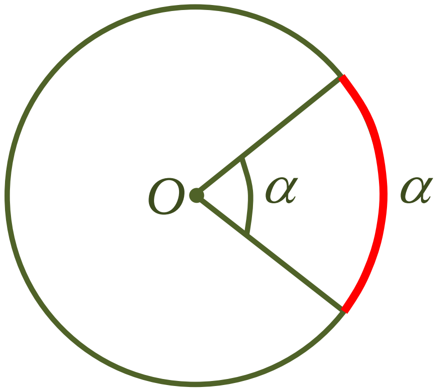
Aula 04– Funções Trigonométricas

Estudo das funções seno, cosseno, tangente, cotangente, secante e cossecante.

Arcos e ângulos



α - ângulo central

α tem a mesma medida do arco de circunferência que ele determina.

A circunferência toda mede 360º.

Grau (º) é um arco unitário cujo comprimento é igual a $\frac{1}{360}$ da circunferência que contém o arco a ser medido.

Radiano (rad) é o arco unitário cujo comprimento é igual ao raio da circunferência que contém o arco a ser medido, ou seja, $\frac{1}{2\pi}$ da circunferência.

Relações

$$2\pi \sim 360^{o} \qquad \pi \sim 180^{o} \qquad \frac{\pi}{2} \sim 90^{o}$$

$$160^{o} \sim ?\,rad$$

$$\begin{array}{lcl} 180^{o} & \text{———} & \pi\, rad \\ 160^{o} & \text{———} & x\, rad \end{array}$$

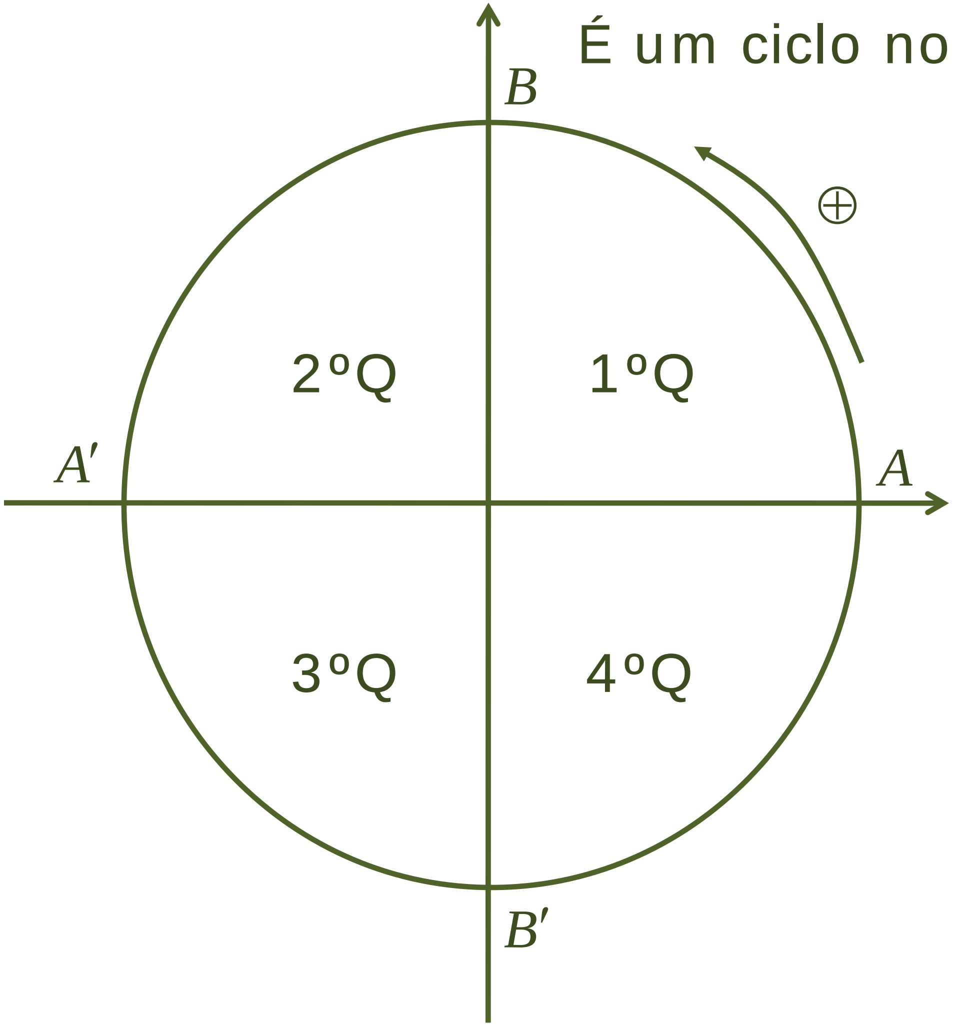
$$x = \frac{160\pi}{180} = \frac{8\pi}{9} rad$$

$$\frac{5\pi}{6} rad \sim ?^{o}$$

$$\begin{array}{lcl} 180^{o} & \text{———} & \pi\, rad \\ x^{o} & \text{———} & \frac{5\pi}{6}\, rad \end{array}$$

$$x = \frac{180 \cdot \frac{5\pi}{6}}{\pi} = 150^{o}$$

Ciclo trigonométrico



É um ciclo no sentido anti-horário (positivo)

Origem: A

medida do raio $= 1$

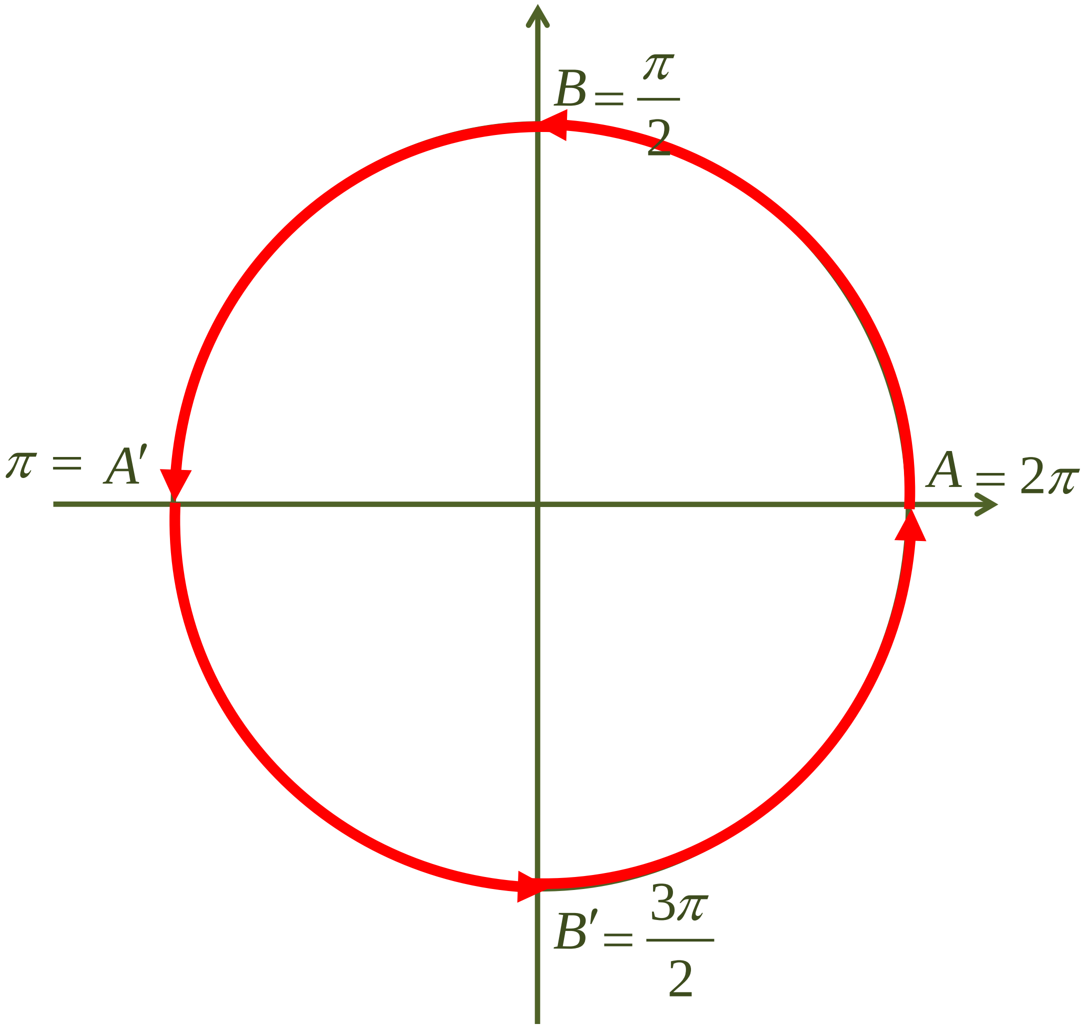
$A(1,0)$

$B(0,1)$

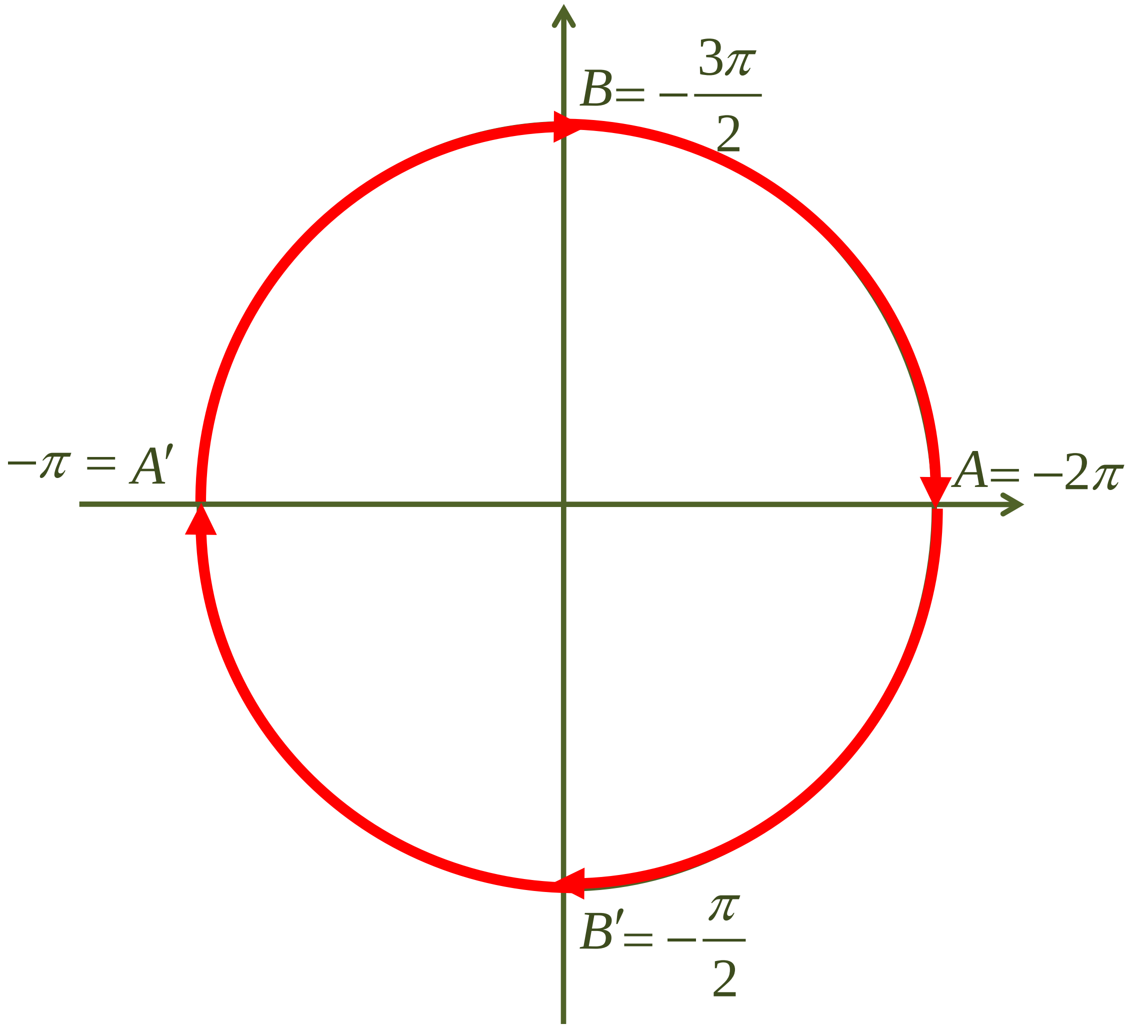
$A'(-1,0)$

$B'(0,-1)$

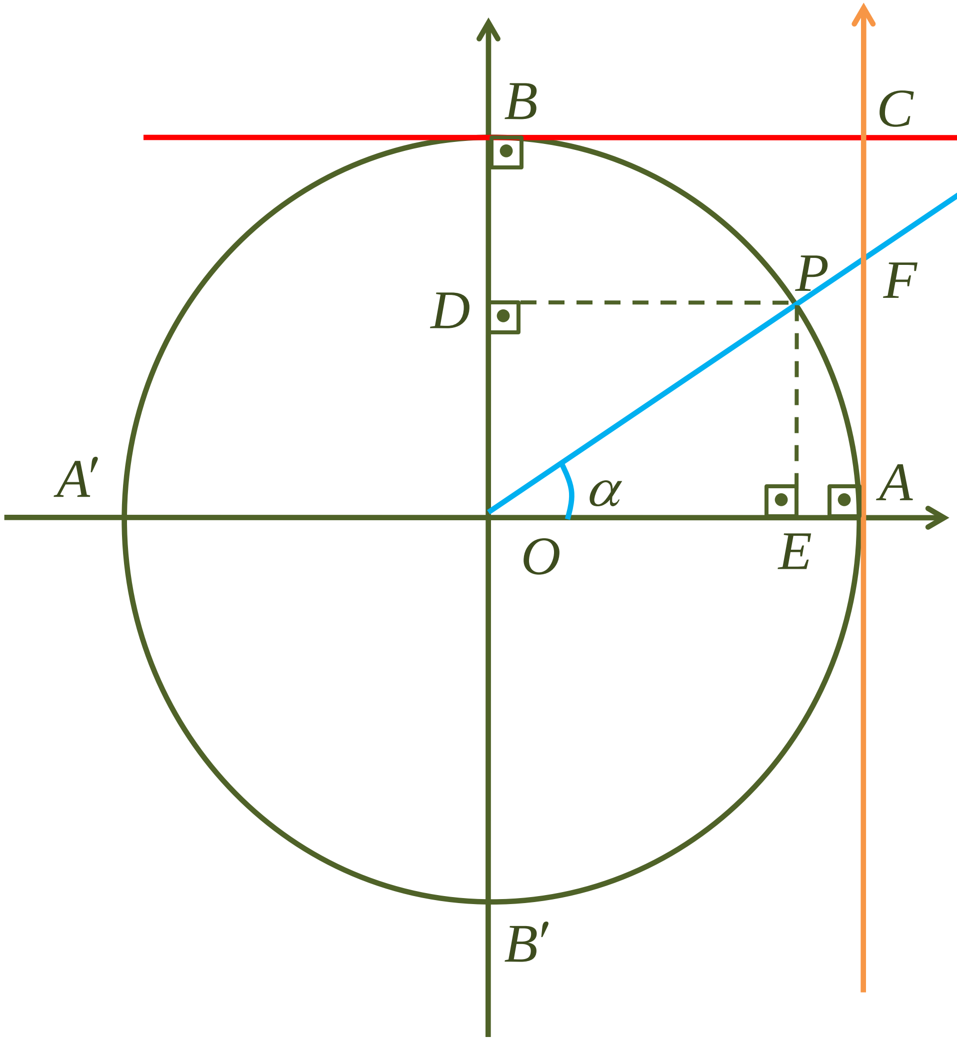
Ângulos (sentido positivo)



Ângulos (sentido negativo)



Funções trigonométricas



$\overrightarrow{OB}$ - eixo dos senos

$\overrightarrow{OA}$ - eixo dos cossenos

$\overrightarrow{AC}$ - eixo das tangentes

$\overrightarrow{BC}$ - eixo das cotangentes

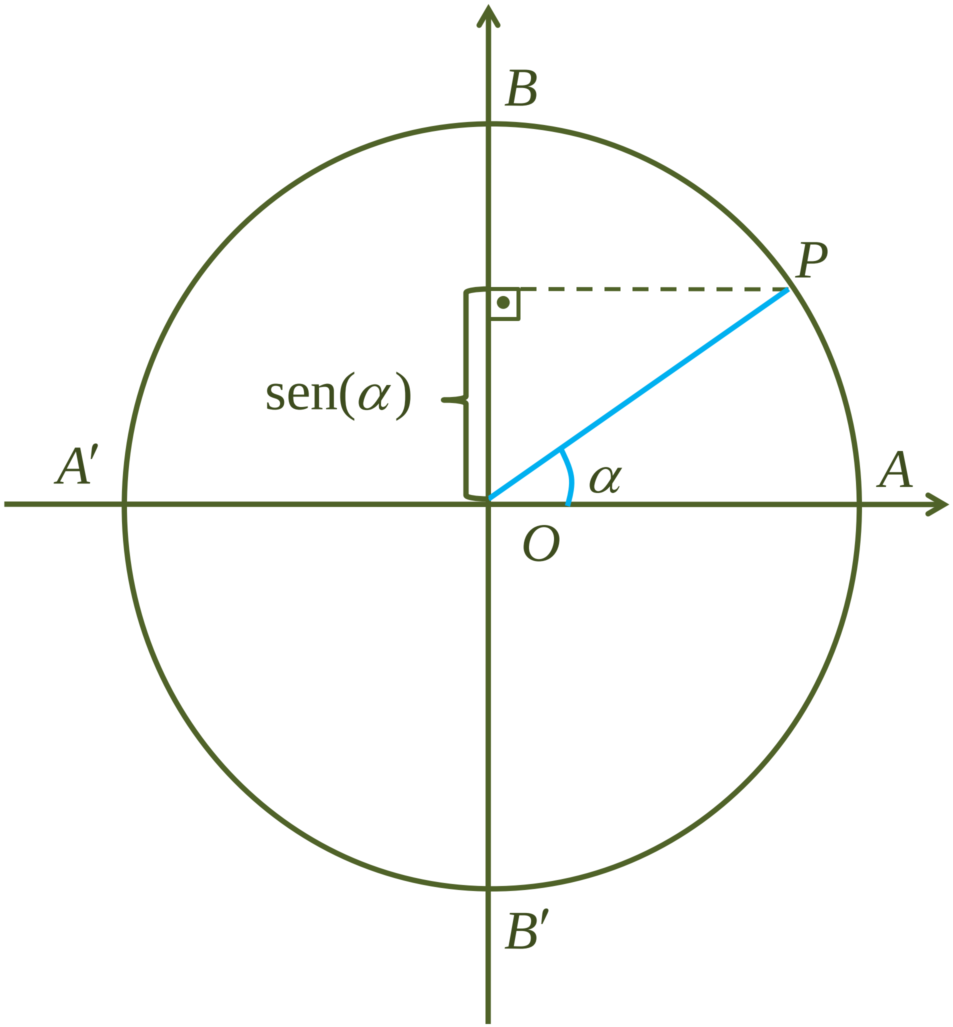
$$\operatorname{sen}(\alpha) = \overrightarrow{OD}$$

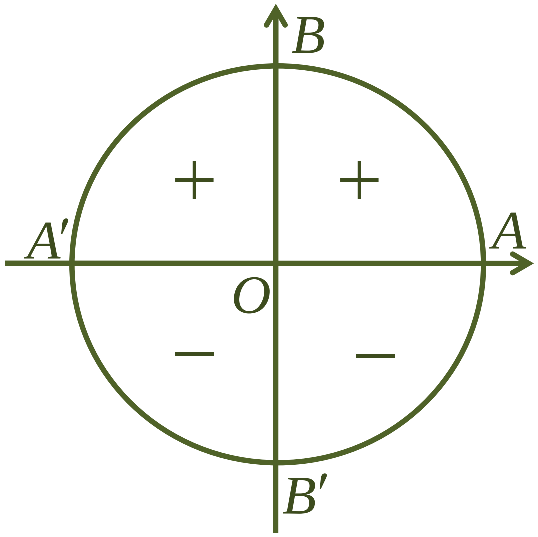
$$\cos(\alpha) = \overrightarrow{OE}$$

$$\operatorname{tg}(\alpha) = \overrightarrow{OF}$$

$$\operatorname{cotg}(\alpha) = \overrightarrow{BG}$$

Função seno





$$\operatorname{sen}(0) = 0$$

$$\operatorname{sen}\frac{\pi}{2} = 1$$

$$\operatorname{sen}\pi = 0$$

$$\operatorname{sen}\frac{3\pi}{2} = -1$$

$$\operatorname{sen}2\pi = 0$$

Propriedades do sen(α)

1) $-1 \leq \operatorname{sen}\alpha \leq 1$ (função limitada)

2) $0 \leq \alpha \leq \dfrac{\pi}{2} \Rightarrow \operatorname{sen}\alpha$ crescente.

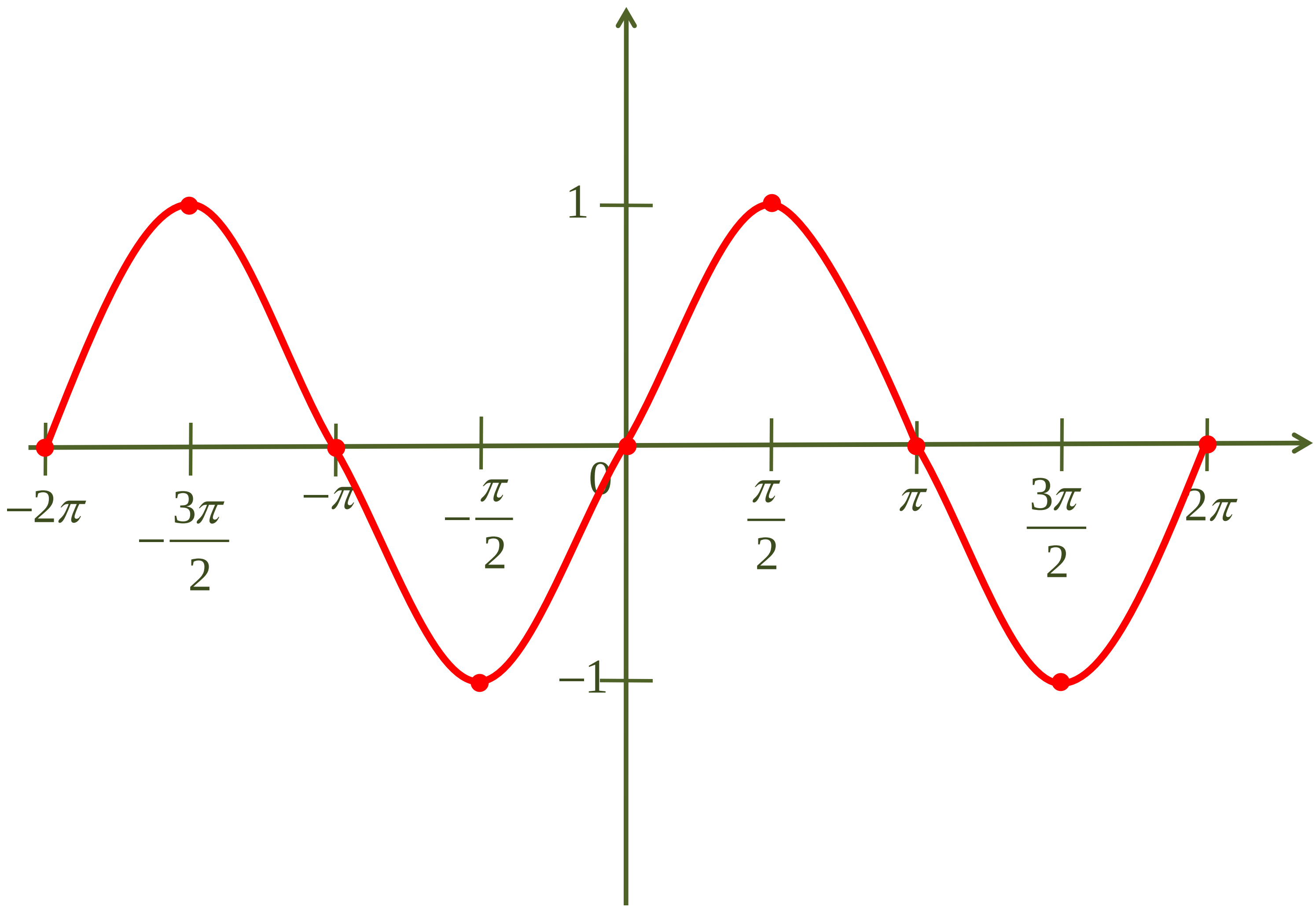
3) $\dfrac{\pi}{2} \leq \alpha \leq \dfrac{3\pi}{2} \Rightarrow \operatorname{sen}\alpha$ decrescente.

4) $\dfrac{3\pi}{2} \leq \alpha \leq \pi \Rightarrow \operatorname{sen}\alpha$ crescente.

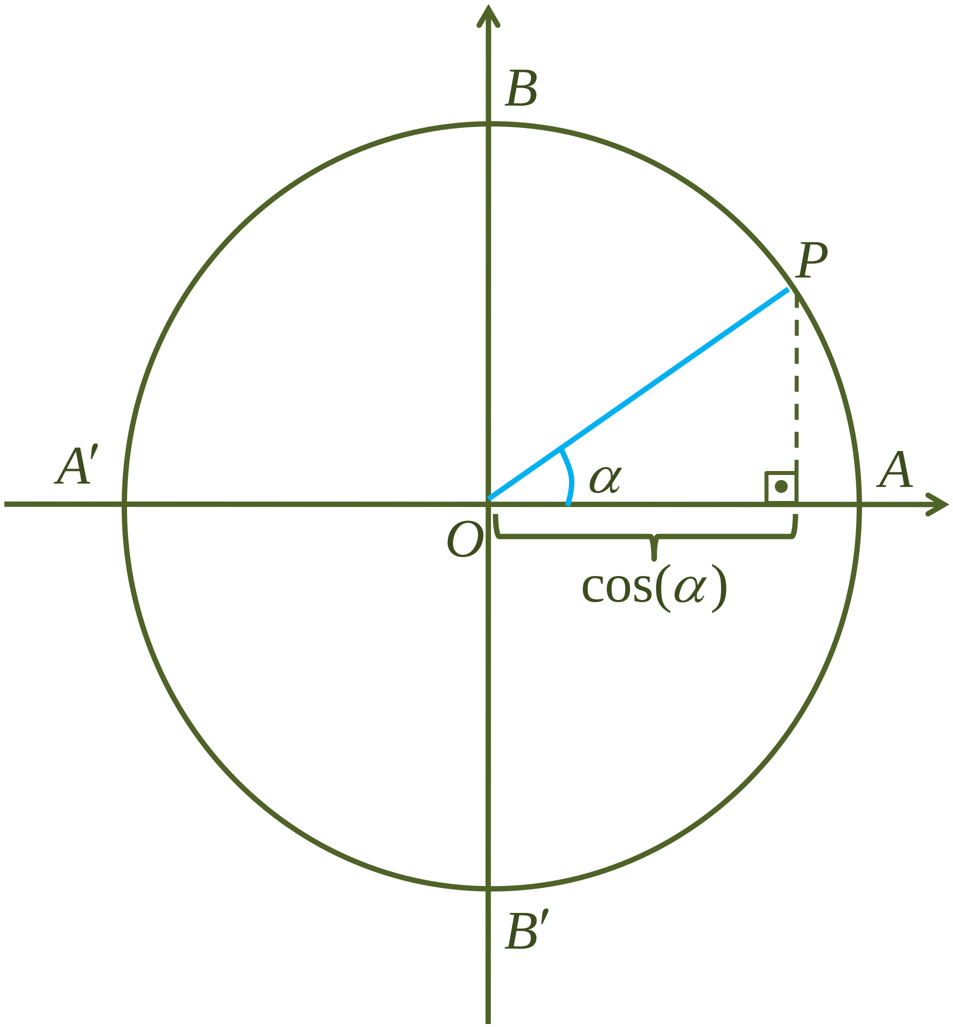
5) A Função é periódica

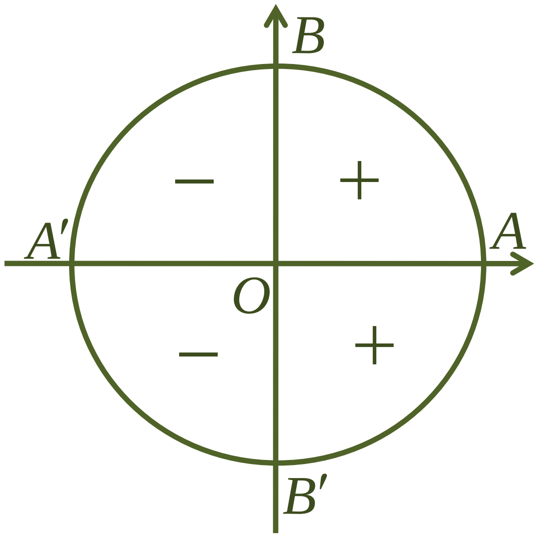
6) $D(f) = \mathbb{R}$

Gráfico de sen(α)



Função cosseno





$$\cos(0) = 1$$

$$\cos\frac{3\pi}{2} = 0$$

$$\cos\frac{\pi}{2} = 0$$

$$\cos 2\pi = 1$$

$$\cos \pi = -1$$

Propriedades do cos(α)

1) $-1 \leq \cos\alpha \leq 1$ (função limitada)

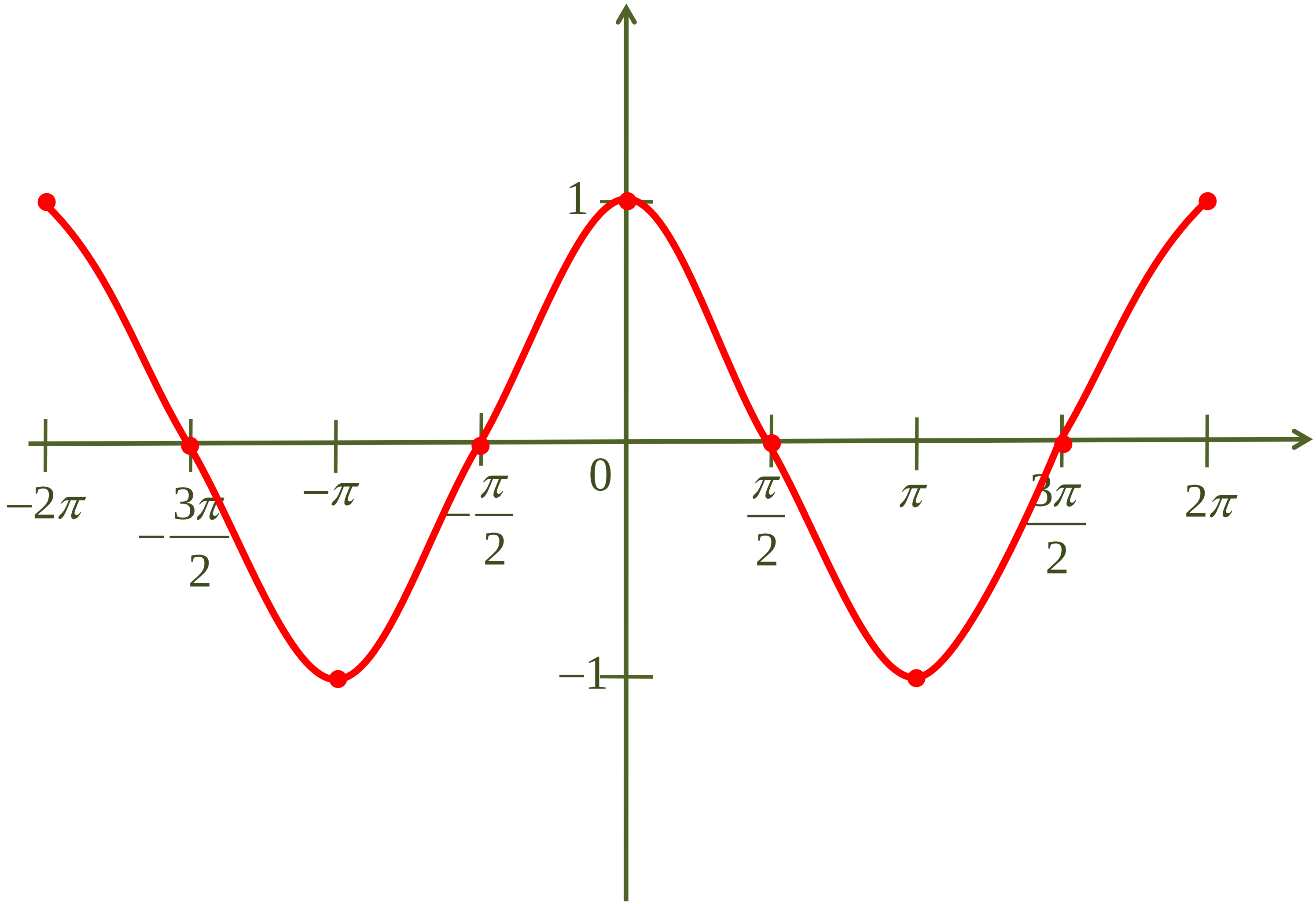
2) $0 \leq \alpha \leq \pi \Rightarrow \cos\alpha$ decrescente.

3) $\pi \leq \alpha \leq 2\pi \Rightarrow \cos\alpha$ crescente.

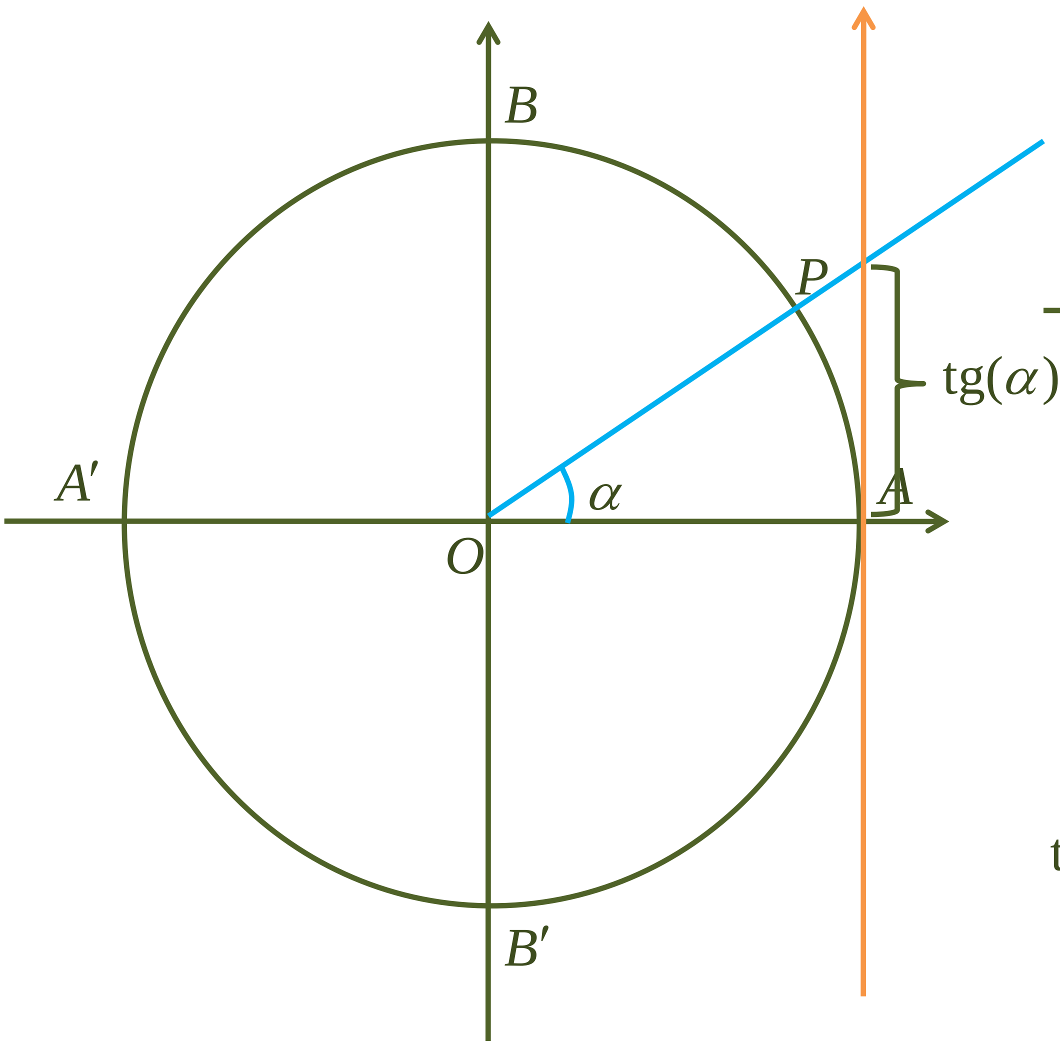
4) A Função é periódica

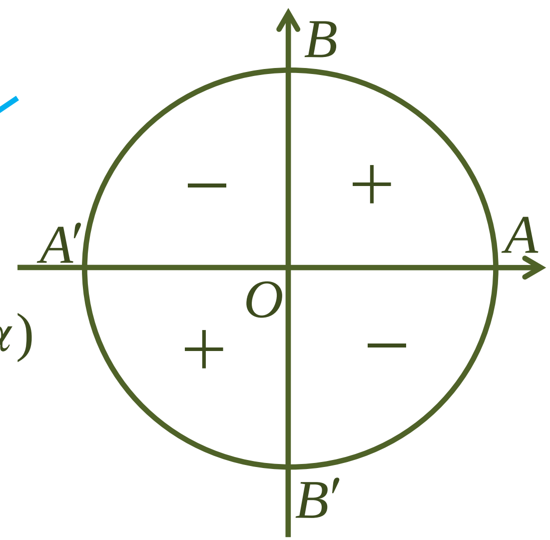
5) $D(f) = \mathbb{R}$

Gráfico de cos(α)



Função Tangente





$$\mathrm{tg}(0) = 0$$

$$\mathrm{tg}\,\frac{\pi}{2} = \nexists$$

$$\mathrm{tg}\,\pi = 0$$

$$\mathrm{tg}\,\frac{3\pi}{2} = \nexists$$

$$\mathrm{tg}\,2\pi = 0$$

Propriedades da tg(α)

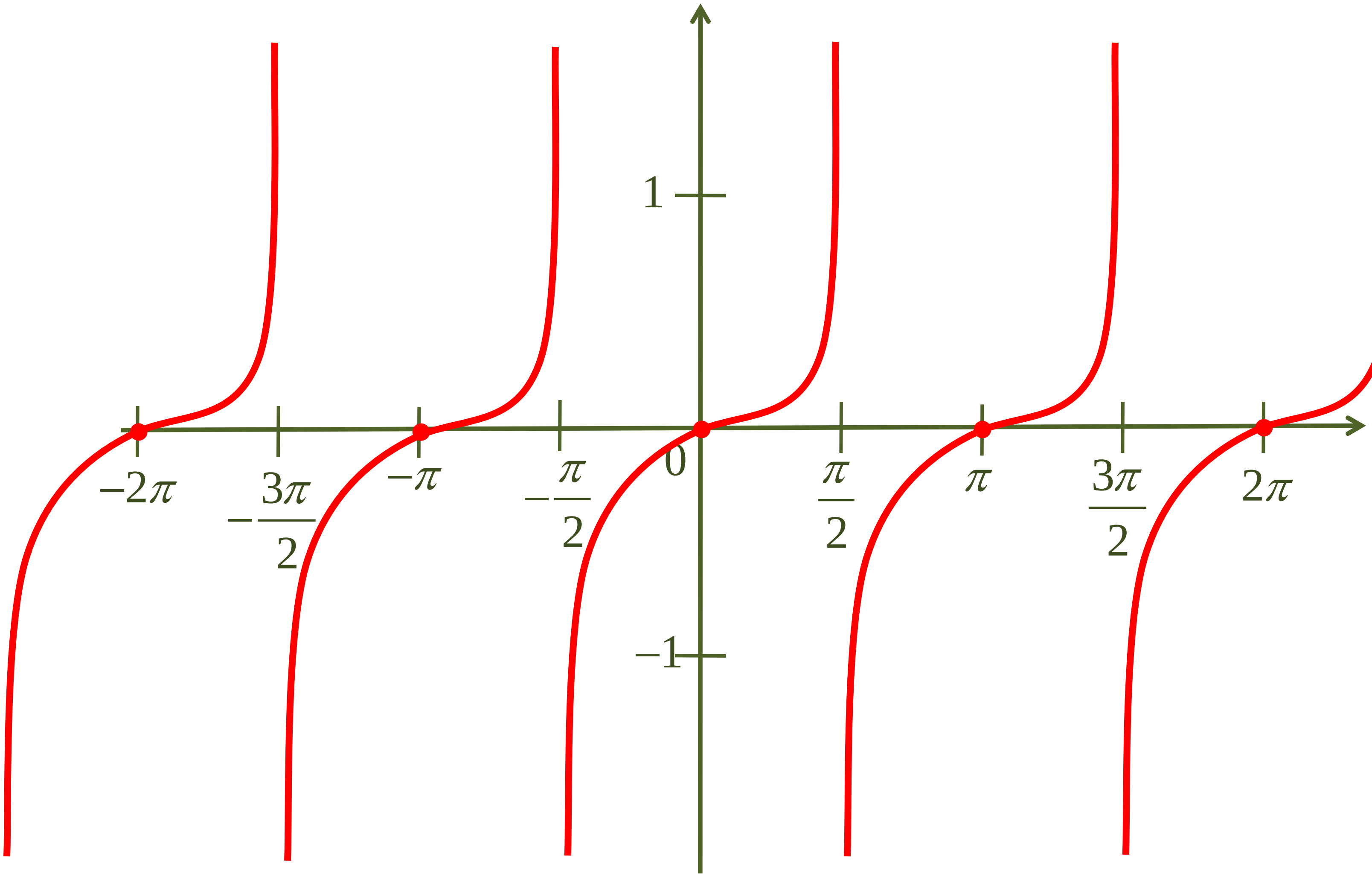
1) $\mathrm{Im}(f) = \mathbb{R}$ (função ilimitada)

2) Monótona crescente em todo seu domínio.

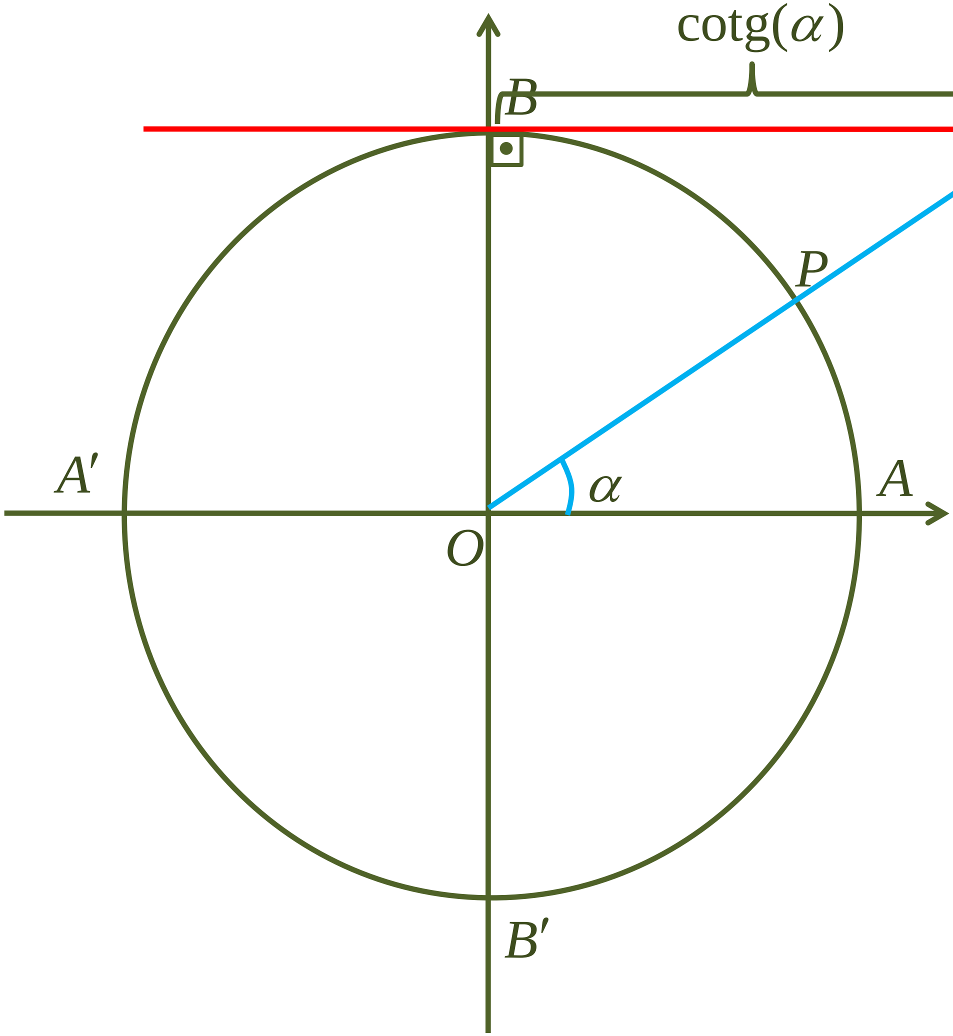
3) A Função é periódica.

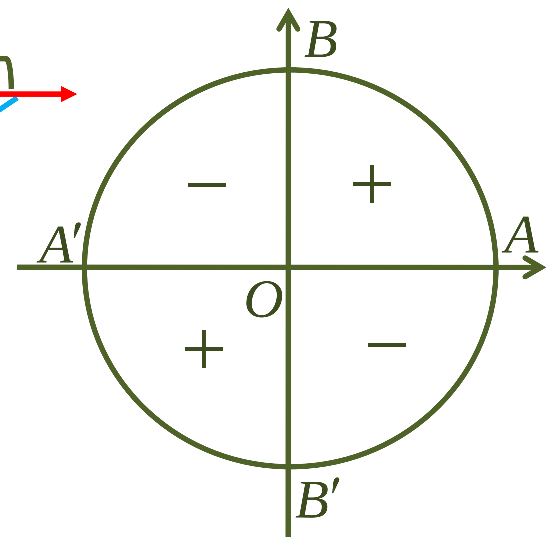
4) $D(f) = \left\{ x \in \mathbb{R} \,/\, x \neq \dfrac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$

Gráfico da tg(α)



Função Cotangente





$$\text{cotg}(0) = \nexists$$

$$\text{cotg}\frac{3\pi}{2} = 0$$

$$\text{cotg}\frac{\pi}{2} = 0$$

$$\text{cotg}\,2\pi = \nexists$$

$$\text{cotg}\,\pi = \nexists$$

Propriedades da cotg(α)

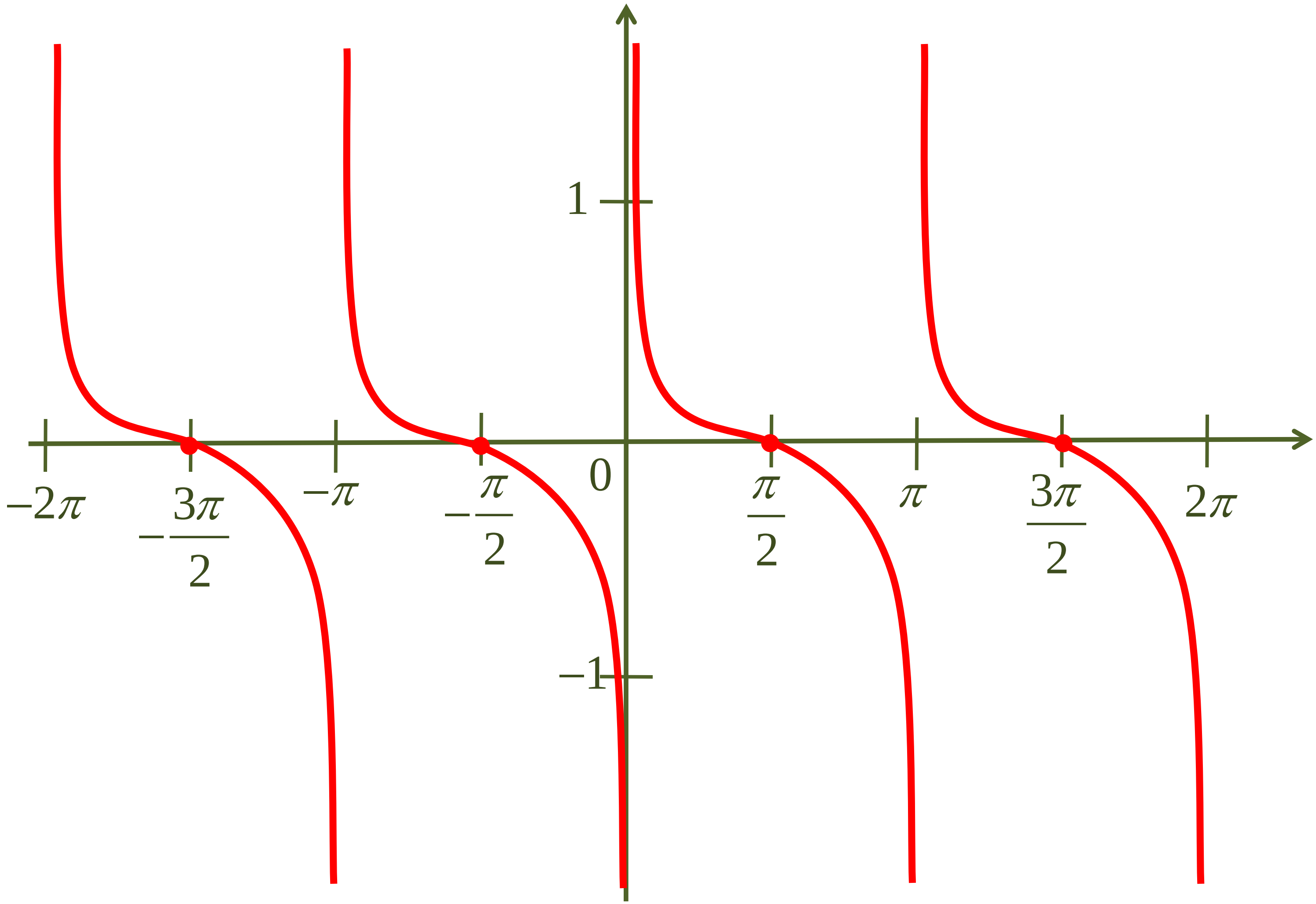
1) $\mathrm{Im}(f) = \mathbb{R}$ (função ilimitada)

2) Monótona decrescente em todo seu domínio.

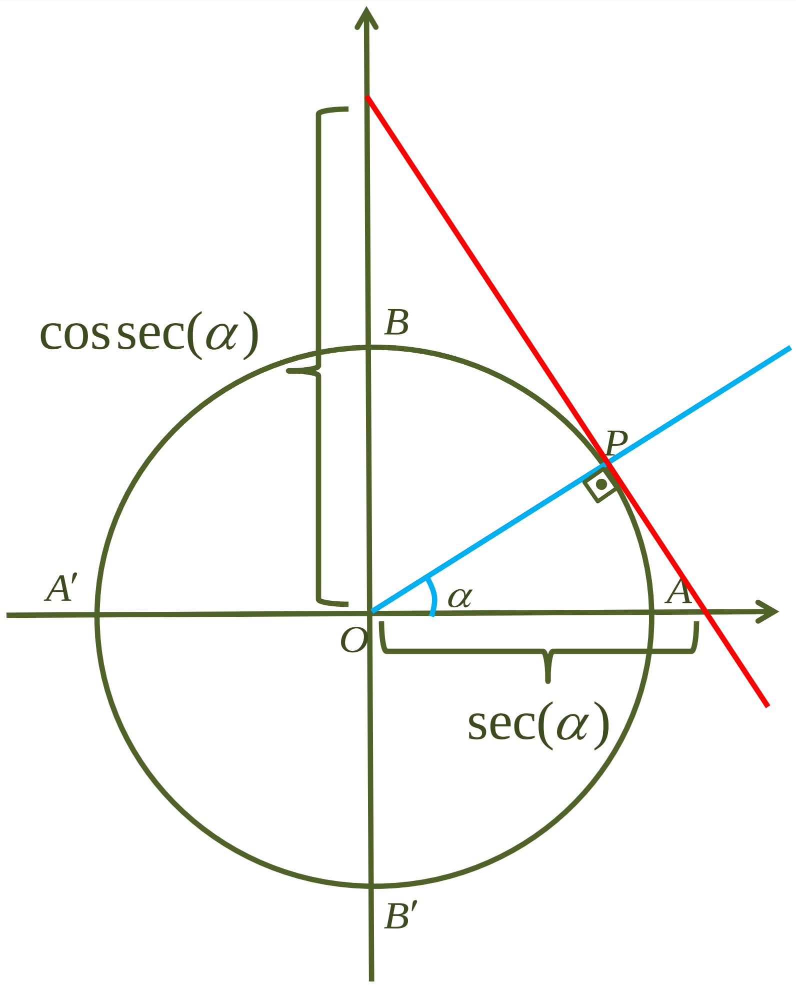
3) A Função é periódica.

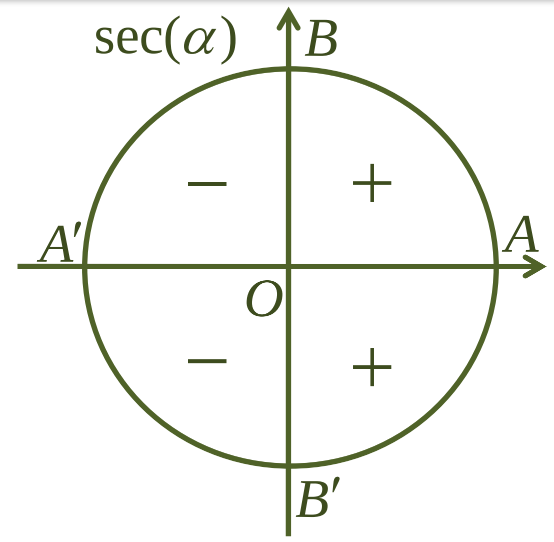
4) $D(f) = \{x \in \mathbb{R} / x \neq k\pi; k \in \mathbb{Z}\}$

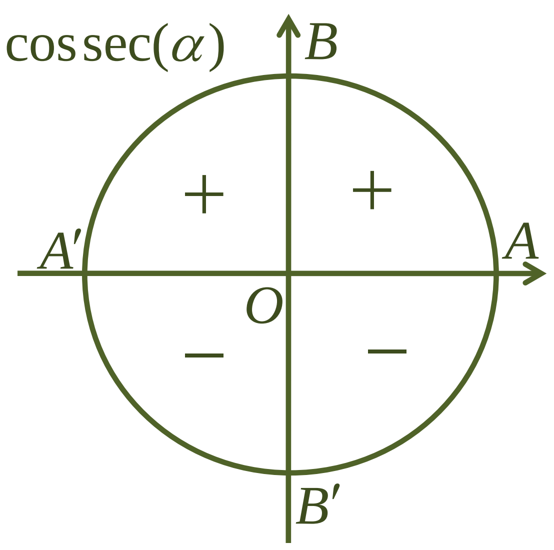
Gráfico de cotg(α)



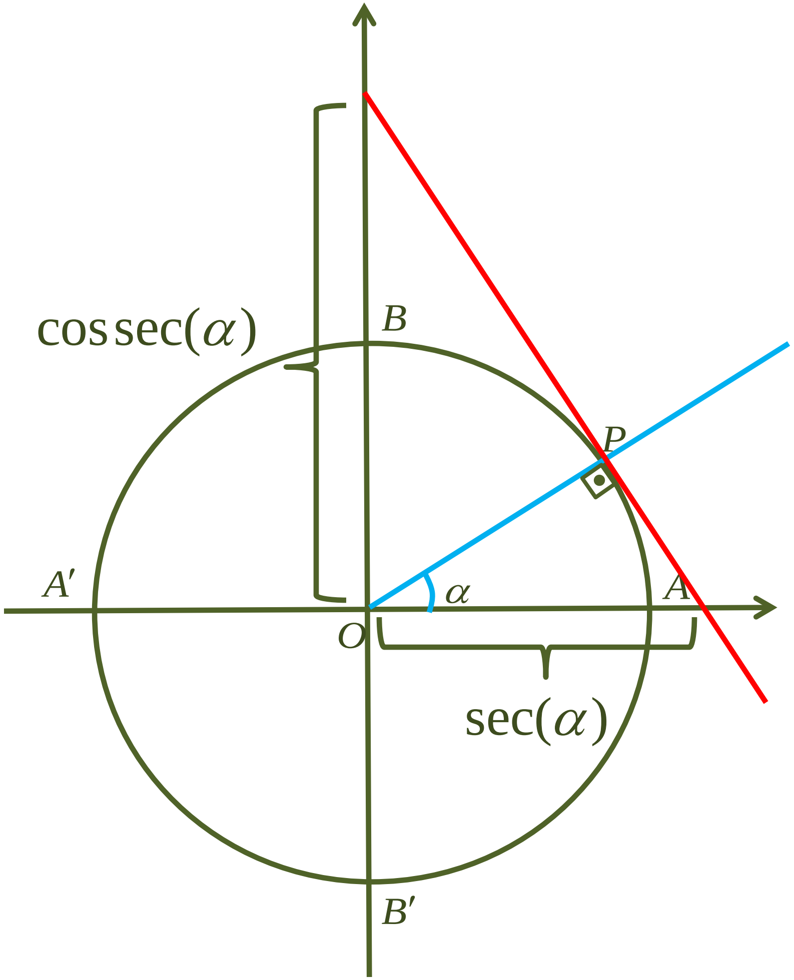
Funções Secante e Cossecante







Funções Secante e Cossecante



$$\sec 0 = 1 \qquad \operatorname{cossec} 0 = \nexists$$

$$\sec \frac{\pi}{2} = \nexists \qquad \operatorname{cossec} \frac{\pi}{2} = 1$$

$$\sec \pi = -1 \qquad \operatorname{cossec} \pi = \nexists$$

$$\sec \frac{3\pi}{2} = \nexists \qquad \operatorname{cossec} \frac{3\pi}{2} = -1$$

$$\sec 2\pi = 1 \qquad \operatorname{cossec} 2\pi = \nexists$$

Prop. da Secante e Cossecante

1) $D(\sec) = \left\{ x \in \mathbb{R} / x \neq \dfrac{\pi}{2} + k\pi; k \in \mathbb{Z} \right\}$ e

$D(\text{cossec}) = \{ x \in \mathbb{R} / x \neq k\pi; k \in \mathbb{Z} \}$

2) $\text{Im}(\sec) = \text{Im}(\text{cossec}) = \mathbb{R} - (-1,1)$

3) Ambas são periódicas.

Gráfico da sec(α)

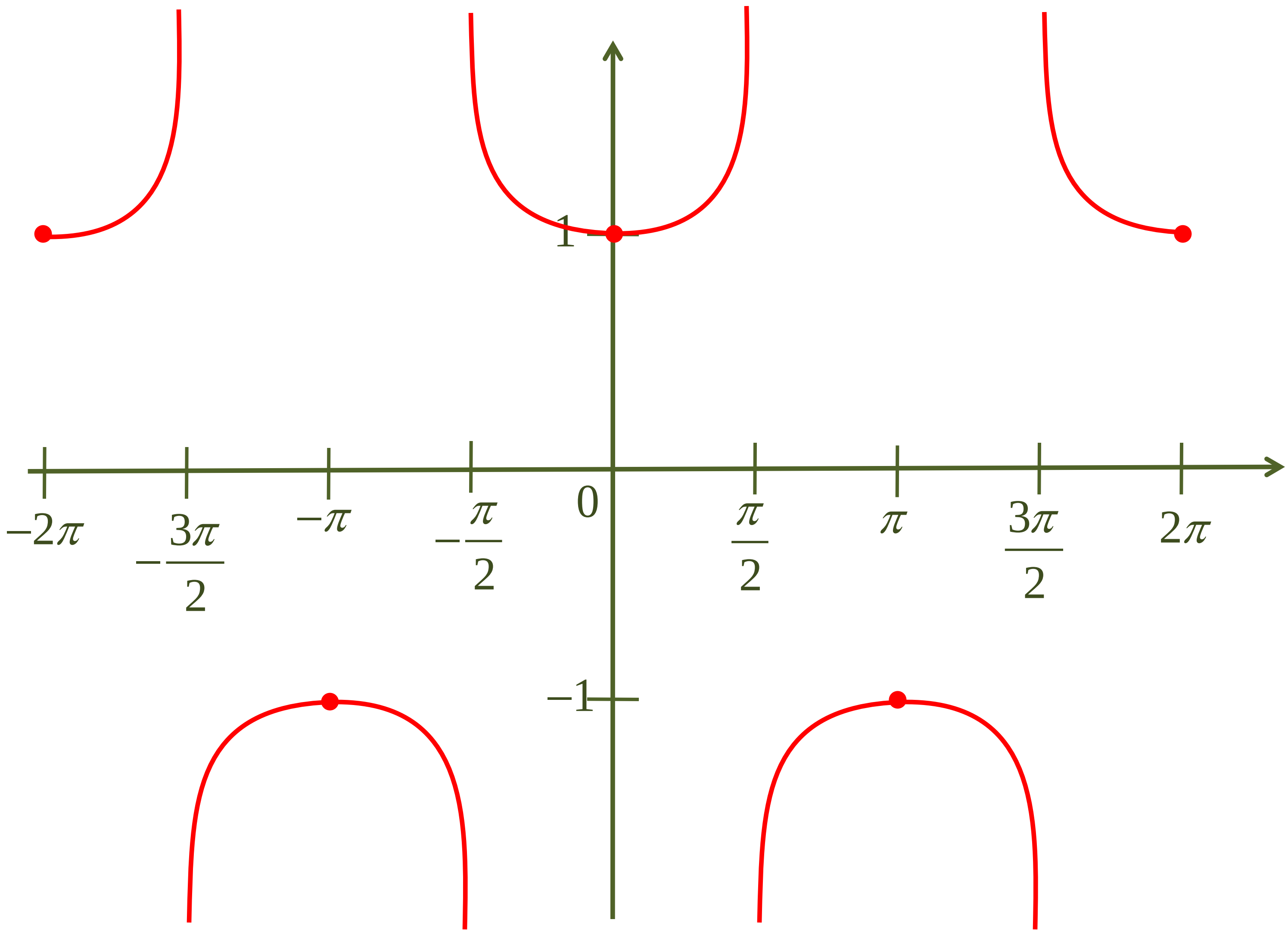
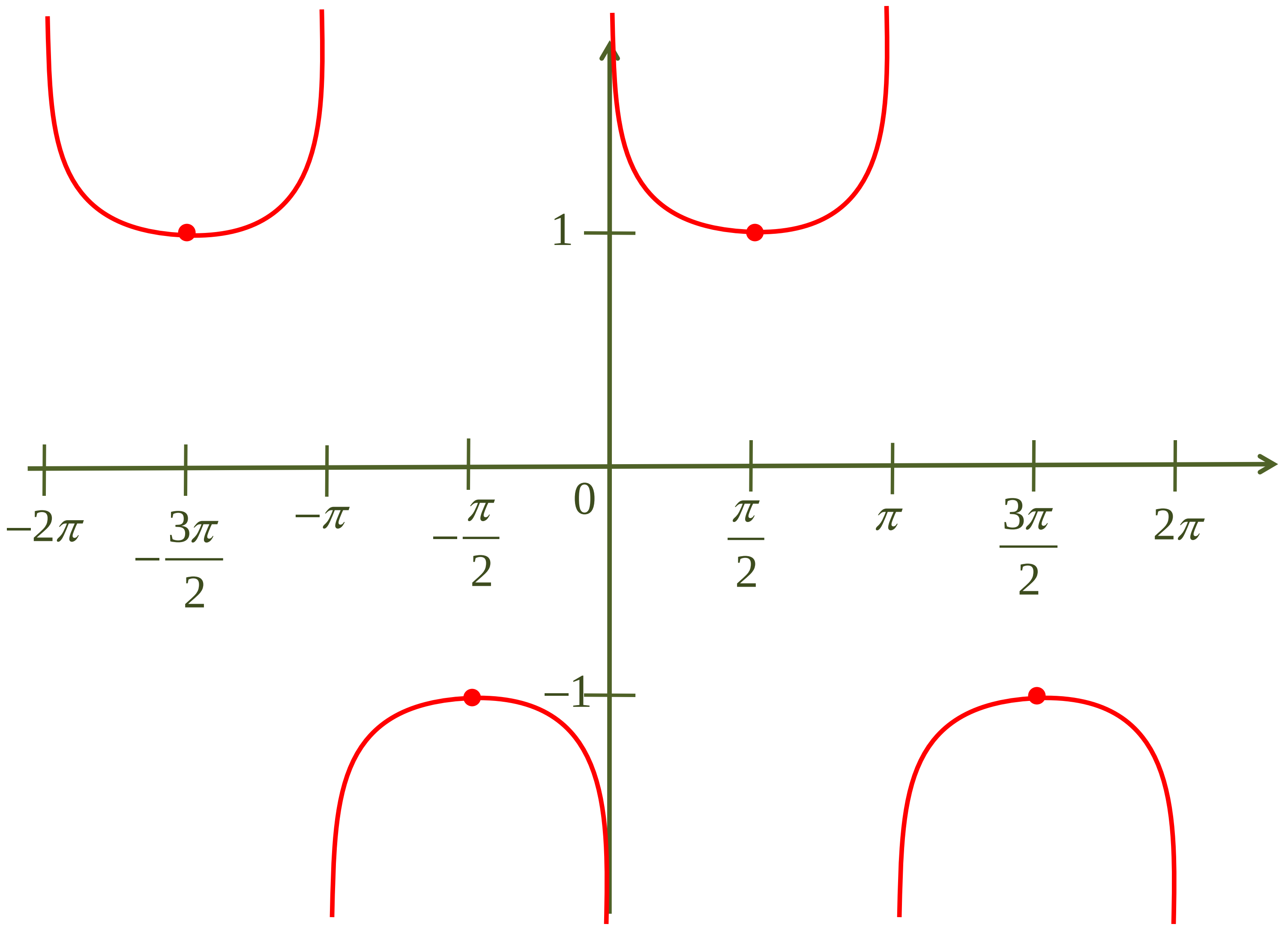
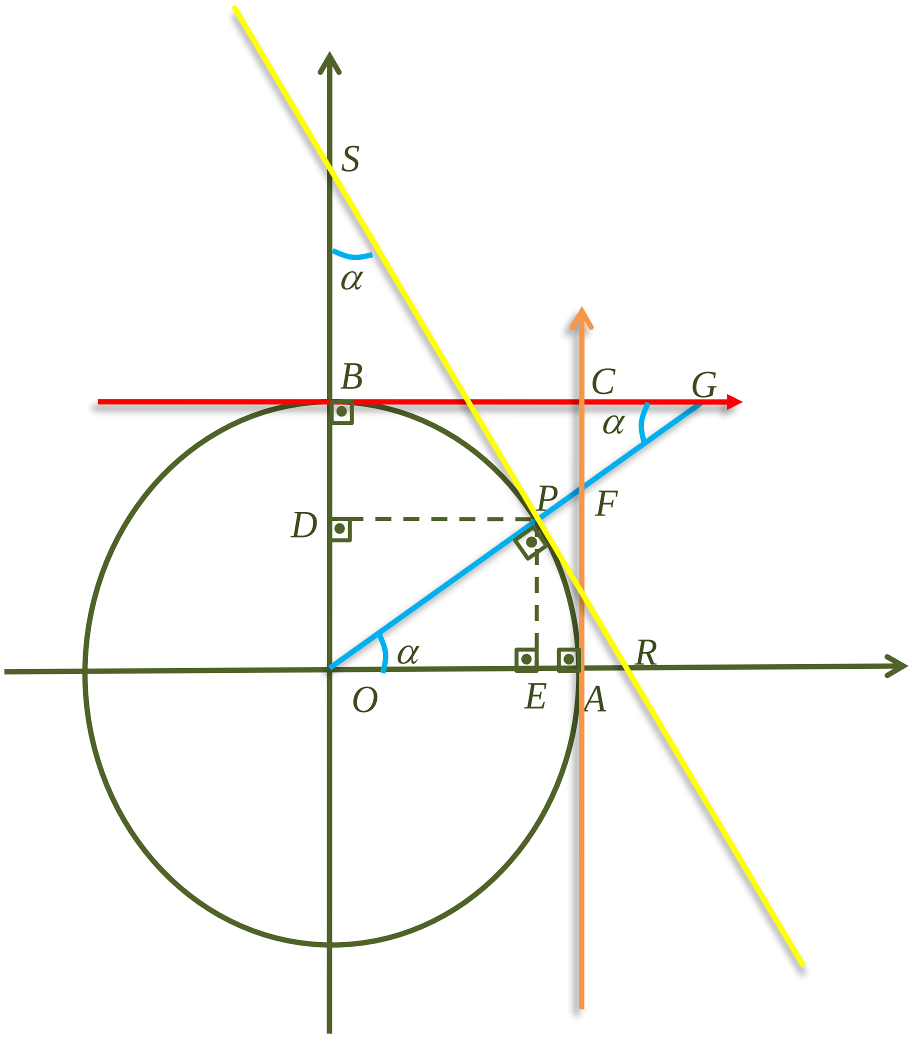


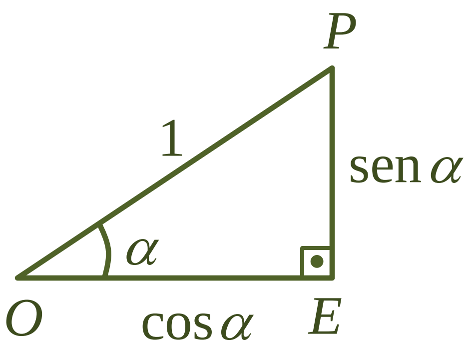
Gráfico da cossec(α)



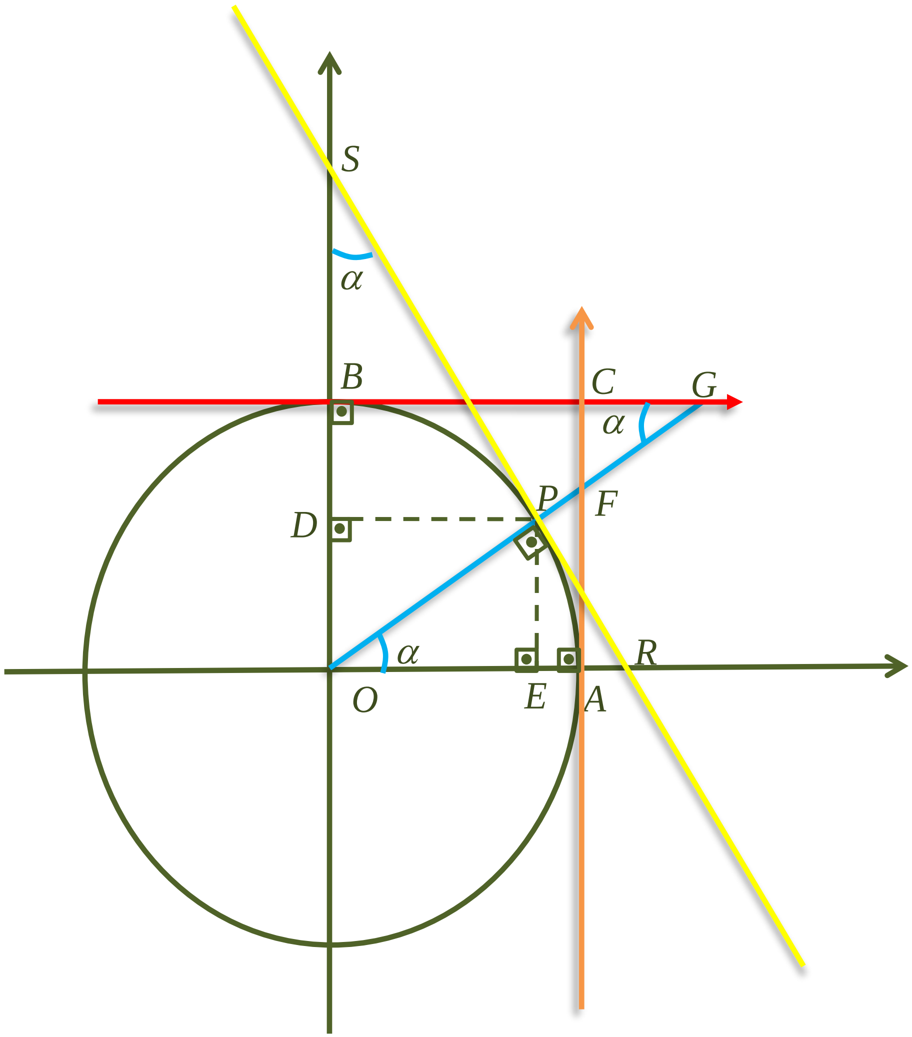
Relações fundamentais (1)

1) $\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

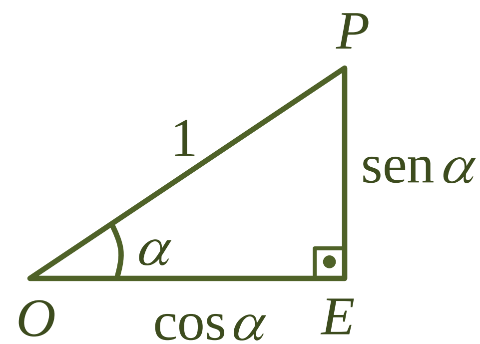


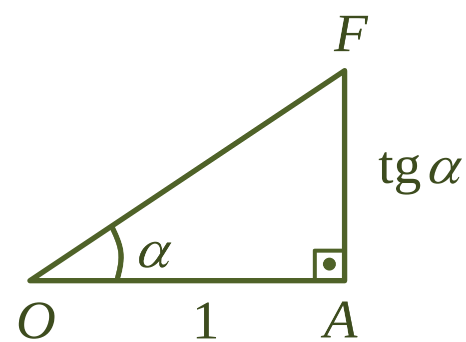


Relações fundamentais (2)



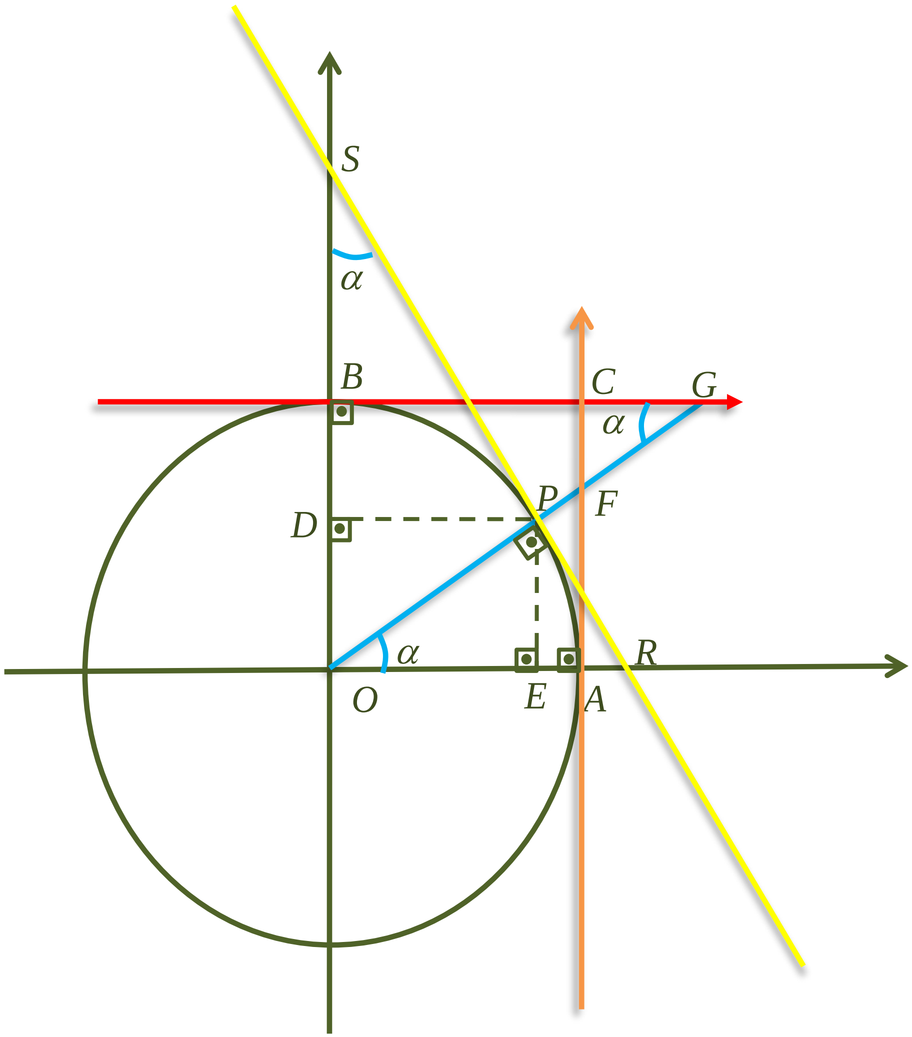
$$1)\ \operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

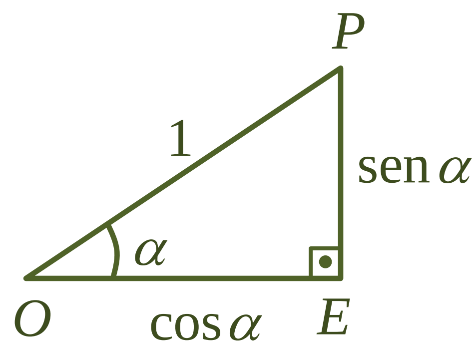


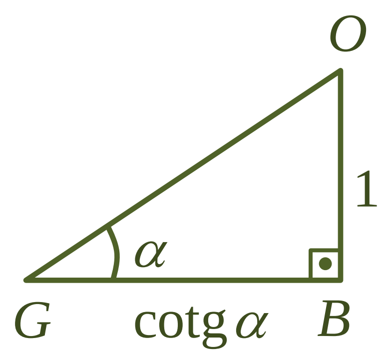


$$\frac{\operatorname{sen}\alpha}{\operatorname{tg}\alpha} = \frac{\cos\alpha}{1} \Rightarrow 2)\ \operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

Relações fundamentais (3)

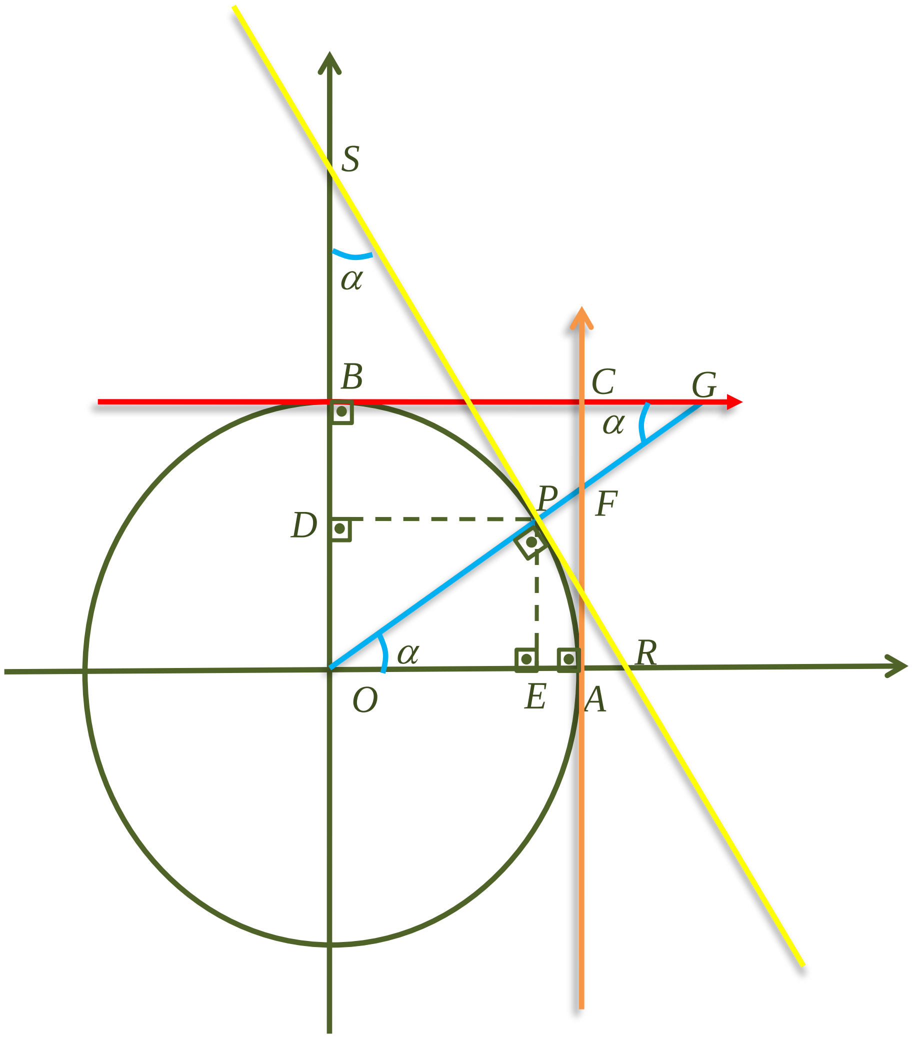


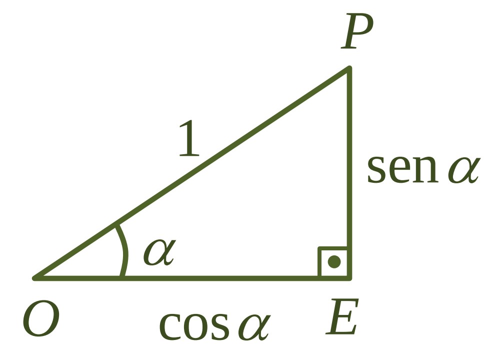


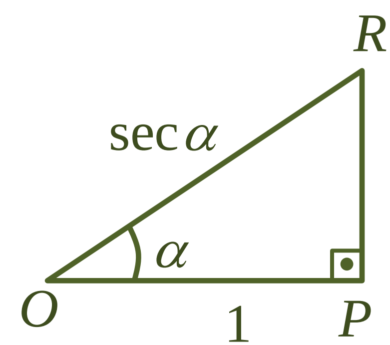


$$\frac{\operatorname{sen}\alpha}{1} = \frac{\cos\alpha}{\operatorname{cotg}\alpha} \Rightarrow 3)\ \operatorname{cotg}\alpha = \frac{\cos\alpha}{\operatorname{sen}\alpha}$$

Relações fundamentais (4)

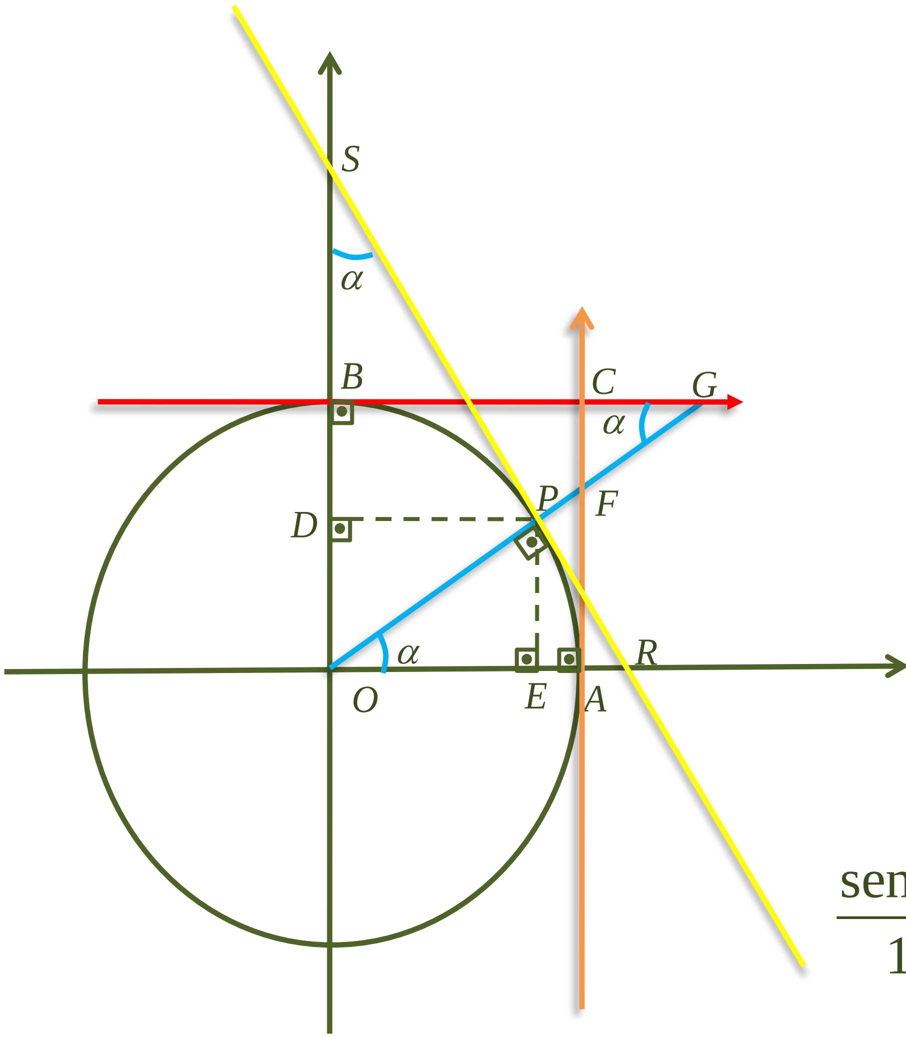


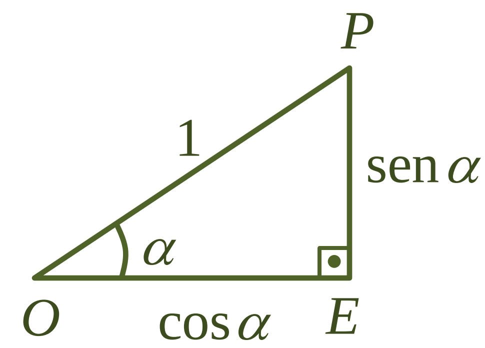


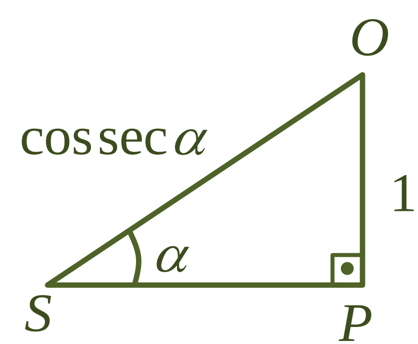


$$\frac{1}{\sec\alpha} = \frac{\cos\alpha}{1} \Rightarrow 4)\ \sec\alpha = \frac{1}{\cos\alpha}$$

Relações fundamentais (5)







$$\frac{\operatorname{sen}\alpha}{1} = \frac{1}{\operatorname{cossec}\alpha} \Rightarrow 5)\ \operatorname{cossec}\alpha = \frac{1}{\operatorname{sen}\alpha}$$

Relações fundamentais (6 e 7)

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1\Rightarrow\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}\Rightarrow\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}+\frac{\cos^2}{\cos^2}=\frac{1}{\cos^2\alpha}$$

$$\Rightarrow \operatorname{tg}^2\alpha+1=\sec^2\alpha\ \Rightarrow 6)\ \sec^2\alpha-\operatorname{tg}^2\alpha=1$$

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1\Rightarrow\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}^2\alpha}=\frac{1}{\operatorname{sen}^2\alpha}\Rightarrow\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha}+\frac{\cos^2}{\operatorname{sen}^2}=\frac{1}{\operatorname{sen}^2\alpha}$$

$$\Rightarrow 1+\operatorname{cotg}^2\alpha=\operatorname{cossec}^2\alpha\ \Rightarrow 7)\ \operatorname{cossec}^2\alpha-\operatorname{cotg}^2\alpha=1$$

Outras relações

8) $\text{sen}(a+b) = \text{sen}\,a \cdot \cos b + \text{sen}\,b \cdot \cos a$

9) $\text{sen}(a-b) = \text{sen}\,a \cdot \cos b - \text{sen}\,b \cdot \cos a$

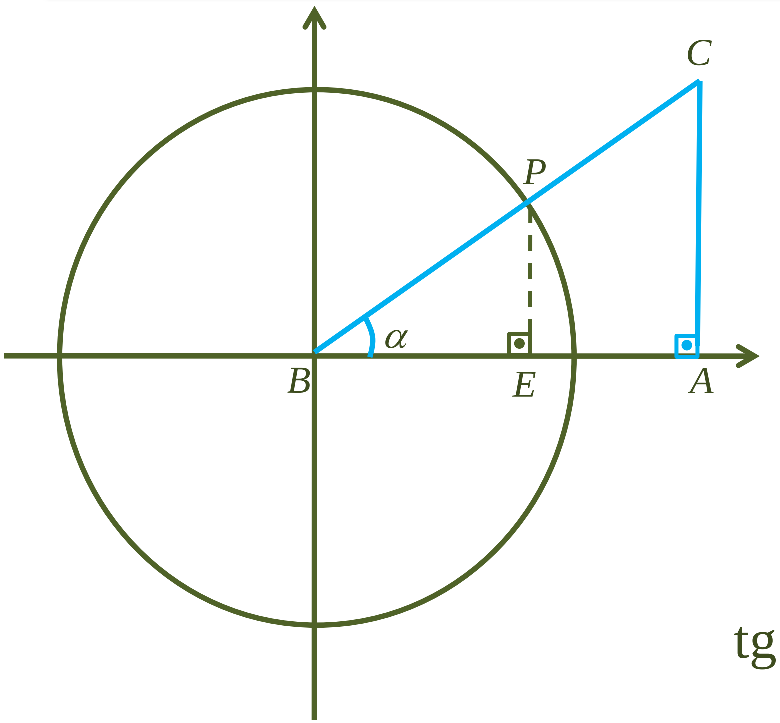
10) $\cos(a+b) = \cos a \cdot \cos b - \text{sen}\,b \cdot \text{sen}\,a$

11) $\cos(a-b) = \cos a \cdot \cos b + \text{sen}\,b \cdot \text{sen}\,a$

12) $\text{sen}(2a) = 2 \cdot \text{sen}\,a \cdot \cos b$

13) $\cos(2a) = \cos^2 a - \text{sen}^2 a$

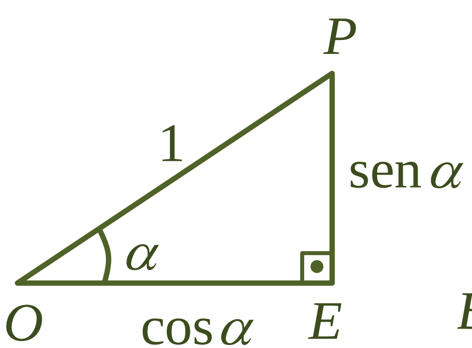
Trigonometria em triângulos

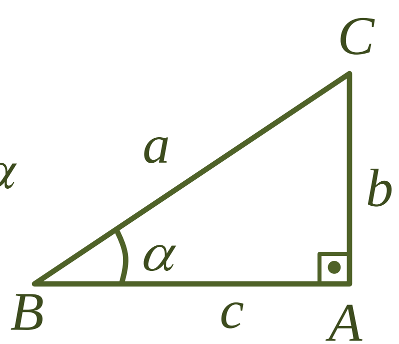


$$\frac{\operatorname{sen}\alpha}{b} = \frac{1}{a} \Rightarrow \operatorname{sen}\alpha = \frac{b}{a} = \frac{\text{cat. oposto}}{\text{hipotenusa}}$$

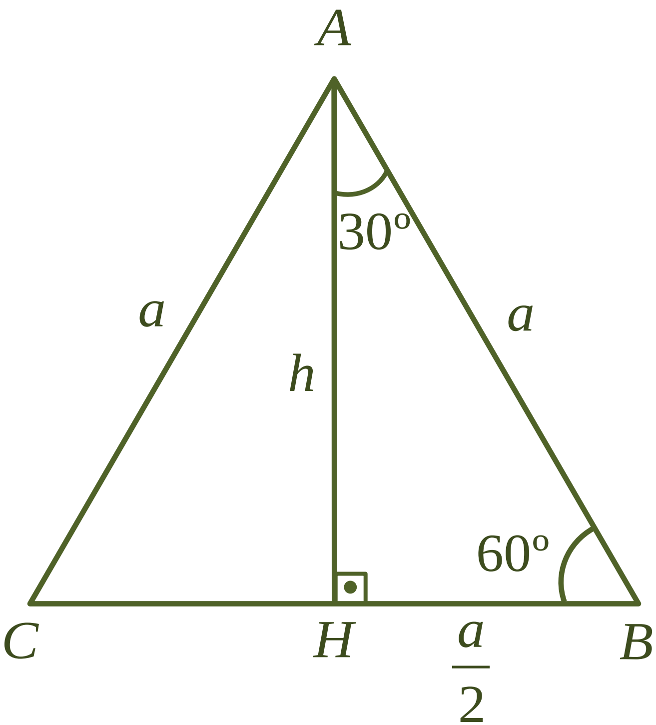
$$\frac{\cos\alpha}{c} = \frac{1}{a} \Rightarrow \cos\alpha = \frac{c}{a} = \frac{\text{cat. adjacente}}{\text{hipotenusa}}$$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\frac{b}{a}}{\frac{c}{a}} \Rightarrow \operatorname{tg}\alpha = \frac{b}{c} = \frac{\text{cat. oposto}}{\text{cat. adjacente}}$$





sen e cos de 30º e 60º



$$a^2 = \left(\frac{a}{2}\right)^2 + h^2 \Rightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$$\therefore h = \frac{a\sqrt{3}}{2}$$

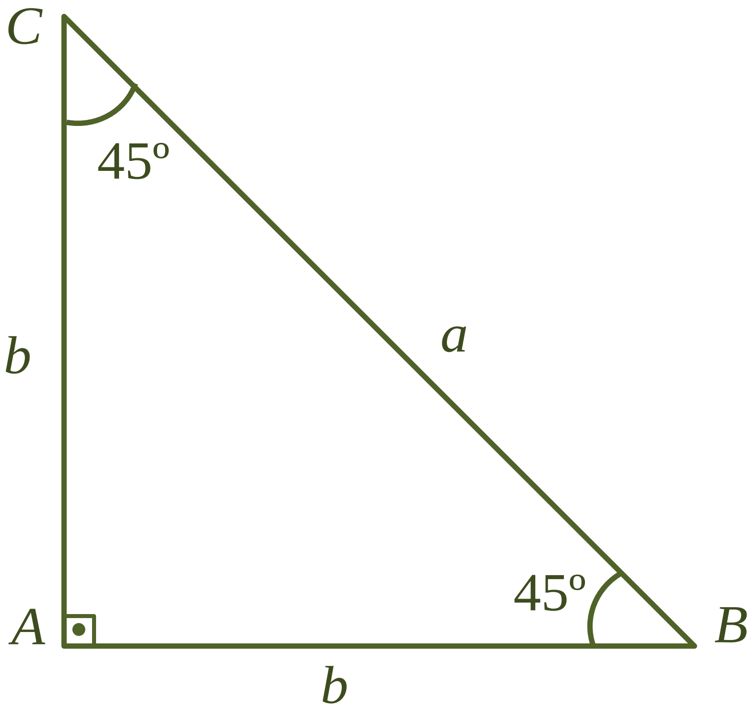
$$\operatorname{sen} 30^{\circ} = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$

$$\cos 30^{\circ} = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{\sqrt{3}}{2}$$

$$\operatorname{sen} 60^{\circ} = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^{\circ} = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$

sen e cos de 45º



$$\operatorname{sen} 45^{\circ} = \frac{\frac{a\sqrt{2}}{2}}{a} = \frac{\sqrt{2}}{2}$$

$$\cos 45^{\circ} = \frac{\frac{a\sqrt{2}}{2}}{a} = \frac{\sqrt{2}}{2}$$

$$a^2 = b^2 + b^2 \Rightarrow b = \frac{a}{\sqrt{2}} = \frac{a\sqrt{2}}{2}$$

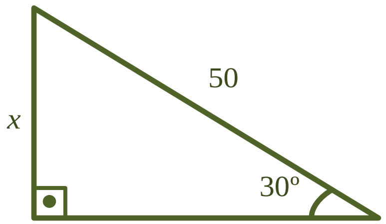
Tabela

Grau	Rad	sen	cos	tg	cotg	sec	cossec
0	0	0	1	0	$\nexists$	1	$\nexists$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90	$\frac{\pi}{2}$	1	0	$\nexists$	0	$\nexists$	1
180	π	0	-1	0	$\nexists$	-1	$\nexists$
270	$\frac{3\pi}{2}$	-1	0	$\nexists$	0	$\nexists$	-1
360	2π	0	1	0	$\nexists$	1	$\nexists$

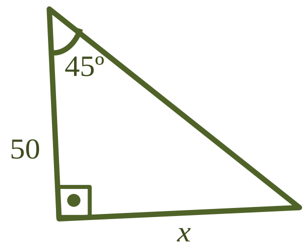
Exemplo

Determine o valor de x:

a)



b)



$$\text{sen}\,30^{\circ} = \frac{x}{50}$$

$$x = 25$$

$$\tan 45^{\circ} = \frac{x}{50}$$

$$x = 50$$

Obrigado !

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