Successive Combination Jet Algorithm

For Hadron Collisions

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Abstract

Jet finding algorithms, as they are used in e^+e^- and hadron collisions, are reviewed and compared. It is suggested that a successive combination style algorithm, similar to that used in e^+e^- physics, might be useful also in hadron collisions, where cone style algorithms have been used previously.

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I. INTRODUCTION

The measurement of jet cross sections has provided useful tests of Quantum Chromodynamics both at hadron colliders and at electron-positron colliders. The observed jets provide a view of the underlying hard quark and gluon interactions that occur at very small distance scales. However, this view is inevitably clouded by the subsequent long distance showering and eventual hadronization of the primary quarks and gluons. Furthermore, since the quarks and gluons carry non-zero color charges and the final hadrons do not, there can be no *unique* association of a jet of hadrons with a single initial quark or gluon. Nevertheless, with a suitable definition of the jet cross section one hopes to minimize the effect of long distance physics and of the inherent jet ambiguities and obtain a fairly precise picture of the short distance dynamics.

Although the basic hard scattering processes studied in hadron-hadron and in electron-positron collisions are much the same, the overall event structure is quite different. In the e^+e^- case the initial state is purely electromagnetic and the entire final state can be thought of as arising from the short distance interaction of the virtual photon with the quarks. In this sense all of the hadrons in the final state are associated with the hard scattering process. In contrast the overall structure of the hadron-hadron case is much more complex. Of the large number of initial state partons, only one "active parton" from each incident hadron participates in the hard scattering process. Thus only a fraction of the hadrons in the final state are to be (loosely) associated with the hard scattering process, with the remainder ascribed to the "underlying event." This second contribution corresponds to the soft interactions of the remaining partons in the incident hadrons and, in first approximation, can be treated as uncorrelated with the hard process. The active partons also produce additional radiation in the form of initial state bremsstrahlung that is not present in e^+e^- events. The underlying event plus the initial state radiation produce the characteristic "beam jets" of hadron collisions: particles with small momenta transverse to the beam axis, but possibly large momenta along the beam axis. The long distance

soft interactions responsible for the observed color singlet hadrons will, of course, result in some degree of dynamical coupling between all of these components. There will also be an essentially kinematical correlation induced by the fact that the jet selection or trigger process will generally be biased to choose events where the beam jets have higher than average global E_T and multiplicity (i.e., the underlying event is noisier than average).

These differences between the event structure of e^+e^- collisions and hadron-hadron collisions have, quite naturally, led to differences in the way jet definitions have been employed in the two cases. One might categorize the differences as follows.

First, the cross sections studied are different. In e^+e^- collisions, where the entire event arises from an initially small number of energetic partons, one typically works with *exclusive* jet cross sections describing the production of exactly n jets and nothing else. In hadron-hadron collisions the practice has been to measure *inclusive* large p_T jet cross sections, that is, cross sections to make n jets with specified properties plus any number of other unobserved jets or particles not in jets.

Second, the variables used are different. For e^+e^- annihilation, one wants to emphasize rotational invariance. Thus the natural variables are energies E and polar angles θ, ϕ. For hadron-hadron collisions, one wants to emphasize invariance under boosts along the beam axis since, in fact, the c.m. frame of the hard scattering is typically moving in the hadron-hadron c.m. frame. Thus the natural variables are transverse momenta p_T or the corresponding "transverse energy" $E_T \equiv E \sin\theta$, azimuthal angle ϕ and pseudo-rapidity $\eta = -\ln(\tan(\theta/2))$.

Third, the jet definitions or algorithms employed to precisely define the (otherwise ambiguous) jets tend to be correspondingly different. As implied above in e^+e^- collisions one normally uses a jet definition that associates *every* final-state hadron *uniquely* with one of the jets. In hadron-hadron collisions producing high p_T jets, only a small fraction of the final state hadrons are associated with the high p_T jets. The other particles present in the event can be thought of as associated with the "beam jets" introduced earlier. One wants to keep the high p_T jets distinct from the hadronic debris in the beam jets. For this reason,

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one has typically used a cone definition [1], which was, in fact, inspired by the original theoretical definition for jets in e^+e^- collisions [2]. A jet in this definition is a set of particles whose momentum vectors lie within a certain angular cone. Such a definition suppresses the effect of the beam jets, since only a small fraction of the low p_T particles in the beam jets will fall into the cone of a high p_T jet. Furthermore, since this contribution is essentially determined by geometry, it is relatively easy to estimate. In the case of e^+e^- collisions, the jet algorithm typically used experimentally is rather of the "successive combination" variety first introduced by the Jade group at DESY [3]. In this kind of definition, one recursively groups sets of particles with "nearby" momenta, as defined by some measure, into larger sets of particles. The initial sets consist of just one particle each. The final sets are the jets.

In this paper, we discuss a jet definition for hadron collisions that makes use of some of the ideas used in jet definitions in e^+e^- collisions. The definition is essentially that proposed by S. Catani, Yu. Dokshitzer, M. Seymour, and B. R. Webber [4], adapted for the measurement of inclusive, rather than exclusive, jet cross sections. We first define the algorithm. Then we comment on some of its features. Finally, we provide some evidence that this definition may have advantages compared to the cone definition that is currently the standard for hadron collisions.

II. THE ALGORITHM

We consider hadron collisions in the hadron-hadron c.m. frame with the z-axis taken in the beam direction. We represent the final state of the collision as consisting of a starting set of "protojets" i with momenta p_i^μ. The starting p_i^μ may be the momenta of individual particles, or each p_i^μ may be the total momentum of the particles whose paths are contained in a small cell of solid angle about the interaction point, as recorded in individual towers of a hadron calorimeter. In either case, we have in mind that the masses $[p_i^\mu p_{i\mu}]^{1/2}$ are small compared to the transverse momenta $p_{i,T}$, so that the p_i^μ are essentially lightlike. Each protojet is characterized by its azimuthal angle ϕ_i, its pseudorapidity $\eta_i = -\ln(\tan(\theta_i/2))$,

and its transverse energy $E_{T,i} = |\vec{p}_{T,i}|$.

Starting with the initial list of protojets, the jet algorithm recursively groups pairs of protojets together to form new protojets. The idea is that protojets with nearly parallel momenta should be joined, so that they will eventually form part of the same jet. The algorithm also determines when, for a particular protojet, joining should cease. This protojet is then labeled a completed "jet" and is not manipulated further.

The algorithm depends on a parameter R, which should be chosen to be of order 1. This parameter is analogous to the cone size parameter in the cone algorithm.

The algorithm begins with a list of protojets as described above and an empty list of completed jets. It then proceeds recursively as follows:

1. For each protojet, define

$$d_i = E_{T,i}^2 \tag{1}$$

   and for each pair of protojets define

$$d_{ij} = \min(E_{T,i}^2, E_{T,j}^2) \, [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2. \tag{2}$$

2. Find the smallest of all the d_i and d_{ij} and label it $d_{\min}$.

3. If $d_{\min}$ is a d_{ij}, merge protojets i and j into a new protojet k with:

$$E_{T,k} = E_{T,i} + E_{T,j} \tag{3}$$

   and

$$\eta_k = [E_{T,i}\eta_i + E_{T,j}\eta_j]/E_{T,k}, \tag{4a}$$

$$\phi_k = [E_{T,i}\phi_i + E_{T,j}\phi_j]/E_{T,k}. \tag{4b}$$

4. If $d_{\min}$ is a d_i, the corresponding protojet i is "not mergable." Remove it from the list of protojets and add it to the list of jets.

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5. Go to step 1.

The procedure continues until there are no more protojets. As it proceeds, it produces a list of jets with successively larger values of $d_i = E_{T,i}^2$.

III. COMMENTS

The algorithm above produces a list containing many jets for each event. However, only the jets with large values of E_T (which are the last to be added to the jet list) are of much physical interest. The jets with smaller E_T are "minijets" or just random debris from the beam jets. This situation is fine for the construction of an inclusive jet cross section. Consider, for instance, the one-jet inclusive cross section $d\sigma/dE_T$ for, say, $E_T = 100$ GeV at $\sqrt{s} = 1800$ GeV. We first note that the high value of E_T tells us that this jet is a signal of a short distance process. Second, we recall that in hadron collisions the probability for a parton collision decreases very quickly as the value of the parton-parton c.m. energy $\sqrt{\hat{s}}$ increases. Thus it is very unlikely that an event with a 100 GeV jet contains other high E_T jets beyond a second jet with $E_T \approx 100$ GeV that is needed to balance the transverse momentum. That is to say, the one jet inclusive cross section is primarily sensitive to the highest E_T jets in hard scattering events, even though the jet list for each event contains many jets covering a wide range of transverse energies.

It is crucial that a jet cross section be "infrared safe." That is, when the cross section is calculated in QCD at the parton level, the cross section must be finite, despite the infrared divergences present in the Feynman diagrams. At the level of the physical hadrons, this means that the cross section is not sensitive to long distance effects. The infrared divergences in Feynman diagrams come from configurations in which a parton emits a soft gluon, with $q^\mu \to 0$, or in which an outgoing parton divides into two collinear partons, or in which an incoming parton emits another parton that carries away a fraction of its longitudinal momentum but no transverse momentum. The probability for one of these configurations to occur is infrared sensitive, and infinite in fixed order perturbation theory. However, unitarity

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dictates that the sum of the probabilities for one of these configurations to happen or not to happen is 1. For this reason, infrared safety is achieved if the measured jet variables do not change when an $E_T \to 0$ parton is emitted or when a parton divides into collinear partons. We note that $d\sigma/dE_T$ and similar jet cross sections produced using the algorithm described in the previous section will have this property. If a parton divides into two partons with collinear momenta, then the algorithm immediately recombines them, producing the same result as if the parton had not divided. Similarly, an $E_T \to 0$ parton may wind up in one of the high E_T jets or it may be left by itself, but in the limit that its E_T tends to zero it does not affect the transverse energy or angle of the high E_T jet.

We have discussed the one jet inclusive cross section. For the two jet inclusive cross section, it would be sensible to pick the two jets in each event that have the highest transverse energies, just as has been done for jets defined with the cone algorithm. By analogy, in defining the one jet inclusive cross section one might be tempted to pick only the most energetic jet in each event. However, at the Born level there are two jets with exactly equal transverse energies. Which one winds up with the most transverse energy is affected by a long-distance process, the emission of very low E_T gluons. Thus the resulting cross section would not be infrared safe.

The algorithm defined above is very similar to the simplest version of the various options discussed by Catani et al. [4], which are generalizations of the "Durham" algorithm [5] for e^+e^- annihilation. The differences arise primarily from questions of emphasis. Catani et al. take the approach that the analysis should be kept as similar as possible to earlier e^+e^- work. Thus they fix the parameter R at the value 1 and focus on *exclusive* jet cross sections. They also introduce two further parameters. The first additional parameter is d_{cut}. When the smallest d_i or d_{ij} is larger than d_{cut}, their recursion halts. The jets that have been generated thus far, all of which have $E_{T,i}^2 < d_{\text{cut}}$, are regarded as part of the beam jet. The remaining protojets, all of which have $E_{T,i}^2 > d_{\text{cut}}$, are treated as resulting from the hard scattering process. These protojets are then resolved into the final "jets" in direct analogy to the e^+e^- case using a resolution parameter y_{cut}. This is a sensible way to define an *exclusive*

jet cross section in analogy to the e^+e^- case but now including the beam jets. For instance, one might measure in this way a cross section $\sigma_3(d_{\text{cut}}, y_{\text{cut}})$ to produce exactly three high E_T jets plus the beam jets. However, we prefer to maintain a similarity with the previous cone algorithm work in hadron-hadron collisions. The jet definition in the preceding section is intended to define *inclusive* jet cross sections in terms of the single angular resolution parameter R (which plays a role similar to y_{cut}). For example, the two jet inclusive cross section $d\sigma/dM_{JJ}$ defined in this way is a function of only the jet-jet invariant mass M_{JJ} and R. It would be an additional complication if it also depended on the parameters d_{cut} and y_{cut}.

We note, however, that an exclusive n-jet cross section can be defined using the algorithm in Section 2. In this case, one needs a jet hardness parameter to play the role of d_{cut}. A convenient choice is to count only jets with transverse energies above a cutoff $E_{T,\text{cut}}$. Except for the issue of variable R, this is essentially the $y_{\text{cut}} = 1$ scenario of Catani et al.

The function d_{ij} given in eq. (2) measures how "nearby" the pair of protojets (i, j) is. As in other algorithms of the successive combination type, the idea is to combine first the protojets that are "nearest", and thus have the smallest d_{ij}. There are, of course, other possibilities for the function d_{ij}, the measure of "nearness". For instance, one might use the invariant pair mass [for massless protojets i and j],

$$d_{i,j}^M = M_{ij}^2 = 2E_{T,i}E_{T,i}[\cosh(\eta_i - \eta_j) - \cos(\phi_i - \phi_j)]. \tag{5}$$

For small $|\eta_i - \eta_j|$ and $|\phi_i - \phi_j|$ this is

$$d_{i,j}^M \approx E_{T,i}E_{T,j}[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]. \tag{6}$$

Such a choice for $d_{i,j}$ yields an algorithm analogous to the "Jade" version of the successive combination algorithm used in e^+e^- annihilation. The corresponding algorithm with the factor $E_{T,i}E_{T,j}$ replaced by $\min(E_{T,i}^2, E_{T,j}^2)$, as in eq. (2), is analogous to the "Durham" algorithm [5] in e^+e^- annihilation as noted earlier. A discussion of the relative merits of these choices in the context of e^+e^- annihilation may be found in [6]. The Durham algorithm has been also discussed in the context of lepton-hadron collisions in [7].

IV. COMPARISON WITH CONE ALGORITHM

Now consider the algorithm advocated here. At any stage in the operation of the algorithm, the two protojets i and j with the smallest value of d_{ij} are merged if d_{ij} of eq. (2) is less than the smaller of $E_{T,i}^2$ and $E_{T,j}^2$. That is, they are merged if

$$\sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} < R. \tag{7}$$

Thus the issue of which protojets to merge first depends on transverse energies and angles, but the issue of whether to merge two protojets or to declare that they cannot be merged is solely a question of the angle between them.

The merging condition in eq. (7) makes it clear that the present algorithm is in fact not so different from a cone algorithm. The latter is typically defined [1] in terms of the particles n whose momenta $\vec{p_n}$ lie within a cone centered on the jet axis (η_J, ϕ_J) in pseudorapidity η and azimuthal angle ϕ,

$$\sqrt{(\eta_n - \eta_J)^2 + (\phi_n - \phi_J)^2} < R. \tag{8}$$

The jet angles (η_J, ϕ_J) are the averages of the particles' angles,

$$\eta_J = \sum_{n \in \text{cone}} p_{T,n} \eta_n / E_{T,J} , \tag{9a}$$

$$\phi_J = \sum_{n \in \text{cone}} p_{T,n} \phi_n / E_{T,J} , \tag{9b}$$

with

$$E_{T,J} = \sum_{n \in \text{cone}} p_{T,n} , \tag{10}$$

analogous to eqs. (4) and (3), respectively. This process is iterated until the cone center matches the jet center (η_J, ϕ_J) computed in eq. (9).

The definition of the jet axis given in eqs. (9) and (10) is chosen to be simple when expressed in the natural variables (E_T, η, ϕ). Of course, other choices could also yield infrared safe jet definitions. Thus this definition should be regarded as a convenient convention. This

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convention has been continued in the merging conditions, eqs. (3) and (4), of the successive combination algorithm.

While the successive combination algorithm never assigns a particle to more than one jet, this is not the case for the cone algorithm as defined so far. It is possible for jet cones to overlap, so that one particle is contained in more than one jet. This issue was discussed in [8] in the context of the order α_s^3 perturbative calculation. At this order it is possible for two one-parton jets to lie within the cone of a two parton jet. In the calculation [9], such jets are "merged." That is, the two parton jet is kept and the one parton subjets are not considered as being legitimate jets on their own. In a physical event, with many more particles, the merging question is more serious, and a criterion for merging must be part of the experimental algorithm [10].

The difference between cone and successive combination algorithm jets is apparent even in the simplest example of merging two partons (or hadrons) to make a jet. In the cone algorithm [1], two partons i and j are merged if each falls within an angular distance R of the jet axis defined by eq. (9b). The parton with the smaller E_T, call it i, is farther from the jet axis. In the cone algorithm, the limit on angular separation between this parton and the jet J (including the partons i and j) has the simple form

$$\sqrt{(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2} < R \,. \tag{11}$$

The corresponding relation in terms of the angular separation of the two partons is

$$\sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} < \frac{E_{T,J}}{E_{T,J} - E_{T,i}} R \,. \tag{12}$$

The limit on the right hand side lies in the range $R < \{E_{T,J}/[E_{T,J} - E_{T,i}]\}R \leq 2R$ since $0 < E_{T,i} \leq E_{T,J}/2$. Thus configurations are possible with two equally energetic partons located near opposite edges of the cone, and nothing in the center of the cone. These are precisely the configurations where the merging issue arises in order α_s^3 perturbation theory.

In the successive combination algorithm, it is the separation between the two partons (or protojets) that has the simple limit of eq. (7). It is this angle between i and j, not

that between i and the final jet direction, that controls the question of merging. The corresponding relation for the angular separation between parton i (the lower E_T parton), and the jet J in order for merging to occur is

$$\sqrt{(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2} < \frac{E_{T,J} - E_{T,i}}{E_{T,J}} R \,, \tag{13}$$

where the right hand side is in the range $R/2 \le [E_{T,J} - E_{T,i}]/E_{T,J} < R$. Thus the lower-$E_T$ parton can be far from the jet axis, up to a maximum separation R, while the higher E_T parton must be closer to the jet axis.

Because of the difference between eq. (11) and eq. (13), the average distribution of transverse energy within jets depends on which algorithm one uses. With the successive combination algorithm, there is less transverse energy near the edge of the allowed angular region than there is with the cone algorithm. This is illustrated in Fig. 1. The quantity plotted is the order α_s^3 perturbative result for the average transverse energy fraction as a function of distance from the center of the jet, for jets with $E_T = 100$ GeV at $\sqrt{s} = 1800$ GeV for $R = 1$. The histogram represents the E_T fraction in angular annuli, r to $r + 0.1$, where r is the distance from the jet center, $r = \sqrt{(\eta - \eta_J)^2 + (\phi - \phi_J)^2}$. Thus the sum over all bins, $r < R$ including $0 < r < 0.1$, should equal unity. Note also that the calculation yields energy outside of the cone, $r > R$. This arises in the perturbative calculation from configurations with only one parton in the cone (the jet) but with another parton nearby.

The cone algorithm, in principle, requires one to find all possible solutions of the cone matching conditions, eqs. (8), (9) and (10), before beginning the merging algorithm. In experimental applications [10], however, one may begin the search for valid jet cones using "initiator" calorimeter cells, i.e., cells with E_T above some threshold value. In this case, possible jets that consist of two widely separated subjets may not be recognized because there is no initiator cell between the subjets. Thus, the two widely separated jets may not be merged. The theoretical study [8] suggests that the jet finding algorithm used by the CDF group is likely not to merge two subjets when their separation is greater than a value R_{sep}, which appears to be about $1.3R$ in practice as is consistent with CDF studies [10].

In this language, the successive combination algorithm, when applied to two parton jets, corresponds to the limiting case $R_{sep} = R$.

At higher orders in perturbation theory or for more realistic jets containing many particles, it is eq. (11) that survives in the cone algorithm case and eq. (7) for the successive combination algorithm. Thus cone jets always have well defined, smooth boundaries, although the amount of E_T near the edge will depend in practice on how the issue of the merging of overlapping jets is handled. For the successive combination algorithm there is no "merging question." It is automatically dealt with by the algorithm as all particles are assigned to a unique jet. Furthermore, only small E_T particles can be as far as R from the jet axis. However, there is a price for this simplicity. Particles with very small E_T can be very far from the jet axis ($> R$). Thus jet boundaries can be complicated. Here, the issue is intertwined jets rather than overlapping jets.

Does it follow that the successive combination algorithm is better in some sense than the cone algorithm, or is it just different? The criteria that we will examine are 1) estimated size of higher order perturbative corrections to a jet cross section calculated using fixed order perturbation theory, 2) estimated size of corrections to the calculated cross section arising from "power suppressed" or "hadronization" effects, and 3) simplicity and definiteness of the algorithm.

We turn first to the estimated size of perturbative corrections. We consider as an example the one jet inclusive cross section $d\sigma/dE_T d\eta$ at $\sqrt{s} = 1800$ GeV, averaged over the rapidity range $0.1 < |\eta| < 0.7$ used by the CDF Collaboration where the comparison of theory with data has been very successful [9,12]. Again we calculate this cross section at order α_s^3 according to both jet definitions, using the methods described in [8] and [9]. The question is then, how big are the order α_s^4 corrections? Of course, we can only make estimates. One way to do this is to examine the dependence of the calculated cross section on the renormalization and factorization scale μ. If we could calculate to order α_s^4, then the renormalization group guarantees that some of the α_s^4 terms would cancel most of the μ dependence that occurs in the α_s^3 cross section (so that the derivative of the cross section with respect to μ would

then be of order α_s^5). Thus the α_s^4 contributions to the cross section are likely to be at least as large as the difference between the cross section calculated with a "best guess" for μ, which we take to be $\mu = E_T/2$, and the cross section calculated with the alternative choices $\mu = E_T/4$ or $\mu = E_T$. In Fig. 2a, we show the cross section at $E_T = 100$ GeV as a function of the clustering parameter R calculated according to both algorithms for these three values of μ.

This rather busy graph becomes remarkably simple, as in Fig. 2b, if we rescale the parameter R in the two cases. Let the clustering parameter in the successive combination algorithm, eq. (2), be called R_{comb} for the present purposes and plot the cross section versus

$$R' = R_{\mathrm{comb}}. \tag{14}$$

For comparison, we label the clustering parameter in the case of the cone algorithm, eq. (8), as R_{cone} and in Fig. 2b we display the cone results as a function of a scaled parameter

$$R' = 1.35 \times R_{\mathrm{cone}}. \tag{15}$$

We see from the graph that, for each value of μ, the calculated cross sections are almost identical as long as we identify $R_{\mathrm{comb}} \approx 1.35 \times R_{\mathrm{cone}}$. Thus, for instance, a jet cross section calculated or measured with the cone algorithm using the standard value $R_{\mathrm{cone}} = 0.7$ should be compared to a jet cross section with the successive combination algorithm using $R_{\mathrm{comb}} = 0.945 \approx 1.0$.

In Fig. 3 we plot the order α_s^3 inclusive jet cross section as defined by the successive combination algorithm with $R_{\mathrm{comb}} = 1.0$ versus the jet E_T at $\sqrt{s} = 1800$ GeV taking $\mu = E_T/2$. The corresponding cross section calculated using the cone algorithm with $R_{\mathrm{cone}} = 0.7$ is nearly identical (see, for example, Fig. 1 of [12]), and is not shown as it would not be distinguishable. In the range $E_T/4 < \mu < E_T$, 10 GeV $< E_T < 500$ GeV, we find that the two algorithms give order α_s^3 theoretical cross sections that agree to within 10%.

We conclude from this comparison that the successive combination algorithm is neither better or worse than the cone algorithm, at least as judged according to this μ-dependence

standard. We also note that the argument [8] that $R_{\mathrm{cone}} \approx 0.7$ is a particularly stable and thus sensible regime for comparing fixed order perturbation theory with experiment is now translated into $R_{\mathrm{comb}} \approx 1.0$. This is just the regime studied by Catani et al. [4].

We now examine the question of higher order corrections from a point of view that relates directly to the reasoning behind the successive combination algorithms: the idea of putting parton showers back together to reconstruct the parent parton. We suggest that jet cross sections defined with the cone algorithm may have larger higher order perturbative corrections than do jet cross sections defined with the successive combination algorithm because of what may be called "edge of the cone" effects. Consider the application of the cone algorithm to two partons, 1 and 2, with roughly equal transverse energies and an angular separation of approximately $2R_{\mathrm{cone}}$, the troublesome configuration mentioned earlier. These two partons are near the edge of the region in which they can form an allowable cone jet. Suppose that parton 2 splits into two partons, 2a and 2b, that each have substantial transverse energy. If the angle separating partons 2a and 2b is infinitesimal, then they will both fit into the jet cone. The resulting jet (1,2a,2b) will have the same direction and transverse energy as the jet (1,2) that one obtains if the splitting did not occur. However, if the angle separating partons 2a and 2b is small but not infinitesimal, it can very well be that the three partons (1,2a,2b) cannot fit into a cone to form a single jet. Since the matrix element for such a parton splitting is large (although not infinite), one may worry that there will be corresponding large order α_s^4 corrections to jet cross sections calculated with the cone algorithm. What is the situation with the successive combination algorithm? Here we should consider partons 1 and 2 separated by an angle of approximately R_{comb}, near the edge of the region in which they can form an allowable jet. If parton 2 splits into partons 2a and 2b with a small angular separation, then the successive combination algorithm will combine them together into a protojet $2'$ that approximates parton 2. As long as the angle separating partons 2a and 2b is not too large, the protojet $2'$ will have jet parameters close enough to those of parton 2 that the algorithm will then combine $2'$ with parton 1. On the basis of this argument, we expect that order α_s^4 corrections to jet cross

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sections calculated with the successive combination algorithm may be smaller than with the cone algorithm. For similar reasons, we expect also that jet cross sections calculated with the successive combination algorithm will also exhibit smaller corrections attributable to the final combination of partons into hadrons. Unfortunately, from the order α_s^3 perturbative calculation we cannot determine the magnitude of these edge of the cone effects. [1]

Finally, we comment on the relative merits of the two algorithms from the point of view of simplicity and definiteness. Here the cone algorithm appears at first to have the advantage. With the cone algorithm, a jet consists simply of all the particles whose momentum vectors fit into an apparently well defined and regular cone centered on the jet axis. This is a simple and appealing idea. The successive combination algorithm, in contrast, takes more effort to define and does not yield regular jet shapes in the $\eta - \phi$ plane. However, in the application of the cone algorithm, one quickly discovers that ambiguous cases with overlapping jet cones arise. What do you do when two or more jets contain particles in common? The algorithm must be expanded to cover these cases. Unfortunately, there are many ways to proceed, and none of them is particularly simple. Thus the cone algorithm actually consists of a simple part that works beautifully for joining two partons into a jet and a complicated part that one is forced to use in order to deal with real-world multi-hadron events. With the successive combination algorithm, the recursive steps require some thought to define, but once they are set, the whole algorithm is complete.

V. SUMMARY

We have discussed a jet definition for inclusive jet measurements in hadron collisions that makes use of ideas previously applied to jet definitions for e^+e^- collisions. The algorithm

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[1]This is clearly a subject for Monte Carlo study as in [4]. Unfortunately the cone algorithm used in that analysis is quite different from that described here. In particular, the cone algorithm in [4] is not infrared safe so that the results presented are difficult to interpret.

identifies a jet by successively combining "nearby" pairs of particles or protojets. The concept of "nearby" is measured in (E_T, η, ϕ) space and involves a limit R_{comb} on angular separations that is very similar to the usual cone algorithm parameter R_{cone}. We find that, calculated perturbatively, the inclusive jet cross section that results from the new algorithm with parameter R_{comb} is essentially identical to the cone algorithm result with $R_{\mathrm{cone}} = R_{\mathrm{comb}}/1.35$. While the final geometry of a jet defined by the successive combination algorithm is likely to be more complex than that from the cone algorithm, the former definition has the advantage that there is no problem with overlapping jets as there is in the cone case. We have also presented a qualitative argument that, due to "edge of the cone" effects, cross sections calculated with the successive combination algorithm are likely to exhibit smaller higher order and hadronization corrections. Only further detailed experimental and theoretical studies can demonstrate whether the successive combination type algorithm has quantitative advantages over other algorithms.

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FIGURES

FIG. 1.   Fraction of jet E_T in angular annuli r to $r+0.1$ comparing the cone algorithm with the the successive combination case. In both cases the jet has $R = 1.0$, $E_T = 100$ GeV, $\sqrt{s} = 1800$ GeV, $0.1 < |\eta_J| < 0.7$ with renormalization/factorization scale $\mu = E_T/2$ and the structure functions of HMRS(B) [11].

FIG. 2.   Order α_s^3 inclusive jet cross section for $E_T = 100$ GeV, $\sqrt{s} = 1800$ GeV, averaged over η_J in the range $0.1 < |\eta_J| < 0.7$ with the structure functions of HMRS(B) [11] for the two algorithms specified in the text. The curves for the cone algorithm are: $\mu = E_T$ (solid), $\mu = E_T/2$ (dot-dash), $\mu = E_T/4$ (dot-dot-dot-dash); for the successive combination algorithm: $\mu = E_T$ (long dash), $\mu = E_T/2$ (medium dash), $\mu = E_T/4$ (short dash). a) Standard case plotted versus $R = R_{\text{cone}} = R_{\text{comb}}$. b) Same as a) except that the cone algorithm is plotted versus $R' = 1.35 R_{\text{cone}}$ while the successive combination case has $R' = R_{\text{comb}}$.

FIG. 3.   Order α_s^3 inclusive jet cross section as defined by the successive combination algorithm with $R_{\text{comb}} = 1.0$ versus the jet E_T for $\sqrt{s} = 1800$ GeV, $\mu = E_T/2$, averaged over η_J in the range $0.1 < |\eta_J| < 0.7$ with the structure functions of HMRS(B) [11].

a)