Estimation of the Black-Scholes model

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Introduction

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t$$

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Financial Mathematics History

1995 : Bibby and Sorensen
Martingale estimating functions for discretely observed diffusion processes

1973 : Black and Scholes
The Pricing of Options and Corporate Liabilities

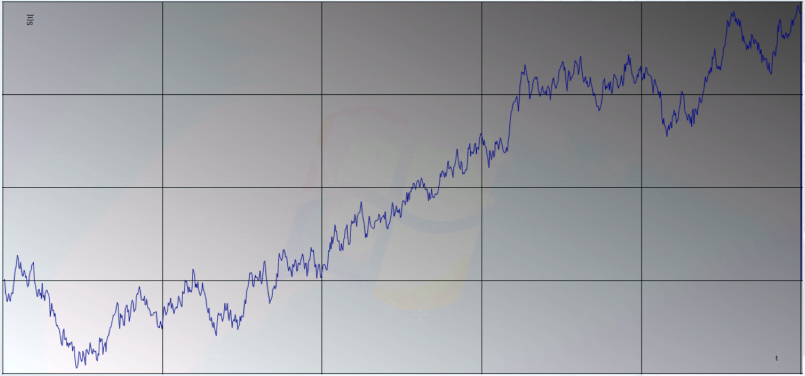
1923 : Wiener
Differential space

1900 : Bachelier
The Theory of Speculation

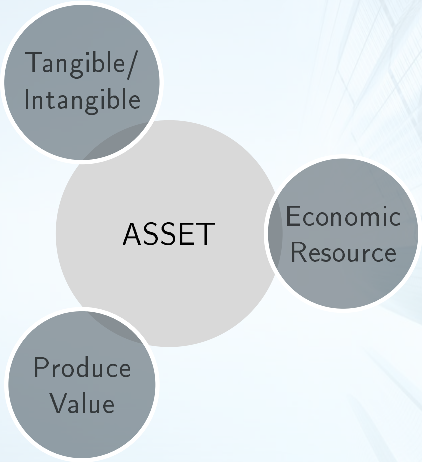
Brownian Motion

- Gaussian process
  - Stochastic : depending on time and randomness
  - Process distribution : Gaussian distribution
- Continuous path
- Covariance : $cov(B_s, B_t) = t \wedge s$

Brownian Motion Simulation



Asset


Tangible/
Intangible
ASSET
Economic
Resource
Produce
Value

Black-Scholes Equation

Stochastic differential equation :

$$\begin{cases} \mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t \\ S_0 > 0 \end{cases}$$

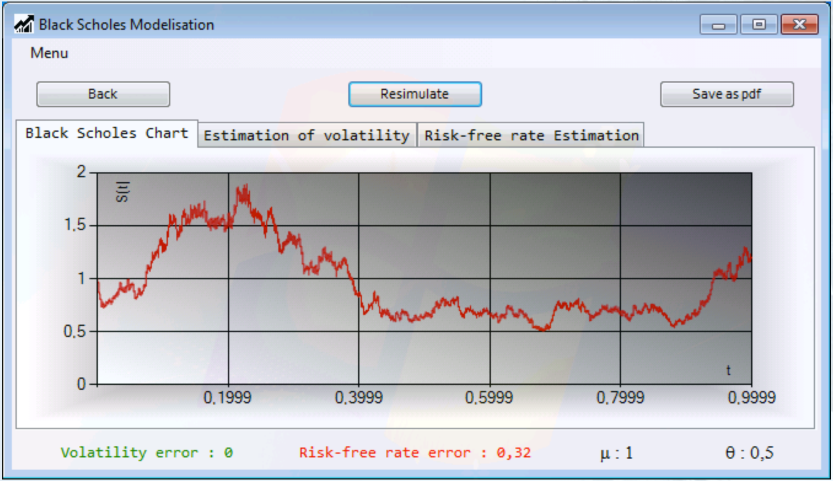
- S_t : price of the asset at a time t
- μ : drift
- σ : volatility

Black-Scholes Solution

$$S_t = S_0 \exp(\theta t + \sigma B_t)$$

- S_t : price of the asset at a time t
- t : time until maturity
- B : Brownian motion
- θ : risk-free rate $(\mu - \frac{\sigma^2}{2})$
- σ : volatility

Black-Scholes Simulation

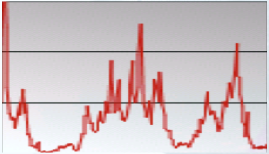


Volatility

$$S_t = S_0 \exp(\theta t + \sigma B_t)$$

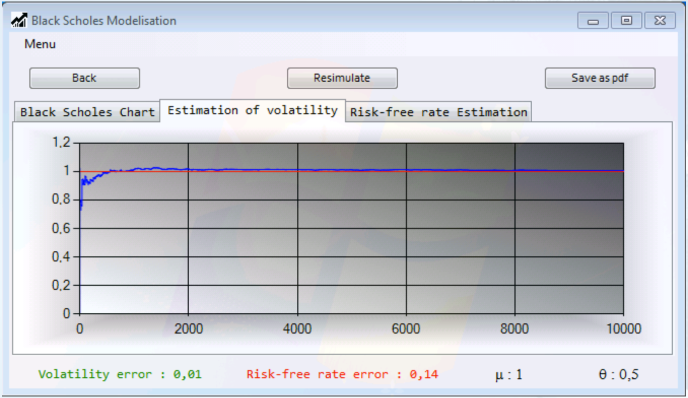
Measure of the amplitude of the financial asset variations

- HIGH Volatility
  - risky investment



- LOW Volatility
  - asset with few risks

Volatility Estimation



$$\hat{\sigma}_n^2 = \frac{1}{n-1}\left(\sum_{k=0}^{n-1}\frac{\mid X_{t_{k+1}} - X_{t_k} \mid^2}{t_{k+1} - t_k} - \frac{\mid X_T - X_0 \mid^2}{T}\right)$$

Risk-Free Rate

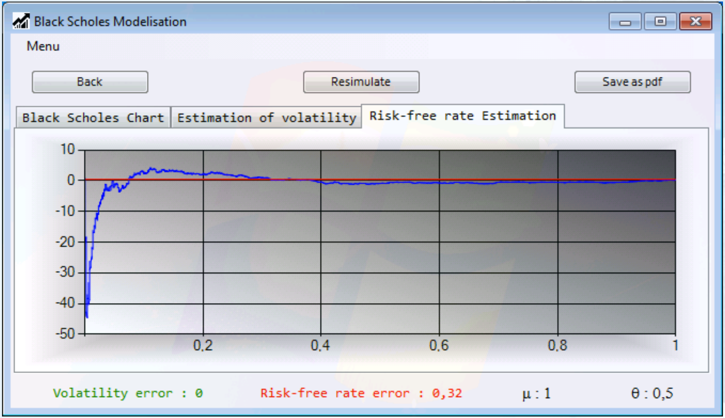
$$S_t = S_0 \exp(\theta t + \sigma B_t)$$

The theoretical rate of return of an investment with no risk of financial loss

$$\theta = \mu - \frac{\sigma^2}{2}$$

- μ : drift -> trend
- σ : volatility -> noise

Risk-Free Rate Estimation



$$\widehat{\theta}_T = \frac{X_T - X_0}{T}$$

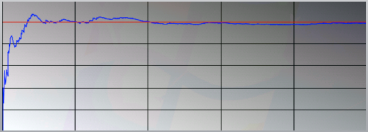
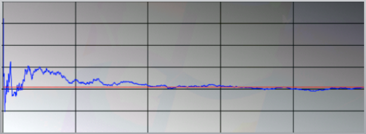
Drift

$$\begin{cases} \mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t \\ S_0 > 0 \end{cases}$$

$$\theta = \mu - \frac{\sigma^2}{2}$$

Trends that controls the deterministic component of the process

Relevance Of Estimation

Volatility	Risk-free rate
Number of Observations +++	Long Time +++
	
Volatility error : 0,01	Risk-free rate error : 0,02

Application

Thank you for your attention !