MTH 2245 SF&PDE
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PINN for solving PDE and Its application in Alzheimer's disease

Abstract

- Alzheimer's disease (AD) is a neurodegenerative disorder characterized by the accumulation of misfolded tau proteins in the brain, crucial to disease progression.This study integrates Physics-Informed Neural Networks (PINNs) with symbolic regression to model the dynamics of tau proteins using reaction-diffusion equations.By applying our approach, we successfully predicted the reaction term in the equation, revealing a faster rate of tau misfolding in individuals with Alzheimer's compared to healthy controls.

Problem definition

- Alzheimer's disease (AD) involves the accumulation of misfolded tau proteins in the brain, which can be modeled using a reaction-diffusion equation (PDE). The concentration of tau proteins c(t,x) evolves over time, described by:

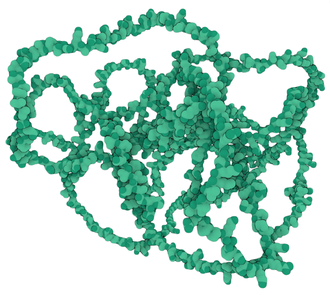
$$\frac{\partial C}{\partial t} = \nabla \cdot (D \cdot \nabla C) + f(C)$$

- We discretize the reaction-diffusion model on a weighted graph G, where each node i represents a brain region and the edges represent connections between them. This gives a discretized equation for the change in tau protein concentration ci at each region over time:

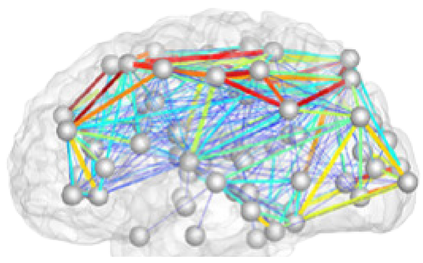
$$\frac{dc_i}{dt} = h_i^k(t, C) + f(c_i) \rightarrow i = 1, \dots, N$$

Literature review

- Chronological Structure: Early studies focused on predefined reaction terms to model biological processes, But in our approach we use multiple reaction terms for more precise prediction.

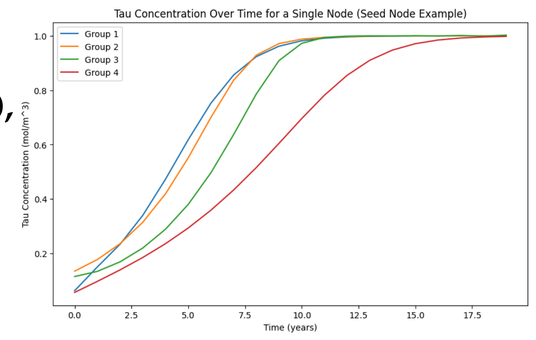


- Methodological Structure: (PINNs) Proven effective for solving PDEs with missing terms, though applications to tau protein dynamics are still emerging.



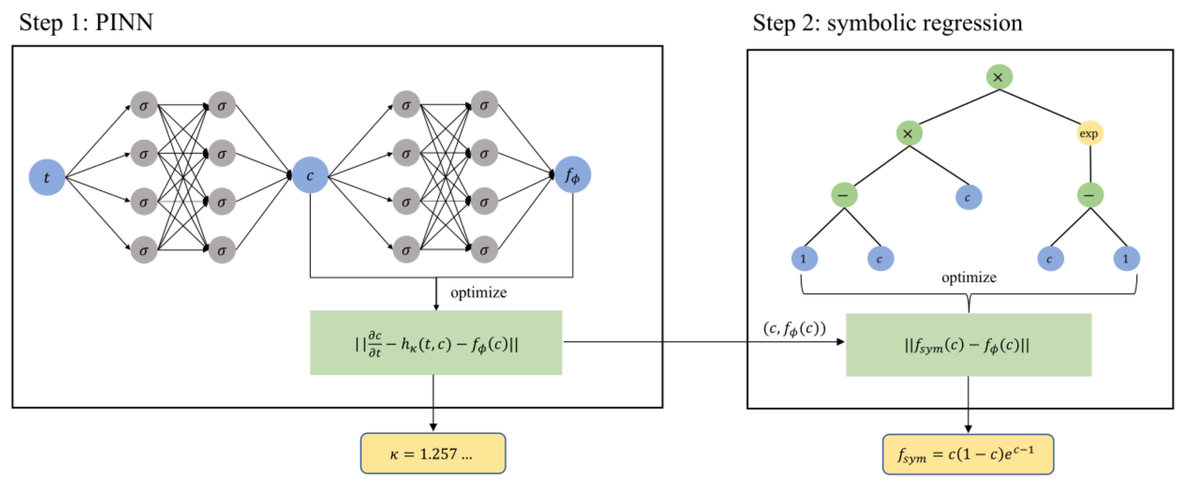
Data description

- In this study, we modeled tau concentration for 76 subjects, divided into four groups, using synthetic data.
- Parameters k, αi, and αij were assigned distributions BoundNormal(1, 0.52),N(0.6, 0.12), and N(αi,0.22), respectively.
- Initial tau concentrations, ci(0), were sampled from a normal distribution based on real data.
- Tau concentrations were sampled at tk= k for k = 0 to 20 years.



Methodology

- Our approach consists of two main branches PINN and Symbolic Regression.



- For discovering f is formulated as follows: first infer c, f, and κ from data of c and the physics defined in this equation using PINNs, and then find the analytic expression of f using symbolic regression.

$$\frac{dC_i}{dt} = -K \sum_{j=1}^{N} L_{ij} C_j + \alpha f(C_i) \rightarrow i = 1, ..$$

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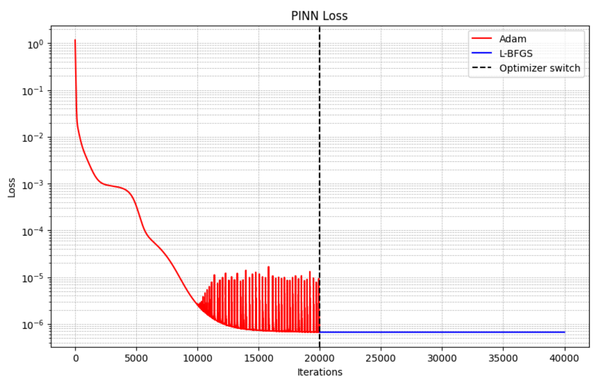
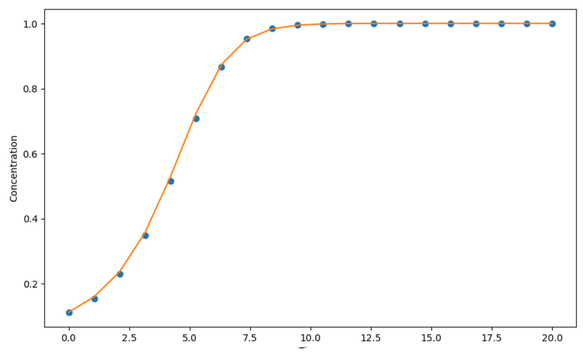
Results

- PINNs and symbolic regression accurately predicted tau protein concentrations, demonstrating alignment with synthetic data.
- Integrated physical laws into the loss function for efficient training without numerical methods.
- Used Adam Optimizer (20,000 epochs) for initial training and L-BFGS Optimizer (20,000 epochs) to ensure convergence.
- Identified interpretable reaction term f(c) using symbolic regression.





κ	α	f_{sym}
0.29	0.15	$8.908 \cdot c^3 - 23.446 \cdot c^2 + 15.667 \cdot c + 0.792$

Conclusion

- We demonstrated the effectiveness of combining PDE-based modeling with machine learning.
- We predicted tau concentrations and patterns using synthetic data.
- We used symbolic regression and PINNs to model tau dynamics in Alzheimer's.

Future work

- Enhance tau dynamics model with PET and MRI data.
- Visualize tau trends and disease progression using a web-based platform.

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Scan QR code for references