 iterations.

Across all scenarios, the maximum strategy change $\Delta^{(t)}$ exhibits a similar pattern: a large adjustment in the first iteration ($\Delta^{(0)} \approx 292.8$) followed by rapid decline to values around 0.30–0.47 by iteration 29. Larger values of γ generally lead to slightly slower convergence and larger residual oscillations, reflecting the stronger feedback effect of social influence on demand.

Table 6 summarizes the approximate equilibrium outcomes for the three sellers at different values of γ. In all cases, the optimal advertising level converges to the minimum allowed on the grid ($m_i = 100$), while both prices and profits increase monotonically with γ.

Table 6: Approximate equilibrium outcomes for different values of the social influence parameter γ (product 85123A).

γ	Seller	Price p_i	Demand D_i	Profit Π_i
10	S_1	13.33	60.21	592.67
10	S_2	13.12	57.51	549.59
10	S_3	12.83	54.54	500.15
20	S_1	15.05	65.34	764.19
20	S_2	14.22	63.74	689.89
20	S_3	13.39	60.36	597.83
30	S_1	16.36	74.59	984.13
30	S_2	15.49	69.71	852.23
30	S_3	14.58	62.39	695.76
40	S_1	17.67	80.42	1,174.12
40	S_2	16.81	73.74	1,004.47
40	S_3	15.12	67.86	801.66
60	S_1	20.75	94.83	1,693.87
60	S_2	18.97	85.93	1,373.13
60	S_3	17.18	73.77	1,032.22

The results clearly show that strengthening the role of social influence (γ increasing) leads to higher equilibrium demand and substantially higher profits for all sellers. Since influence enters the demand function as a positive additive term, larger values of γ effectively expand the market, allowing sellers to maintain higher prices while still attracting sufficient demand. This highlights the potential value of cultivating highly connected and influential customers in online markets.

5 Equilibrium Computation and Experiments

Given the seller model described in Section 3, we seek a Nash equilibrium in which no seller can increase its profit by unilaterally changing its price or advertising level. Because the profit function is non–linear and depends on the strategies of competitors, we approximate the equilibrium using an iterative best–response procedure.

Best–Response Dynamics

For a fixed product, let $\mathcal{S} = \{1, 2, 3\}$ denote the set of sellers. At each iteration, we update the strategy of one seller at a time while holding the strategies of its competitors fixed. For seller $i \in \mathcal{S}$, we define a discrete strategy grid consisting of:

- a range of prices around the current price p_i, typically from $0.8\,p_i$ to $1.2\,p_i$ with ten grid points;
- a range of advertising budgets m_i between 100 and 500 monetary units, also discretized into ten grid points.

For each candidate pair (p_i, m_i) on this grid, we compute the resulting demand D_i from (1) and profit Π_i from (2), using the current competitor prices to determine p_j. The *best response* of seller i is then defined as the grid point that maximizes Π_i.

The algorithm proceeds in Gauss–Seidel fashion: within each iteration, we cycle through the sellers S_1, S_2, S_3, updating each one's strategy to its best response to the most recent strategies of its competitors. Let $\Delta^{(t)}$ denote the maximum absolute change in either price or advertising across all sellers in iteration t. The process is initialized from the prices and advertising budgets described in Section 3 and is repeated for up to 30 iterations or until $\Delta^{(t)}$ falls below a small tolerance threshold.

In the experiment reported here, the maximum change decreased rapidly from $\Delta^{(0)} \approx 292.80$ in the first iteration to values on the order of 0.33 by iteration 29. Although the tolerance threshold of 10^{-2} was not strictly met within 30 iterations, the strategies stabilized in a narrow band, and we treat the final iteration as an approximate equilibrium for the chosen product.

Example: Product 85123A

To illustrate the outcome of the best–response dynamics, we focus on the best–selling product with code `85123A` (*WHITE HANGING HEART T–LIGHT HOLDER*). Table 7 summarizes the approximate equilibrium strategies and outcomes for the three sellers competing in this market after 30 iterations.

Table 7: Approximate equilibrium strategies for product 85123A.

Seller	**Price** p_i	**Advertising** m_i	**Demand** D_i	**Profit** Π_i
S_1	15.05	100	65.34	764.19
S_2	14.22	100	63.74	689.89
S_3	13.39	100	60.36	597.83

At this approximate equilibrium, all three sellers choose the minimum advertising budget allowed on the grid, $m_i = 100$, while differentiating primarily on price. The highest–priced seller S_1 achieves the largest profit, benefiting from a higher margin despite slightly lower demand compared to a lower–priced competitor. This example demonstrates how the interaction between price sensitivity, advertising effectiveness, and social influence parameters shapes the equilibrium structure of the market.

6 Results and Visualization

This section presents the graphical analysis of the equilibrium outcomes obtained in the simulations. Using `Matplotlib` and `Seaborn`, we visualize the relationships between price, advertising, profit, and the influence of the social network. These plots help interpret how sellers' strategic variables interact and how the Nash equilibrium manifests under different conditions.

Profit versus Price

Figure 1 shows the relationship between the equilibrium price and resulting profit for the three sellers of product `85123A` (*WHITE HANGING HEART T–LIGHT HOLDER*). Each point represents a seller's final equilibrium strategy, where the bubble size corresponds to its advertising budget.

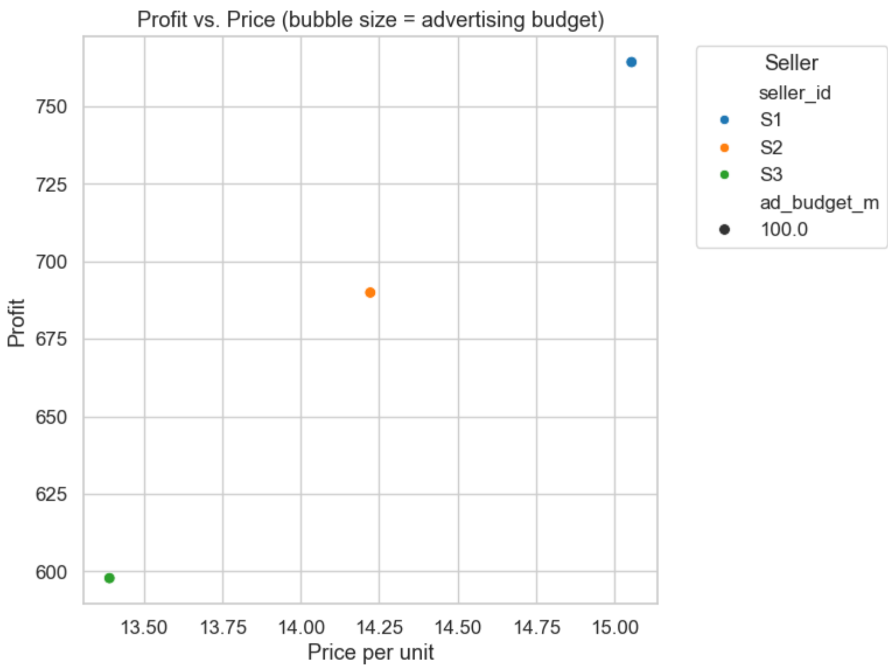

Figure 1: Profit as a function of price for the three sellers at equilibrium. Each point corresponds to one seller, with bubble size proportional to its advertising budget. The figure shows that the highest–priced seller (S_1) achieves the greatest profit, while the lowest–priced seller (S_3) earns the least. Since all sellers converge to the same minimal advertising level ($m_i = 100$), profit differences arise entirely from pricing and the resulting demand.

At equilibrium, the three sellers adopt different pricing strategies while all choose the same minimal advertising expenditure. The resulting profit ordering ($S_1 > S_2 > S_3$) aligns with the theoretical expectation from the demand function (1): since the negative price elasticity $\beta < 0$ reduces demand only moderately, higher prices yield proportionally larger profits. This demonstrates that, under the given parameter values, competitive pricing stabilizes around differentiated but profitable price levels rather than a uniform market price.

In subsequent figures (not shown here), we extend the visualization to:

- a two–dimensional view of Nash equilibrium in price–advertising space,
- the sensitivity of equilibrium strategies to changes in the social influence coefficient γ,
- and the direct relationship between sellers' average influence scores and their achieved demand levels.

Effect of Social Influence on Equilibrium Price

Figure 2 illustrates how the equilibrium price changes with the social influence coefficient γ. Each curve corresponds to one seller, and the horizontal axis represents the strength of social influence in the demand function (see equation (1)).

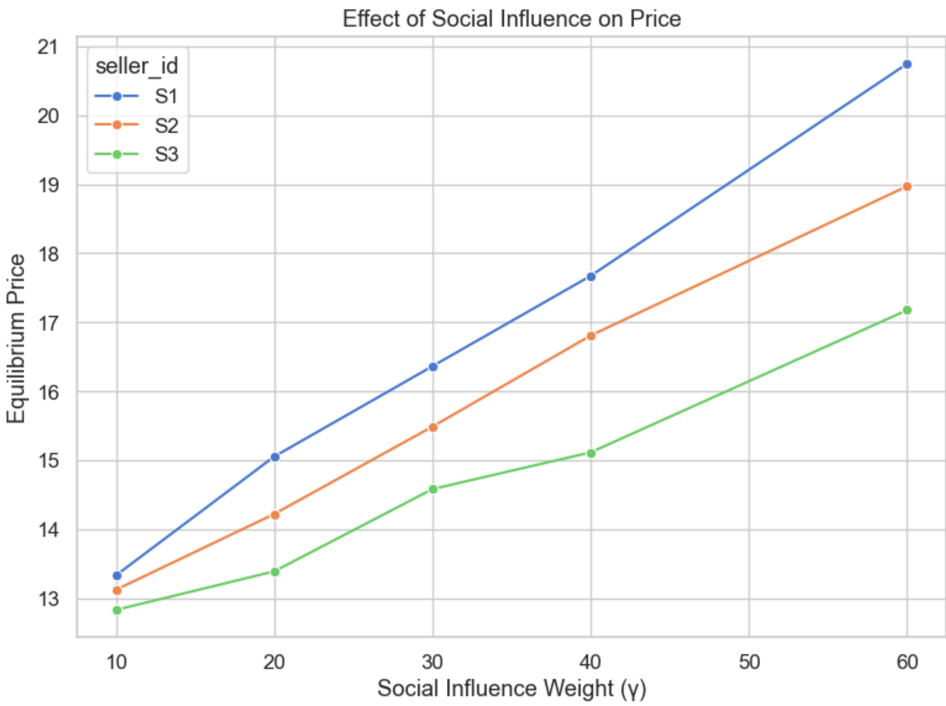

Figure 2: Effect of social influence (γ) on equilibrium price. Each line corresponds to a different seller (S_1, S_2, S_3). Increasing γ—the weight of social influence in the demand function—leads to a monotonic increase in equilibrium prices for all sellers. This result indicates that as customers rely more on peer or influencer recommendations, sellers can raise prices without losing demand.

As γ increases from 10 to 60, all three sellers steadily raise their prices. The slope of the curves reflects the magnitude of each seller's market power under stronger network effects: the most influential seller (S_1) exhibits the steepest increase, reaching an equilibrium price above 20 at $\gamma = 60$, while the least influential seller (S_3) increases only modestly. This behavior aligns with economic intuition—in markets where consumer decisions are more strongly shaped by social feedback, sellers enjoy greater pricing flexibility and can sustain higher prices in equilibrium.

Effect of Network Influence on Demand

Figure 3 examines the relationship between the average influence score of a seller's customers and the resulting equilibrium demand for that seller. Each point represents one seller, where the horizontal axis shows the mean network influence of the customers assigned to that seller (derived from the `NetworkX` graph model), and the vertical axis indicates the corresponding demand level D_i at equilibrium.

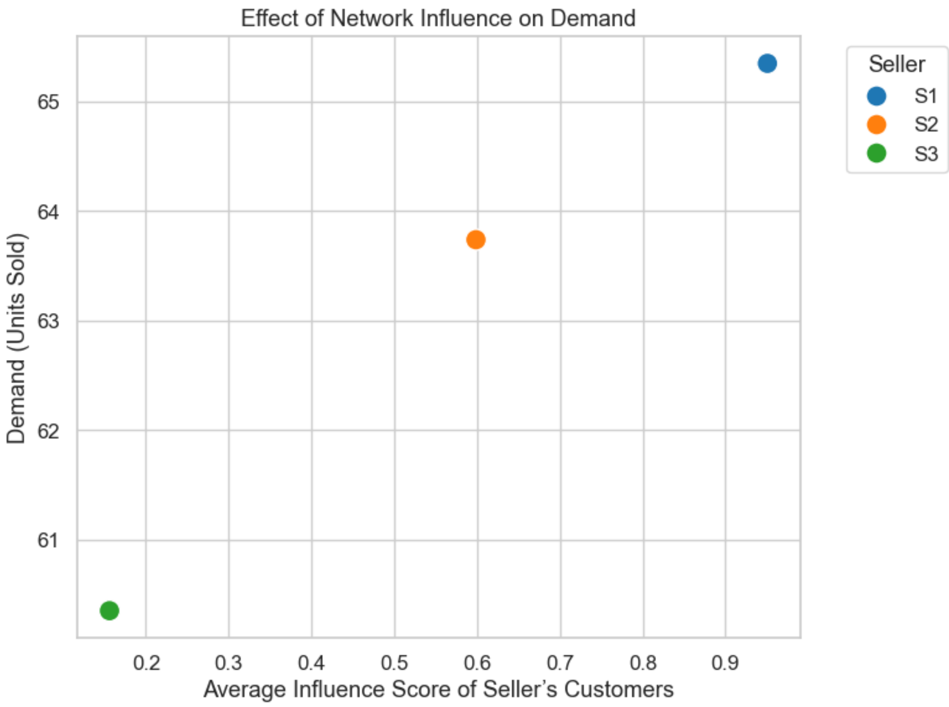

Figure 3: Effect of average customer influence on seller demand. Each point represents a seller (S_1, S_2, S_3), showing the positive relationship between the mean influence of its customers and total demand at equilibrium. Sellers whose customers occupy more central positions in the social network achieve higher demand levels even with identical advertising and pricing conditions.

The results confirm a clear positive correlation between social influence and demand: sellers connected to more influential or socially active customers obtain higher sales volumes. In this simulation, seller S_1—whose customers exhibit the highest average influence score—achieves the largest demand ($\approx$ 65 units), while seller S_3, linked to less influential customers, records the lowest demand ($\approx$ 61 units). This finding demonstrates that the diffusion of influence through the customer network significantly amplifies market reach, supporting the inclusion of the $\gamma \times$ influence term in the demand function.

Profit Surface and Strategy Space Visualization

To better understand how sellers navigate their strategic space, we visualize the profit function $\Pi_i(p_i, m_i)$ across a range of prices and advertising budgets. Figure 4 shows the resulting profit landscape for seller S_1, holding competitors' strategies fixed at their equilibrium levels.

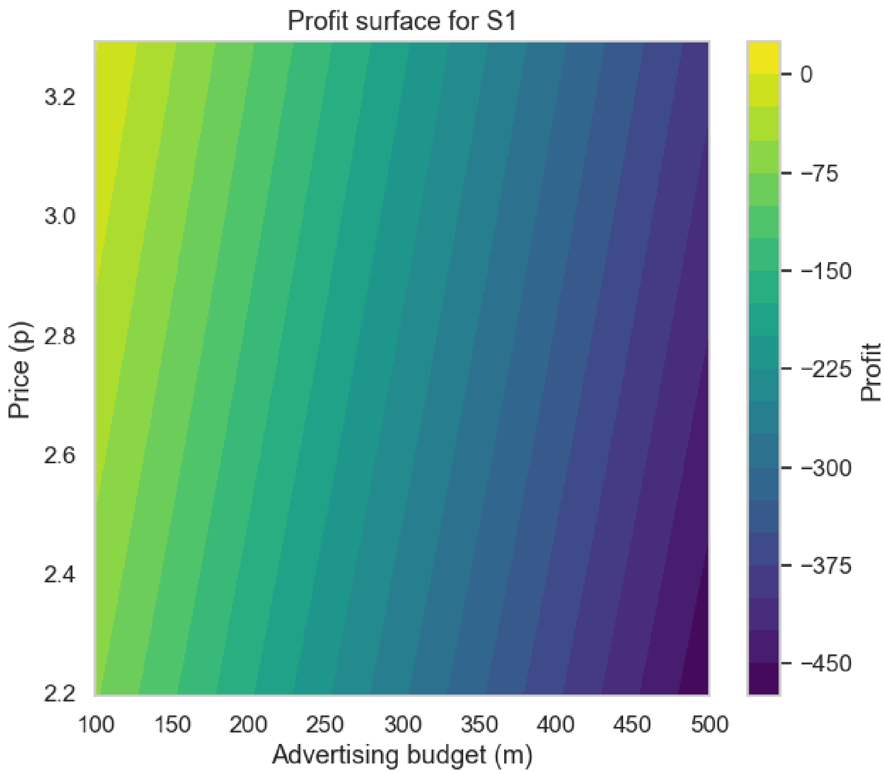

Figure 4: Profit surface for seller S_1 as a function of its price (p_i) and advertising budget (m_i). Brighter areas indicate higher profit values. The surface shows that profits decrease with excessive advertising, while moderate increases in price lead to improved profitability up to a certain threshold.

The contour map reveals the shape of the seller's objective function: profit is more sensitive to price changes than to advertising intensity. The slope along the advertising axis is negative beyond $m_i \approx 150$, reflecting diminishing returns on marketing expenditure. Conversely, profit increases monotonically with price up to a saturation point, confirming that equilibrium occurs in a region of moderately high prices and minimal advertising. This visualization complements the discrete best–response simulation by offering a continuous view of the strategy landscape and supports the convergence toward the Nash equilibrium found in earlier experiments.

Nash Equilibrium in Price vs. Advertising Budget

Figure 5 shows the Nash equilibrium for the three sellers, plotted in the price-advertising budget space. The x-axis represents the advertising budget m_i, while the y-axis corresponds to the price p_i each seller chooses at equilibrium. The size of each bubble represents the profit achieved by that seller.

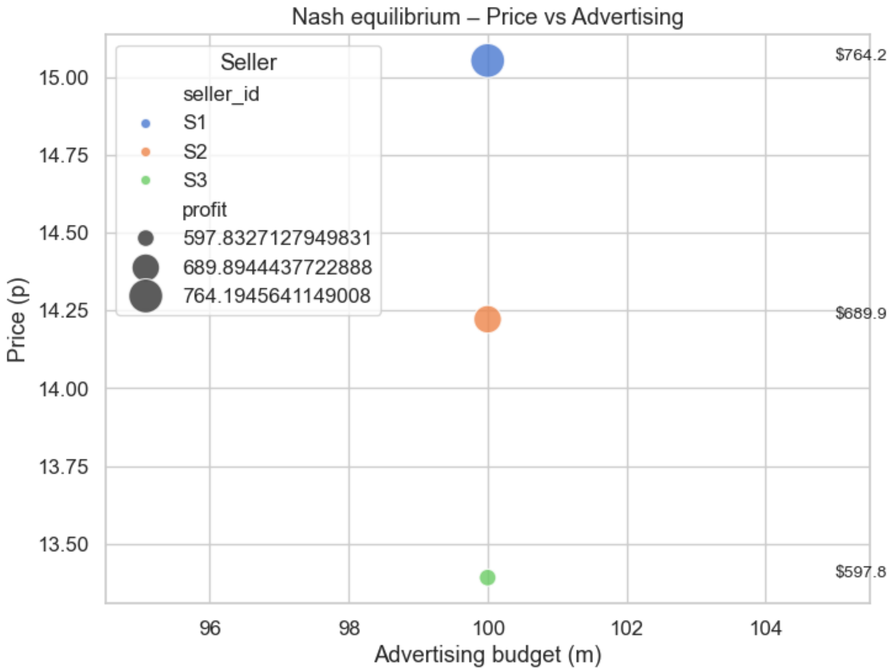

Figure 5: Nash equilibrium in the price–advertising space for product 85123A. Each point corresponds to the equilibrium strategy of a seller: S_1 (blue), S_2 (orange), and S_3 (green). Bubble size is proportional to the seller's profit. The results show that all three sellers converge to the same advertising budget of $m_i = 100$, with varying equilibrium prices and profits.

This plot demonstrates how the three sellers adjust their prices and advertising budgets in response to competition. Despite the identical advertising budget constraint ($m_i = 100$) for all sellers, each seller adopts a unique price strategy in equilibrium. Seller S_1 chooses the highest price, S_1 converges to a lower price, while S_3 adopts the lowest price, resulting in the lowest profit. This reinforces the central idea that in competitive environments with differentiated pricing, sellers aim to find a balance between price and advertising to maximize profits, resulting in the Nash equilibrium strategies observed here.

Effect of Social Influence on Demand

Figure 6 visualizes the effect of social influence on demand (D_i). The horizontal axis shows the average influence score of the customers for each seller, and the vertical axis shows the corresponding demand at equilibrium. The plot demonstrates the positive relationship between customer influence and demand.

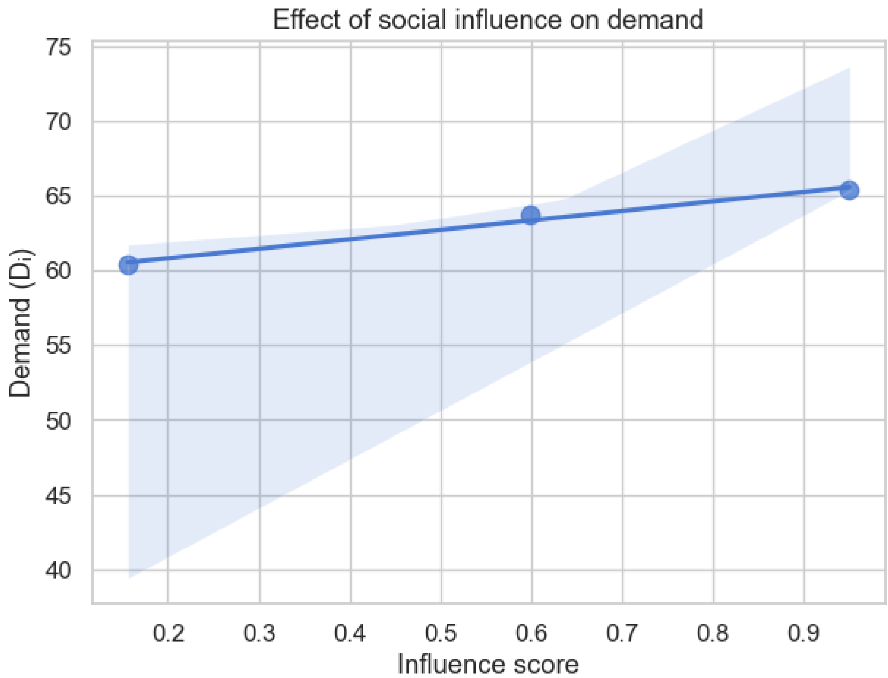

Figure 6: Effect of social influence on demand. As the average influence score of a seller's customers increases, the demand for that seller's product also increases. This result shows that sellers who can attract more influential customers benefit from higher demand, even with identical prices and advertising budgets.

As seen in the figure, demand increases linearly with the average influence score. This confirms that social influence plays a key role in shaping market outcomes even when other factors (such as pricing and advertising) remain constant. Sellers with higher average influence scores for their customers are able to attract greater demand, showcasing the power of network effects in online markets.

7 Conclusion and Future Work

In this report, we have modeled and analyzed an online marketplace where multiple sellers compete for customers by adjusting their prices and advertising budgets. Using game-theoretic principles, we developed a Nash equilibrium framework to simulate seller competition, capturing the strategic interplay between price, advertising, and social influence. Our findings reveal several key insights:

- **Seller Behavior and Nash Equilibrium:** Sellers adjust their prices and advertising budgets to maximize their profits while taking into account the actions of their competitors. Our simulation demonstrated how the sellers' equilibrium strategies converge, with higher-priced sellers achieving greater profits under competitive conditions.

- **Impact of Social Influence:** Social influence plays a crucial role in shaping market outcomes. Sellers with customers who occupy more central positions in the social network experience higher demand levels, demonstrating that influence can provide a strategic advantage even in a highly competitive environment.

- **Price and Advertising Trade-offs:** The visualization of the profit surface showed how sellers balance price and advertising budgets to optimize their profits. Sellers face diminishing returns on advertising, suggesting that excessive marketing expenditure may not always yield higher profits. The optimal strategy typically involves a moderate price with minimal advertising expenditure.

- **Network Effects on Demand:** Our analysis of how the average influence score of a seller's customers affects demand highlighted the importance of customer networks in online markets. Sellers whose customers are more socially influential experience increased demand, further emphasizing the role of network effects in market competition.

Future Work

While the current model provides valuable insights into seller competition and social influence in online markets, several extensions and improvements can be made:

- **Dynamic Pricing and Advertising:** In this study, prices and advertising budgets were treated as static strategies. Future work could extend the model to allow for dynamic pricing, where sellers continuously adjust their strategies based on market conditions, such as changes in competitor behavior or shifts in consumer preferences.

- **Customer Heterogeneity:** The current model assumes that all customers behave similarly. Incorporating customer heterogeneity (e.g., varying willingness to pay or different price sensitivities) would make the model more realistic and allow for a deeper understanding of how different customer segments impact the equilibrium.

- **Network Structure and Evolution:** The current network structure is static, but in real-life markets, customer networks evolve over time. Future research could explore the dynamics of network growth and the evolution of influence over time, as well as the effects of viral marketing and information diffusion.

- **Incorporating External Factors:** The model could be extended to account for external factors, such as seasonal demand fluctuations or changes in competitor strategies. Including such variables would provide a more comprehensive view of online market dynamics.

- **Multiple Products and Product Bundling:** Future extensions could explore competition between sellers offering multiple products or product bundles. This would allow for a more nuanced analysis of pricing strategies across different product categories and the potential for cross-selling.

Overall, this work lays the foundation for further exploration of competitive dynamics in online marketplaces, with applications in e-commerce strategy, digital marketing, and social network analysis. By extending the model to incorporate more complex behaviors, the insights derived from this study could be applied to real-world market design and strategy optimization.