The Cedilleum Language Specification
Syntax, Typing, Reduction, and Elaboration

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July 3, 2018

1   Syntax

id			identifiers for definitions
u			term variables
X			type variables
κ			kind variables
x	::=	$id \mid u \mid X$	non-kind variables
y	::=	$x \mid \kappa$	all variables

Figure 1: Identifiers

Identifiers   Figure 1 gives the metavariables used in our grammar for identifiers. We consider all identifiers as coming from two distinct lexical "pools" – regular identifiers (consisting of identifiers id given for modules and definitions, term variables u, and type variables X) and kind identifiers κ. In Cedilleum source files (as in the parent language Cedille) kind variables should be literally prefixed with κ – the suffix can be any string that would by itself be a legal non-kind identifier.

p	::=	u	variables
		$\lambda\,u.\,p$	functions
		$p\;p'$	applications
		$\mu\langle u_{rec}\;u_{out}\;u_z\rangle\;e_s\;\{pcase^*\}$	fixed-point and pattern matching
		$\mu'\;p_s\;p^*$	auxiliary pattern matching
$pcase$	::=	$\mid u\;u^*\mapsto p$	

Figure 2: Untyped terms

Untyped Terms   The grammar of pure (untyped) terms the untyped λ-calculus augmented with a primitives for combination fixed-point and pattern-matching definitions (and an auxiliary pattern-matching construct).

Modules and Definitions   All Cedilleum source files start with production *mod*, which consists of a module declaration, a sequence of import statements which bring into scope definitions from other source files, and a sequence of *commands* defining terms, types, and kinds. As an illustration, consider the first few lines of a hypothetical `list.ced`:

mod	::=	**module** id **.** $imprt^*$ cmd^*	module declarations
$imprt$	::=	**import** id **.**	module imports
cmd	::=	$defTermOrType$	definitions
		$defDataType$	
		$defKind$	
$defTermOrType$	::=	$id\ checkType^?\ =\ t$ **.**	term definition
		$id : K = T$ **.**	type definition
$defKind$	::=	$\kappa = K$	kind definition
$defDataType$	::=	**data** $id\ param^*$ **:** $K = constr^*$ **.**	datatype definitions
$checkType$	::=	**:** T	annotation for term definition
$param$	::=	$(x : C)$	
$constr$	::=	**|** $id : T$	

Figure 3: Modules and definitions

```
module list .

import nat .
```

Imports are handled first by consulting a global options files known to the Cedilleum compiler (on *nix systems ˜/.cedille/options) containing a search path of directories, and next (if that fails) by searching the directory containing the file being checked.

Term and type definitions are given with an identifier, a classifier (type or kind, resp.) to check the definition against, and the definition. For term definitions, giving classifier (i.e. the type) is optional. As an example, consider the definitions for the type of Church-encoded lists and two variants of the nil constructor, the first with a top-level type annotation and the second with annotations sprinkled on binders:

```
cList : ⋆ → ⋆
      = λ A : ⋆ . ∀ X : ⋆ . (A → X → X) → X → X .

cNil  : ∀ A : ⋆ . cList · A
      = Λ A . Λ X . λ c . λ n . n .
cNil' = Λ A : ⋆ . Λ X : ⋆ . λ c : A → X → X . λ n : X . n .
```

Kind definitions are given without classifiers (all kinds have super-kind $\square$), e.g. κfunc = ⋆ → ⋆

Inductive datatype definitions take a set of *parameters* (term and type variables which remain constant throughout the definition) well as a set of *indices* (term and type variables which *can* vary), followed by zero or more constructors. Each constructor begins with "|" (though the grammar can be relaxed so that the first of these is optional) and then an identifier and type is given. As an example, consider the following two definitions for lists and vectors (length-indexed lists).

```
data List (A : ⋆) : ⋆ =
  | nil  : List
  | cons : A → List → List
  .
data Vec (A : ⋆) : Nat → ⋆ =
  | vnil  : Vec Z
  | vcons : ∀ n : Nat . A → Vec n → Vec (S n)
  .
```

$$
\begin{array}{lll}
S & ::= & \square & \text{sole super-kind} \\
& & K & \text{kinds} \\
C & ::= & K & \text{types} \\
& & T & \text{types} \\
K & ::= & \Pi\, x : C\, .\, K & \text{explicit product} \\
& & C \to K & \text{kind arrow} \\
& & \star & \text{the kind of types that classify terms} \\
& & (K) & \\
\\
T & ::= & \Pi\, x : T\, .\, T & \text{explicit product} \\
& & \forall\, x : C\, .\, T & \text{implicit product} \\
& & \lambda\, x : C\, .\, T & \text{type-level function} \\
& & T \Rightarrow T' & \text{arrow with erased domain} \\
& & T \to T' & \text{normal arrow type} \\
& & T \cdot T' & \text{application to another type} \\
& & T\, t & \text{application to a term} \\
& & \{\, p \simeq p'\} & \text{untyped equality} \\
& & (T) & \\
& & X & \text{type variable} \\
& & \bullet & \text{hole for incomplete types} \\
\end{array}
$$

Figure 4: Kinds and types

Types and Kinds In Cedilleum, the expression language is stratified into three main "classes": kinds, types, and terms. Kinds and types are listed in Figure 4 and terms are listed in Figure 5 along with some auxiliary grammatical categories – s groups terms and types together as "subjects" resp. of typing and kinding, and C groups types and kinds together as classifiers. In both of these figures, the constructs forming expressions are listed from lowest to highest precedence – "abstractors" ($\lambda\, \Lambda\, \Pi\, \forall$) bind most loosely and parentheses most tightly. Associativity is as-expected, with arrows ($\to \Rightarrow$) and applications being left-associative and abstractors being right-associative.

The language of kinds and types is similar to that found in the Calculus of Implicit Constructions[1]. Kinds are formed by dependent and non-dependent products (Π and $\to$) and a base kind for types which can classify terms ($\star$). Types are also formed by the usual (dependent and non-dependent) products (Π and $\to$) and also *implicit* products ($\forall$ and $\Rightarrow$) which quantify over erased arguments (that is, arguments that disappear at run-time). Π-products are only allowed to quantify over terms as all types occurring in terms are erased at run-time, but $\forall$-products can quantify over types *and* terms because terms can be erased. Meanwhile, non-dependent products ($\to$ and $\Rightarrow$) can only "quantify" over terms because non-dependent type quantification does not seem particularly useful. Besides these, Cedilleum features type-level functions and applications (with term and type arguments), and a primitive equality type for untyped terms. Last of all is the "hole" type ($\bullet$) for writing partial type signatures or incomplete type applications. There are term-level holes as well, and together the two are intended to help facilitate "hole-driven development": any hole automatically generates a type error and provides the user with useful contextual information.

We illustrate with another example: what follows is a module stub for **DepCast** defining dependent casts – intuitively, functions from $a : A$ to $B\, a$ that are also equal[2] to identity – where the definitions `CastE` and `castE` are incomplete.

```
module DepCast .

CastE ◁ Π A : ⋆ . (A → ⋆) → ⋆ = ● .
```

---

[1]Cite

[2]Module erasure, discussed below

`castE ⊲ ∀ A : ⋆ . ∀ B : A → ⋆ . CastE · A · B ⇒ Π a : A . B a = • .`

s	$::=$	t	term
		T	type
t	$::=$	$\lambda\, x\, C^?.\, t$	normal abstraction
		$\Lambda\, x\, C^?.\, t$	erased abstraction
		$[\ defTermOrType\ ]\ \text{-}\ t$	let definitions
		$\rho\ t\ \text{-}\ t'$	equality elimination by rewriting
		$\phi\ t\ \text{-}\ t'\ \{t''\}$	type cast
		$\chi\ T\ \text{-}\ t$	check a term against a type
		$\delta\ \text{-}\ t$	ex falso quodlibet
		$\theta\ t\ t'^*$	elimination with a motive
		$t\ t'$	applications
		$t\ \text{-}t'$	application to an erased term
		$t\ \cdot T$	application to a type
		$\beta\ \{t\}$	reflexivity of equality
		$\varsigma\ t$	symmetry of equality
		$\mu\langle u, X, u', u''\rangle\ t\ motive^?\ \{\ case^*\ \}$	type-guarded pattern match and fixpoint
		$\mu'\ t\ motive^?\ \{\ case^*\ \}$	auxiliary pattern match
		u	term variable
		(t)	
		$\bullet$	hole for incomplete term
$case$	$::=$	$\mid id\ vararg^* \mapsto t$	pattern-matching cases
$vararg$	$::=$	u	normal constructor argument
		$\text{-}u$	erased constructor argument
		$\cdot X$	type constructor argument
$class$	$::=$	$:\ C$	
$motive$	$::=$	$@\ T$	motive for induction

Figure 5: Annotated Terms

Terms Terms can be explicit and implicit functions (resp. indicated by λ and Λ) with optional classifiers for bound variables, let-bindings, applications $t\ t'$, $t\ \text{-}t'$, and $t\ \cdot T$ (resp. to another term, an erased term, or a type). In addition to this there are a number of useful operators that will be discussed in more detail (and whose purpose will become more apparent) in Section 3.

2   Erasure

The definition of the erasure function given in Figure 6 takes the annotated terms from Figures 4 5 to the untyped terms of Figure 2. The last two equations indicate that any type or erased arguments in the the zero or more *vararg*'s of pattern-match case are indeed erased.

3   Type System

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[4]Where we assume t does not occur anywhere in T

[4]Where $\mathtt{tt} = \lambda\, x.\, \lambda\, y.\, x$ and $\mathtt{ff} = \lambda\, x.\, \lambda\, y.\, y$

$$
\begin{aligned}
|x| &= x \\
|\star| &= \star \\
|\square| &= \square \\
|\beta\ \{t\}| &= |t| \\
|\delta\ t| &= |t| \\
|\chi\ T^?\text{-}\ t| &= |t| \\
|\theta\ t\ t'^*| &= |t|\ |t'^*| \\
|\varsigma\ t| &= |t| \\
|t\ t'| &= |t|\ |t'| \\
|t\ \text{-}t'| &= |t| \\
|t\ \cdot T| &= |t| \\
|\rho\ t\ \text{-}\ t'| &= |t'| \\
|\forall\,x{:}C.\,C'| &= \forall\,x{:}|C|.\,|C'| \\
|\Pi\,x{:}C.\,C'| &= \Pi\,x{:}|C|.\,|C'| \\
|\lambda\,u{:}T.\,t| &= \lambda\,u.\,|t| \\
|\lambda\,u.\,t| &= \lambda\,u.\,|t| \\
|\lambda\,X{:}K.\,C| &= \lambda\,X{:}|K|.\,|C| \\
|\Lambda\,x{:}C.\,t| &= |t| \\
|\phi\ t\ \text{-}\ t'\ \{t''\}| &= |t''| \\
|[x = t : T]|\ \text{-}\ t'| &= (\lambda\,x.\,|t'|)\ |t| \\
|[X = T : K]\ \text{-}\ t| &= |t| \\
|\{t \simeq t'\}|| &= \{|t| \simeq |t'|\} \\
|\mu\langle u, X, u', u''\rangle\ t\ motive^?\ \{case^*\}| &= \mu\langle u, u', u''\rangle\ t\ \{|case^*|\} \\
|\mu'\ t\ motive^?\ \{case^*\}| &= \mu'\ t\ \{|case^*|\} \\[1em]
|id\ vararg^* \mapsto t| &= id\ |vararg^*|\ \mapsto |t| \\[1em]
|\text{-}u| &= \\
|\cdot T| &=
\end{aligned}
$$

Figure 6: Erasure for annotated terms

$$
\overline{\Gamma \vdash \star : \square}
$$

$$
\frac{\Gamma \vdash C : S \quad \Gamma, y : C \vdash C' : S' \quad Var(y, S)}{\Gamma \vdash \Pi\,y{:}C.\,C' : S'}
\qquad
\frac{\Gamma \vdash C : S \quad \Gamma, y : C \vdash C' : \star \quad Var(y, S)}{\Gamma \vdash \forall\,y{:}C.\,C' : \star}
$$

$$
\frac{FV(p\ p') \subseteq dom(\Gamma)}{\Gamma \vdash \{p \simeq p'\} : \star}
\qquad
\overline{\Gamma \vdash \kappa : \Gamma(\kappa)}
\qquad
\overline{\Gamma \vdash X : \Gamma(X)}
$$

$$
\frac{\Gamma \vdash \Pi\,x{:}C.\,K : \square \quad \Gamma, x{:}C \vdash T : K}{\Gamma \vdash \lambda\,x{:}C.\,T : \Pi\,x{:}C.\,K}
\qquad
\frac{\Gamma \vdash T : \Pi\,x{:}K.\,K' \quad \Gamma \vdash T' : K}{\Gamma \vdash T\ \cdot T' : [T'/x]K'}
\qquad
\frac{\Gamma \vdash T : \Pi\,x{:}T'.\,K \quad \Gamma \vdash_{\Downarrow} t : T'}{\Gamma \vdash T\ t : [t/x]K}
$$

Figure 7: Kind and sort checking for Cedilleum $\boxed{\Gamma \vdash C : S}$

$$\frac{}{\Gamma \vdash_\delta u : \Gamma(u)}$$

$$\frac{\Gamma \vdash T : K \quad \Gamma, x{:}T \vdash_\delta t : T'}{\Gamma \vdash_\delta \lambda x{:}T.t : \Pi x{:}T.T'}$$

$$\frac{\Gamma, x{:}T \vdash_\Downarrow t : T'}{\Gamma \vdash_\Downarrow \lambda x.t : \Pi x{:}T.T'}$$

$$\frac{\Gamma \vdash C : S \quad x \notin FV(|t|) \quad \Gamma, x{:}C \vdash_\delta t : T}{\Gamma \vdash_\delta \Lambda x{:}C.t : \forall x{:}C.T}$$

$$\frac{x \notin FV(|t|) \quad \Gamma, x{:}C \vdash_\delta t : T}{\Gamma \vdash_\Downarrow \Lambda x.t : \forall x{:}C.T}$$

$$\frac{\Gamma \vdash_\Uparrow t : \Pi x{:}T'.T \quad \Gamma \vdash_\Downarrow t' : T'}{\Gamma \vdash_\delta t\ t' : [t'/x]T}$$

$$\frac{\Gamma \vdash_\Uparrow t : \forall X{:}K.T' \quad \Gamma \vdash T : K}{\Gamma \vdash_\delta t \cdot T : [T/X]T'}$$

$$\frac{\Gamma \vdash_\Uparrow t : \forall x{:}T'.T \quad \Gamma \vdash_\Downarrow t' : T'}{\Gamma \vdash_\delta t \text{ -}t' : [t'/x]T}$$

$$\frac{\Gamma \vdash_\Uparrow t : T' \quad |T'| =_\beta |T|}{\Gamma \vdash_\Downarrow t : T}$$

$$\frac{\Gamma \vdash T : K \quad \Gamma \vdash_\Downarrow t : T \quad \Gamma, id = t{:}T \vdash_\delta t' : T'}{\Gamma \vdash_\delta [id : T = t] \text{ -} t' : T'}$$

$$\frac{\Gamma \vdash_\Uparrow t : T \quad \Gamma, id = t{:}T \vdash_\delta t' : T'}{\Gamma \vdash_\delta [id = t] \text{ -} t' : T'}$$

$$\frac{\Gamma \vdash_\Uparrow t : \{t_1 \simeq t_2\} \quad \Gamma \vdash_\Uparrow t' : [t_1/x]\,T}{\Gamma \vdash_\delta \rho\ t \text{ -} t' : [t_2/x]\,T}\ {}_3$$

$$\frac{\Gamma \vdash K : \square \quad \Gamma \vdash T : K \quad \Gamma, id = T{:}K \vdash_\delta t' : T'}{\Gamma \vdash_\delta [id : K = T] \text{ -} t' : T'}$$

$$\frac{\Gamma \vdash \{t' \simeq t'\} : \star}{\Gamma \vdash_\Downarrow \beta\{t\} : \{t' \simeq t'\}}$$

$$\frac{\Gamma \vdash_\delta t : \{t_1 \simeq t_2\}}{\Gamma \vdash_\delta \varsigma\ t : \{t_2 \simeq t_1\}}$$

$$\frac{\Gamma \vdash_\Downarrow t : \{|t_1| \simeq |t_2|\} \quad \Gamma \vdash_\delta t_1 : T}{\Gamma \vdash_\delta \phi\ t \text{ -} t_1\ \{t_2\} : T}$$

$$\frac{\Gamma \vdash_\Downarrow t : T}{\Gamma \vdash_\Uparrow \chi\ T \text{ -} t : T}$$

$$\frac{\Gamma \vdash_\Downarrow t : \{\mathtt{tt} \simeq \mathtt{ff}\}}{\Gamma \vdash_\Downarrow \delta \text{ -} t : T}\ {}_4$$

$$\frac{\Gamma \vdash_\Uparrow t : \mathtt{??} \quad \Gamma \vdash_? t'^* : \mathtt{??}}{\Gamma \vdash_\Downarrow \theta\ t\ t'^* : T}$$

Figure 8: Type checking for Cedilleum