Ising Model

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1.2D Ising Model

I followed the approach mentioned in the text by creating an `IsingModel` class. The class requires initial conditions including either, Network size, or A predefined spin configuration along with the available spin states, interaction coefficient (J), and temperature (T).
The class contains three key methods:

1. `energy()`: Computes the total energy of the system using periodic boundary conditions.

2. `delta_E(i,j)`: Calculates the energy difference ΔE that would result from flipping the spin at position (i, j). This implementation uses the efficient neighbor-counting approach discussed in the text.

3. `MCstep(N,s,visualize)`: Performs N Monte Carlo steps according to the Metropolis algorithm. When enabled via the `visualize` parameter, it displays the spin configuration at intervals of s steps (see code for implementation details).

For computational efficiency, the class precomputes all possible Boltzmann factors $e^{-\beta\Delta E}$ for the allowed energy changes. The implementation handles both random initialization and predefined spin configurations.

after defining the class as above i used a loop and gradually decrease Tempreture from 100 to 0.1 with step 0.1. at each tempreture i perform 300 Monte Carlo step and finally after all these steps at each tempreture i plot the network state, i used red color for negetive spins and blue for positive spins, the results are as follows:

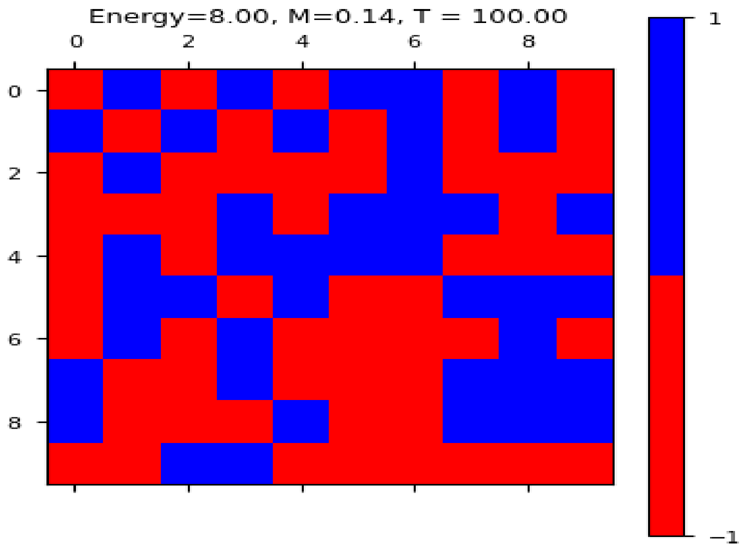

Figure 1

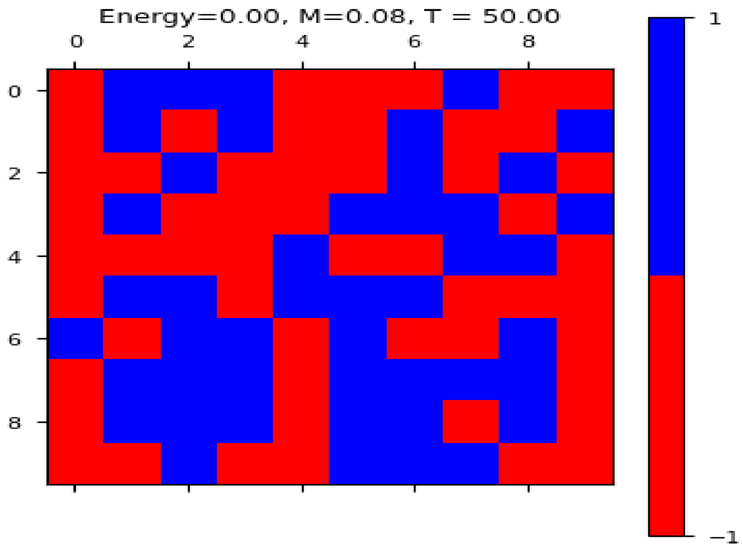

Figure 2

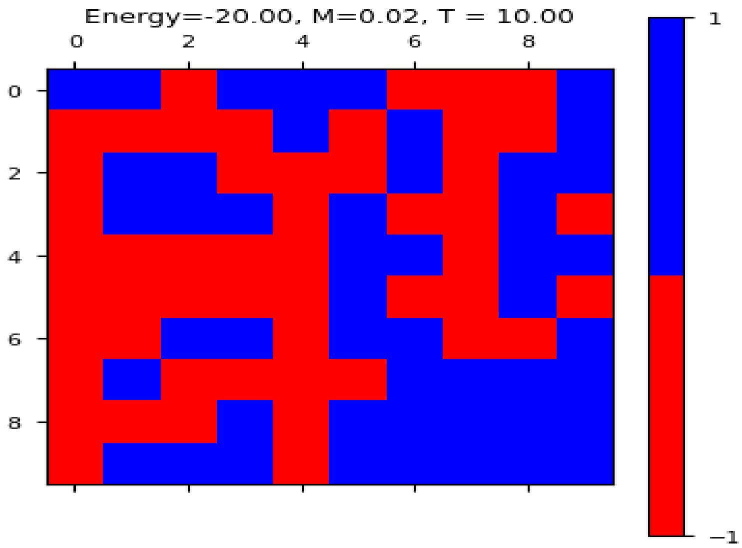

Figure 3

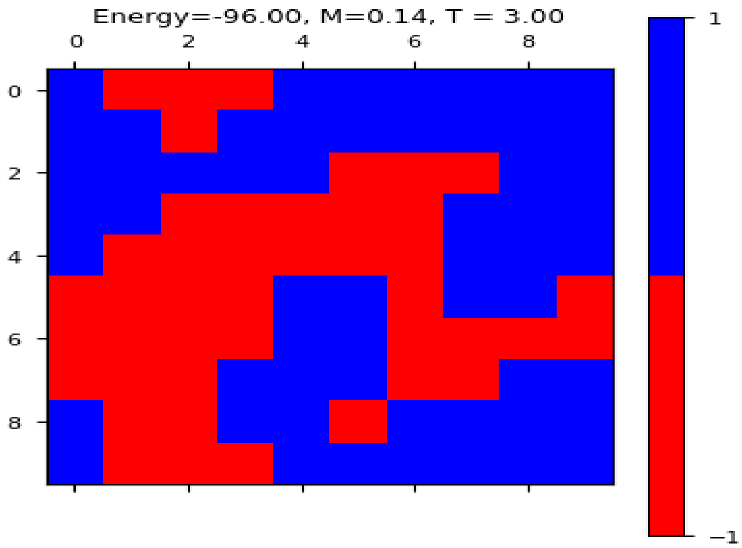

Figure 4

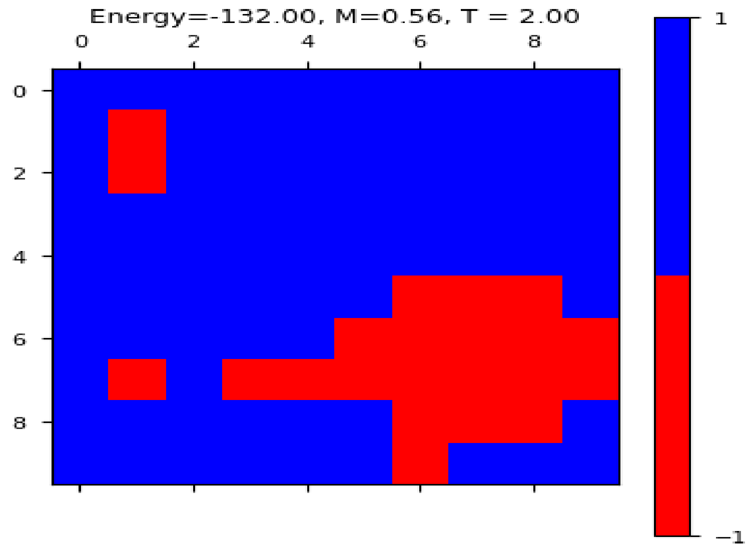

Figure 5

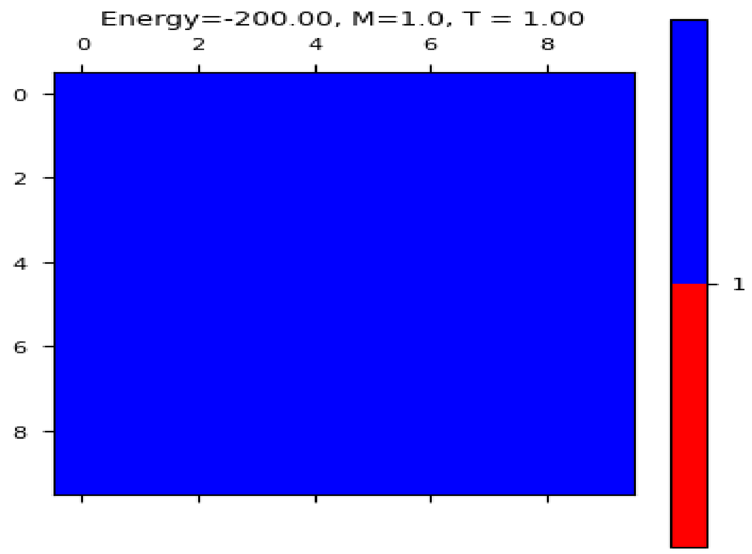

Figure 6

as expected at low tempreture all spins are in the same direction,you can see the state of the network at other tempreture aswell(check the code).now i plot magntization versus T and versus 1/T:

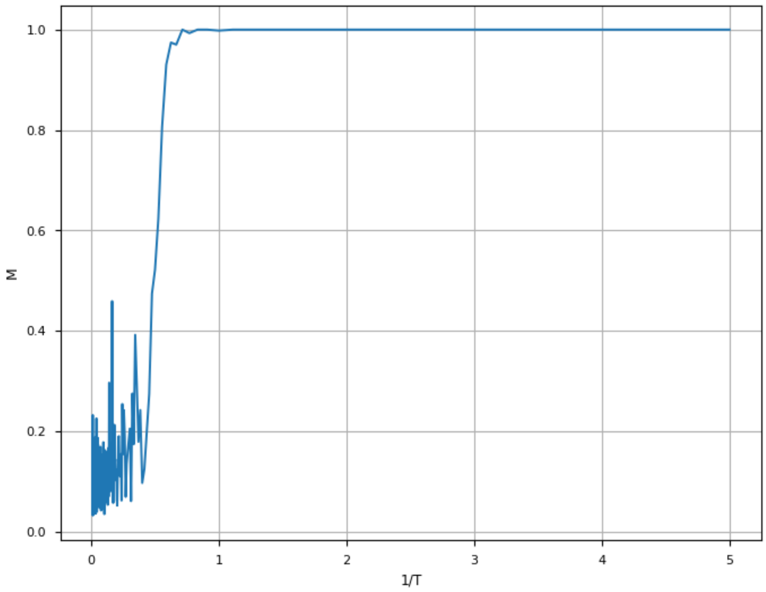

Figure 7

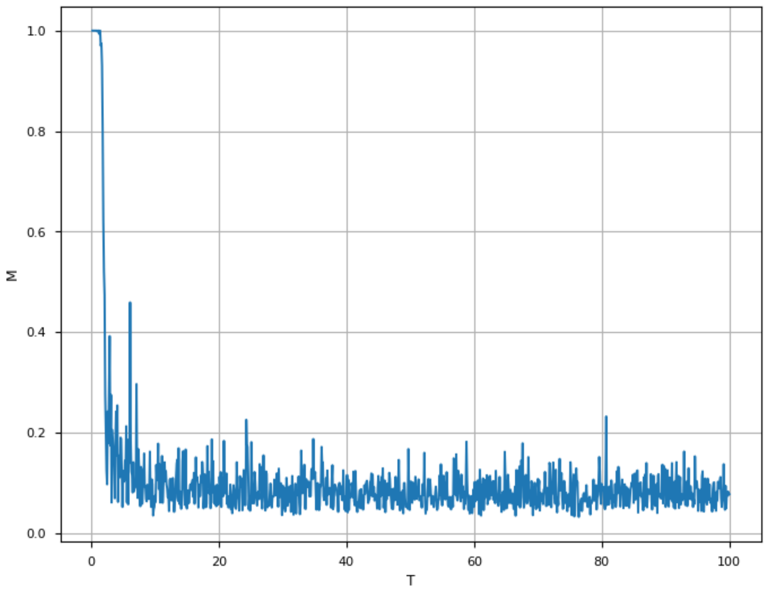

Figure 8

2.magnetic susceptibility

I have to prove:

$$\chi = \frac{\sigma_m^2}{k_B T}$$

We know:

$$\chi = \left(\frac{\partial \langle M \rangle}{\partial H}\right)_{H=0}$$

for calculating this derivative i apply small external magnetic field (H), So:

$$Z = \sum_{\text{states}} e^{-\beta(E_0 - HM)} = \sum_{\text{states}} e^{-\beta E_0} e^{\beta HM}$$

$$\langle M \rangle = \frac{1}{Z} \sum M e^{-\beta E_0} e^{\beta HM}$$

because H is very small i can expand every term using tylor expansion:

$$e^{\beta HM} \approx 1 + \beta HM$$

$$\sum M e^{-\beta E_0}(1 + \beta HM) = \sum M e^{-\beta E_0} + \beta H \sum M^2 e^{-\beta E_0}$$

$$= Z_0 \langle M \rangle_0 + \beta H Z_0 \langle M^2 \rangle_0$$

Simillarly for Z We can write:

$$Z = Z_0(1 + \beta H \langle M \rangle_0)$$

So:

$$\langle M \rangle = \frac{Z_0 \langle M \rangle_0 + \beta H Z_0 \langle M^2 \rangle_0}{Z_0(1 + \beta H \langle M \rangle_0)}$$

$$= \frac{\langle M \rangle_0 + \beta H \langle M^2 \rangle_0}{1 + \beta H \langle M \rangle_0}$$

$$\frac{1}{1 + \beta H \langle M \rangle_0} \approx 1 - \beta H \langle M \rangle_0$$

$$\langle M \rangle \approx \left(\langle M \rangle_0 + \beta H \langle M^2 \rangle_0\right)(1 - \beta H \langle M \rangle_0)$$

$$\langle M \rangle \approx \langle M \rangle_0 + \beta H \langle M^2 \rangle_0 - \beta H \langle M \rangle_0^2$$

$$\langle M \rangle \approx \langle M \rangle_0 + \beta H \left(\langle M^2 \rangle_0 - \langle M \rangle_0^2\right)$$

$$\chi = \left(\frac{\partial \langle M \rangle}{\partial H}\right)_{H=0} = \beta \left(\langle M^2 \rangle_0 - \langle M \rangle_0^2\right)$$

$$\chi = \frac{\sigma_m^2}{k_B T}$$

3.Calculation of some physical properties using 2D ising model

To solve this question, I first modified my previous *IsingModel* class. I saved the energy history and defined a method, *corr_len*, for calculating the correlation length using the following formula:

$$C(l) = \frac{\langle S_{i,j}S_{i,j+l}\rangle_{ij} - \langle S_{i,j}\rangle\langle S_{i,j+l}\rangle}{\sigma^2}$$

I faced several challenges:

- At high temperatures, correlation lengths were very large. I handled this by setting different thresholds for valid computed $C(l)$ values and on σ^2 of the network.
- At each temperature, I reported the average of these correlation lengths as the correlation length of the network at that temperature.

I calculated the Heat Capacity and χ using their standard formulas:

$$\chi = \frac{\sigma_m^2}{k_B T}$$

$$C_V = \frac{\sigma_E^2}{k_B T}$$

the results are as follows:

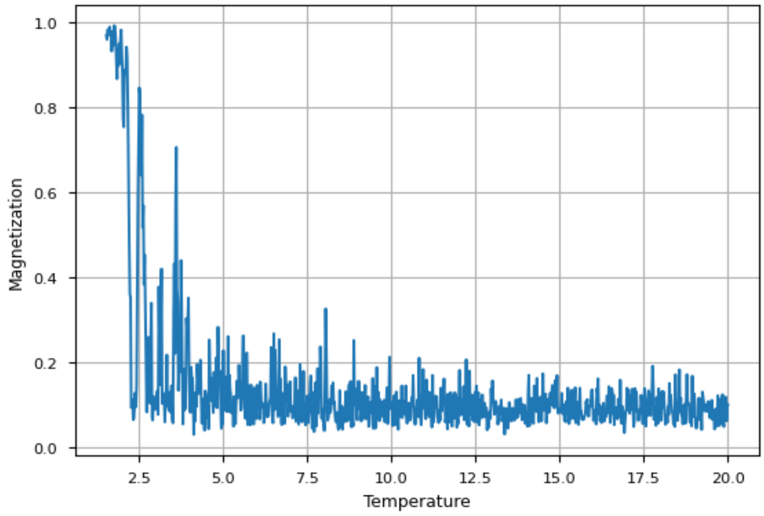

Figure 9

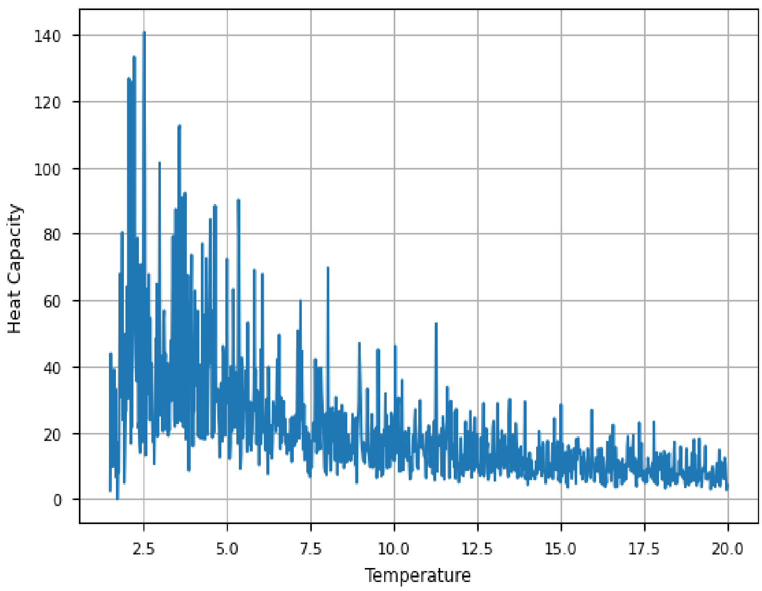

Figure 10

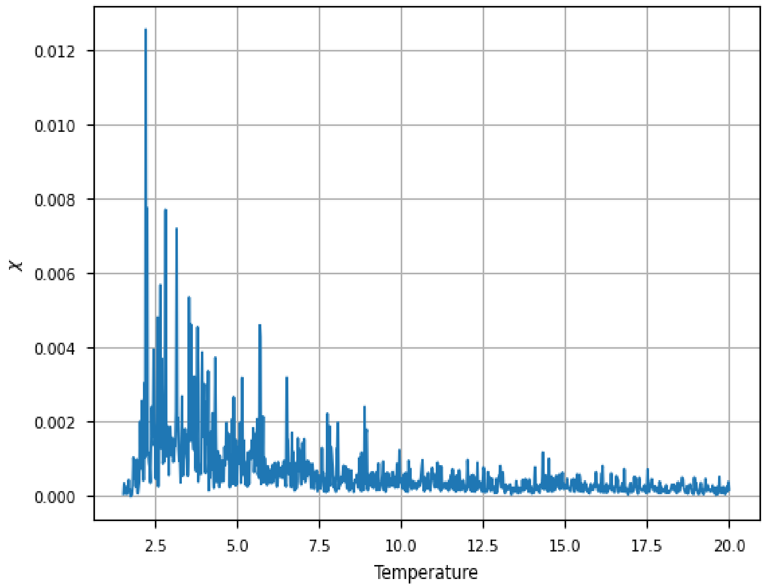

Figure 11

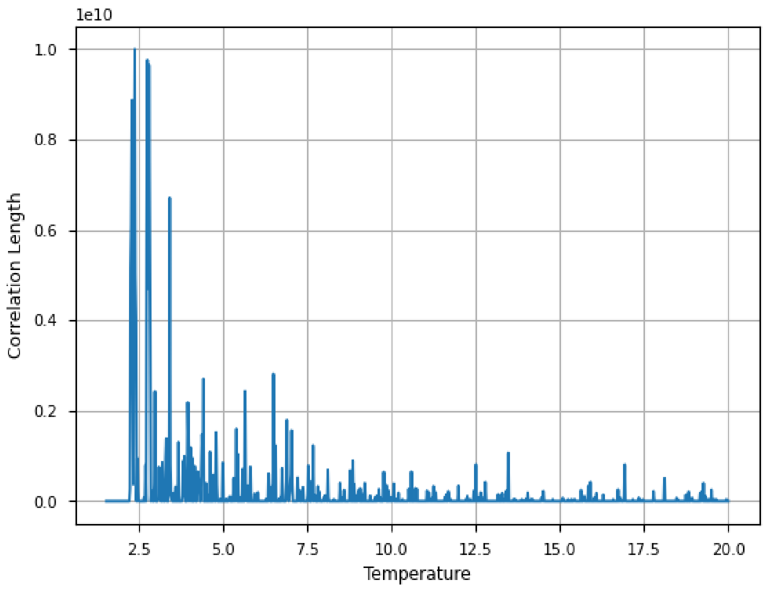

Figure 12

I used $J = 1$ and $k_B = 1$, so the critical temperature is 2.27 under these conditions. While some noise remains visible in the results, all plots clearly show a sharp peak around $T \approx 2.4$, which is very close to the critical temperature.

Now let's examine how these parameters vary with network size (from 5 to 24). While the code for generating the plots is available, here I present only their maximum values across different network sizes:

Heat Capacity	χ	M	Correlation length	Max Correlation Temp	Network size
2.29e+01	2.87e-02	1.00e+00	0.00e+00	1.50e+01	5.0
4.39e+01	2.28e-02	9.72e-01	0.00e+00	1.50e+01	6.0
2.51e+01	1.34e-02	9.84e-01	0.00e+00	1.50e+01	7.0
4.37e+01	1.68e-02	1.00e+00	0.00e+00	1.50e+01	8.0
4.36e+01	8.06e-03	1.00e+00	0.00e+00	1.50e+01	9.0
8.07e+01	1.56e-02	9.93e-01	1.46e+09	4.60e+00	10.0
1.32e+02	1.05e-02	1.00e+00	8.59e+09	3.20e+00	11.0
7.80e+01	3.69e-03	9.95e-01	8.05e+09	2.50e+00	12.0
7.17e+01	2.32e-03	9.94e-01	4.35e+09	2.70e+00	13.0
1.66e+02	4.05e-03	1.00e+00	3.00e+09	4.90e+00	14.0
1.41e+02	1.28e-03	1.86e-01	1.00e+10	1.90e+00	15.0
1.55e+02	2.13e-03	8.20e-01	1.00e+10	2.20e+00	16.0
2.11e+02	1.12e-03	9.65e-01	1.00e+10	2.40e+00	17.0
8.43e+01	1.09e-03	4.36e-01	8.81e+09	2.60e+00	18.0
1.56e+02	7.98e-04	1.79e-01	1.00e+10	1.50e+00	19.0
1.52e+02	1.04e-03	6.87e-01	1.00e+10	3.30e+00	20.0
2.58e+02	6.77e-04	3.02e-01	1.00e+10	1.90e+00	21.0
3.43e+02	1.02e-03	8.40e-01	1.00e+10	2.30e+00	22.0
6.87e+02	6.53e-04	5.46e-01	1.00e+10	2.10e+00	23.0
2.42e+02	3.91e-04	1.83e-01	1.00e+10	2.20e+00	24.0

Figure 13

Here are the values for different network sizes. The temperature for small networks shows slightly unexpected results, likely due to finite-size effects. For larger networks, the deviation decreases progressively and approaches the expected critical temperature $T_c = 2.27$.

To calculate the critical exponents ν, γ, and β, I will use log-log plots of the network size versus $M_{\max}$ and $\chi_{\max}$. The results for these values are as follows:

$$\nu = 16$$

$$\gamma = 5.4$$

$$\beta = 100$$

These results are unsatisfactory! The discrepancy likely stems from the calculation of $T_c(L)$.

To determine c_0, I fitted a linear model to C_v versus $\ln(|T - T_c|)$. For this network size, $T_c = 2.45$. Using this approach, we obtain:

$$c = 0.54$$

which is very close to the theoretical value of 0.4945, with approximately 10% error.