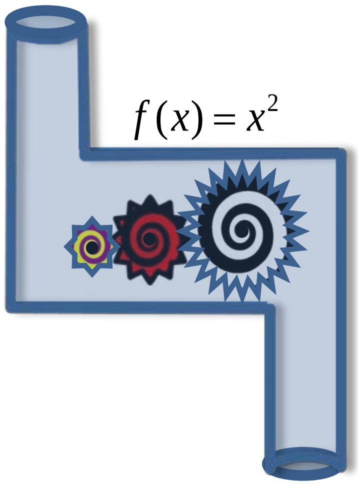
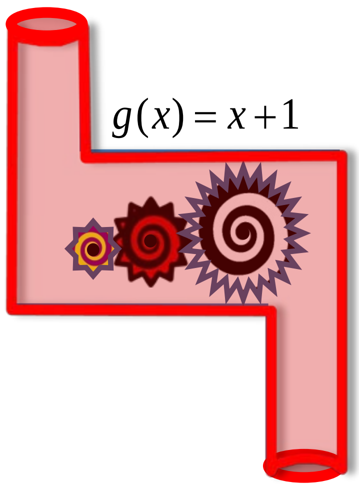
Aula 06

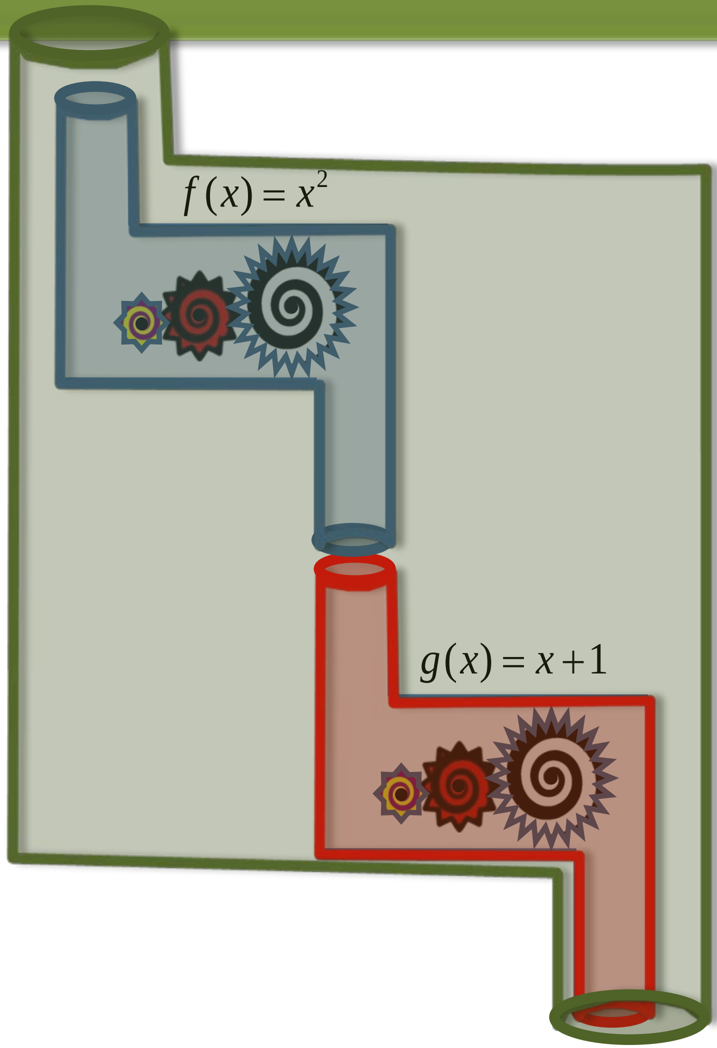
Função composta e função inversa.

Função Composta





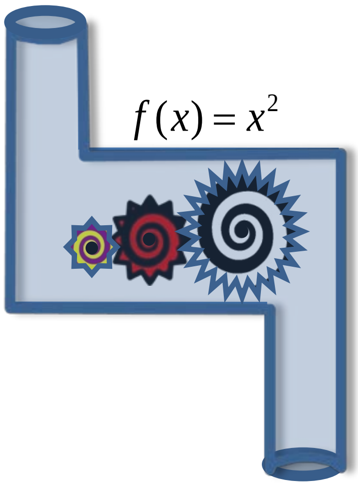
Função Composta

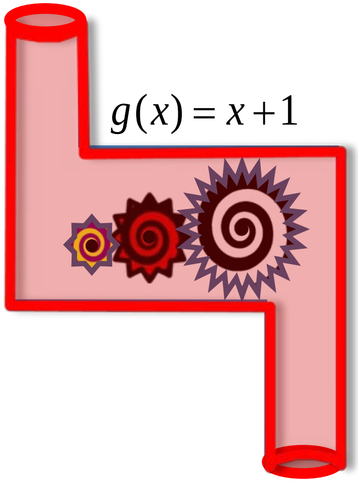


$$g(f(x)) = g(x^2)$$

$$g(f(x)) = x^2 + 1$$

Função Composta



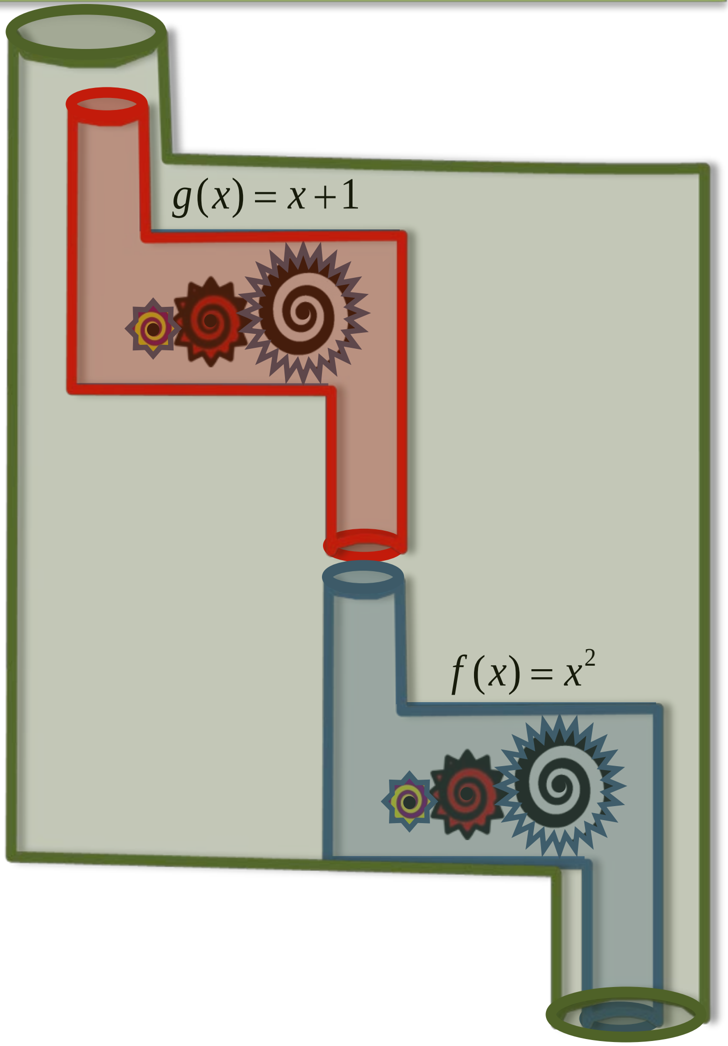


Função Composta

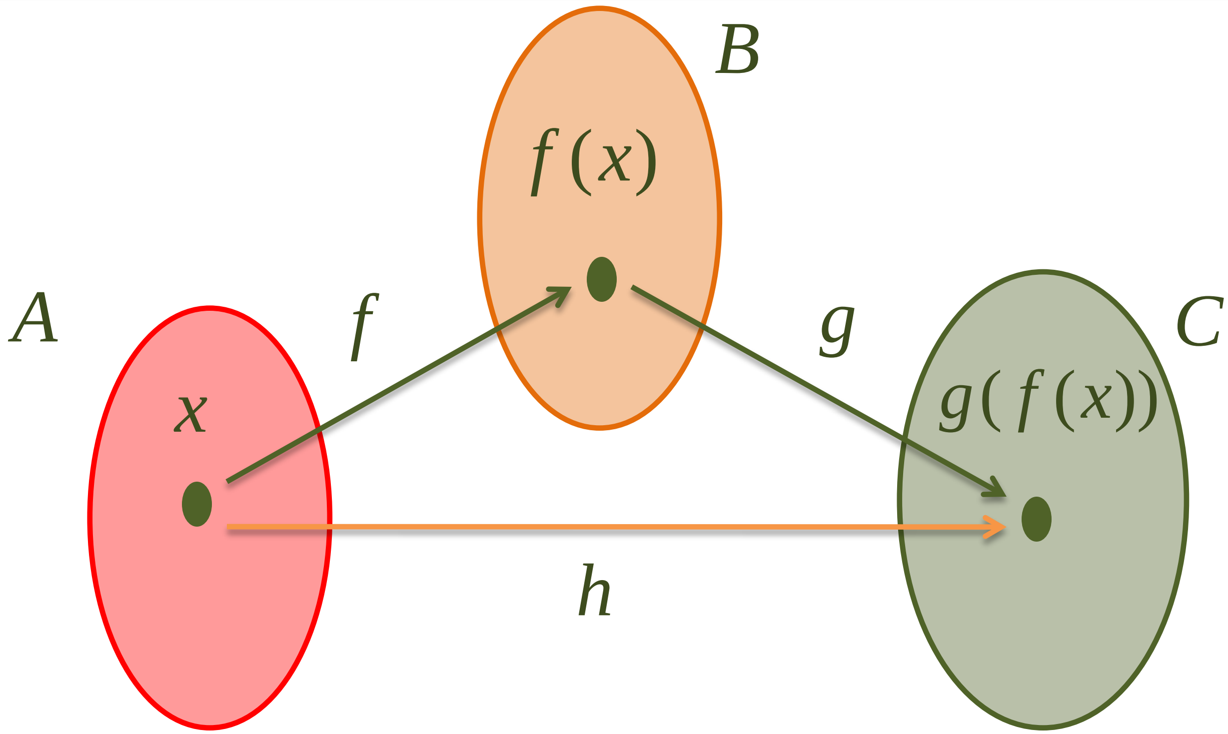
$$f(g(x)) = f(x+1)$$

$$f(g(x)) = (x+1)^2$$

$$f(g(x)) = x^2 + 2x + 1$$

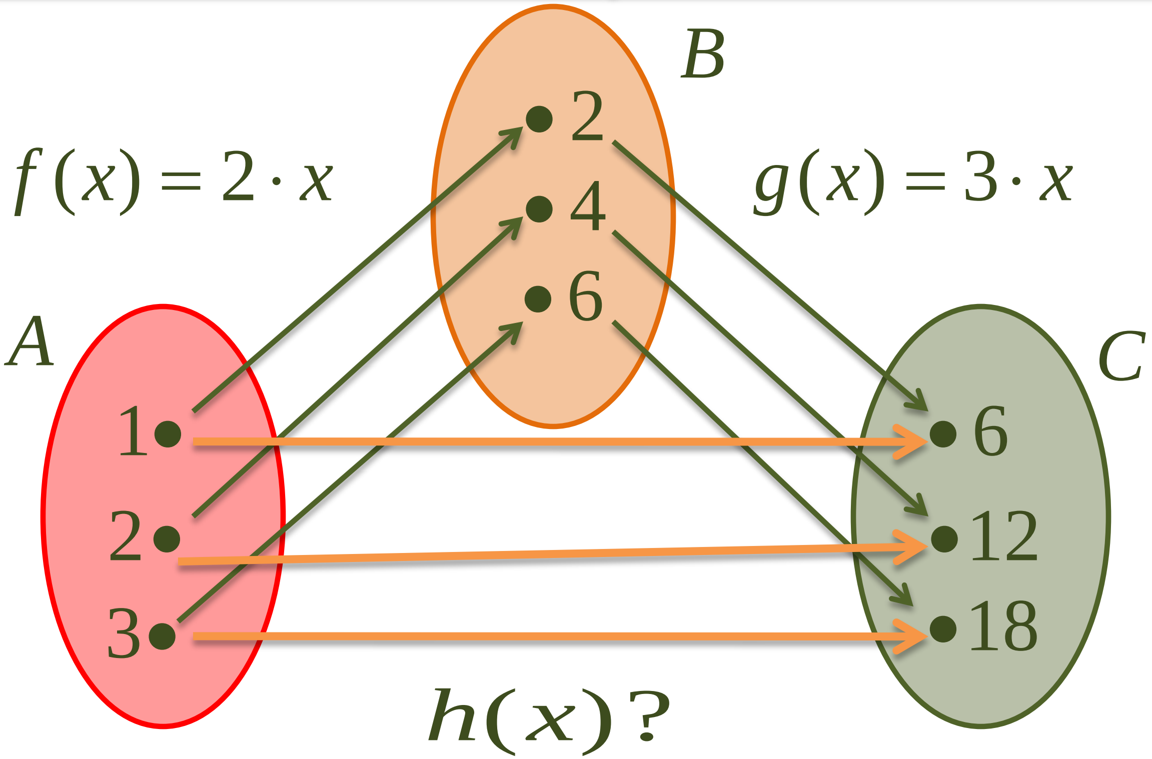


Função Composta



$$h(x) = g(f(x)) = (g \circ f)(x)$$

Exemplo 1



$$g(x) = 3 \cdot x$$

$$g(f(x)) = 3 \cdot f(x) = 3 \cdot 2 \cdot x = 6 \cdot x$$

$$\therefore h(x) = 6 \cdot x$$

Exemplo 2

Sejam f e g funções reais. Determine $\left(f \circ g\right)(x)$ e $\left(g \circ f\right)(x)$ em cada caso:

a) $f(x) = 2x - 3$ e $g(x) = x^2 - 3$

$$\left(f \circ g\right)(x) = 2x^2 - 9 \qquad \left(g \circ f\right)(x) = 4x^2 - 12x + 6$$

b) $f(x) = \dfrac{x}{x+1}$ e $g(x) = 9 - x^2$

$$\left(f \circ g\right)(x) = \frac{x^2 - 9}{x^2 - 10} \qquad \left(g \circ f\right)(x) = 9 - \frac{x^2}{(x+1)^2}$$

Exemplo 3

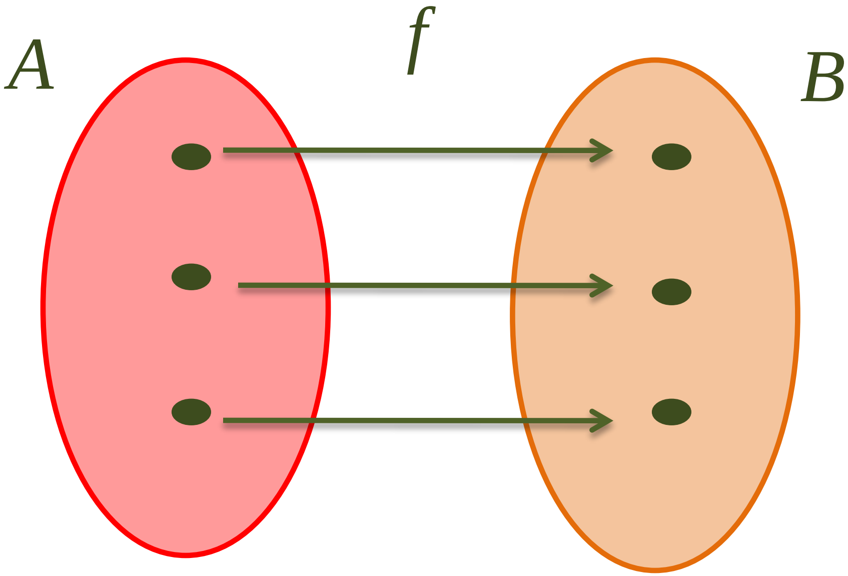
Encontre $f(x)$ e $g(x)$ de modo que a função possa ser escrita como $h(x) = f(g(x))$:

a) $h(x) = \sqrt{x^2 - 5x}$ $\quad f(x) = \sqrt{x}$ e $g(x) = x^2 - 5x$

b) $h(x) = |3x - 2|$ $\quad f(x) = |x|$ e $g(x) = 3x - 2$

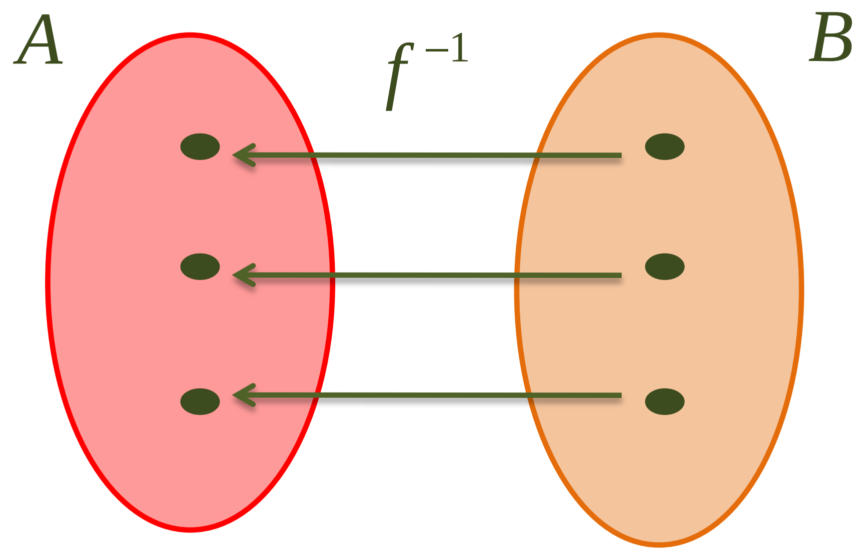
c) $h(x) = \dfrac{1}{x^3 - 5x + 3}$ $\quad f(x) = \dfrac{1}{x}$ e $g(x) = x^3 - 5x + 3$

Função Inversa



$$f : A \to B \text{ bijetora}$$

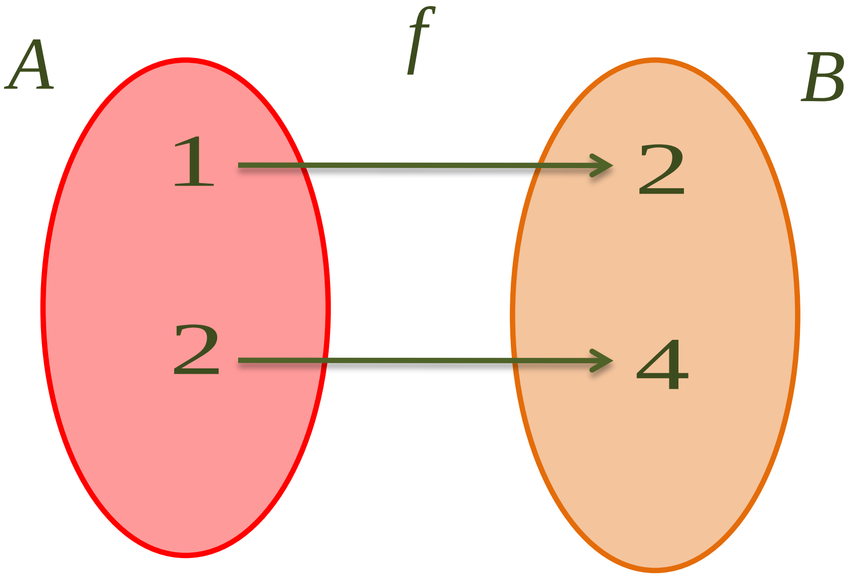
$$D(f) = A \quad \mathrm{Im}(f) = B$$



$$f^{-1} : B \to A$$

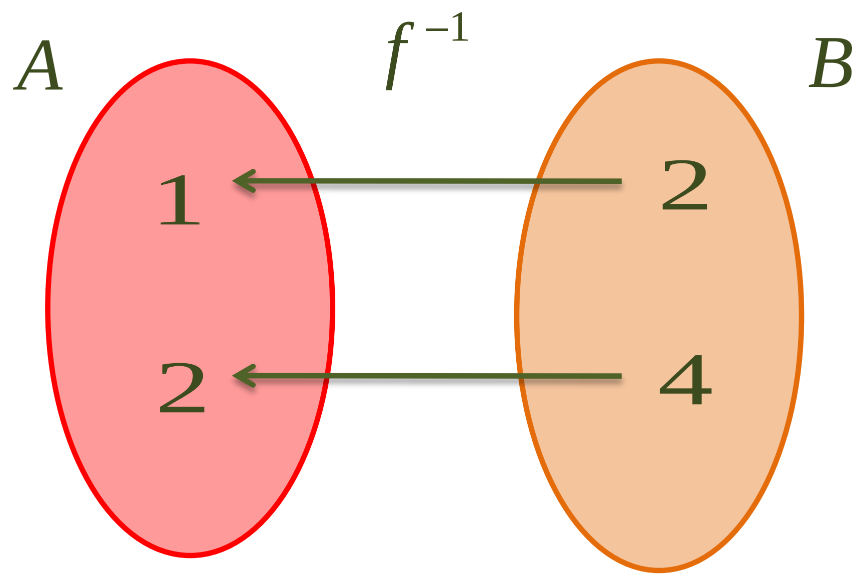
$$D(f^{-1}) = B \quad \mathrm{Im}(f^{-1}) = A$$

Exemplo



$$f : A \to B$$
$$x \mapsto y = 2x$$

$$f^{-1} : B \to A$$
$$x \mapsto y = \frac{x}{2}$$



Como obter a função inversa?

$$y = 2 \cdot x$$

I) $x = 2 \cdot y$

II) $2 \cdot y = x \Leftrightarrow \underbrace{y = \frac{x}{2}}_{\text{inversa } f^{-1}}$

$$\therefore f^{-1}(x) = \frac{x}{2}$$

Exemplo 4

A fórmula para converter a temperatura Celsius x em temperatura Kelvin é $k(x) = x + 273{,}16$.

A fórmula para converter a temperatura Fahrenheit em temperatura Celsius é

$$c(x) = \frac{5(x-32)}{9}.$$

a) Encontre $c^{-1}(x)$. Para que serve esta fórmula?

b) Encontre $(k \circ c)(x)$. Para que serve esta fórmula?

Solução

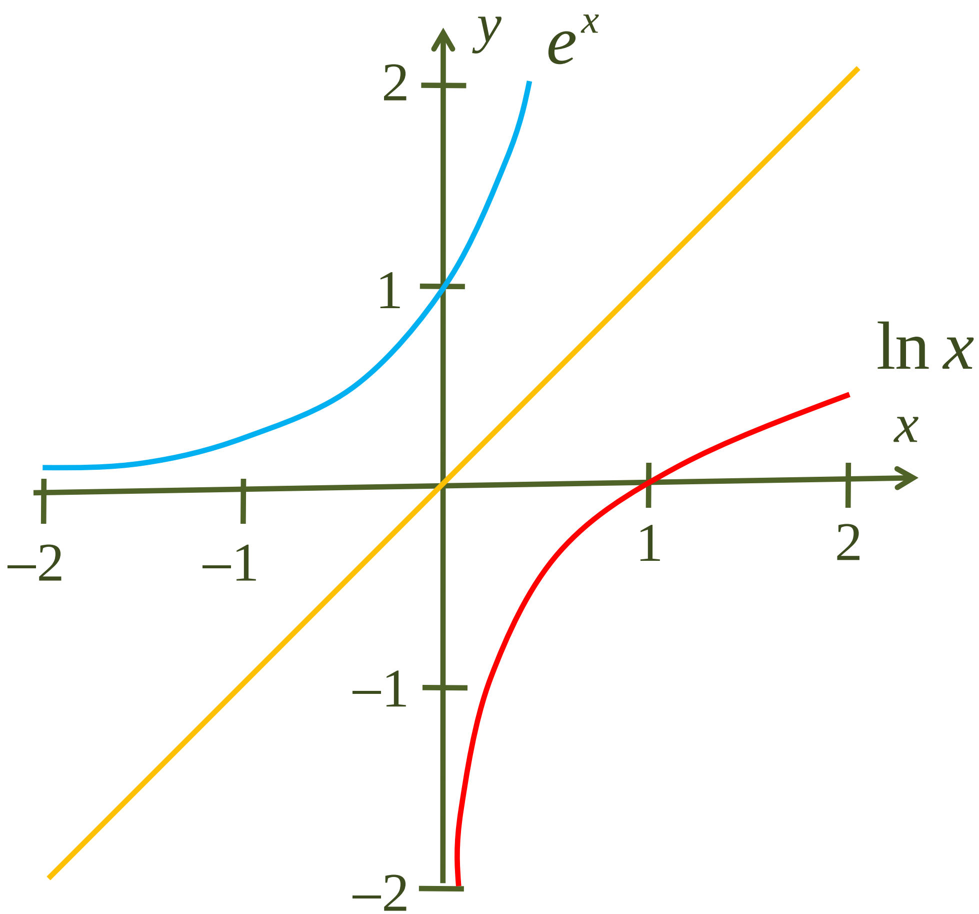
a) $c^{-1}(x) = \frac{9}{5}x + 32$

Converte de Celsius para Fahrenheit.

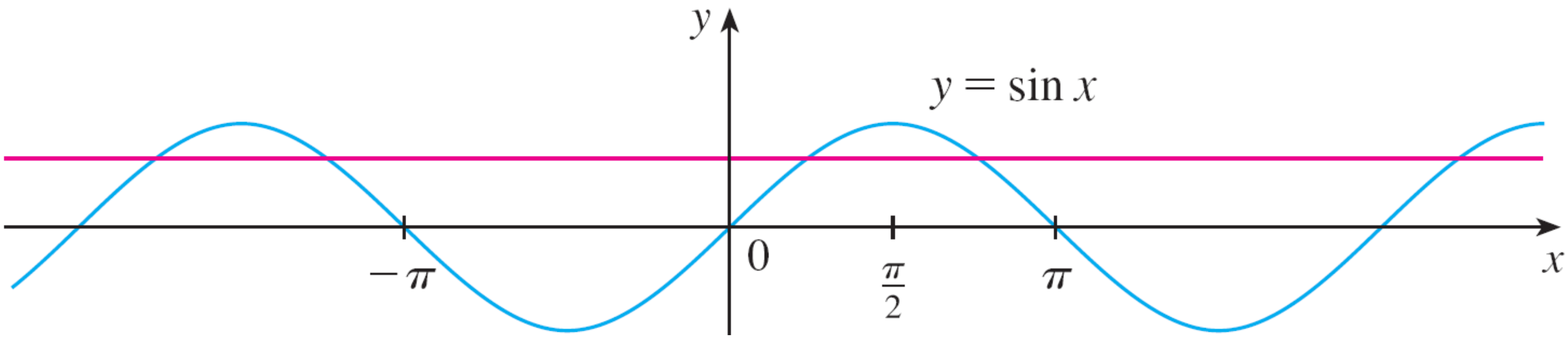
b) $(k \circ c)(x) = \frac{5}{9}x + 255,38$

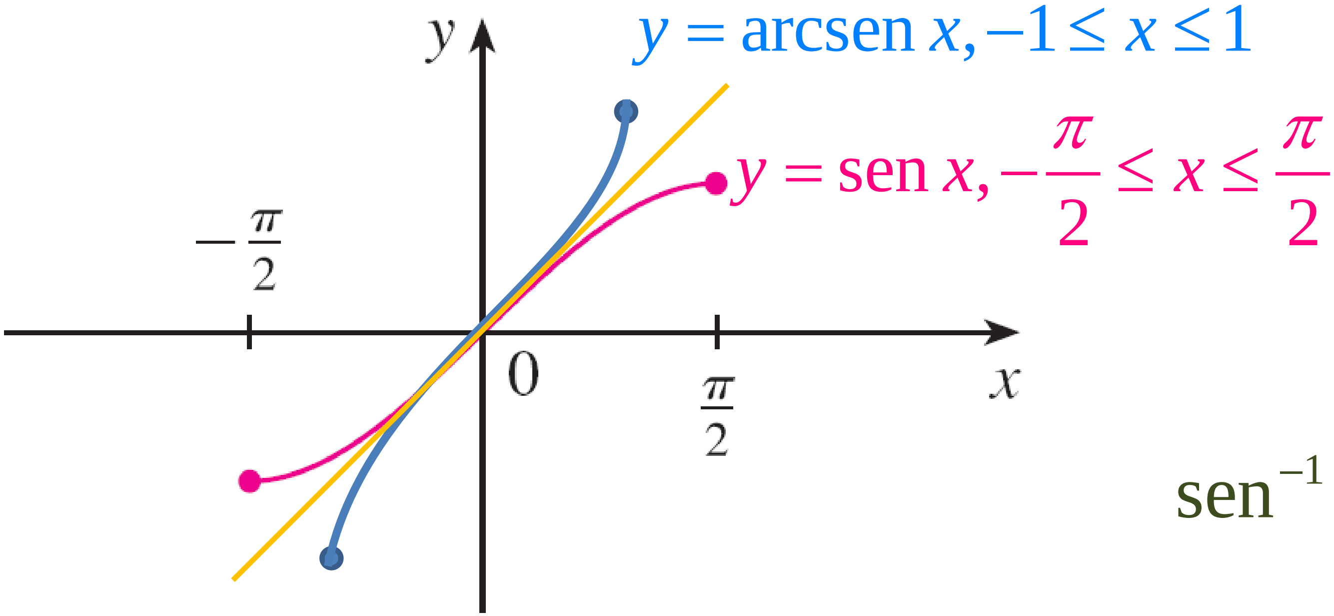
Converte de Fahrenheit para Kelvin.

Exemplo 5



$arc\,\text{sen}\; x$





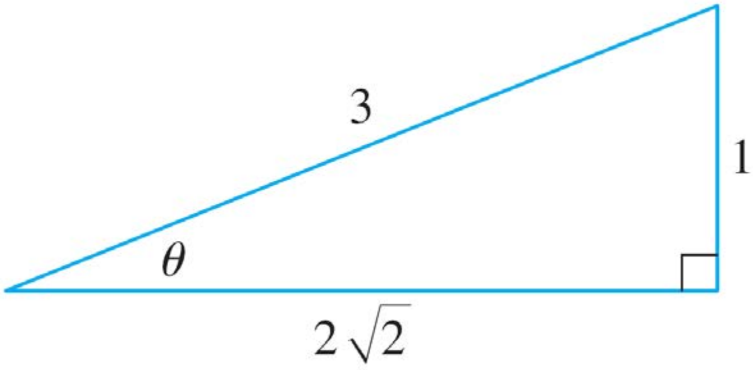
$$\text{sen}^{-1} x \neq \frac{1}{\text{sen}\, x}$$

Exemplo 6

Calcule (a) $\arcsen(1/2)$ (b) $\operatorname{tg}(\arcsen(1/3))$

a) $\arcsen(1/2) = \pi/6$, pois $\operatorname{sen}(\pi/6) = 1/2$

b) Seja $\theta = \arcsen(1/3) \Rightarrow \operatorname{sen}\theta = 1/3$



$$\Rightarrow \operatorname{tg}\theta = \frac{1}{2\sqrt{2}}$$

$$\therefore \operatorname{tg}(\arcsen(1/3)) = \frac{1}{2\sqrt{2}}$$

Mais inversas trigonométricas

$$\cos^{-1}x = y \quad \Longleftrightarrow \quad \cos y = x \quad \text{e} \quad 0 \leqslant y \leqslant \pi$$

$$\tan^{-1}x = y \quad \Longleftrightarrow \quad \tan y = x \quad \text{e} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$y = \csc^{-1}x \;\; (|x| \geqslant 1) \quad \Longleftrightarrow \quad \csc y = x \quad \text{e} \quad y \in (0, \pi/2] \cup (\pi, 3\pi/2]$$

$$y = \sec^{-1}x \;\; (|x| \geqslant 1) \quad \Longleftrightarrow \quad \sec y = x \quad \text{e} \quad y \in [0, \pi/2) \cup [\pi, 3\pi/2)$$

$$y = \cot^{-1}x \;\; (x \in \mathbb{R}) \quad \Longleftrightarrow \quad \cot y = x \quad \text{e} \quad y \in (0, \pi)$$

Obrigado !

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