$ABCD-A_1B_1C_1D_1$

$A(1,2,3),B(4,5,6),C(5,6,4)$ D_1 _____

> $(2+\frac{3\sqrt{6}}{2},3-\frac{3\sqrt{6}}{2},1)$ $(2-\frac{3\sqrt{6}}{2},3+\frac{3\sqrt{6}}{2},1)$

A,B,C

$\overrightarrow{AD}\parallel\overrightarrow{BC}$ D $(2,3,1)$

$A_1B_1C_1D_1$ $ABCD$ D_1

D_1 $D_1(a,b,c)$ $DD_1\perp$ $ABCD$

$DD_1\perp AB$, $DD_1\perp BC$ 0

$$\begin{cases}3(a-2)+3(b-3)+3(c-1)=0\\ a-2+b-3-2(c-1)=0\end{cases}$$

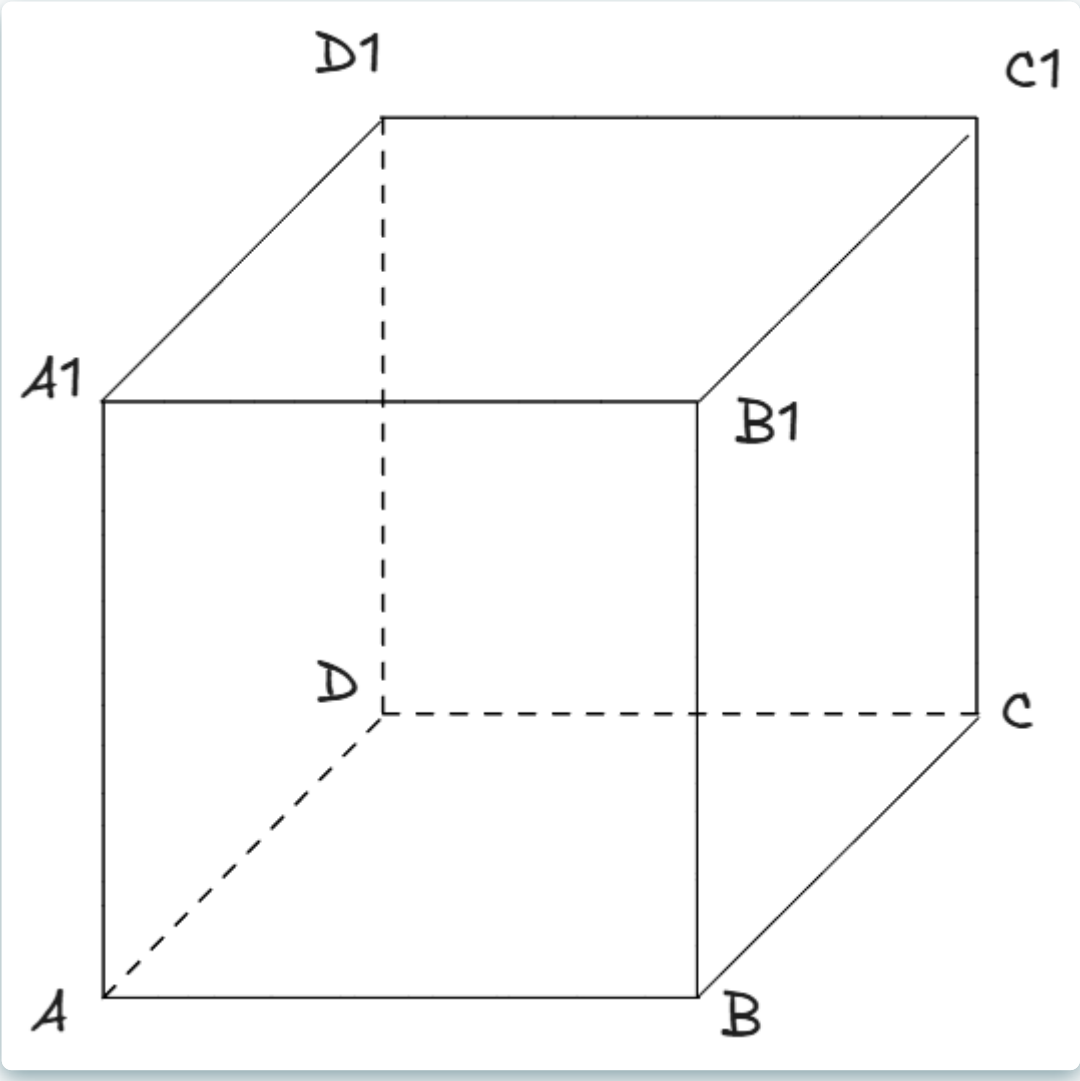
DD_1 $ABCD$ $|DD_1|=|AB|=3\sqrt{3}$

$$\sqrt{(a-2)^2+(b-3)^2+(c-1)^2}=3\sqrt{3}$$

a,b,c

$ABCD-A_1B_1C_1D_1$ 1 P Q A_1D

BD_1 $|PQ|$ _____



$\frac{\sqrt{6}}{6}$

P, Q $|PQ|$ P, Q

$Ax + By + C = 0$

$Q(x, y, z)$ Q BD_1 $\overrightarrow{BQ} \parallel \overrightarrow{BD_1}$

$$(x-1, y-1, z) \parallel (-1-1, 1)$$

$$\frac{x-1}{-1}=\frac{y-1}{-1}=\frac{z}{1}$$

BD_1

$=k$

$$\frac{x-1}{-1}=\frac{y-1}{-1}=\frac{z}{1}=k$$

$x=1-k,y=1-k,z=k$ Q $(1-k,1-k,k)$
x,y,z k k

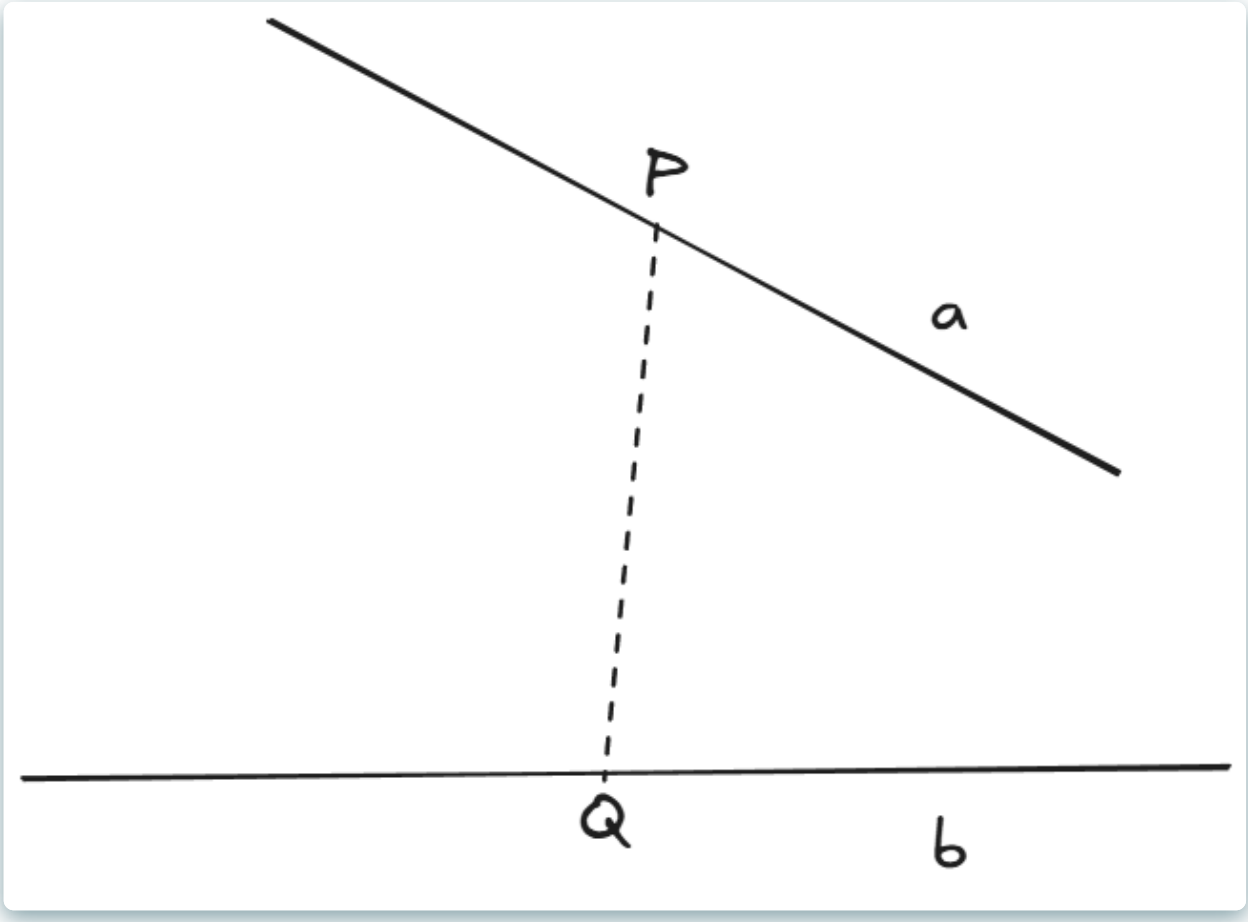
P P
$(t,0,t)$ t A_1D A_1D

$|PQ|$

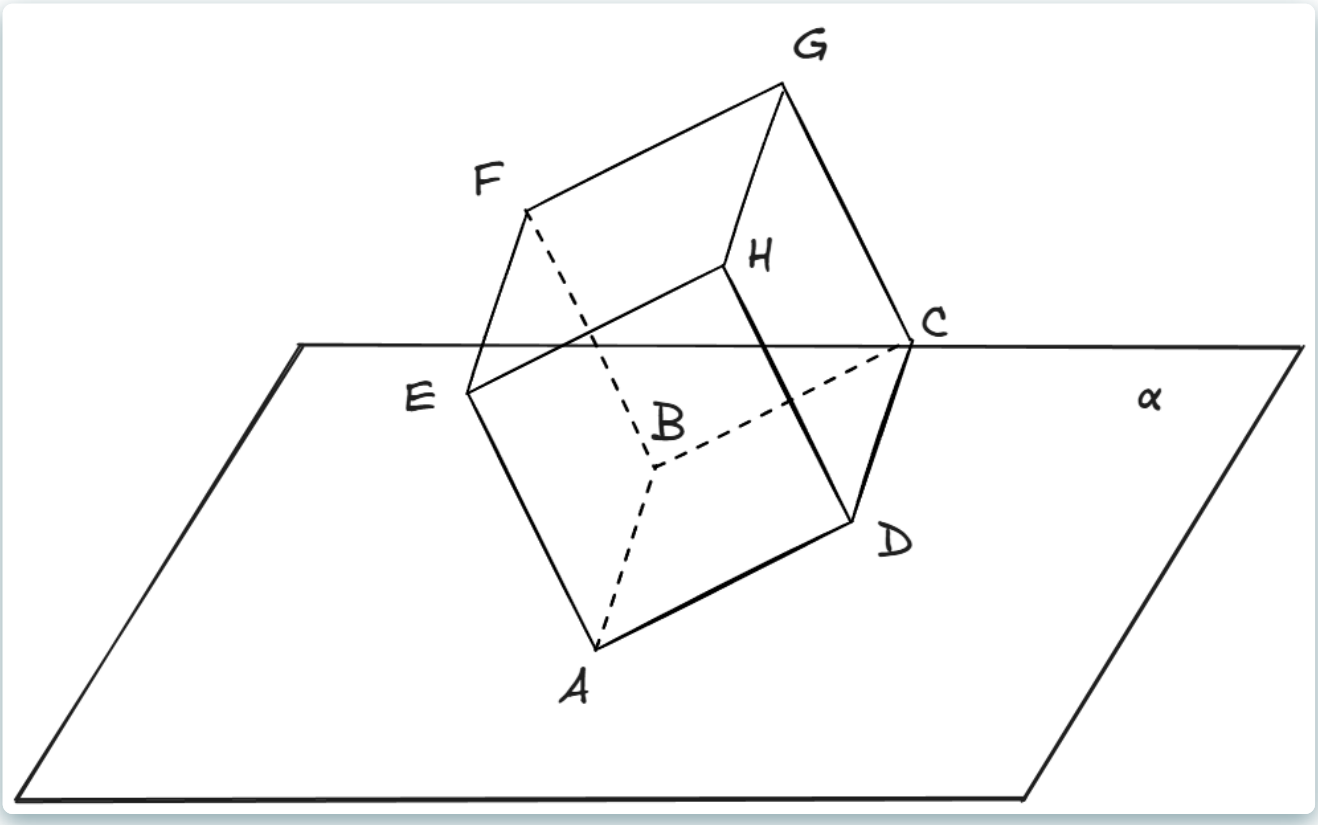
$$\begin{aligned}|PQ|&=\sqrt{(1-k-t)^2+(1-k)^2+(k-t)^2}\\&=\sqrt{3k^2+2t^2-4k-2t+2}\\&=\sqrt{3(k-\frac{2}{3})^2+2(t-\frac{1}{2})^2+\frac{1}{6}}\\&\geq\sqrt{\frac{1}{6}}\\&=\frac{\sqrt{6}}{6}\end{aligned}$$

a, b a, b PQ a, b
PQ a, b

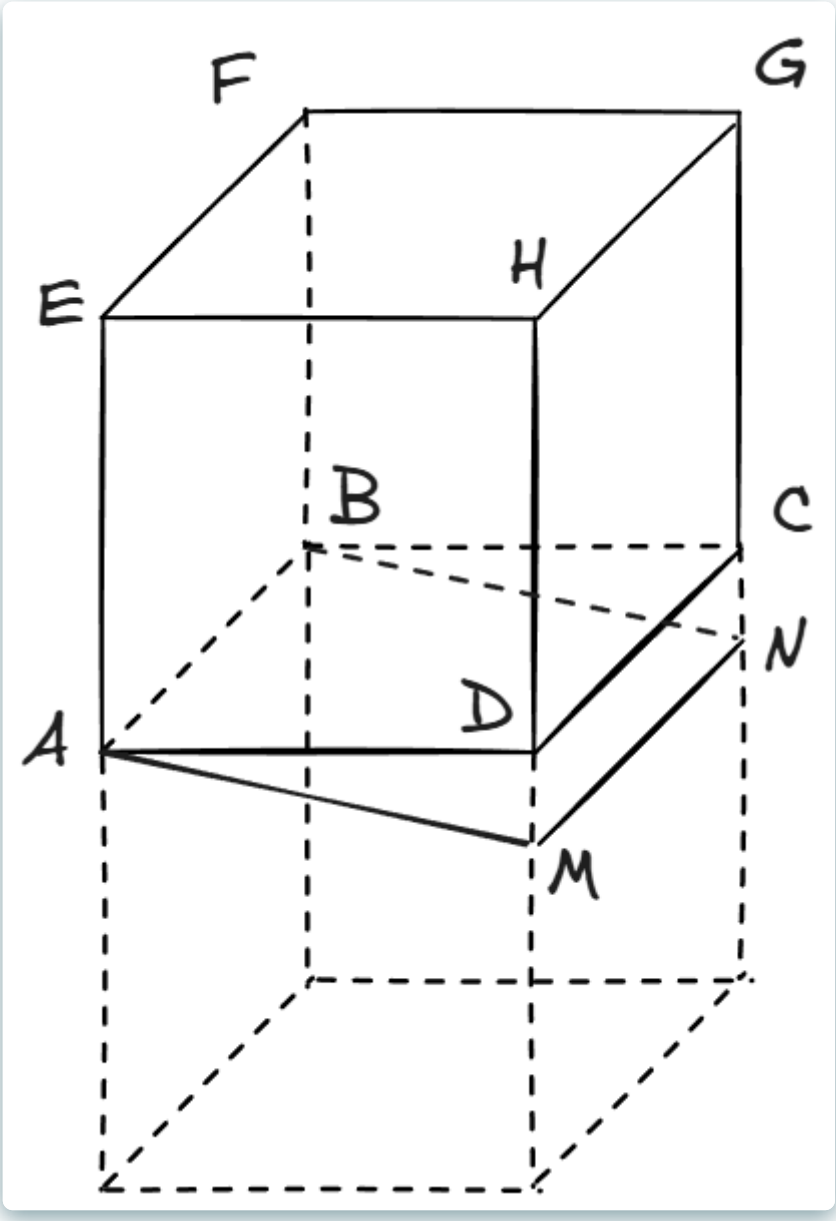
A_1D BD_1



$ABCD - EFGH$ AB α $ABCD$ α
30° $|AB| = 1$ G α



$\frac{\sqrt{3}+1}{2}$



$ABNM$ α

$\angle MAD = \angle NBC = 30^\circ$

$$Ax + By + Cz + D = 0$$

$$Ax + By + C = 0$$

z

(x_0, y_0, z_0) $Ax + By + Cz + D = 0$

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

$ABNM$

B $ABNM$

$Ax + By + Cz + D = 0$ $B(0,0,0), A(1,0,0), N(0,1,-\frac{\sqrt{3}}{3})$

$$\begin{cases} D = 0 \\ A + D = 0 \\ B - \dfrac{\sqrt{3}}{3}C + D = 0 \end{cases}$$

$A = 0, B = \frac{\sqrt{3}}{3}C, D = 0$ $\frac{\sqrt{3}}{3}Cy + Cz = 0$
$y + \sqrt{3}z = 0$

$G(0,1,1)$ $ABNM$

$$d = \frac{|1 + \sqrt{3}|}{\sqrt{0^2 + 1^2 + \sqrt{3}^2}} = \frac{1 + \sqrt{3}}{2}$$

- $Ax + By + Cz + D = 0$ $\overrightarrow{n} = (A, B, C)$
- $\overrightarrow{n} = (A, B, C)$ $Ax + By + Cz + D = 0$
  D
- $Ax + By + Cz + D_1 = 0$ $Ax + By + Cz + D_2 = 0$

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2 + C^2}}$$