$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a,b>0)$ $(a,0)$ $3c$ c

H2

$\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$

---

a,b

- $a>b$ $(-c,0),(c,0)$ $(a,0)$
- $a<b$ $(0,c),(0,-c)$ $(a,0)$

$M(1,1)$ $-\frac{1}{2}$ $C:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ A,B
M AB C _____

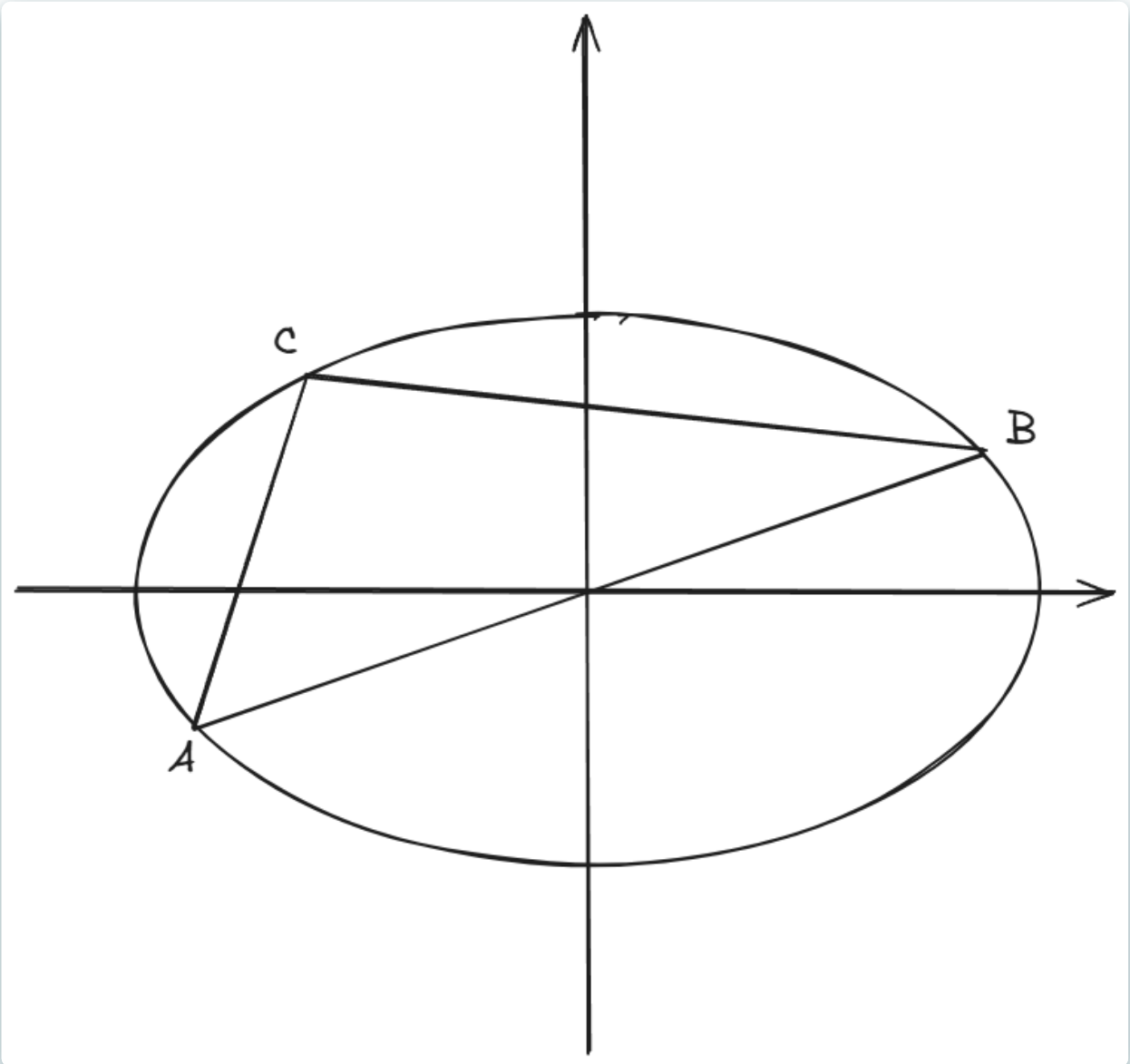
H2

$\frac{\sqrt{2}}{2}$

$C:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>0,b>0)$ AB AB C AC k_1 BC k_2

$$k_1k_2=e^2-1$$

e



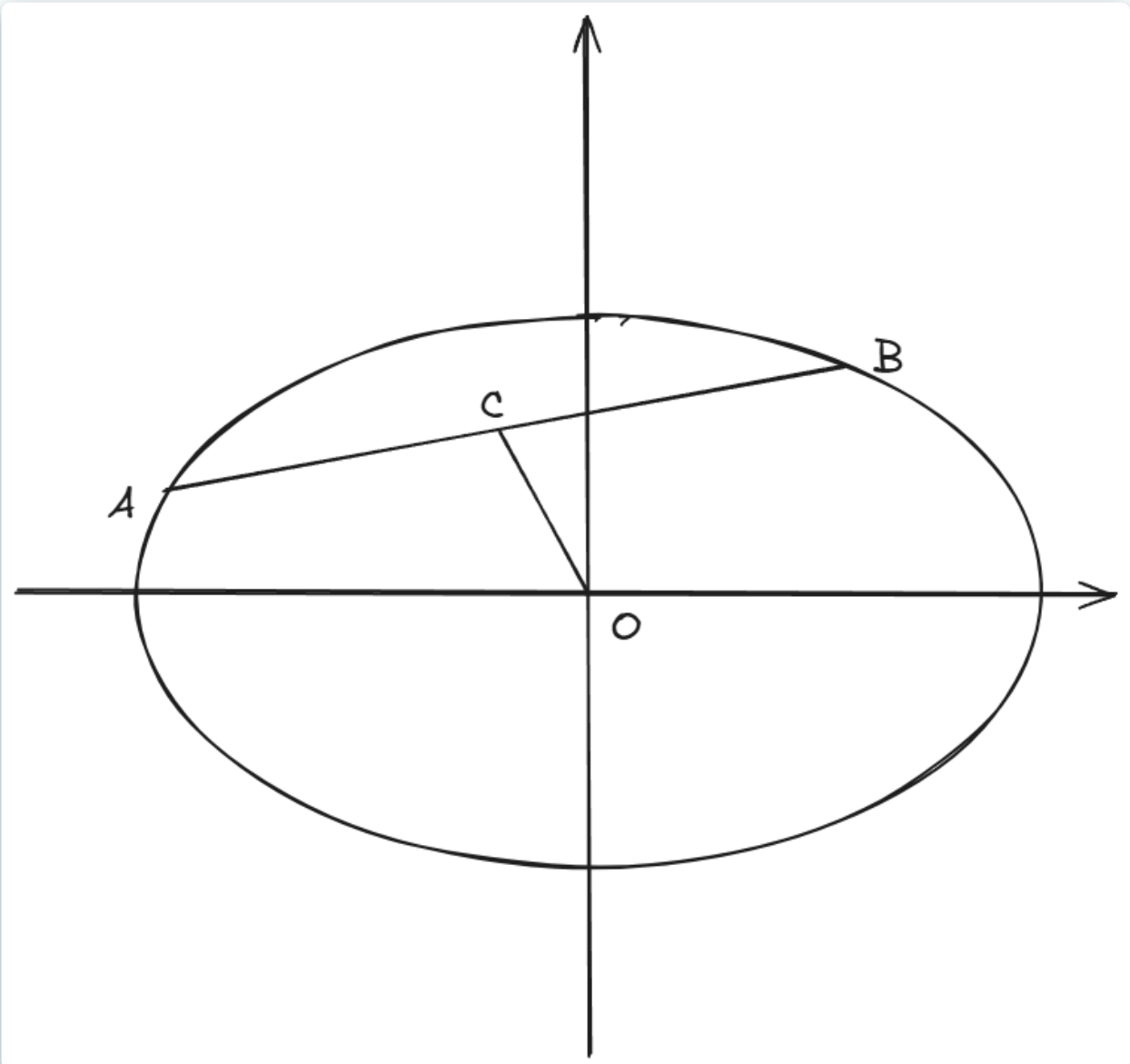
$k_1k_2=-1$ $e=0$

$k_1k_2=0^2-1=-1$

$k_1k_2=e^2-1$

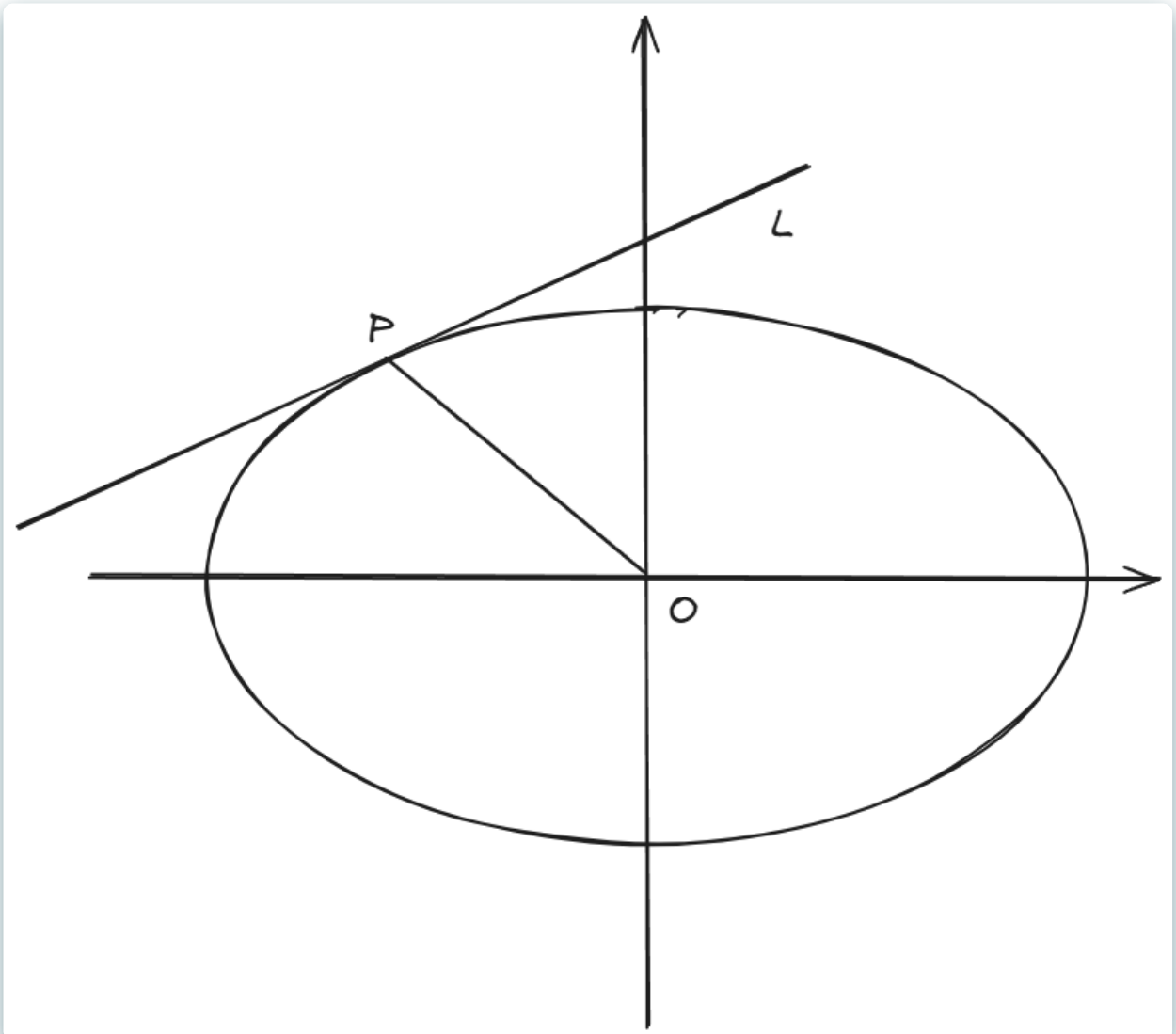
$C:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>0,b>0)$ O AB C AB AB k_1 OC k_2

$$k_1k_2=e^2-1$$



$C:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>0,b>0)$ O l P OP k_1 l k_2

$$k_1k_2=e^2-1$$



$-\frac{1}{2} \times 1 = e^2 - 1$

$e = \frac{\sqrt{2}}{2}$

$C : \frac{x^2}{2} + y^2 = 1$ l x y A, B O OAB

H2

$\sqrt{2}$

$C:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a,b>0)$ $P(x_0,y_0)$
$l:\frac{x_0x}{a^2}+\frac{y_0y}{b^2}=1$ P l C l P C

1.
2.
3.

$\frac{x_0x}{2}+y_0y=1$ (x_0,y_0) $(\frac{2}{x_0},0),(0,\frac{1}{y_0})$

$$S=\frac{1}{x_0y_0}$$

(x_0,y_0)

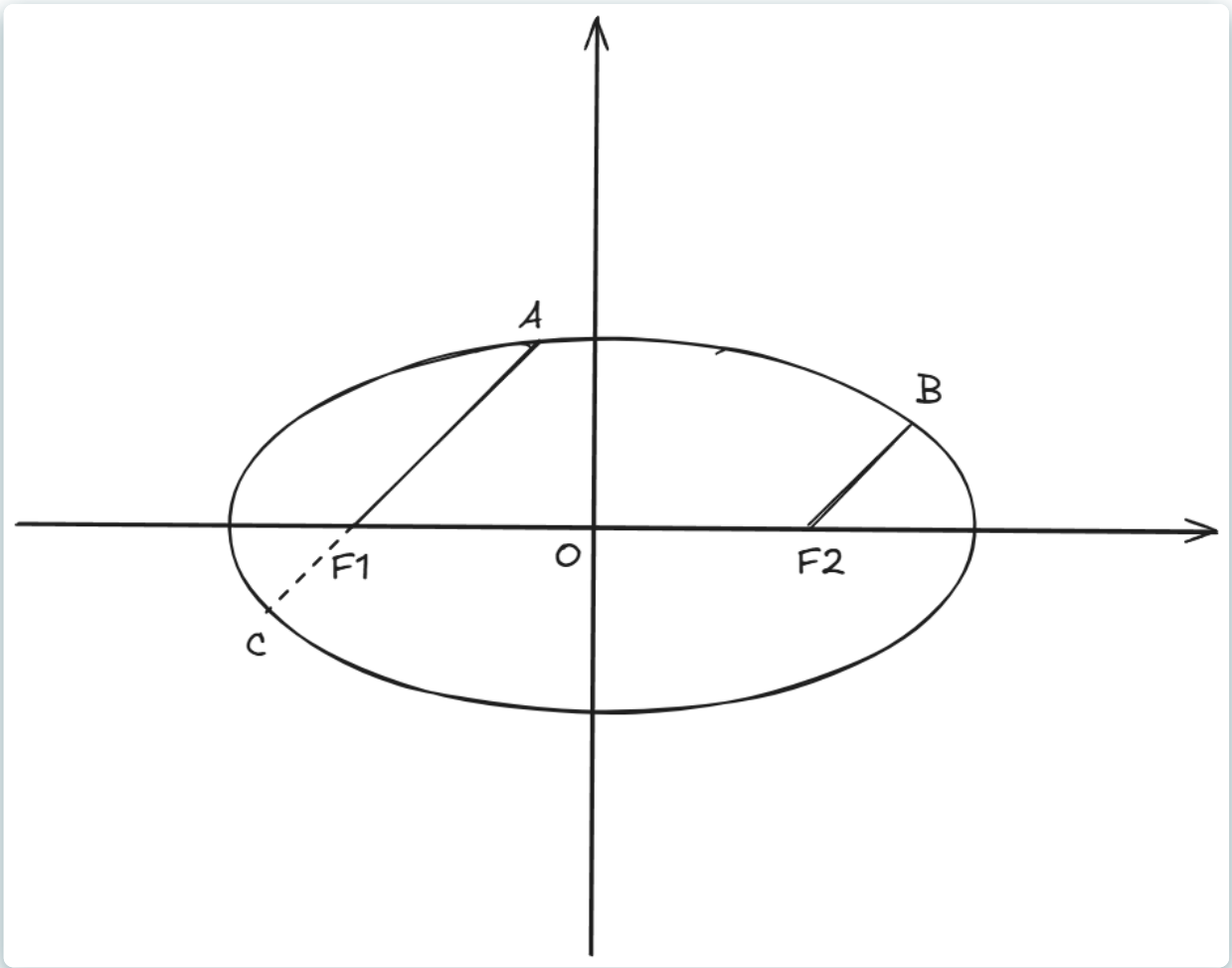
$$\frac{x_0^2}{2}+y_0^2=1\geq\sqrt{2}x_0y_0$$

$x_0y_0\leq\frac{\sqrt{2}}{2}$ $S\geq\sqrt{2}$

F_1, F_2 $\frac{x^2}{3} + y^2 = 1$ A, B

H2 $\overrightarrow{F_1A} = 5\overrightarrow{F_2B}$ A _____

> $(0, \pm 1)$



F_2B F_1C

$A(x_1, y_1), C(x_2, y_2)$ F_1 $(-\sqrt{2}, 0)$

AC $y = k(x + \sqrt{2})$

$$\begin{cases} y = k(x+\sqrt{2}) \\ \dfrac{x^2}{3} + y^2 = 1 \end{cases}$$

$$(1+3k^2)x^2 + 6\sqrt{2}k^2x + 6k^2 - 3 = 0$$

$x_1 + x_2 = -\frac{6\sqrt{2}k^2}{1+3k^2}$ $x_1x_2 = \frac{6k^2-3}{1+3k^2}$ $\overrightarrow{F_1A} = 5\overrightarrow{F_2B}$

$x_1 + 5x_2 = -6\sqrt{2}$ $y_1 + 5y_2 = 0$ $x_1 = 0$

A $(0, \pm 1)$

▶

$$\begin{cases} \dfrac{x_1^2}{3} + y_1^2 = 1 & (9) \\ \dfrac{x_2^2}{3} + y_2^2 = 1 & (10) \end{cases}$$

$$\frac{\frac{y_1+y_2}{2}}{\frac{x_1+x_2}{2}} \cdot \frac{y_1 - y_2}{x_1 - x_2} = -\frac{1}{3}$$

AC

F_1 AC AC

$(\frac{x_1+5x_2}{6}, \frac{y_1+5y_2}{6})$

5 25

$$\begin{cases} \dfrac{x_1^2}{3} + y_1^2 = 1 & (11) \\ \dfrac{(5x_2)^2}{3} + (5y_2)^2 = 25 & (12) \end{cases}$$

$$\frac{(x_1-5x_2)(x_1+5x_2)}{3}+(y_1-5y_2)(y_1+5y_2)=-24$$

F_1 $(\frac{x_1+5x_2}{6},\frac{y_1+5y_2}{6})$

$(-c,0)$ $(-\sqrt{2},0)$ $x_1+5x_2=-6\sqrt{2},y_1+5y_2=0$

$$x_1-5x_2=6\sqrt{2}$$

$x_1+5x_2=-6\sqrt{2}$ $x_1=0$ A

$(0,\pm1)$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ F_1,F_2 $A(x_0,y_0)$

$$|AF_1|=a+ex_1$$
$$|AF_2|=a-ex_2$$

$$|AF_1|=\sqrt{3}+\frac{\sqrt{6}}{3}x_1$$
$$|CF_1|=\sqrt{3}+\frac{\sqrt{6}}{3}x_2$$

$|AF_1|=5|CF_1|$ $\sqrt{3}+\frac{\sqrt{6}}{3}x_1=5\sqrt{3}+\frac{5\sqrt{6}}{3}x_2$

$x_1-5x_2=6\sqrt{2}$

$x_1+5x_2=-6\sqrt{2}$ $x_1=0$

$x^2+y^2=r^2$

$$\begin{cases} x=r\sin\theta \\ y=r\cos\theta \end{cases}$$

r θ r θ

x

x,y

$l:ax+by+c=0$

$$\begin{cases} x=r\sin\theta+x_0 \\ y=r\cos\theta+y_0 \end{cases}$$

(x_0,y_0) O' r P O'

$|r|$ θ

x

(x_0,y_0) (x_0,y_0)

$F_1(-\sqrt{2},0)$ AC

$$\begin{cases} x=r\sin\theta-\sqrt{2} \\ y=r\cos\theta \end{cases}$$

$$\frac{(r\sin\theta-\sqrt{2})^2}{3}+(r\cos\theta)^2=1$$

$$(1+2\cos^2\theta)\cdot r^2-2\sqrt{2}\sin\theta\cdot r-1=0$$

r

r_1, r_2 A C F_1 r_1, r_2

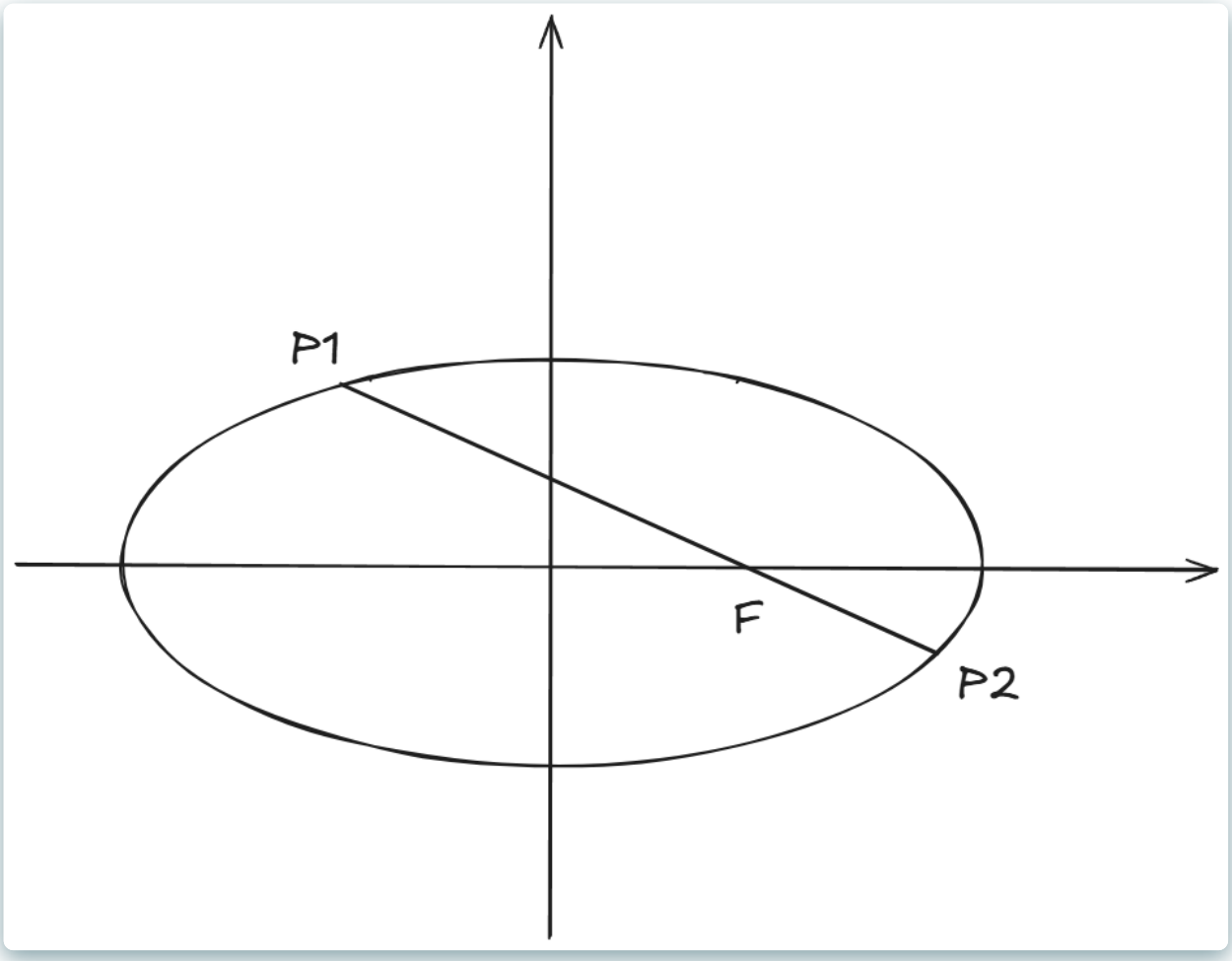
AF_1, CF_1 $|r_1| = 5|r_2|$

$r_1 + r_2 = \frac{2\sqrt{2}\sin\theta}{1+2\cos^2\theta}$ $r_1 r_2 = -\frac{1}{1+2\cos^2\theta}$ r_1, r_2, θ

$C: \frac{x^2}{4} + \frac{y^2}{3} = 1$ F l $P_i(x_i, y_i)(i = 1, 2)$

H2 $y_1 > y_2$ $\overrightarrow{P_1F} = 2\overrightarrow{FP_2}$ l $|P_1P_2|$

$l: y = -\frac{\sqrt{5}}{2}(x - 1)$ $|P_1P_2| = \frac{\sqrt{5}}{2}$



F P_1P_2

$$\begin{cases} x = r\cos\theta + 1 \\ y = r\sin\theta \end{cases}$$

$3x^2 + 4y^2 = 12$

$$3(r\cos\theta + 1)^2 + 4r^2\sin^2\theta = 12$$
$$(3 + \sin^2\theta)\cdot r^2 + 6\cos\theta \cdot r - 9 = 0$$

$P_1F = 2P_2F$ r_1, r_2 P_1, P_2 F

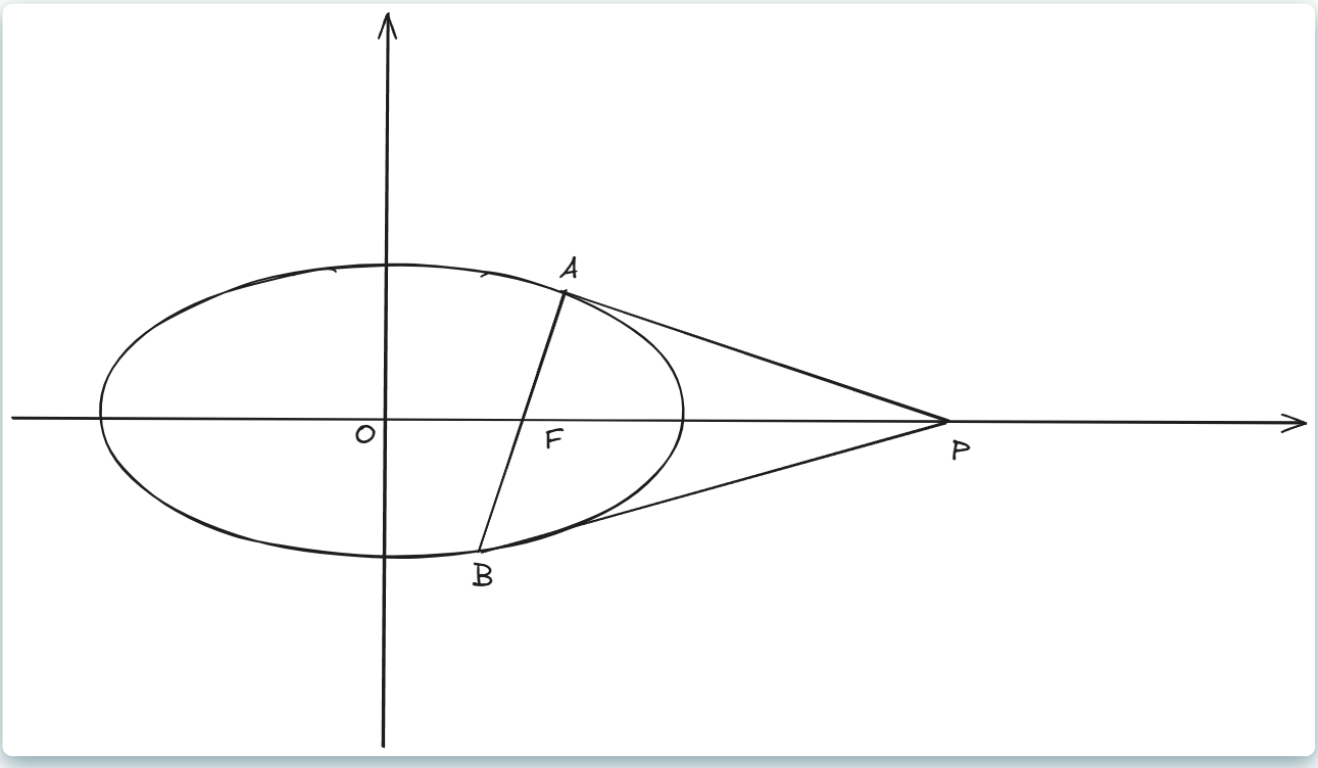
$|r_1| = P_1F$ $|r_2| = P_2F$ $|r_1| = 2|r_2|$

$r_1 + r_2 = -\frac{6\cos\theta}{3+\sin^2\theta}$ $r_1r_2 = -\frac{9}{3+\sin^2\theta}$

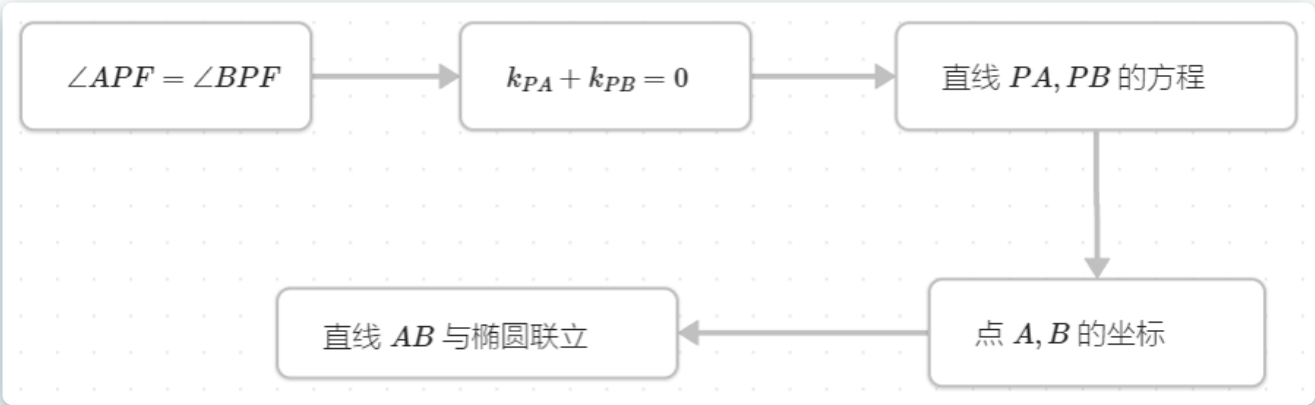
$\tan\theta = -\frac{\sqrt{5}}{2}, r_1 = \frac{\sqrt{5}}{3}, r_2 = -\frac{\sqrt{5}}{6}$

$\Gamma : \frac{x^2}{4} + \frac{y^2}{3} = 1$ Γ F P $(4, 0)$ F
AB Γ A, B $\angle APF = \angle BPF$

H2



$\angle APF, \angle BPF$
$\angle APF = \angle BPF$ $k_{PA} + k_{PB} = 0$
PA, PB P A, B
A, B F AB
A, B



F $x = 4$

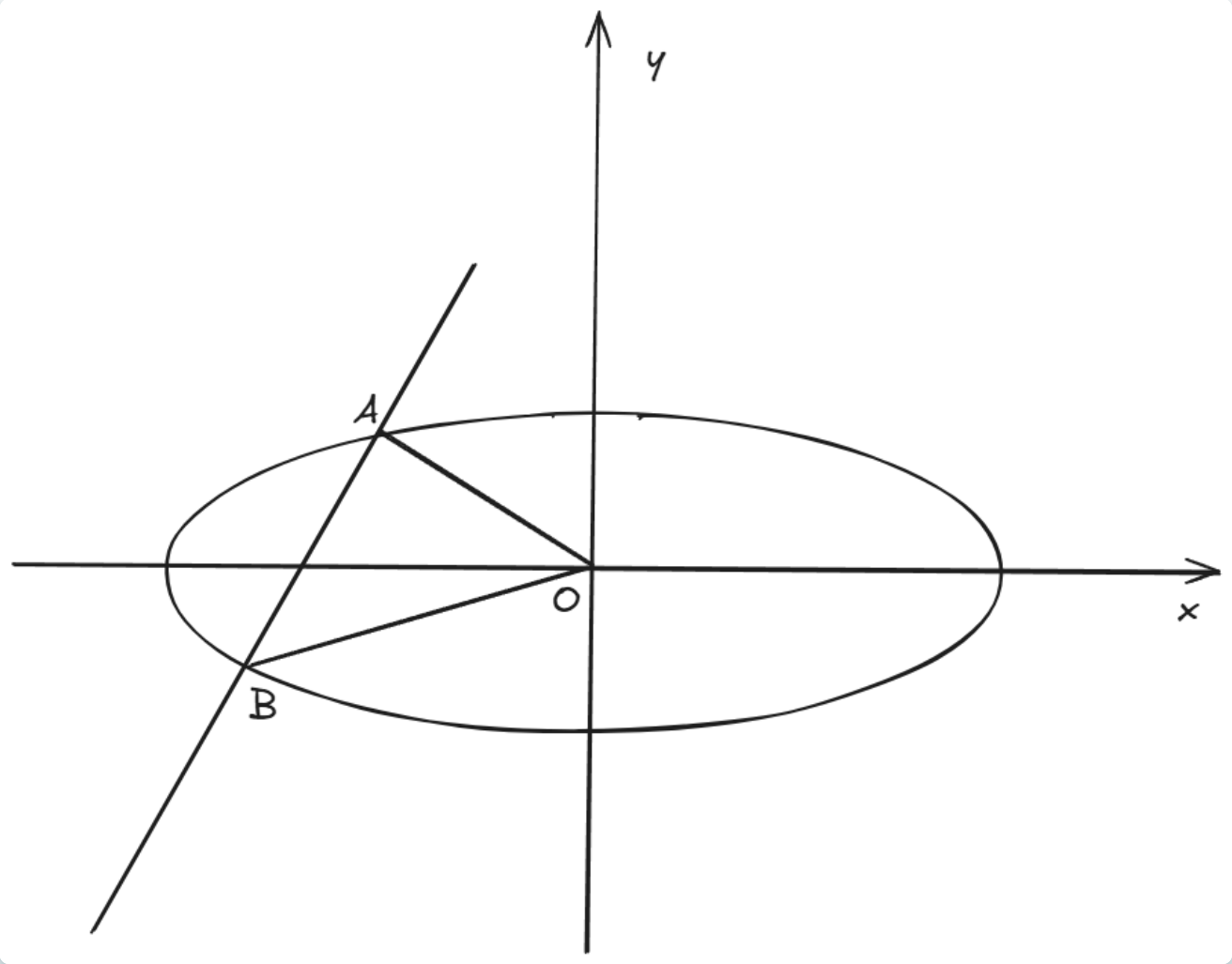
P x

$\Gamma : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a, b > 0)$ l Γ A, B

O OAB S_{OAB}

H2

$\frac{ab}{2}$



x $a > b$

S_{OAB} AB O
AB AB

AB $y = kx + m$
AB x AB $x = my + t$
AB y

AB $y = kx + m$

$$S_{OAB} = \frac{ab|m|\sqrt{a^2k^2 + b^2 - m^2}}{a^2k^2 + b^2}$$

$$S_{OAB} \le \frac{ab\frac{m^2+a^2k^2+b^2-m^2}{2}}{a^2k^2+b^2}$$
$$= \frac{ab}{2}$$

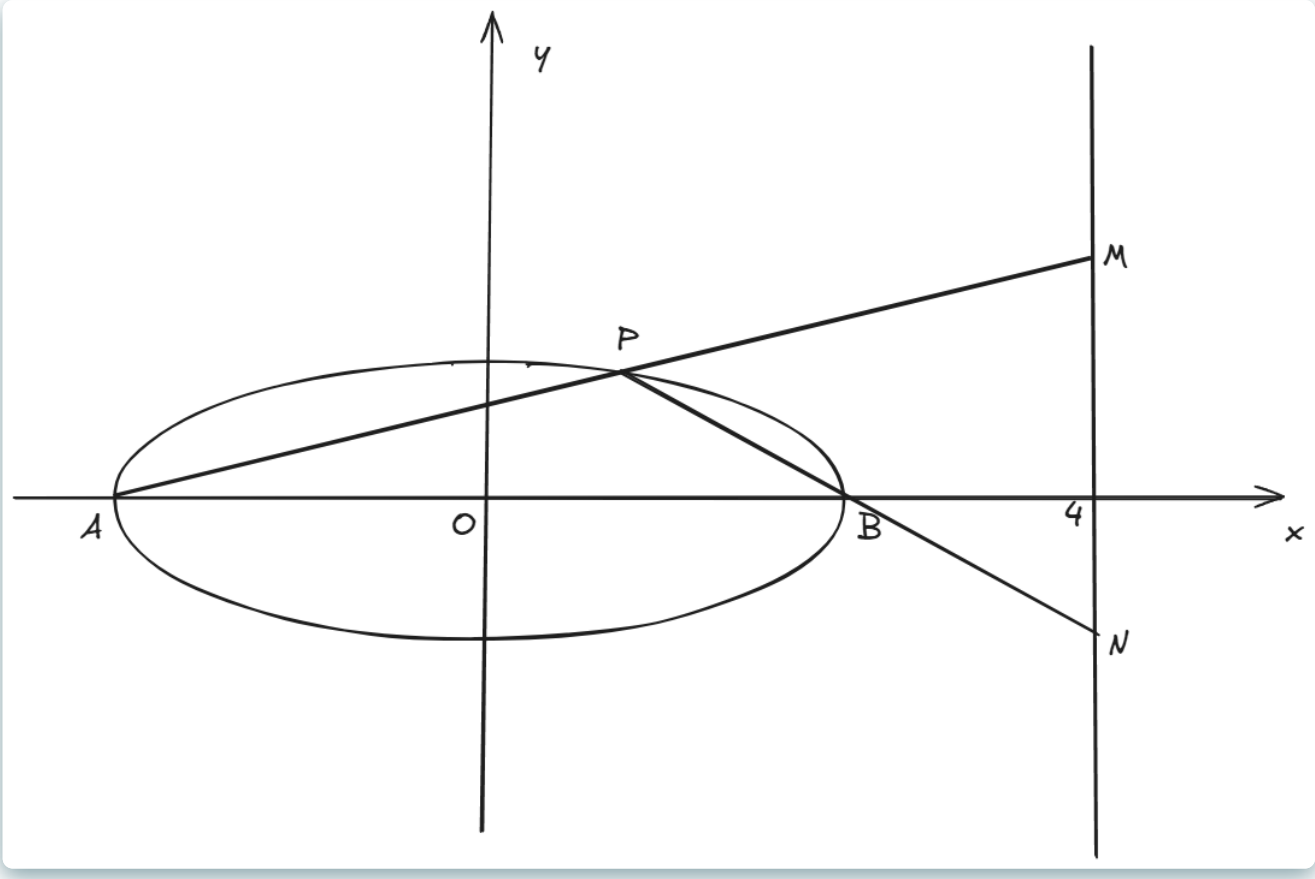
$$|m| = \sqrt{a^2k^2+b^2-m^2}$$

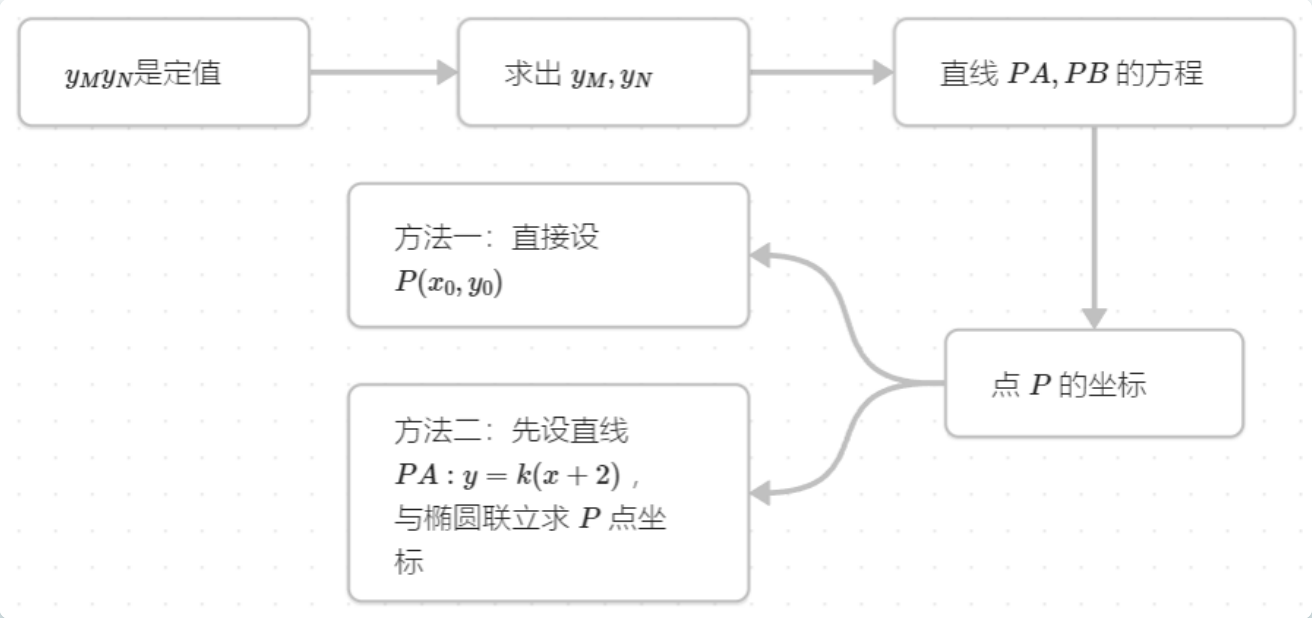
S_{OAB}

> $y = kx + m$ $= \sqrt{k^2+1}|x_1 - x_2| = \sqrt{(\frac{1}{k})^2+1}|y_1 - y_2|$
>
> $x = my + t$ $= \sqrt{m^2+1}|y_1 - y_2| = \sqrt{(\frac{1}{m})^2+1}|x_1 - x_2|$

$\Gamma : \frac{x^2}{4} + \frac{y^2}{3} = 1$ A B P Γ A B PA PB $x = 4$ M N M N

H2





AB $k_{PA}k_{PB}=e^2-1=-\frac{3}{4}$ PA $y=k(x+2)$ PB $y=-\frac{3}{4k}(x-2)$ M,N $(4,6k),(4,-\frac{3}{2k})$ -9

P

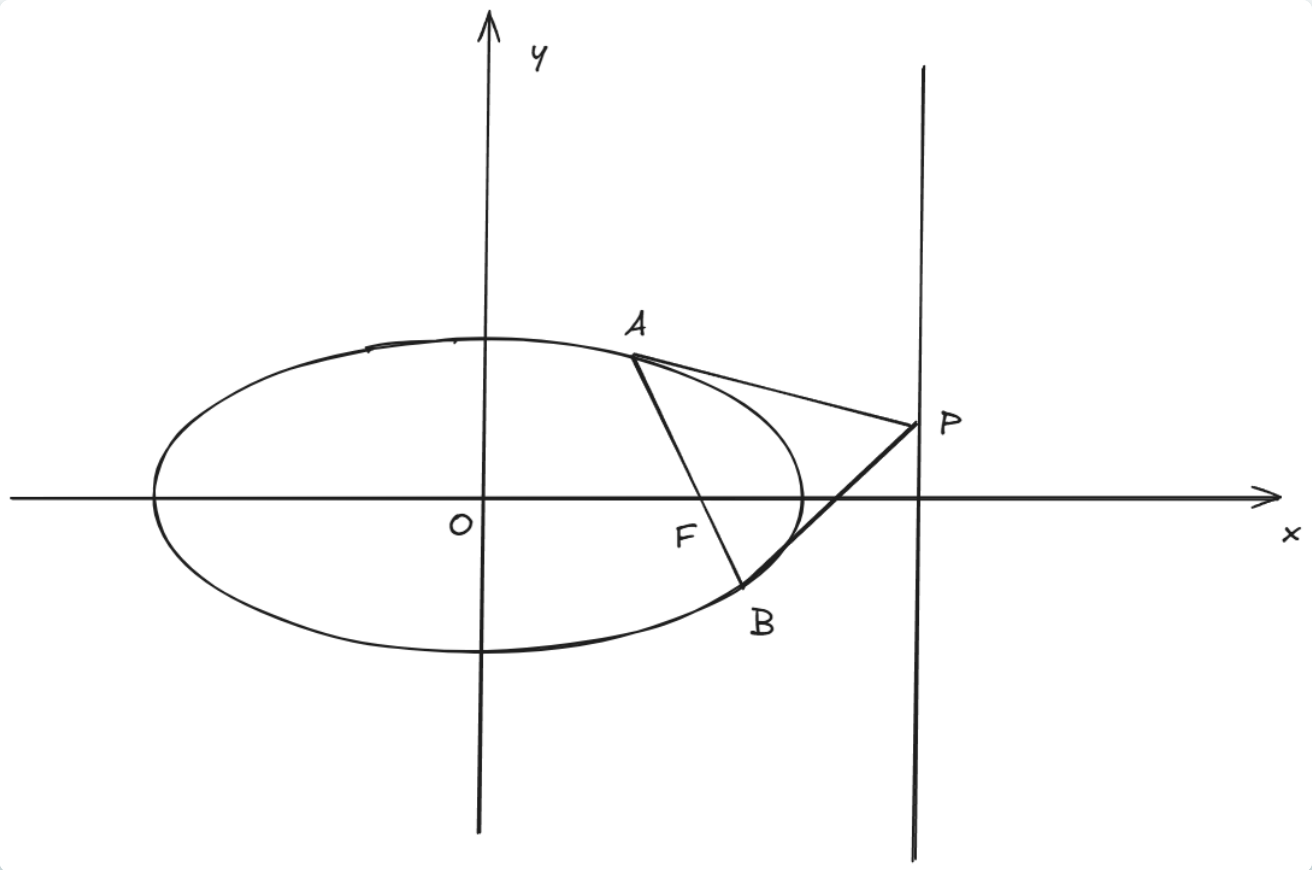
$\Gamma : \frac{x^2}{4} + \frac{y^2}{3} = 1$ F F Γ A, B
Γ A, B P

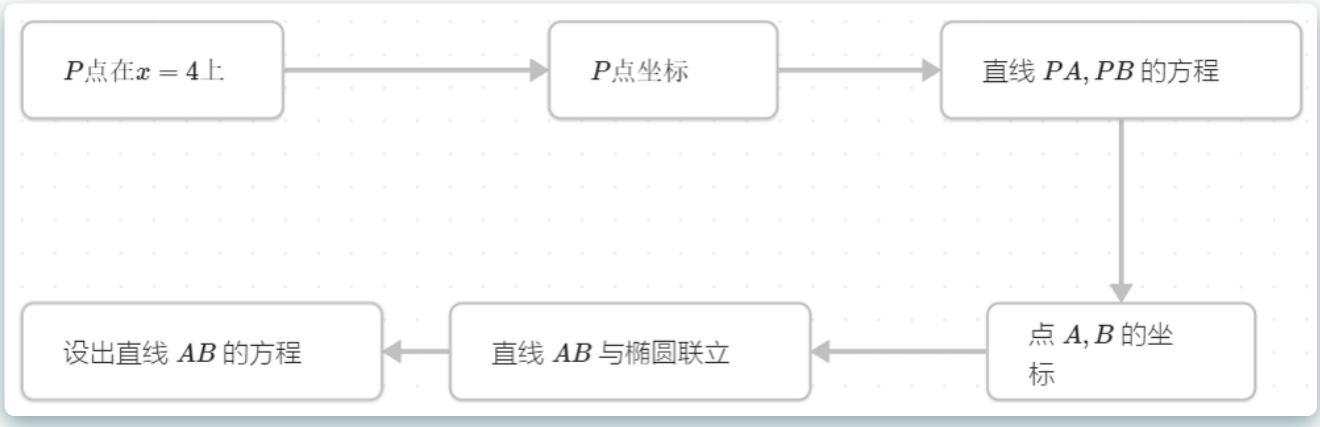
H2

P $x = 4$

$PA \perp PB$

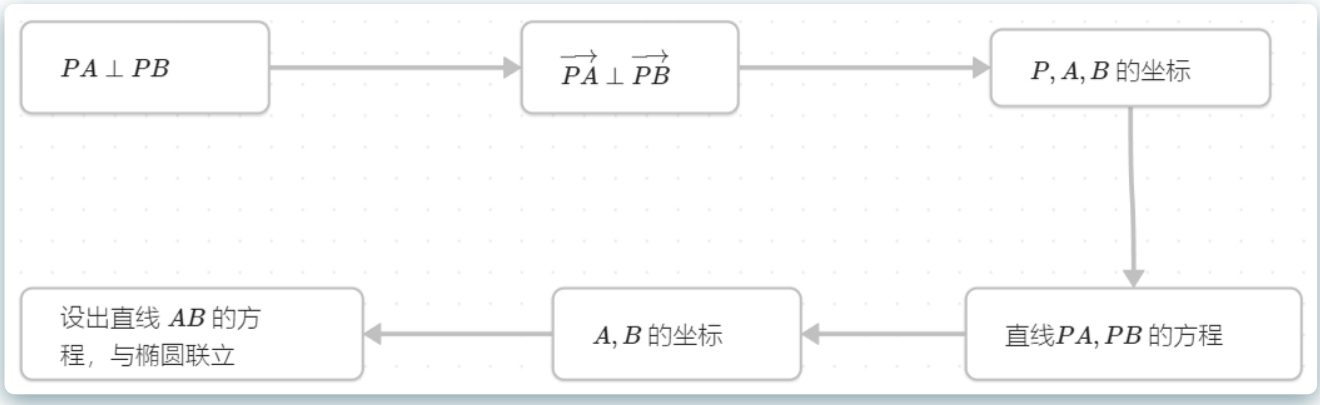
$PF \perp AB$





P 4 P (m,n)

$m=4$



P, A, B

$\overrightarrow{PA} \perp \overrightarrow{PB}$