Inventory Control Project

Operations Research Module: 21SCIB04I

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Part 1

Application Description

The Inventory of any business, has to be properly controlled as it has a great influence on it, since inventory decisions are jointly considered with production decisions that determines the net profit. An Inventory holds materials that can be of either independent demand (The product sold as a whole to customers, e.g. Car) or of dependent demand (The parts that the business works on manufacturing to form their main sold product, e.g. Car Parts).

The focus will be on forecasting the independently demanded materials, that are classified into three priority levels (using ABC Analysis), upon which the cost differs and thus the profit is altered.

The EOQ Analysis (Economic Order Quantity), one of the oldest classical production scheduling models, aims for minimizing the total holding costs (price of carrying items over time, directly proportional to number of items held) and ordering costs (cost of the process of placing certain order, constant) of inventory products.

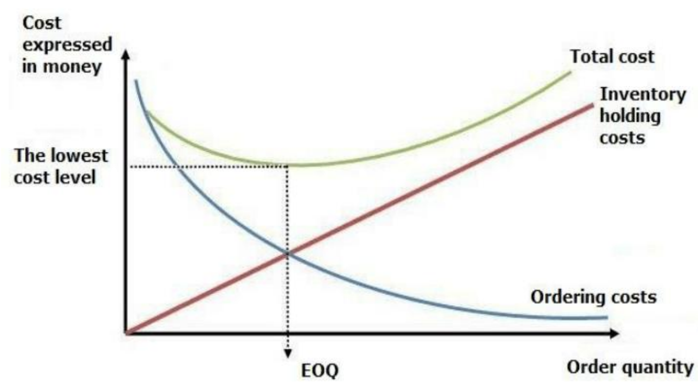

Figure 1. Minimizing costs using EOQ [1].

According to the demand frequency for products, **restocking costs** are determined and hence maintained to stabilize or even increase profit. This can be achieved by planning in advance to restock when market (suppliers') prices are the lowest then storing them till selling season, where demands are at their highest. Simply because, **holding costs** are usually low in relation to shortage costs.

Problem Statements

Optimized Objectives

Constraints

	Problem Statement	Optimized Objective Detailed equations are found on the next section.	Constraints Detailed equations are found ordered on the next section.
1st LP Model [2]	Assume that a business is required to produce the following number of products in the next 4 days: • Day1: 100 • Day2: 150 • Day3: 200 • Day4: 170 Goal is minimizing the holding cost ($1 per item / day) of and maximizing the profit at the same time. Inventory cost (/holding cost) is recalculated everyday according to the number of remaining items to be stored, since the beginning of production, without the day's sales made. IC = (BegInventory + newProduction – day's Sales) * holding cost	Unit production costs differ according to the day it was manufactured at as follows: • Day1: $9 • Day2: $12 • Day3: $10 • Day4: $12 Thus, an optimized objective might be producing all required production in the day with least production cost (fixation of price) and storing them till the planned day of selling (Inventory cost $1/unit/day).	• Follow Inventory Balancing constraint. • Fulfill demanded production amounts at the moment of sales. (will already be covered by nature through covering the other constraints) • Non-negative inventory quantities. • Non-negative production quantities.

	Problem Statement	Optimized Objective Detailed equations are found on the next section.	Constraints Detailed equations are found ordered on the next section.
2nd LP Model [3]	Assume that the demands (f d(t) ≥ 0 units) of a business's products are known in advance and that these products' prices are fixed if fulfilled from the owned inventory, else will be of variable price based on the time given (p(t) ≥ $0 / unit) if fulfilled from the market (aka: supplier). The products received from the market (x(t) ≥ 0 units) can exceed the demanded, thus, will be stored for future use in the inventory. Goal is to regularly cover the demands, while minimalizing the need for market purchases (x(t)) by using the owned inventory instead.	Minimization of total cost resulting from the number of times the overcapacity demands require the business to purchase from the market to satisfy customers, over a period of time (Restocking costs).	• Constraint that indicates the progression state of the inventory at time t. • Constraint to identify the peak capability of demands coverage from the inventory, in order to determine the amount of market procurement needed. • Constraint for sustaining the inventory size, by always having the inputted market purchases fewer than both the inventory storage capacity, and the demanded units.

	Problem Statement	Optimized Objective Detailed equations are found on the next section.	Constraints Detailed equations are found ordered on the next section.
3rd LP Model [4]	A method to determine safety stock in the case of incomplete information on demand.	Minimizing the current level of the inventory subjected to Probability mass in point x_i till reaches the expected once, the absolute second moment of X second, and the maximum allowed number of items to be in short.	linear program with an infinite number of constraints, that is soon replaced by a large finite subset solved individually to obtain linear program. The method assumes that integral constraints can be transformed into a sequence, with increasing number of evaluation points, of optimization problems and where the integral is replaced by an infinite sum.

Linear Programming Formulation #1

Decision Variables

x_i : number of products of day i , where $i \in 1, 2, \dots, 4$

y_i : number of products needed to be stored on day i , where $i \in 1, 2, \dots, 4$

Objective Function

MIN ↓ $Z = 9x_1 + 12x_2 + 10x_3 + 12x_4 + (1 * y_1) + (1 * y_2) + (1 * y_3) + (1 * y_4)$

Constraints

- Inventory Balancing Constraints

$$x_1 - 100 = y_1$$

$$y_1 + x_2 - 150 = y_2$$

$$y_2 + x_3 - 200 = y_3$$

$$y_3 + x_4 - 170 = y_4$$

- Non-negativity Constraints

$$x_i,\ y_i \geq 0$$

Linear Programming Formulation #2

Decision Variables

$p(t)$: products market (supplier) price on time t

$x(t)$: number of products needed to be purchased from the market on time i

$\forall t \in T$

Objective Function

$$\text{MIN} \downarrow \; Z = \sum_{t \in T} p(t) * x(t)$$

Constraints

- $b(t) = b(t-1) + x(t) - d(t)$

  Where,

  $b(t) \in [0, B] : available\ amount\ of\ instock\ products\ at\ time\ slot\ t$

  $d(t) : total\ count\ of\ demands\ at\ time\ slot\ t$

- $x(t) \geq d(t) - \min\{ p_d,\ b(t-1) \}$

  Where,

  $p_d :\ maximum\ stock\ output\ degree$

- $x(t) \leq d(t) + \max\{ p_c,\ B - b(t-1) \}$

  Where,

  $p_c :\ \text{maximum stock input degree}$

  $B : inventory\ size$

- $0 \leq b(t) \leq B$

Linear Programming Formulation #3

Decision Variables

t = the level of the inventory,
p_i = the probability mass in point x_i,
z_1 = the expected value of X,
z_2 = the absolute second moment of X,
z_3 = the maximum allowed number of items short.

where $(x_i - t)_+$ stands for $\max(x_i - t, 0)$. The decision variables in [P1] are t and p_i ($i = 1 \ldots N$), where N represents the number of discrete points which have been chosen in the experiment.

Objective Function

The optimisation problem might be formulated as:

$$[\text{P1}] \qquad \text{Min } t$$

$$\text{Subject to } \sum_i p_i = 1$$

$$\sum_i x_i p_i = z_1$$

$$\sum_i x_i^2 p_i = z_2$$

$$\sum_i (x_i - t)_+ p_i \leqslant z_3$$

Problem [P1] offers the answer to the following question: what is the minimal amount of inventory so that a distribution with given characteristics exists in which the expected number short maximally equals the value z_3.

Constraints

Table 1
Lower bounds on the stop-loss premium in an interval $[0, b_0]$.

Lower bounds	Conditions
$\mu_{10} - t_0$	$0 \leqslant t_0 \leqslant b'_0$ or $0 \leqslant t_0 \leqslant (\mu_{20} - \mu_{10} b_0)/(\mu_{10} - b_0)$
$(\mu_{20} - \mu_{10} t_0)/b_0$	$b'_0 \leqslant t_0 \leqslant 0'$ or $(\mu_{20} - \mu_{10} b_0)/(\mu_{10} - b_0) \leqslant t_0 \leqslant \mu_{20}/\mu_{10}$
0	$0' \leqslant t_0 \leqslant b_0$ or $\mu_{20}/\mu_{10} \leqslant t_0 \leqslant b_0$

Table 2
Upper bounds on the stop-loss premium in an interval $[0,b_0]$.

Upper bounds	Conditions
$\mu_{10}(\mu_{20}-\mu_{10}t_0)/\mu_{20}$	$t_0 \leqslant 0'/2$ or $t_0 \leqslant \mu_{20}/(2\mu_{10})$
$\left(\mu_{10}-t_0+\sqrt{(\mu_{20}-\mu_{10}^2)+(t_0-\mu_{10})^2}\right)\Big/2$	$0'/2 \leqslant t_0 \leqslant (b_0+b_0')/2$ or $\mu_{20}/(2\mu_{10}) \leqslant t_0 \leqslant (\mu_{20}-b_0^2)/(2(\mu_{10}-b_0))$
$(\mu_{20}-\mu_{10}^2)(b_0-t_0)/\left((\mu_{20}-\mu_{10}^2)+(b_0-\mu_{10})^2\right)$	$(b_0+b_0')/2 \leqslant t_0$ or $\left(\mu_{20}-b_0^2\right)/(2(\mu_{10}-b_0)) \leqslant t_0$

Table 3
Lower bounds on the number of units short for the illustrative example.

Lower bounds	Conditions
$20 - t_0$	$0 \leqslant t_0 \leqslant 13.333$
$12 - 2/5t_0$	$13.333 \leqslant t_0 \leqslant 30$
0	$30 \leqslant t_0 \leqslant 50$

Table 4
Upper bounds on the number of units short for the illustrative example.

Upper bounds	Conditions
$20 - 2/3t_0$	$0 \leqslant t_0 \leqslant 15$
$(20 - t_0 + \sqrt{200 + (t_0 - 20)})/2$	$15 \leqslant t_0 \leqslant 31.667$
$(100 - 2t_0)/11$	$31.667 \leqslant t_0 \leqslant 50$

Table 5
Lower bounds on the safety inventory level.

Lower bounds	Requirements
$30 - 5/2W$	$W \leqslant 6.667$
$20 - W$	$6.667 \leqslant W$

Table 6
Upper bounds on the safety inventory level.

Upper bounds	Requirements
$50 - 11/2W$	$W \leqslant 3.333$
$(50 - W^2 + 20W)/W$	$3.333 \leqslant W \leqslant 10$
$(60 - 3W)/2$	$10 \leqslant W$

The non-linear constraint may be approximated by letting the value of t coincide with one of the x_i-values (so as $N \to \infty$, the approximation takes the correct value). In such a way the constraint in linearised.

Comparison between the 3 LP Formulations

It became quite obvious that there are several aspects to be considered for an inventory to be well controlled in favor of maximizing the profit.

Each of the Linear Programming formulation mentioned above, tackles a part.

For instance, the **first LP formulation model [2]** was concerned with the holding costs of products stored in the inventory till day of retail.

While, the **second LP formulation model [3]** and the **third model [4]**, focused more on planning strategically and wisely for the restocking costs, that happens when the inventory runs out of products in demand.

Yet, one considered the demands and performance measures previously known values, whereas the second managed to deal with the uncertainty of these variables and embed it within the objective function, to produce more accurate results for easing the decision making of amounts to order, inventory refill policy and of review results.

It is clear that, the **third LP formulation model [4] is the way better than the second [3]** for some reasons.

First, it is more <u>efficient to use</u> as usually the lead time is unknown for any business. Though, it can be calculated as a probability of future results based on past given data records.

This leads to the second reason, of <u>consideration of all possible factors</u> affecting the objective.

Third, it uses a <u>large finite set of unrepeated constraints</u> that leads to having <u>more accurate</u> results.

LP Formulation #1 Solved with Simplex Technique

Solution in traditional way

The cons for using Simplex method is the high cost of pivoting operations, but since this is not a large problem, therefore the cost is tolerable.

- Assume that y1, y2, y3, y4 are expressed as x5, x6, x7, x8 respectively.
- Also, constraints'' format will be slightly changed to ease matrix filling, as follows:

Original format	New format
$x_1 - 100 = y_1$	$x_1 - y_1 = 100$
$y_1 + x_2 - 150 = y_2$	$y_1 + x_2 - y_2 = 150$
$y_2 + x_3 - 200 = y_3$	$y_2 + x_3 - y_3 = 200$
$y_3 + x_4 - 170 = y_4$	$y_3 + x_4 - y_4 = 170$

Problem:

Min $Z = 9x_1 + 12x_2 + 10x_3 + 12x_4 + x_5 + x_6 + x_7 + x_8$

subject to

$$\begin{array}{llllllllll} x_1 & & & & - x_5 & & & & = & 100 \\ & x_2 & & & + x_5 & - x_6 & & & = & 150 \\ & & x_3 & & & + x_6 & - x_7 & & = & 200 \\ & & & x_4 & & & + x_7 & - x_8 & = & 170 \end{array}$$

and $x_1, x_2, x_3, x_4\ x_5, x_6, x_7, x_8 \geq 0$;

The problem is converted to standard form by adding slack, surplus and artificial variables as appropriate

1. As the constraint-1 is of type '=' we should add artificial variable A_1

2. As the constraint-2 is of type '=' we should add artificial variable A_2

3. As the constraint-3 is of type '=' we should add artificial variable A_3

4. As the constraint-4 is of type '=' we should add artificial variable A_4

After introducing artificial variables

Min $Z = 9x_1 + 12x_2 + 10x_3 + 12x_4 + x5 + x6 + x7 + x8 + MA_1 + MA_2 + MA_3 + MA_4$

subject to

$$\begin{array}{llllllllllllll}
x_1 & & & & - x5 & & & & + A_1 & & & & = & 100 \\
& x_2 & & & + x5 & - x6 & & & & + A_2 & & & = & 150 \\
& & x_3 & & & + x6 & - x7 & & & & + A_3 & & = & 200 \\
& & & x_4 & & & + x7 & - x8 & & & & + A_4 & = & 170
\end{array}$$

and x_1, x_2, x_3, x_4, x5, x6, x7, x8, $A_1, A_2, A_3, A_4 \geq 0$

Iteration-1		C_j	9	12	10	12	1	1	1	1	M	M	M	M	
B	***C_B***	***X_B***	***x_1***	***x_2***	***x_3***	***x_4***	***x5***	***x6***	***x7***	***x8***	***A_1***	***A_2***	***A_3***	***A_4***	**MinRatio** ***$\frac{X_B}{x_1}$***
A_1	M	100	**(1)**	0	0	0	-1	0	0	0	1	0	0	0	$\frac{100}{1}=100\rightarrow$
A_2	M	150	0	1	0	0	1	-1	0	0	0	1	0	0	---
A_3	M	200	0	0	1	0	0	1	-1	0	0	0	1	0	---
A_4	M	170	0	0	0	1	0	0	1	-1	0	0	0	1	---
Z=620M		***Z_j***	***M***	***M***	***M***	***M***	**0**	**0**	**0**	***-M***	***M***	***M***	***M***	***M***	
		Z_j-C_j	M-9$\uparrow$	M-12	M-10	M-12	-1	-1	-1	-M-1	0	0	0	0	

Positive maximum Z_j-C_j is M-9 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 100 and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is 1.

Entering $=x_1$, Departing $=A_1$, Key Element $=1$

R_1(new)=R_1(old)

R_2(new)=R_2(old)

R_3(new)=R_3(old)

R_4(new)=R_4(old)

Iteration-2		C_j	9	12	10	12	1	1	1	1	M	M	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**x_6**	**x_7**	**x_8**	**A_1**	**A_2**	**A_3**	**A_4**	**MinRatio** **$\frac{X_B}{x_3}$**
x_1	9	100	1	0	0	0	-1	0	0	0	1	0	0	0	---
A_2	M	150	0	1	0	0	1	-1	0	0	0	1	0	0	---
A_3	M	200	0	0	**(1)**	0	0	1	-1	0	0	0	1	0	2001=200→
A_4	M	170	0	0	0	1	0	0	1	-1	0	0	0	1	---
$Z=520M+900$		**Z_j**	**9**	**M**	**M**	**M**	**$M-9$**	**0**	**0**	**$-M$**	**9**	**M**	**M**	**M**	
		Z_j-C_j	0	$M-12$	$M-10$↑	$M-12$	$M-10$	-1	-1	$-M-1$	$-M+9$	0	0	0	

Positive maximum Z_j-C_j is $M-10$ and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 200 and its row index is 3. So, the leaving basis variable is A_3.

∴ The pivot element is 1.

Entering $=x_3$, Departing $=A_3$, Key Element $=1$

R_3(new)$=R_3$(old)

R_1(new)$=R_1$(old)

R_2(new)$=R_2$(old)

R_4(new)$=R_4$(old)

Iteration-3		C_j	9	12	10	12	1	1	1	1	M	M	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**x_6**	**x_7**	**x_8**	**A_1**	**A_2**	**A_3**	**A_4**	**MinRatio** **$\frac{X_B}{x_5}$**
x_1	9	100	1	0	0	0	-1	0	0	0	1	0	0	0	---
A_2	M	150	0	1	0	0	**(1)**	-1	0	0	0	1	0	0	$\frac{150}{1}=150$ →
x_3	10	200	0	0	1	0	0	1	-1	0	0	0	1	0	---
A_4	M	170	0	0	0	1	0	0	1	-1	0	0	0	1	---
$Z=320M+2900$		**Z_j**	**9**	**M**	**10**	**M**	**$M-9$**	**$-M+10$**	**$M-10$**	**$-M$**	**9**	**M**	**10**	**M**	
		Z_j-C_j	0	$M-12$	0	$M-12$	$M-10$↑	$-M+9$	$M-11$	$-M-1$	$-M+9$	0	$-M+10$	0	

Positive maximum Z_j-C_j is $M-10$ and its column index is 5. So, the entering variable is x_5.

Minimum ratio is 150 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is 1.

Entering $=x_5$, Departing $=A_2$, Key Element $=1$

R_2(new)=R_2(old)

R_1(new)=R_1(old) + R_2(new)

R_3(new)=R_3(old)

R_4(new)=R_4(old)

Iteration-4		C_j	9	12	10	12	1	1	1	1	M	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	A_1	A_2	A_3	A_4	**MinRatio** $\frac{X_B}{x_7}$
x_1	9	250	1	1	0	0	0	-1	0	0	1	1	0	0	---
x_5	1	150	0	1	0	0	1	-1	0	0	0	1	0	0	---
x_3	10	200	0	0	1	0	0	1	-1	0	0	0	1	0	---
A_4	M	170	0	0	0	1	0	0	**(1)**	-1	0	0	0	1	1701=170→
$Z=170M+4400$		**Z_j**	**9**	**10**	**10**	**M**	**1**	**0**	**M-10**	**-M**	**9**	**10**	**10**	**M**	
		Z_j-C_j	0	-2	0	M-12	0	-1	M-11↑	-M-1	-M+9	-M+10	-M+10	0	

Positive maximum Z_j-C_j is M-11 and its column index is 7. So, the entering variable is x_7.

Minimum ratio is 170 and its row index is 4. So, the leaving basis variable is A_4.

∴ The pivot element is 1.

Entering $=x_7$, Departing $=A_4$, Key Element $=1$

R_4(new)=R_4(old)

R_1(new)=R_1(old)

R_2(new)=R_2(old)

R_3(new)=R_3(old) + R_4(new)

Iteration-5		C_j	9	12	10	12	1	1	1	1	M	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	A_1	A_2	A_3	A_4	**MinRatio**
x_1	9	250	1	1	0	0	0	-1	0	0	1	1	0	0	
x_5	1	150	0	1	0	0	1	-1	0	0	0	1	0	0	
x_3	10	370	0	0	1	1	0	1	0	-1	0	0	1	1	
x_7	1	170	0	0	0	1	0	0	1	-1	0	0	0	1	
$Z=6270$		**Z_j**	**9**	**10**	**10**	**11**	**1**	**0**	**1**	**-11**	**9**	**10**	**10**	**11**	
		Z_j-C_j	0	-2	0	-1	0	-1	0	-12	-M+9	-M+10	-M+10	-M+11	

Since all $Z_j\text{-}C_j \leq 0$

Hence, **optimal solution** is arrived with value of variables as:

x_1=250, x_2=0, x_3=370, x_4=0, x_5=150, x_6=0, x_7=170, x_8=0

Min Z=6270

Solution using Excel Solver

[Excel Formulas Solution\Simplex Solution for first LP Formula model using Excel Solver.xlsx]

	A	B	C	D	E
1	DECISION VARIABLES				
2	x1	250			
3	x2	0			
4	x3	370			
5	x4	0			
6	y1	150			
7	y2	0			
8	y3	170			
9	y4	0			
10					
11	OBJECTIVE FUNCTION				
12					
13	Minimize	6270			
14					
15	CONSTRAINTS				
16			Inequality		
17	1	150	=	150	
18	2	0	=	0	
19	3	170	=	170	
20	4	0	=	0	
21	5	250	>=	0	
22	6	0	>=	0	
23	7	370	>=	0	
24	8	0	>=	0	
25	9	150	>=	0	
26	10	0	>=	0	
27	11	170	>=	0	
28	12	0	>=	0	
29					

Sheet1

Dual Form of the solution

Converting the standard problem into a dual formed problem is way too useful because, the dual problem has fewer number of constraints than that of the original one, consequently it will reduce the number of steps needed to reach the optimal value (n-variables in standard form = m constraints in Dual form).

Nevertheless, in our case we have got high number of variables, which will be soon the number of constraints in the dual for, that is why it is unnecessary to make this conversion.

MAX ↑ $Z^* = 100y_1 + 150y_2 + 200y_3 + 170y_4$

subject to

$$\begin{aligned}
y_1 &\le 9 \\
y_2 &\le 12 \\
y_3 &\le 10 \\
y_4 &\le 12 \\
-y_1 + y_2 &\le 1 \\
-y_2 + y_3 &\le 1 \\
-y_3 + y_4 &\le 1 \\
-y_4 &\le 1
\end{aligned}$$

Since $1st, 2nd, 3rd, 4th$ constraints in the standard problem are equalities, the corresponding dual variables y_1, y_2, y_3, y_4 will be unrestricted in sign.

Sensitivity Analysis

[Excel Formulas Solution\first LP Formula model Sensitivity Analysis using Excel Solver.xlsx]

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [first LP Formula model Sensitivity Analysis using Excel Solver.xlsx]Sheet1
Report Created: 20/04/2022 21:38:13

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	x1	250	0	9	2	1
B3	x2	0	0	12	1000000000000000000000000000000.00	2
B4	x3	370	0	10	1	12
B5	x4	0	0	12	1000000000000000000000000000000.00	1
B6	y1	150	0	1	2	1
B7	y2	0	0	1	1000000000000000000000000000000.00	1
B8	y3	170	0	1	1	12
B9	y4	0	0	1	1000000000000000000000000000000.00	12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B17		150	9	0	1000000000000000000000000000000.00	250
B18		0	10	0	1000000000000000000000000000000.00	150
B19		170	10	0	1000000000000000000000000000000.00	370
B20		0	11	0	1000000000000000000000000000000.00	170
B21		250	0	0	250	1000000000000000000000000000000.00
B22		0	2	0	150	0
B23		370	0	0	370	1000000000000000000000000000000.00
B24		0	1	0	170	0
B25		150	0	0	150	1000000000000000000000000000000.00
B26		0	1	0	370	0
B27		170	0	0	170	1000000000000000000000000000000.00
B28		0	12	0	1000000000000000000000000000000.00	0

Microsoft Excel 16.0 Limits Report
Worksheet: [first LP Formula model Sensitivity Analysis using Excel Solver.xlsx]Sheet1
Report Created: 20/04/2022 21:38:14

Cell	Objective Name	Value
B13	Minimize	6270

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
B2	x1	250	250	6270	250	6270
B3	x2	0	0	6270	0	6270
B4	x3	370	370	6270	370	6270
B5	x4	0	0	6270	0	6270
B6	y1	150	150	6270	150	6270
B7	y2	0	0	6270	0	6270
B8	y3	170	170	6270	170	6270
B9	y4	0	0	6270	0	6270

Part 2

Optimization Problem Description

Problem Statement

My travel agency managed to reserve a booth in a special event to distribute exclusive offers for this month, among some people, for marketing purposes. We have two types of tour offers, one for singles and other for families.
I want to decide on the number of single tours offers and family bundles offers that is to be made available by having them printed and distributed during the event.

The single tour requires half (2 hours) the time of the family tour (4 hours), from the agency's representative to make its arrangements.
Each family bundle requires only $5 for paper work and tickets printing, while single offers cost at least $10 each.

We make a profit of $50 on single tours purchases and $30 for family bundles purchases.

The printing fees monthly budget is $200 and the maximum number of work hours of the 3 representatives working with me in the agency, is 144 hours per month.

Optimized Objective

Maximize the profit out of the tour offers sold during the event, through determining the number of tours to offer of each type (single and family).

Constraints

- Time constraints to ensure the total time needed for arranging the sold tours do not exceed the actual total working hours of the available employees in my agency.
- Cost constraint to avoid exceeding the budget set for paper work and printings of the tours to be offered.
- Nonnegativity constraints because number of offerings cannot be negative .

Problem Linear Programming Formulation

Decision Variables

$x_1 : number\ of\ single\ offers\ to\ make\ available\ \boldsymbol{in\ the\ event\ for}\ this\ month$

$x_2 : number\ of\ family\ bundle\ offers\ to\ make\ available\ in\ the\ event\ for\ this\ month$

Objective Function

$MAX \uparrow \quad Z = 50\, x_1 + 30\, x_2$

Constraints

$2\, x_1 + 4\, x_2 \leq 144 \qquad , arrangements\ time\ constraint$

$10\, x_1 + 5\, x_2 \leq 200 \qquad , costs\ constraint$

$x_1, x_2 \geq 0 \qquad , nonnegativity\ constraints$

Solution Using Simplex Technique

Excel Formulas Solution\Simplex Solution for report's part 2 using Excel Solver.xlsx

	A	B	C	D
1	DECISION VARIABLES			
2	x1	20		
3	x2	40		
4				
5	OBJECTIVE FUNCTION			
6				
7	Maximize	2200		
8				
9	CONSTRAINTS			
10			Inequality	
11	time	200	<=	200
12	cost	400	<=	400
13	nonneg	20	>=	0
14	nonneg	40	>=	0

The results revealed that even though the single tours are a value add ($50 income for each) to the total profit of my business, and even though they are arranged in less time than the family bundles, they are yet needed to be offered less due to their high paper work costs and other factors.

Henceforth, the **optimal solution** is to offer 20 of the single tours (x1) and 40 of the family bundle tours (x2), to profit $2200.

Dual Form of the solution

MIN ↓ $Z^* = 144y_1 + 200y_2$

subject to

$$2y_1 + 10y_2 \geq 50$$
$$4y_1 + 5y_2 \geq 30$$

and $y_1, y_2 \geq 0$

References (IEEE)

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[3] L. Yang, M. H. Hajiesmaili, R. Sitaraman, A. Wierman, E. Mallada, and W. S. Wong, "Online linear optimization with inventory management constraints," Proceedings of the ACM on Measurement and Analysis of Computing Systems, 01-Mar-2020. [Online]. Available: https://dl.acm.org/doi/abs/10.1145/3379482. [Accessed: 18-Apr-2022].

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