Hidden Markov Model

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Course Code: Pattern Recognition

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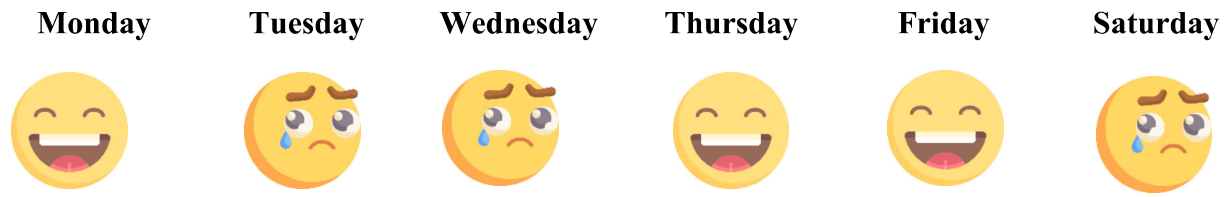
Hidden Markov Model

A Hidden Markov Model is a statistical approach that is frequently use for modelling sequence data. In HMM, probabilities are not known but outcomes are known. It is used to predict the sequence of hidden variables from a set of observed variables.

Scenario:

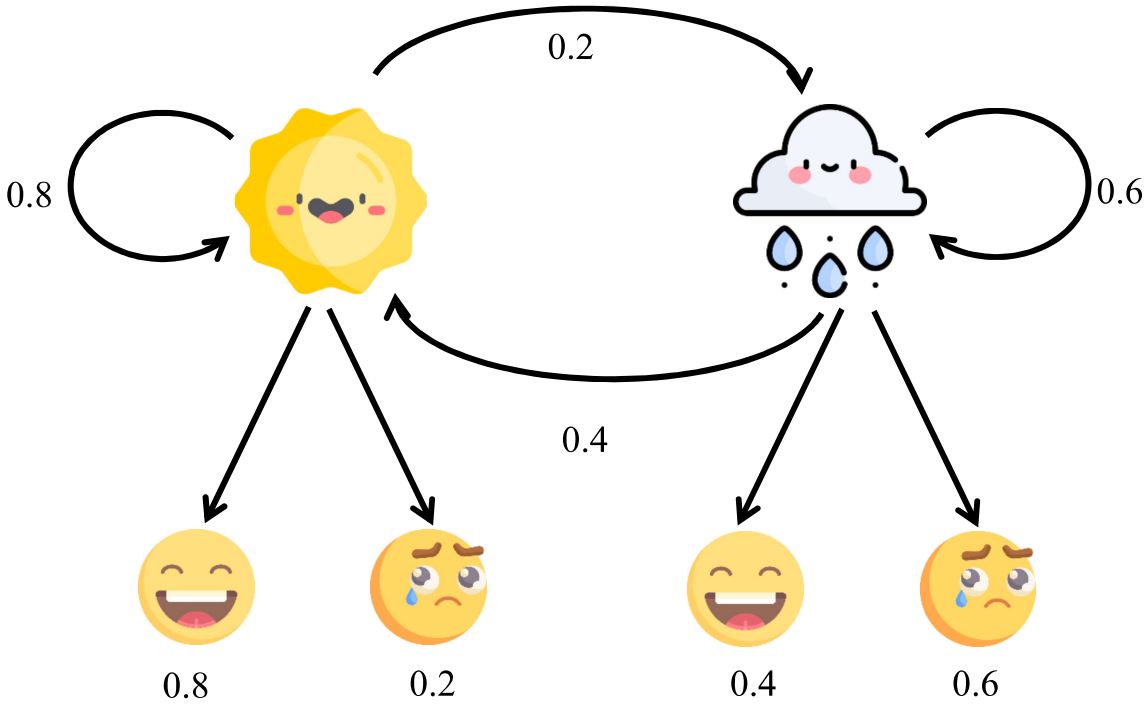
Bob is a person whose mood depends on weather. If today is sunny, then Bob is Happy. But if today is rainy, then Bob is Grumpy. Here Bob's moods are given for a week and we have to find the weather for respective day.



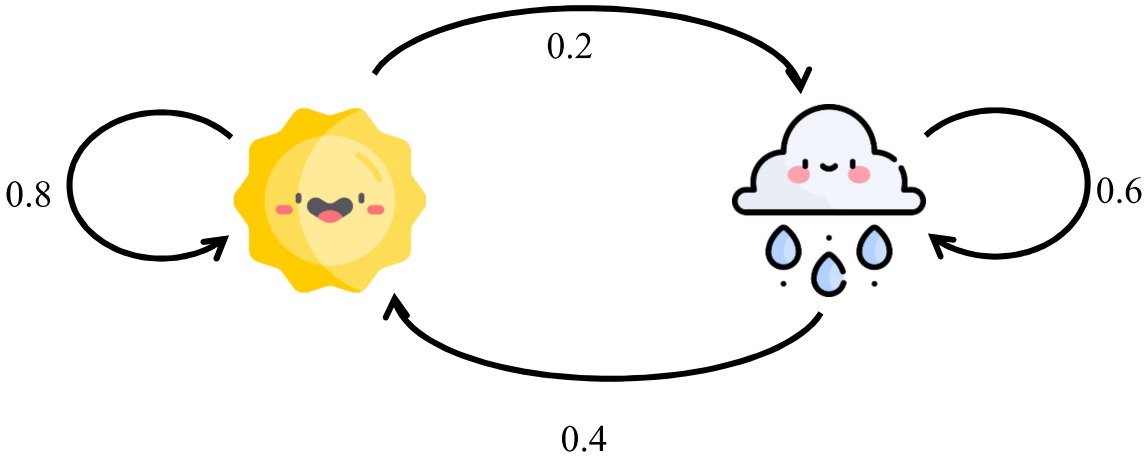
Before solving the real problem, we will find:

- Transition Diagram for a Single Day
- Transition probabilities for a Single Day
- Bayes Theorem:
    - Probabilities of mood given weather
    - Probabilities of weather given mood

As we already know that



Transition Diagram for a Single Day:



$S = 0.8S + 0.4R$ $R = 0.2\,S + 0.6\,R$

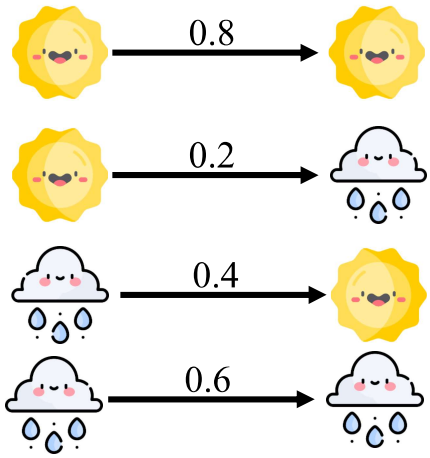
$$S + R = 1$$

After solving system of equations, we will get

$$S = 2/3$$

$$R = 1/3$$

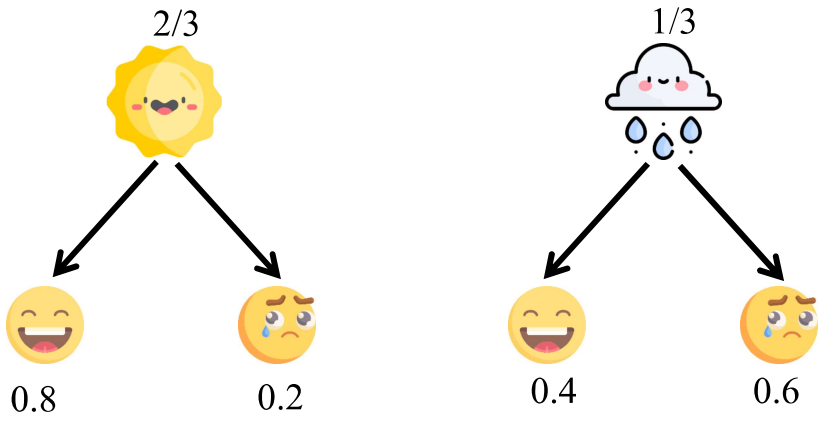
Transition Probability:



Bayes Theorem

The prior probability tell us that there is 2/3 probability to be sunny day and 1/3 probability of rainy day. Then depending on the day, the mood of Bob varies as:

- If today is Sunny day then probability of happy is **0.8** and Grumpy is **0.2.**
- If today is Rainy day then probability of happy is **0.4** and Grumpy is **0.6.**



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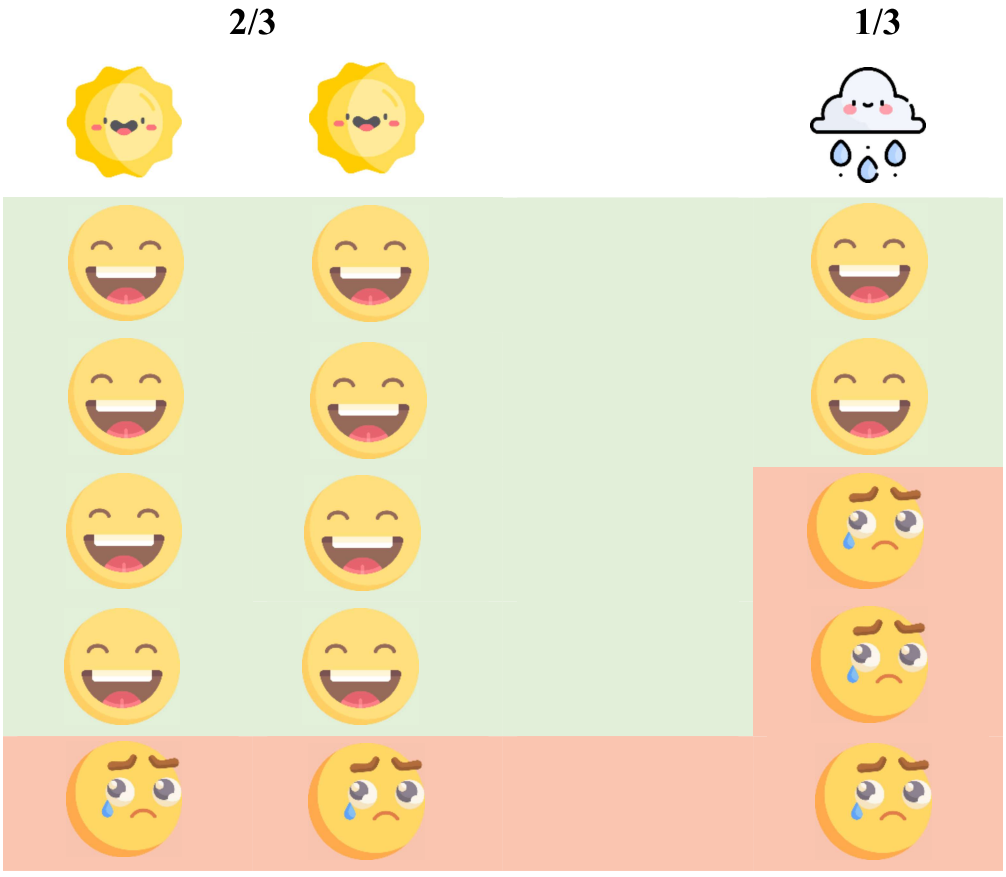
Mood of Bob for 15 days (i.e. 10 days of sunny and 5 days of Rainy) is given as:



Probabilities when Mood is given

If **Happy** is given then:

P ( | ) = 8/10 =0.8

P ( | ) = 2/10 =0.2

If **Grumpy** is given then:

P ( | ) = 2/5 =0.4

P ( | ) = 3/5 =0.6

Probabilities when Weather is given

If **Sun** is given then:

P ( | ) = 8/10 =0.8

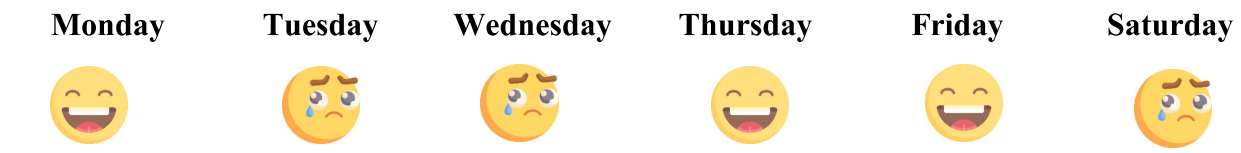
P ( | ) = 2/10 =0.2

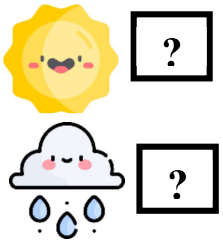
If **Rainy** is given then:

P ( | ) = 2/5 =0.4

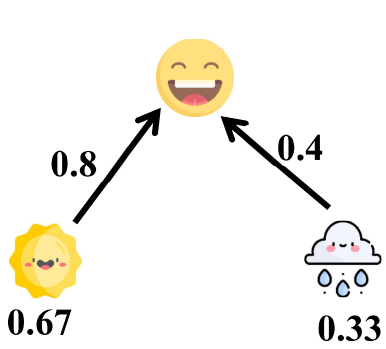
P ( | ) = 3/5 =0.6

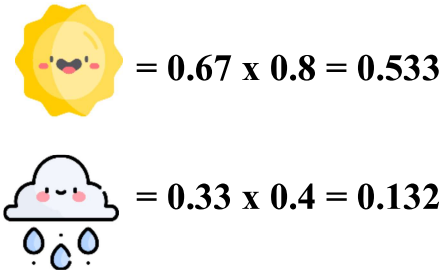
Now coming back to real problem:

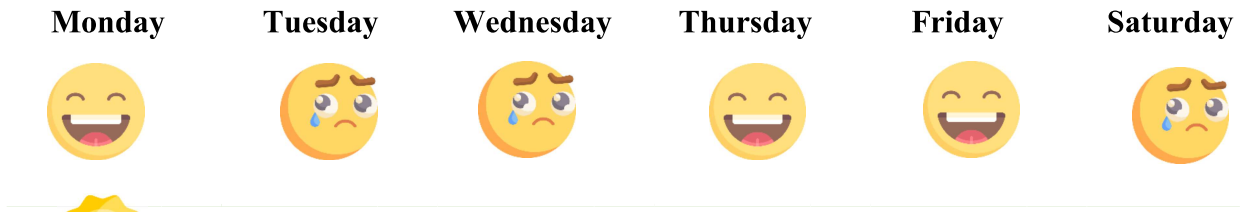




Here we have to find what is the probability of weather on Saturday if Bob is Grumpy?



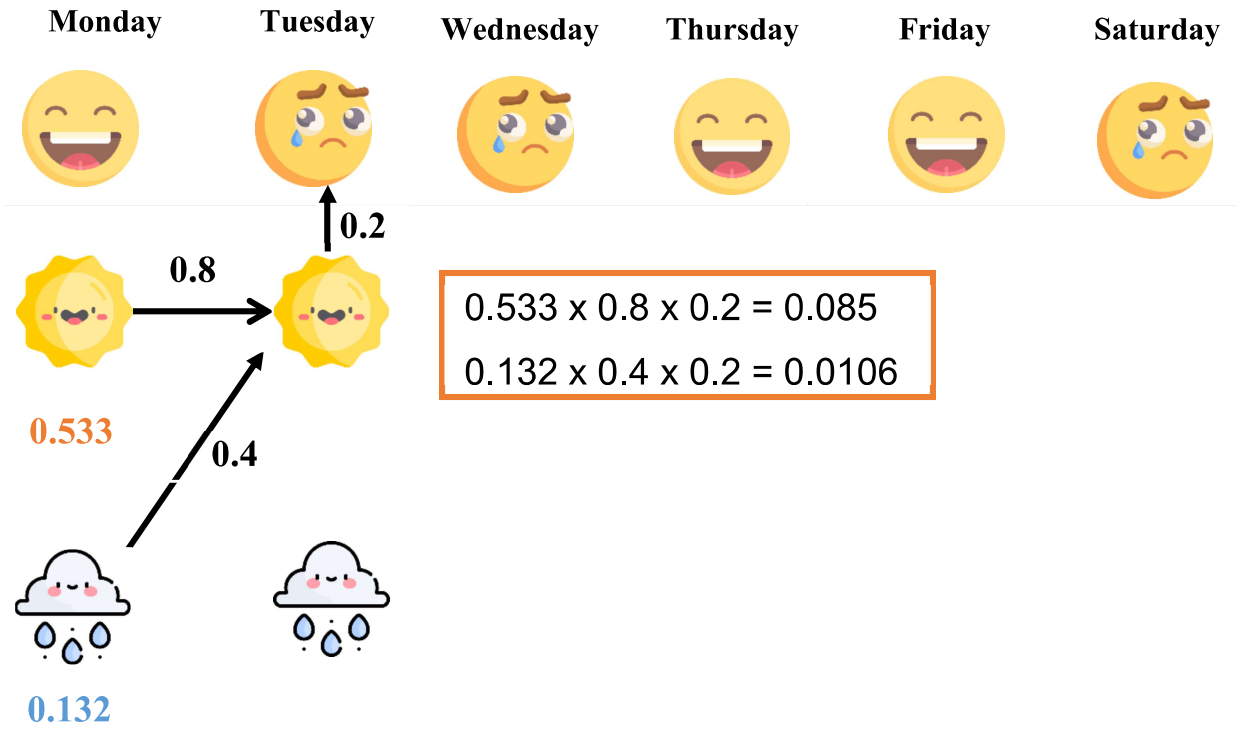


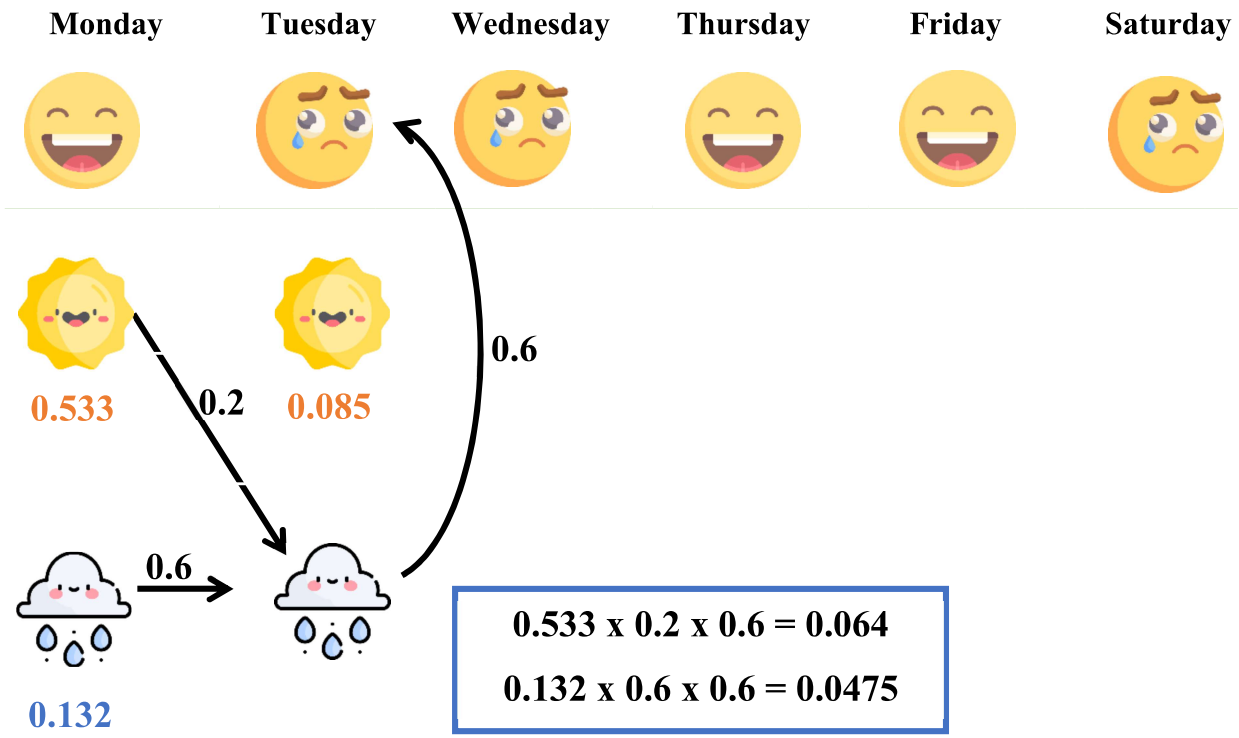


0.533

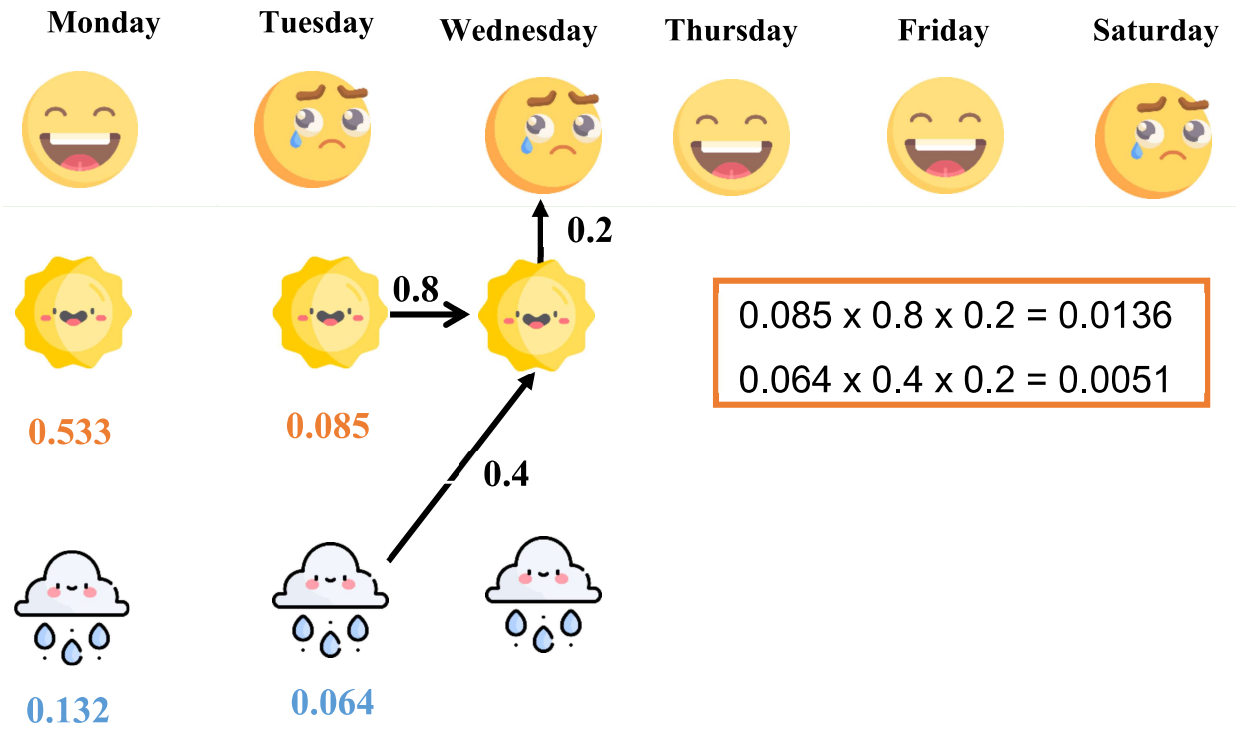
0.132

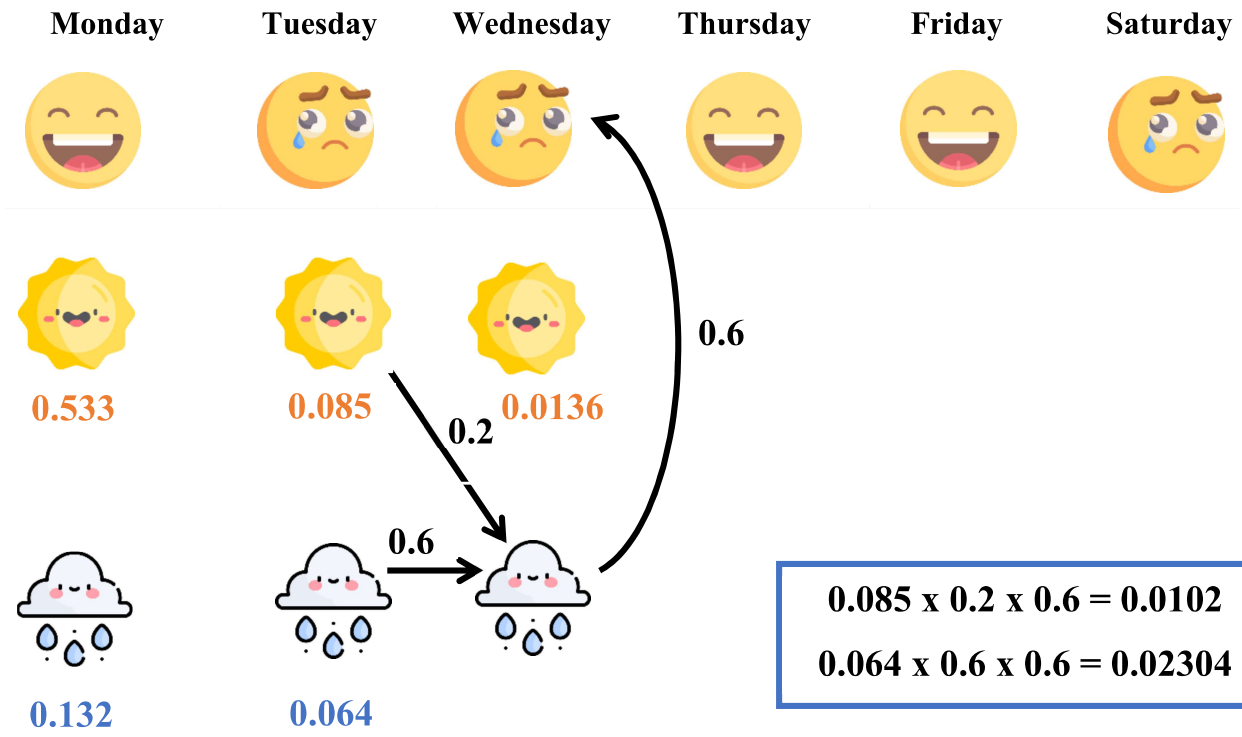
Tuesday


Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.2
0.8
0.533 x 0.8 x 0.2 = 0.085
0.132 x 0.4 x 0.2 = 0.0106
0.533
0.4
0.132

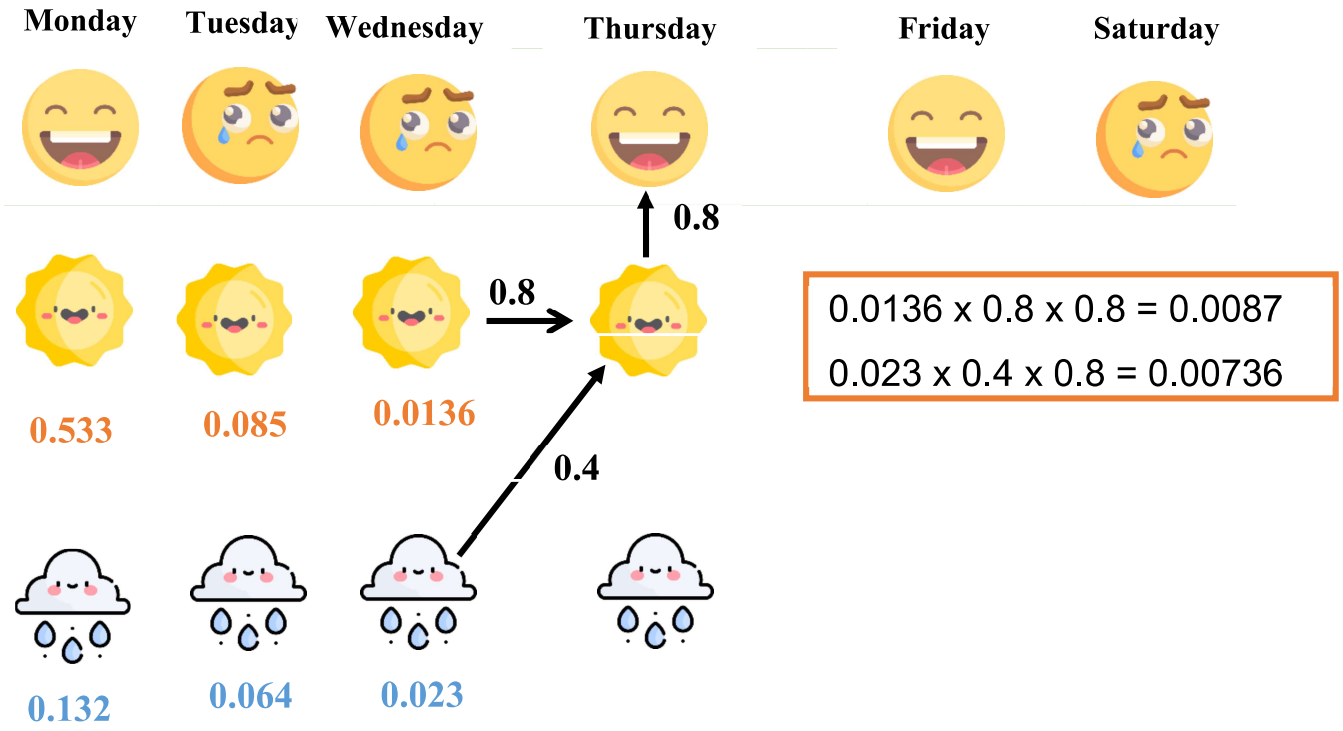

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.533
0.2
0.085
0.6
0.6
0.132
0.533 x 0.2 x 0.6 = 0.064
0.132 x 0.6 x 0.6 = 0.0475

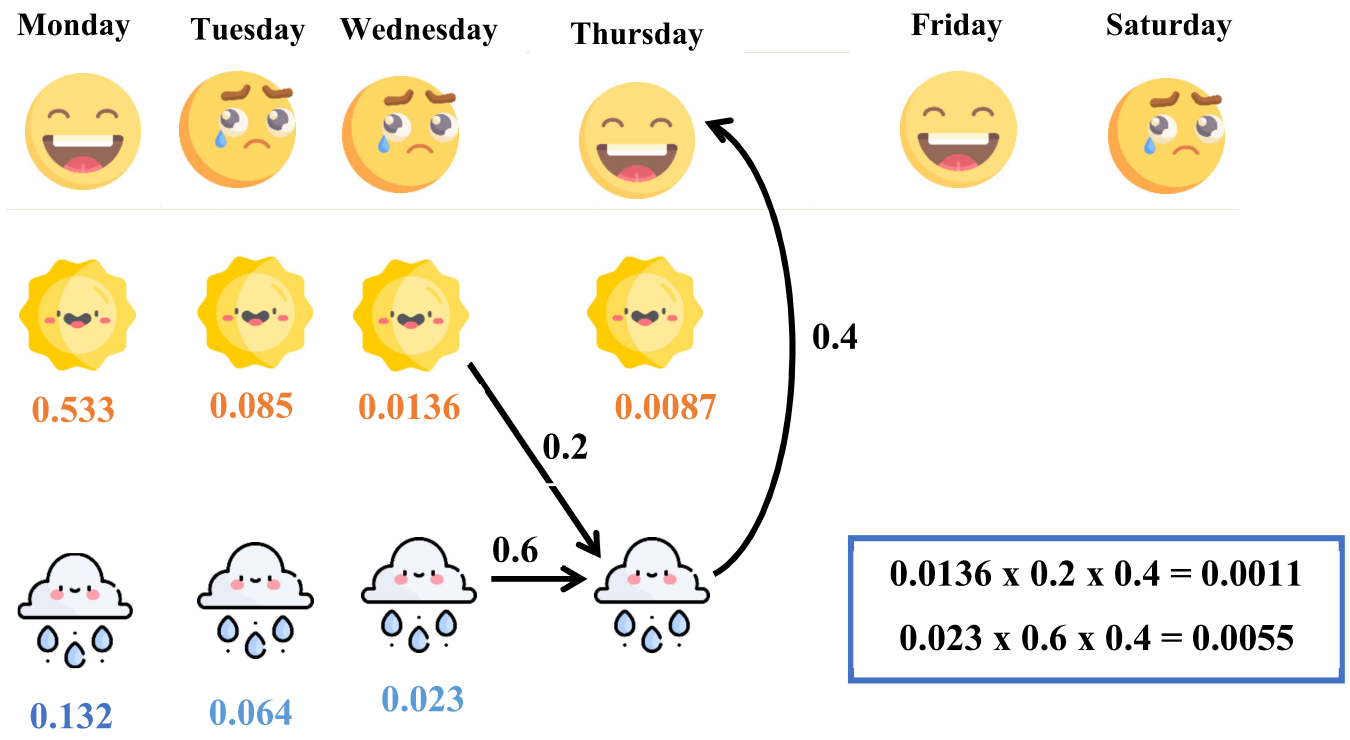
Wednesday


Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.2
0.8
0.4
0.533
0.085
0.132
0.064
0.085 x 0.8 x 0.2 = 0.0136
0.064 x 0.4 x 0.2 = 0.0051

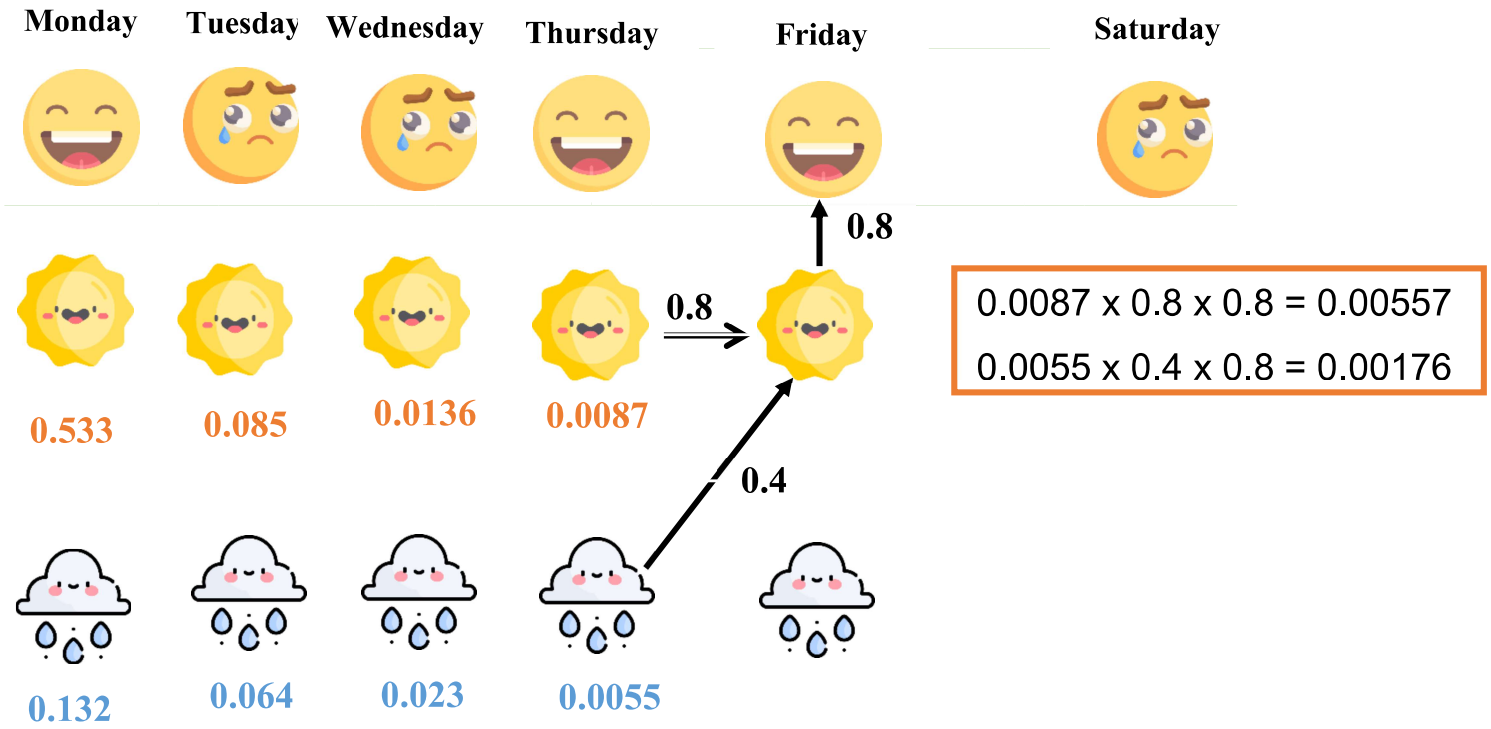

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.6
0.2
0.6
0.533
0.085
0.0136
0.132
0.064
0.085 x 0.2 x 0.6 = 0.0102
0.064 x 0.6 x 0.6 = 0.02304

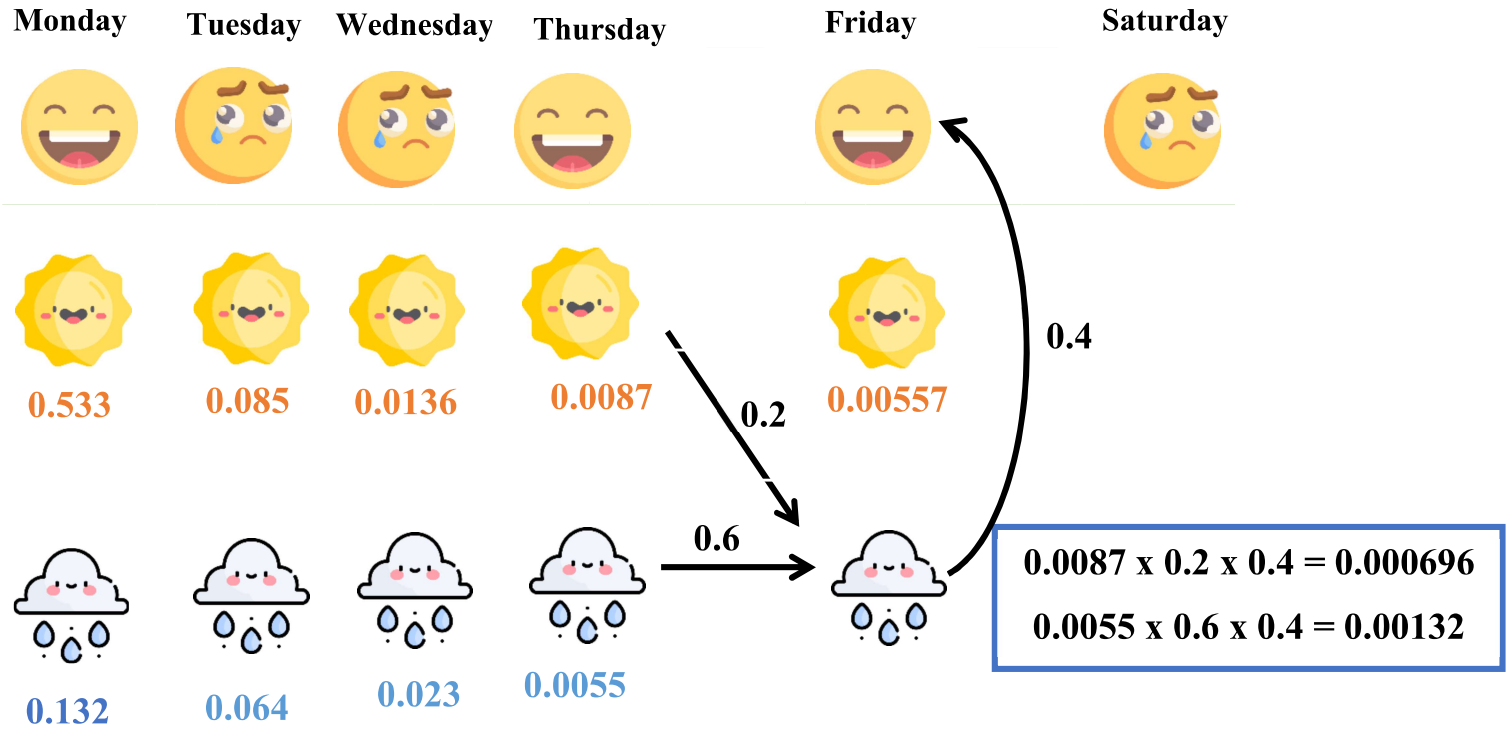
Thursday


Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.8
0.8
0.4
0.533
0.085
0.0136
0.132
0.064
0.023
0.0136 x 0.8 x 0.8 = 0.0087
0.023 x 0.4 x 0.8 = 0.00736

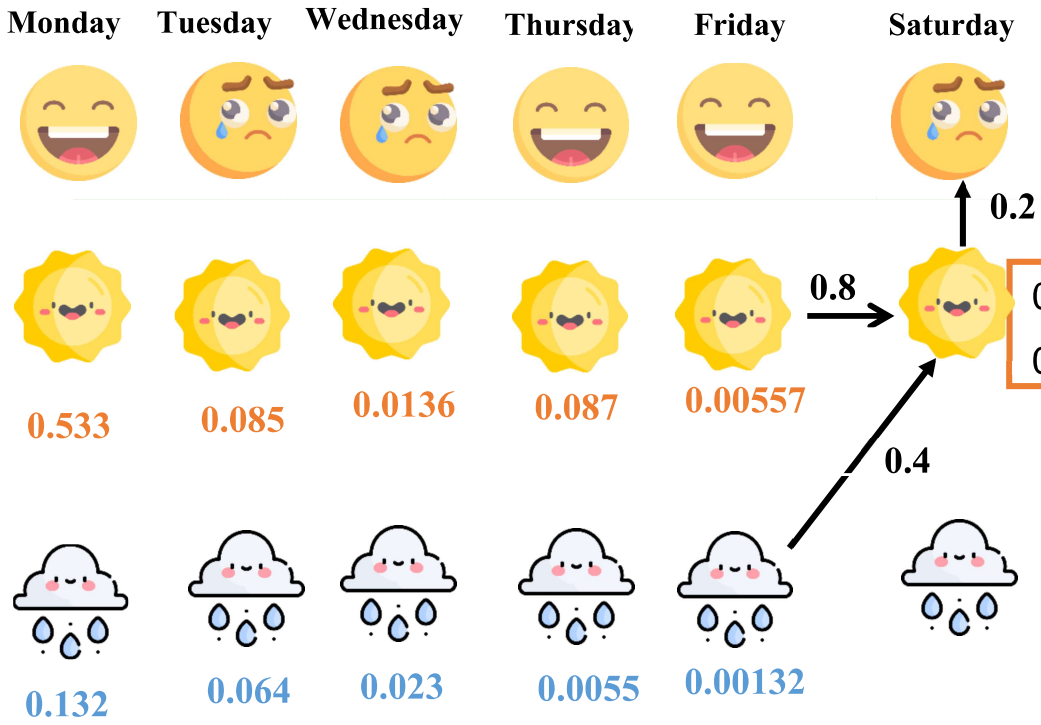

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.4
0.2
0.6
0.533
0.085
0.0136
0.0087
0.132
0.064
0.023
0.0136 x 0.2 x 0.4 = 0.0011
0.023 x 0.6 x 0.4 = 0.0055

Friday


Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.8
0.8
0.4
0.533
0.085
0.0136
0.0087
0.132
0.064
0.023
0.0055
0.0087 x 0.8 x 0.8 = 0.00557
0.0055 x 0.4 x 0.8 = 0.00176

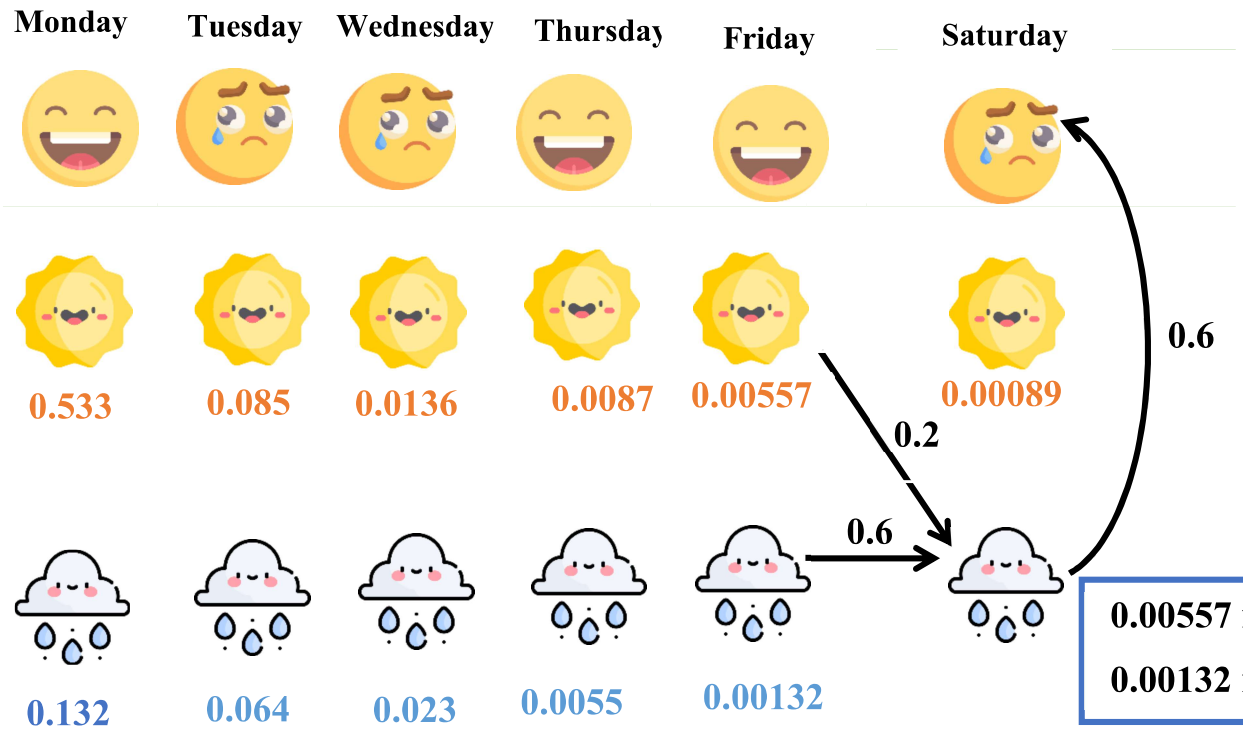

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
0.4
0.2
0.6
0.533
0.085
0.0136
0.0087
0.00557
0.132
0.064
0.023
0.0055
0.0087 x 0.2 x 0.4 = 0.000696
0.0055 x 0.6 x 0.4 = 0.00132

Saturday



0.00557 x 0.8 x 0.2 = 0.00089

0.00132 x 0.4 x 0.2 =0.000106



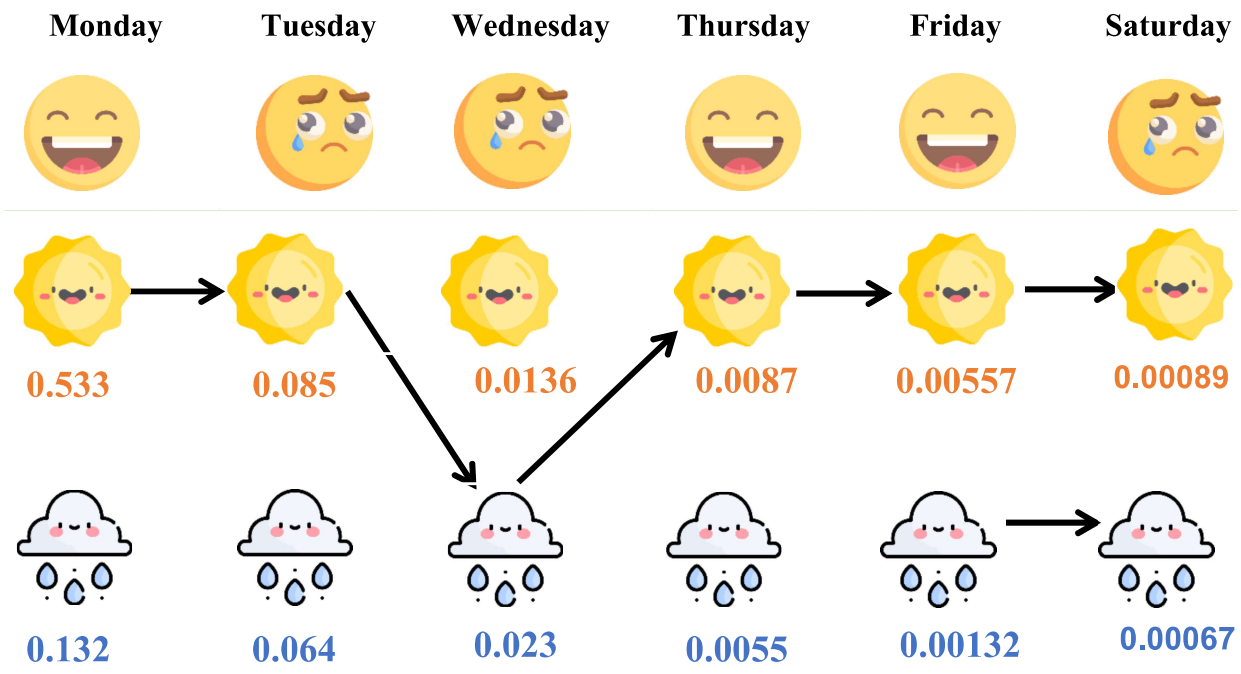
0.00557 x 0.2 x 0.6 = 0.00067

0.00132 x 0.6 x 0.6=0.000475

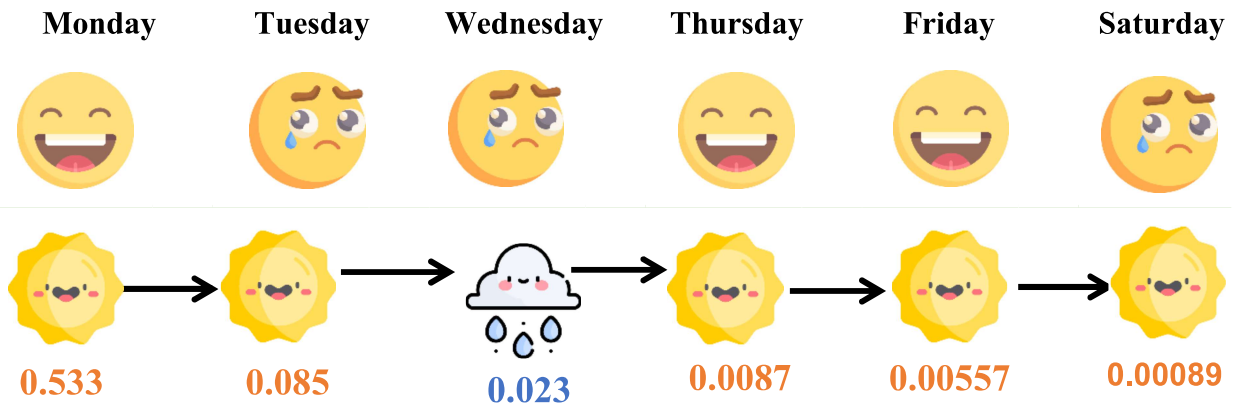
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					
					
0.533	0.085	0.0136	0.0087	0.00557	0.00089
					
0.132	0.064	0.023	0.0055	0.00132	0.00067

		
Monday	0.67 x 0.8 = **0.533**	0.33 x 0.4 = **0.132**
Tuesday	0.533 x 0.8 x 0.2 = **0.085** 0.132 x 0.4 x 0.2 = 0.0106	0.533 x 0.2 x 0.6 = **0.064** 0.132 x 0.6 x 0.6 = 0.0475
Wednesday	0.085 x 0.8 x 0.2 = **0.0136** 0.064 x 0.4 x 0.2 = 0.0051	0.085 x 0.2 x 0.6 = 0.0102 0.064 x 0.6 x 0.6 = **0.023**
Thursday	0.0136 x 0.8 x 0.8 = **0.0087** 0.023 x 0.4 x 0.8 = 0.00736	0.0136 x 0.2 x 0.4 = 0.0011 0.023 x 0.6 x 0.4 = **0.0055**
Friday	0.0087 x 0.8 x 0.8 = **0.00557** 0.0055 x 0.4 x 0.8 = 0.00176	0.0087 x 0.2 x 0.4 = 0.000696 0.0055 x 0.6 x 0.4 = **0.00132**
Saturday	0.00557 x 0.8 x 0.2 = **0.00089** 0.00132 x 0.4 x 0.2 =0.000106	0.00557 x 0.2 x 0.6 = **0.00067** 0.00132 x 0.6 x 0.6 = 0.000475

Select the weather with higher probabilities



Result



Short Explanation of Baum-Welch algorithm

Baum-Welch Algorithm is the special case of expectation-maximization algorithm (E-M Algorithm). It is used to find the unknown parameters of a hidden Markov model (HMM). The forward and the backward formulas gives the expected hidden states given the observed data and the set of parameter matrices before-tuned.