

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a, b > 0) \qquad (a, 0) \qquad 3c \qquad c$$

H2

$$\frac{1}{4} \qquad \frac{1}{3} \qquad \frac{1}{2}$$

$$a, b$$

$$\blacktriangleright \qquad a > b \qquad (-c, 0), (c, 0) \qquad (a, 0)$$

$$\blacktriangleright \qquad a < b \qquad (0, c), (0, -c) \qquad (a, 0)$$

$$\begin{array}{ccccc} M(1, 1) & & -\frac{1}{2} & & C : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0) & & A, B \\ M & & AB & & C & & \end{array}$$

H2

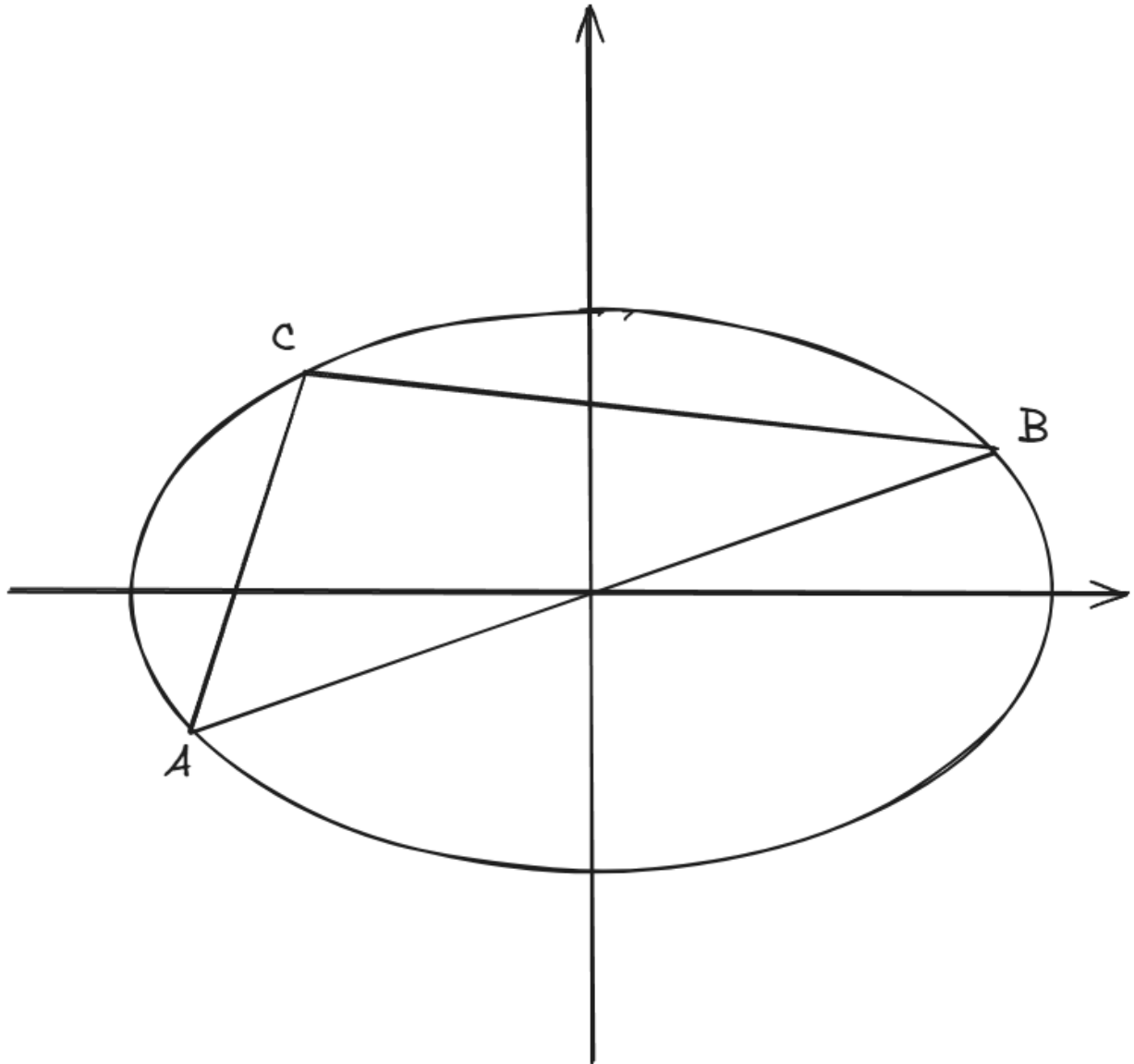
$$\frac{\sqrt{2}}{2}$$

$$C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > 0, b > 0)$$

	AB	AB
C	AC	BC
	k_1	k_2

$$k_1 k_2 = e^2 - 1$$

e



$$k_1 k_2 = -1$$

$$e = 0$$

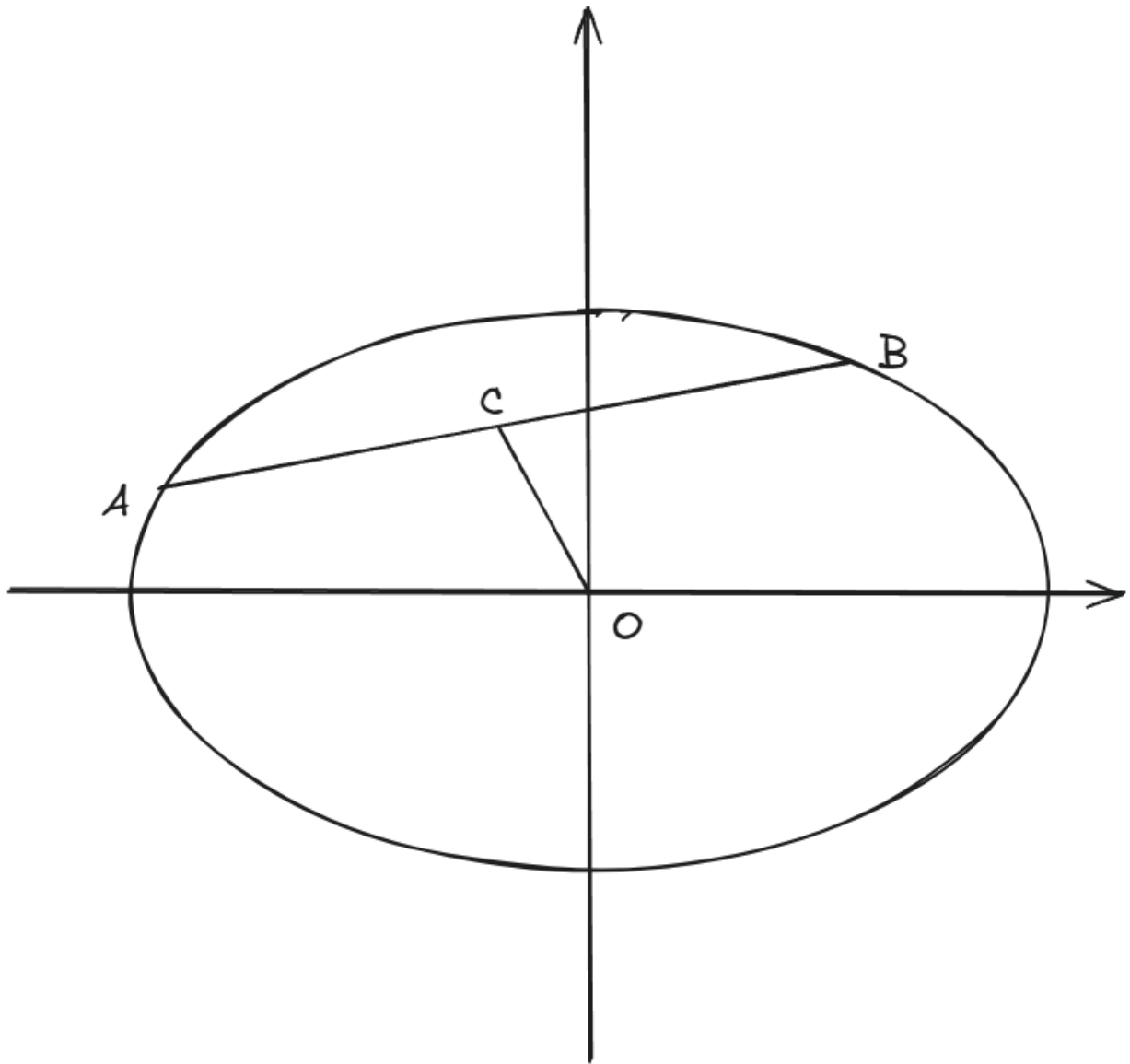
$$k_1 k_2 = 0^2 - 1 = -1$$

$$k_1 k_2 = e^2 - 1$$

$$C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > 0, b > 0) \quad O \quad AB \quad C$$

$$AB \quad AB \quad k_1 \quad OC \quad k_2$$

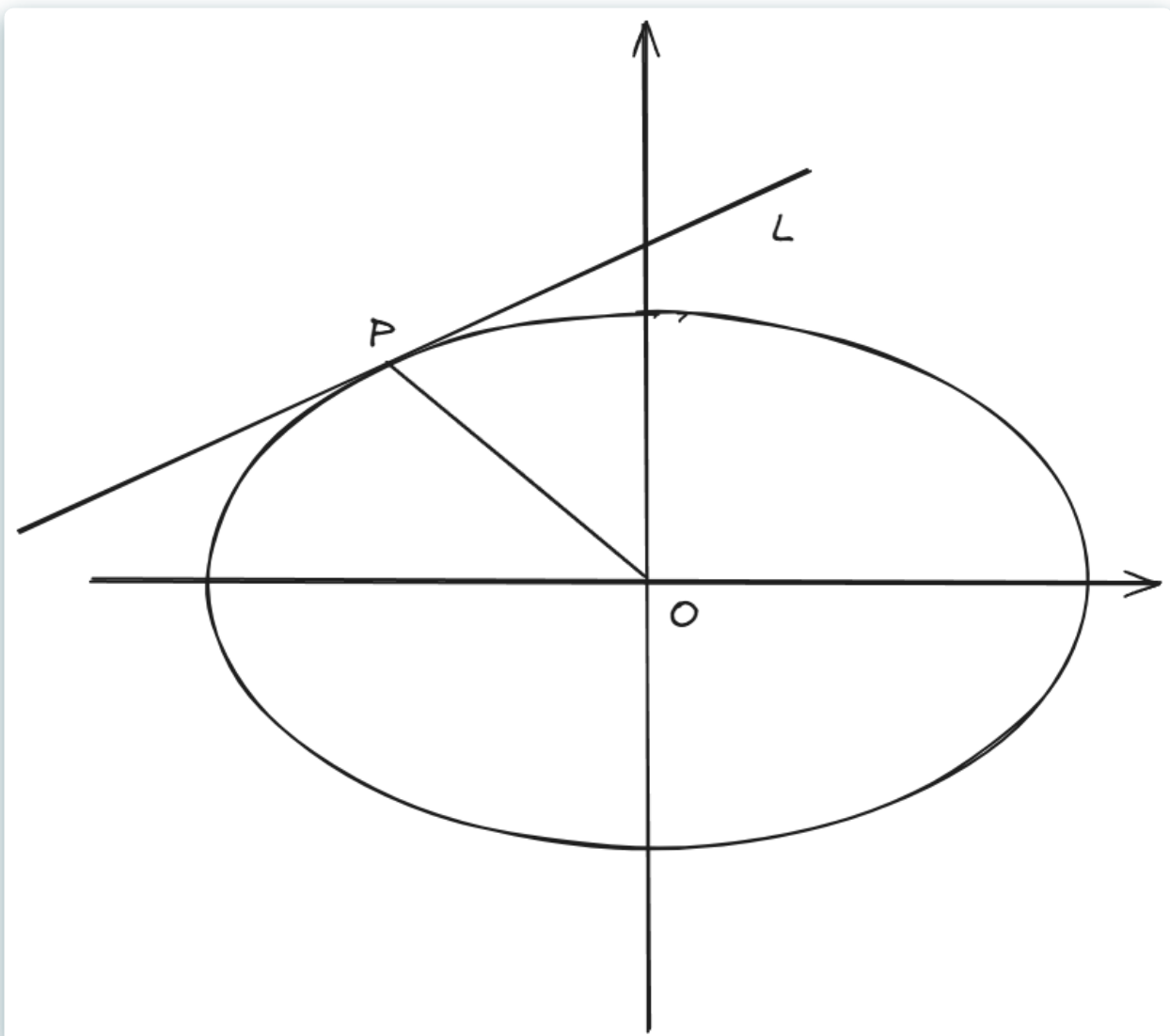
$$k_1 k_2 = e^2 - 1$$



$$C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > 0, b > 0) \quad O \quad l$$

$$P \quad OP \quad k_1 \quad l \quad k_2$$

$$k_1 k_2 = e^2 - 1$$



$$e = \frac{\sqrt{2}}{2} \qquad -\frac{1}{2} \times 1 = e^2 - 1$$

$$C : \frac{x^2}{2} + y^2 = 1 \qquad l \qquad x \qquad y \qquad A, B \qquad O$$

OAB

H2

$$\sqrt{2}$$

$$l : \frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1 \qquad C : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a, b > 0) \qquad P(x_0, y_0)$$

$$P \qquad l \qquad C \qquad l \qquad P \qquad C$$

1.

2.

3.

$$\frac{x_0x}{2} + y_0y = 1 \qquad (x_0, y_0)$$

$$\qquad \qquad \qquad (\frac{2}{x_0}, 0), (0, \frac{1}{y_0})$$

$$S = \frac{1}{x_0y_0}$$

$$(x_0, y_0)$$

$$\frac{x_0^2}{2} + y_0^2 = 1 \geq \sqrt{2}x_0y_0$$

$$x_0y_0 \leq \frac{\sqrt{2}}{2} \qquad S \geq \sqrt{2}$$

$$\overrightarrow{F_1 A} = 5 \overrightarrow{F_2 B}$$

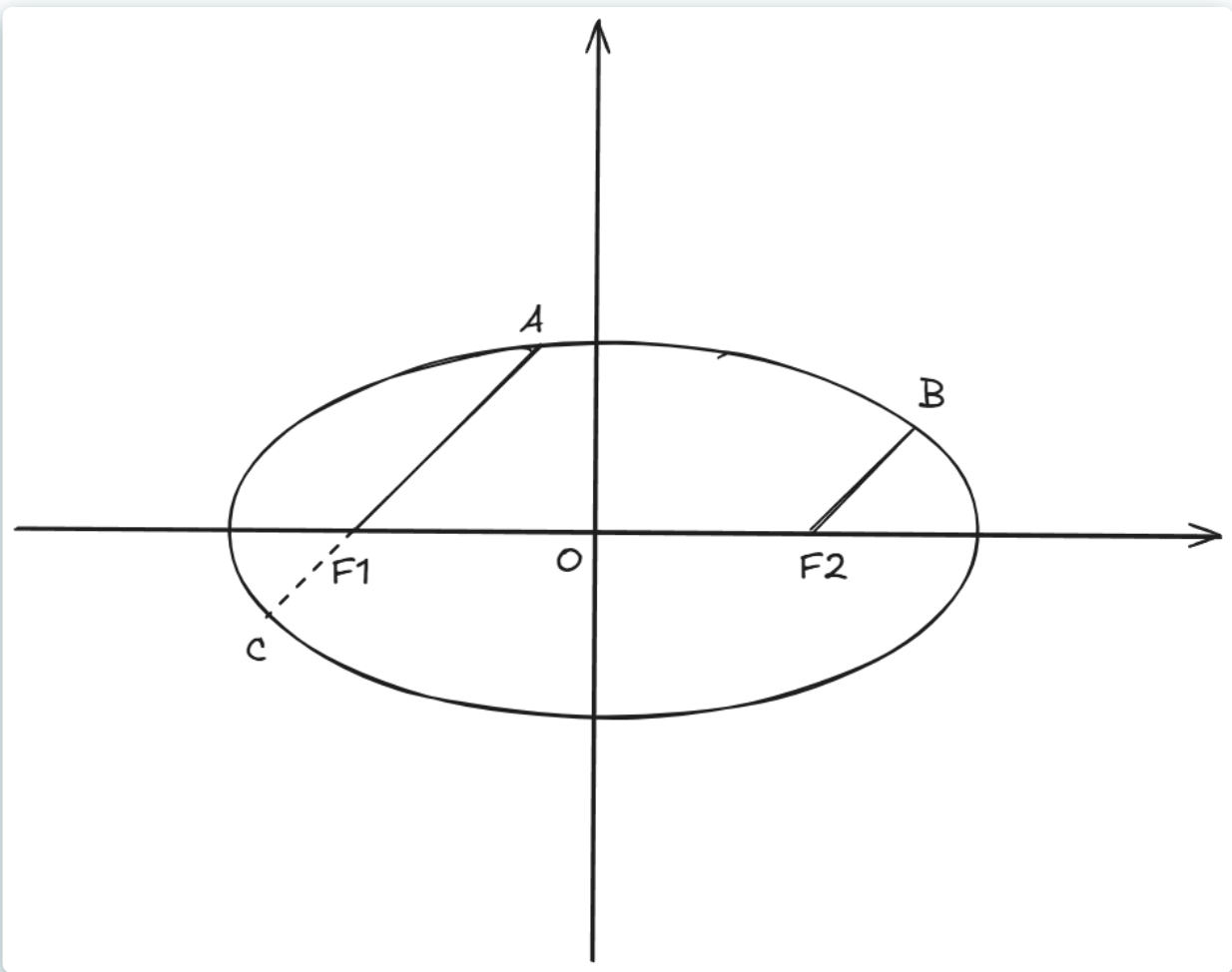
$$\frac{x^2}{3} + y^2 = 1$$

$$A, B$$

H2

A

$$(0, \pm 1)$$



$$A(x_1, y_1), C(x_2, y_2)$$

$$F_1$$

$$F_2 B$$

$$F_1 C$$

$$(-\sqrt{2}, 0)$$

$$AC$$

$$y = k(x + \sqrt{2})$$

$$\left\{\begin{array}{l}y=k(x+\sqrt{2})\\ \frac{x^2}{3}+y^2=1\end{array}\right.$$

$$(1+3k^2)x^2+6\sqrt{2}k^2x+6k^2-3=0$$

$$\begin{array}{lll}x_1+x_2=-\frac{6\sqrt{2}k^2}{1+3k^2}& x_1x_2=\frac{6k^2-3}{1+3k^2}& \overrightarrow{F_1A}=\overrightarrow{5F_2B}\\x_1+5x_2=-6\sqrt{2}& y_1+5y_2=0& x_1=0\\A& (0,\pm 1)& \end{array}$$



$$\left\{\begin{array}{l} \frac{x_1^2}{3}+y_1^2=1 \qquad (9) \\ \frac{x_2^2}{3}+y_2^2=1 \qquad (10) \end{array}\right.$$

$$\frac{\frac{y_1+y_2}{2}}{\frac{x_1+x_2}{2}}\cdot \frac{y_1-y_2}{x_1-x_2}=-\frac{1}{3}$$

$$\begin{array}{ccccc} & & AC & & \\ F_1 & & AC & & AC \\ & & (\frac{x_1+5x_2}{6},\frac{y_1+5y_2}{6}) & & \\ 5 & & 25 & & \end{array}$$

$$\left\{\begin{array}{l} \frac{x_1^2}{3}+y_1^2=1 \qquad (11) \\ \frac{(5x_2)^2}{3}+(5y_2)^2=25 \qquad (12) \end{array}\right.$$

$$\frac{(x_1-5x_2)(x_1+5x_2)}{3}+(y_1-5y_2)(y_1+5y_2)=-24$$

$$\begin{array}{lll} & F_1 & (\frac{x_1+5x_2}{6}, \frac{y_1+5y_2}{6}) \\ (-c, 0) & (-\sqrt{2}, 0) & x_1+5x_2=-6\sqrt{2}, y_1+5y_2=0 \end{array}$$

$$x_1-5x_2=6\sqrt{2}$$

$$\begin{array}{lll} x_1+5x_2=-6\sqrt{2} & x_1=0 & A \\ (0,\pm 1) & & \end{array}$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$F_1, F_2$$

$$A(x_0, y_0)$$

$$|AF_1|=a+ex_1$$

$$|AF_2|=a-ex_2$$

$$|AF_1|=\sqrt{3}+\frac{\sqrt{6}}{3}x_1$$

$$|CF_1|=\sqrt{3}+\frac{\sqrt{6}}{3}x_2$$

$$\begin{array}{lll} |AF_1|=5|CF_1| & \sqrt{3}+\frac{\sqrt{6}}{3}x_1=5\sqrt{3}+\frac{5\sqrt{6}}{3}x_2 & \\ x_1-5x_2=6\sqrt{2} & & \\ x_1+5x_2=-6\sqrt{2} & x_1=0 & \end{array}$$



$$x^2+y^2=r^2$$

$$\begin{cases} x=r\sin\theta \\ y=r\cos\theta \end{cases}$$

$$\begin{array}{ccccc} r & & \theta & & \\ & x & & r & \theta \\ & & x,y & & \end{array}$$

$$l:ax+by+c=0$$

$$\begin{cases} x=r\sin\theta+x_0 \\ y=r\cos\theta+y_0 \end{cases}$$

$$\begin{array}{ccccccc} & (x_0,y_0) & & O' & r & & P & O' \\ & & & |r| & & & \theta & \\ x & & & & & & & \end{array}$$

$$\begin{array}{ccc} & (x_0,y_0) & \\ F_1(-\sqrt{2},0) & AC & (x_0,y_0) \end{array}$$

$$\begin{cases} x=r\sin\theta-\sqrt{2} \\ y=r\cos\theta \end{cases}$$

$$\frac{(r\sin\theta-\sqrt{2})^2}{3}+(r\cos\theta)^2=1$$

$$(1+2\cos^2\theta)\cdot r^2-2\sqrt{2}\sin\theta\cdot r-1=0$$

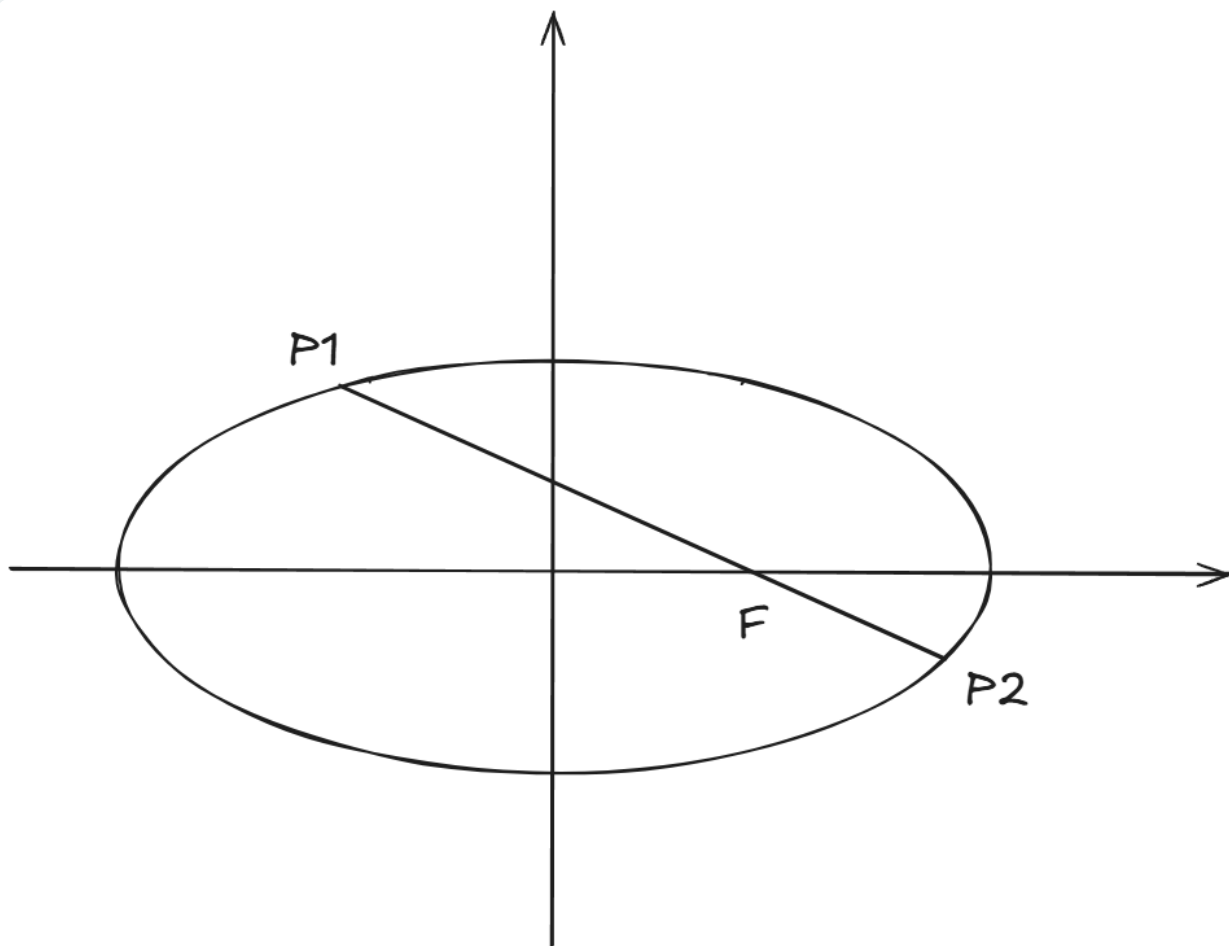
$$\begin{array}{ccccccc}
& & & & & & r \\
r_1,r_2 & & A & C & F_1 & & r_1,r_2 \\
& & AF_1,CF_1 & & & & |r_1|=5|r_2| \\
r_1+r_2=\frac{2\sqrt{2}\sin\theta}{1+2\cos^2\theta} & & & r_1r_2=-\frac{1}{1+2\cos^2\theta} & & & r_1,r_2,\theta
\end{array}$$

5

$$\begin{array}{ccccccc}
& & C:\frac{x^2}{4}+\frac{y^2}{3}=1 & & F & & l \\
& & y_1>y_2 & & \overrightarrow{P_1F}=2\overrightarrow{FP_2} & & l \\
\text{H2} & & & & & & |P_1P_2| \\
& & & & & & P_i(x_i,y_i)(i=1,2)
\end{array}$$

$$l:y=-\frac{\sqrt{5}}{2}(x-1) \qquad |P_1P_2|=\frac{\sqrt{5}}{2}$$

$$$$



F

P_1P_2

$$\begin{cases} x = r \cos \theta + 1 \\ y = r \sin \theta \end{cases}$$

$$3x^2 + 4y^2 = 12$$

$$3(r \cos \theta + 1)^2 + 4r^2 \sin^2 \theta = 12$$

$$(3 + \sin^2 \theta) \cdot r^2 + 6 \cos \theta \cdot r - 9 = 0$$

$$P_1F = 2P_2F$$

r_1, r_2

P_1, P_2

F

$$|r_1| = P_1F$$

$$|r_2| = P_2F$$

$$|r_1| = 2|r_2|$$

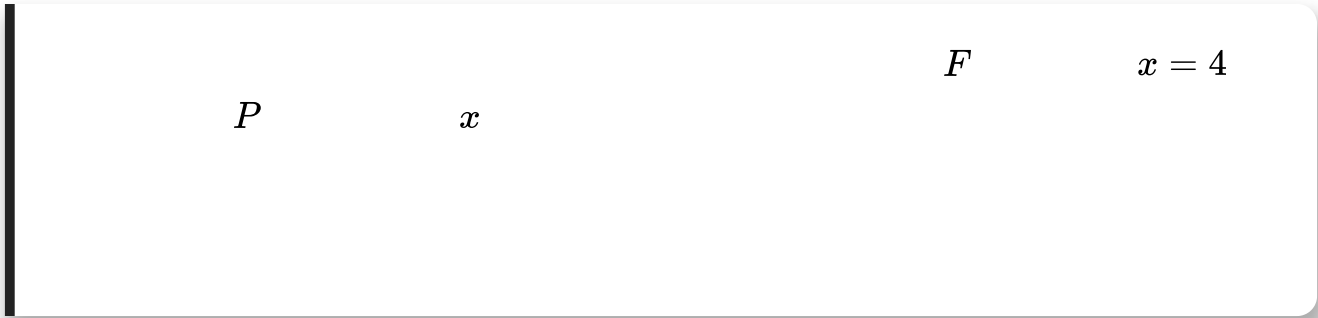
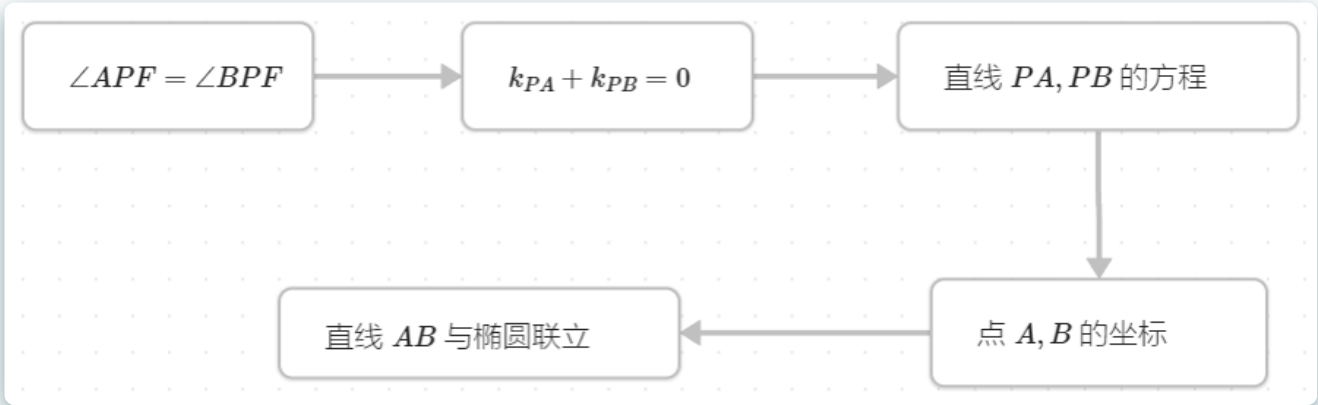
$$r_1 + r_2 = -\frac{6 \cos \theta}{3 + \sin^2 \theta} \quad r_1 r_2 = -\frac{9}{3 + \sin^2 \theta}$$

$$\tan \theta = -\frac{\sqrt{5}}{2}, r_1 = \frac{\sqrt{5}}{3}, r_2 = -\frac{\sqrt{5}}{6}$$

H2

A geometric diagram illustrating the construction of an ellipse. The ellipse has a horizontal major axis and a vertical minor axis, with center O . A focus F is marked on the major axis. An external point P is located on the major axis to the right of the ellipse. Two points A and B are on the upper and lower right portions of the ellipse, respectively. Line segments PA and PB are drawn, representing tangents from P to the ellipse. Line segments FA and FB are also drawn, connecting the focus F to points A and B . A dashed line segment OP is shown, representing the distance from the center to the external point.

AB



$\Gamma : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a, b > 0)$

O

OAB

l

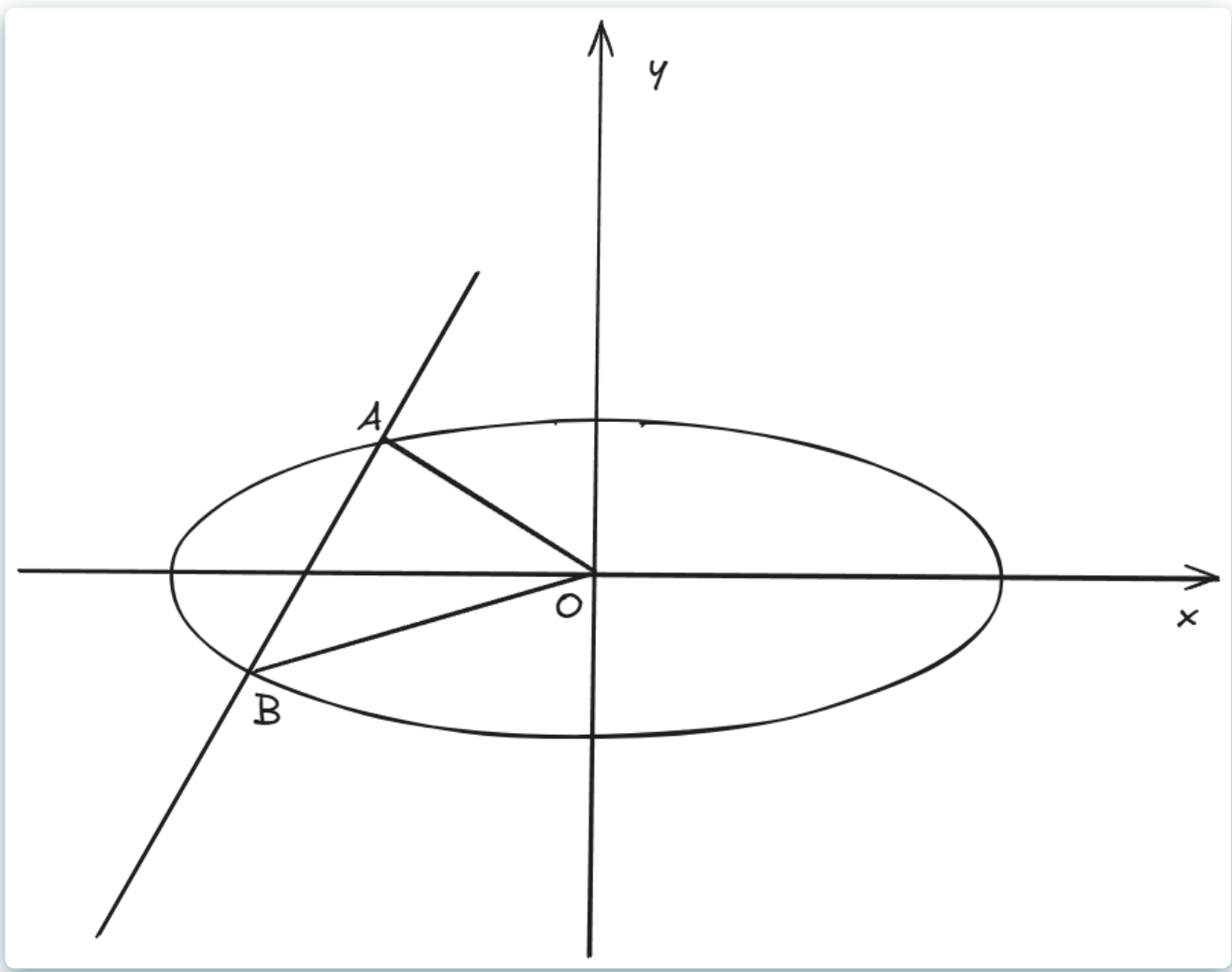
S_{OAB}

Γ

A, B

H2

$\frac{ab}{2}$



$$x \qquad a > b$$

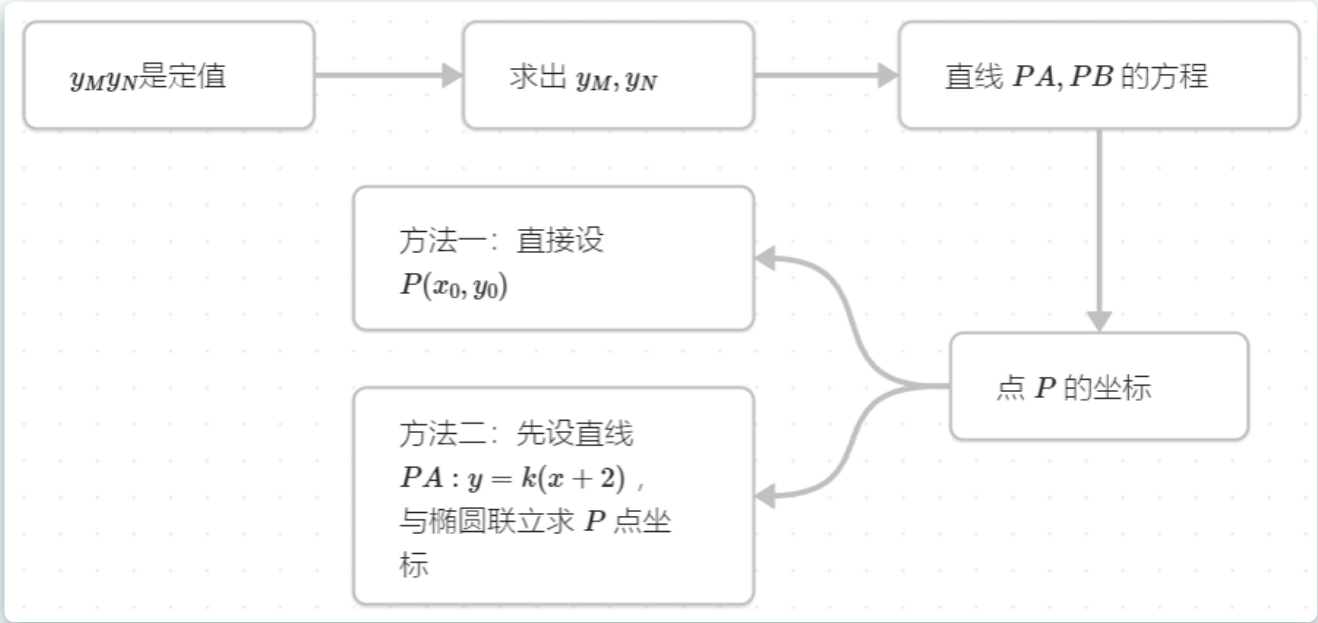
$$AB \qquad S_{OAB} \qquad AB \qquad O$$

$$AB \qquad x \qquad AB \qquad y = kx + m$$

$$AB \qquad y \qquad AB \qquad x = my + t$$

$$AB \qquad y = kx + m$$

$$S_{OAB} = \frac{ab|m|\sqrt{a^2k^2 + b^2 - m^2}}{a^2k^2 + b^2}$$



AB
 M, N

$y = k(x + 2)$
 $(4, 6k), (4, -\frac{3}{2k})$

$k_{PA}k_{PB} = e^2 - 1 = -\frac{3}{4}$
 PB

$y = -\frac{3}{4k}(x - 2)$
 -9

PA

P

H2

$$\Gamma : \frac{x^2}{4} + \frac{y^2}{3} = 1$$

Γ

A, B

P

F

F

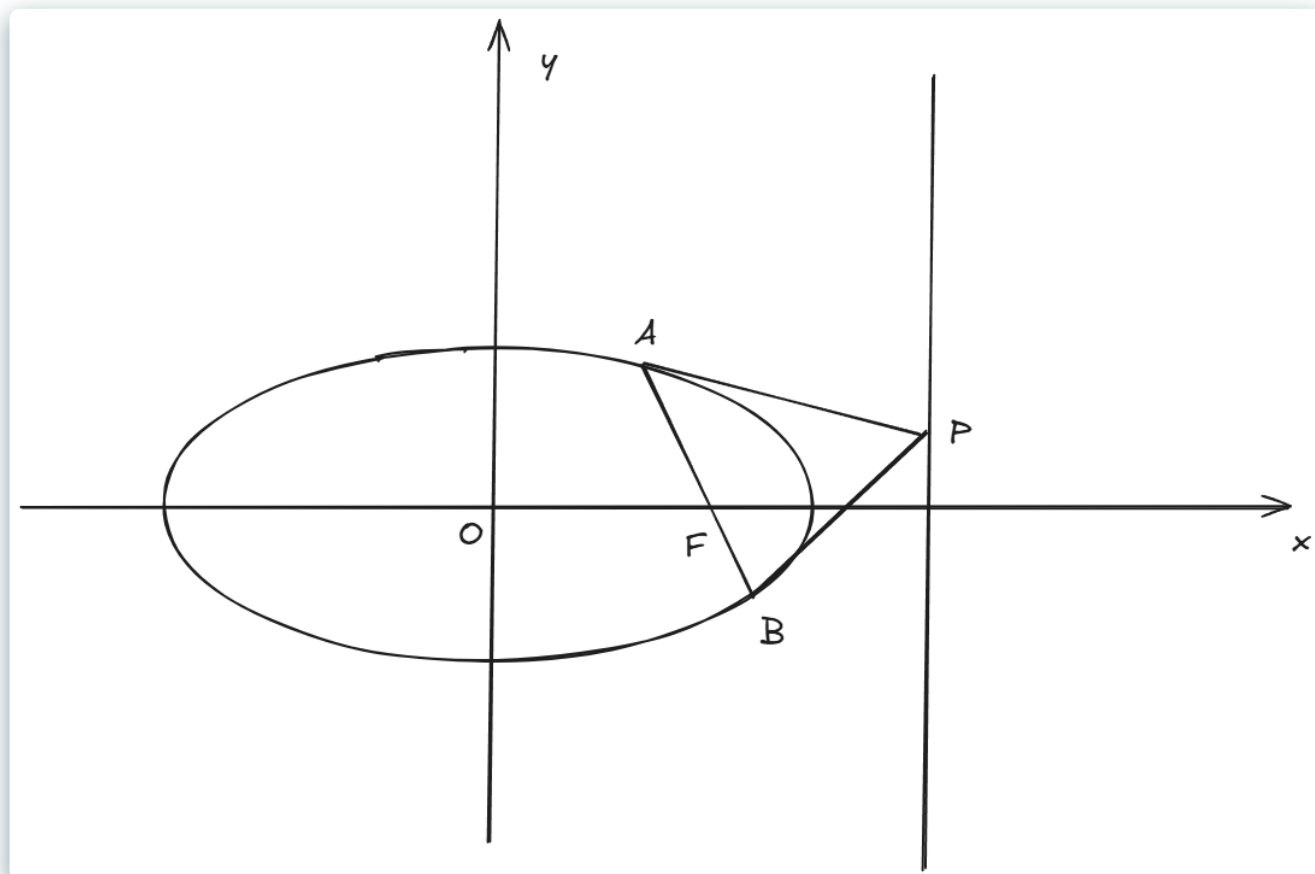
Γ

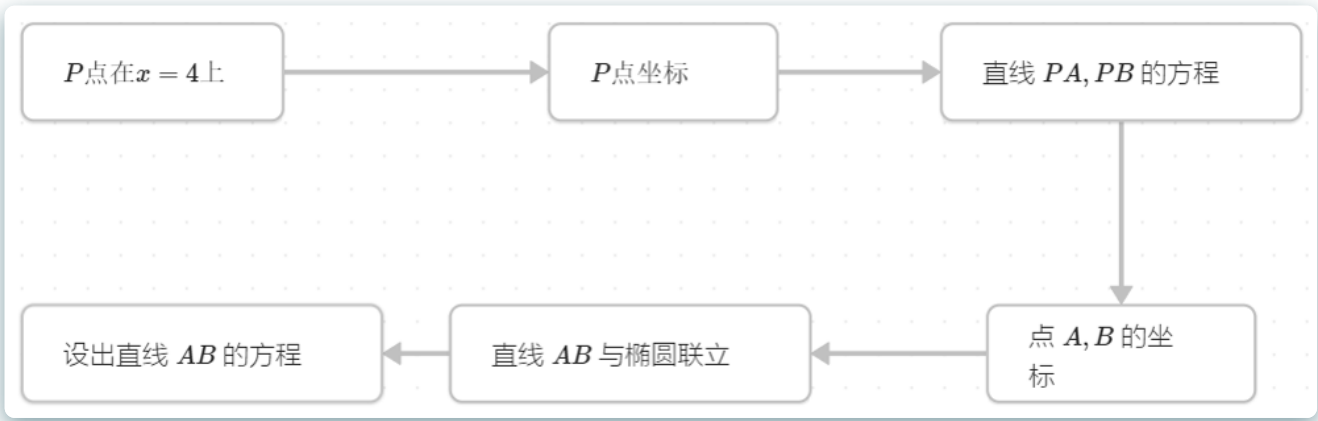
A, B

$$P \quad x = 4$$

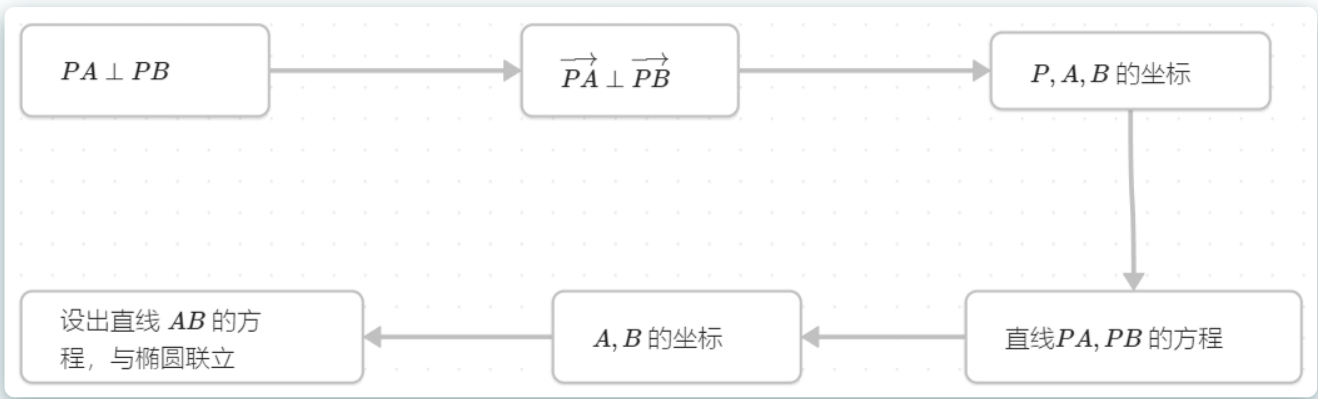
$$PA \perp PB$$

$$PF \perp AB$$





P 4 P (m, n)
 $m = 4$



$\vec{PA} \perp \vec{PB}$ P, A, B