Brownian Motion

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What is Brownian Motion?

- Robert Brown
- Zig zag motion of pollen particles
- Wiener process
- Continuous stochastic process
- Analog of random walk
- Represented by W_t where $t >= 0$

Properties of Brownian Motion

1. If t = 0 then W0 =0
2. Independent Increments
3. Stationary Increments
4. Normal Increments
5. Wt has continuous sample path

Problem 1
Property: Normal Increments

Given that $W_0 = 0$. Calculate $P(W_5 > 2)$.

As we Know that Brownian is one of normal distribution : $N(\mu, \sigma^2) \sim N(0,1)$ where mean=0 and variance=1. Here

$W_5 \sim N(0,5)$

$\mu = 0$

$\sigma = \sqrt{5}$

So

$$P(W_5 > 2) = P\left(\mu > \frac{2-0}{\sqrt{5}}\right)$$

$$= P\left(Z > \frac{2}{\sqrt{5}}\right)$$

$$= P(Z > 0.89)$$

$$= 1 - 0.81327$$

$$= 0.18673$$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189

So Probability is 18.67%.

Problem 2
Property: Independent Increments

Given that $W_0 = 0$. Calculate $P(W_5 > 2 \mid W_1 = 1)$.

Here

$W_5 \sim N(0,5)$

$\mu = 0$

$\sigma = \sqrt{5}$

So

$$P(W_5 > 2 \mid W_1 = 0) = P(W_1 + (W_5 - W_1) > 2 \mid W_1 = 1)$$

$$= P(1 + (W_5 - W_1) > 2 \mid W_1 = 1)$$

$$= P((W_5 - W_1) > 2 - 1 \mid W_1 = 1)$$

$$= P((W_5 - W_1) > 1 \mid W_1 = 1)$$

$$= P((W_5 - W_1) > 1 \mid (W_1 - W_0) = 1)$$

As $(W_5 - W_1)$ and $(W_1 - W_0)$ are independent, So

$$=P((W_5 - W_1) > 1)$$

As $(W_5 - W_1) \sim N(0,4)$

$W_5 \sim N(0,4)$

$\mu = 0$

$\sigma = \sqrt{4}$

$P((W_5 - W_1) > 1) = P(\mu > \frac{1-0}{\sqrt{4}})$

$= P(Z > \frac{1}{\sqrt{4}})$

$= P(Z > 0.5)$

$= 1 - 0.69146$

$= 0.30854$

So Probability is 30.85%.

Z	.00	.01	.02	.03	.04
0.0	.50000	.50399	.50798	.51197	.51595
0.1	.53983	.54380	.54776	.55172	.55567
0.2	.57926	.58317	.58706	.59095	.59483
0.3	.61791	.62172	.62552	.62930	.63307
0.4	.65542	.65910	.66276	.66640	.67003
0.5	$\boxed{.69146}$	.69497	.69847	.70194	.70540
0.6	.72575	.72907	.73237	.73565	.73891
0.7	.75804	.76115	.76424	.76730	.77035
0.8	.78814	.79103	.79389	.79673	.79955
0.9	.81594	.81859	.82121	.82381	.82639

Problem 3
Property: Stationary Increments

Given that $W_0 = 0$. Calculate $P(W_3 > W_2 > W_1)$.

$P(W_3 > W_2 > W_1) = P[\,(W_3 - W_2) > 0 \cap (W_2 - W_1) > 0\,)$

$= P[\,(W_3 - W_2) > 0\,] \,.\, P[\,(W_2 - W_1) > 0\,)]$

Here

$(W_3 - W_2) \sim N(0,1)$, $(W_2 - W_1) \sim N(0,1)$

$\mu = 0$

$\sigma = \sqrt{1} = 1$

$P[\,(W_3 - W_2) > 0\,] = = P(\mu > \frac{0-0}{1})$

$= P(Z > 0)$

$= 1 - 0.5$

$= 0.5$

Z	.00	.01	.02	.03
0.0	.50000	.50399	.50798	.51197
0.1	.53983	.54380	.54776	.55172
0.2	.57926	.58317	.58706	.59095
0.3	.61791	.62172	.62552	.62930
0.4	.65542	.65910	.66276	.66640

$P[\,(W_2 - W_1) > 0\,] = = P(\mu > \frac{0-0}{1})$

$= P(Z > 0)$

$= 1 - 0.5$

$= 0.5$

Z	.00	.01	.02	.03
0.0	$\boxed{.50000}$	.50399	.50798	.51197
0.1	.53983	.54380	.54776	.55172
0.2	.57926	.58317	.58706	.59095
0.3	.61791	.62172	.62552	.62930
0.4	.65542	.65910	.66276	.66640

So

$P(W_3 > W_2 > W_1) = P[\,(W_3 - W_2) > 0\,] \,.\, P[\,(W_2 - W_1) > 0\,)]$

$= 0.5 \times 0.5$

$= 0.25$

So Probability is 25%

Applications of Brownian Motion

- Medical Imaging
- Robotics
- Decision Making

Medical imaging

its application to medical imaging has probably been the most successful and productive.

Brownian motion is used today in two main capacities in this field, classification of tissues and other structures, and also sensitive textures and edges of an image.

difficult to understand the medical images patterns and complexity for this purpose Brownian motion helped us to understand the randomness motions in medical imaging. By treating the natural randomness of the structure as Brownian motion, images of this type can be enhanced by highlighting the natural randomness to overcome the random noise found in the image.

robotics

The idea of robots being able to move around and do things that humans can do is both a fantastical yet popular idea that people harbor concerning life in a futuristic world. It turns out that Brownian motion can help make this dream a reality, at least in assisting robots in moving wherever they want to.

the robot measures depths and elevations around it using a special instrument, then takes this data and recreates a three dimensional map of the land, using Brownian motion to approximate the roughness of the land.

decision making

The most interesting and least obvious application of Brownian motion can be found in the area of decision making. More precisely, Brownian motion can help determine **optimal switching times** in some economic activity that operates on some level of uncertainty.

Example:

An example of such a situation would be the operation of a car manufacturing plant, which might shut down temporarily if prices of steel or other materials needed to build the cars reached a critically high level.This type of problem is known as an optimal switching problem.

Another type of decision making process that Brownian motion can lend assistance to is one where a decision is made when a certain **threshold** requirement is fulfilled,

An example of such a process would be the method of promotion of employees at a company or firm.

Thank You