



Mathematics for A.I.

Linear Programming in Resource Allocation - 1500

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1. Introduction

1.1 Resource Allocation Background

The division of resources among alternative options with the goal of maximizing total return or minimizing total expenses is known as an allocation issue. A collection of resources in predetermined quantities, a series of tasks to be completed, each requiring a certain quantity of resources, and a set of expenses or returns for each task and resource are the components of such issues. The challenge is figuring out how much of each resource to use for each task [1].

1.2 Linear Programming Background

A mathematical model whose needs and aim are represented by linear relationships can be optimized using a technique known as linear programming (LP), also referred to as linear optimization.

To facilitate deeper comprehension, suppose a postman must deliver 6 letters in a day from the post office (located at point A) to different houses (P, Q, R, S, T). The distance between the houses is indicated on the lines as given in 'Figure 1 – Distance between'. If the postman wants to find the shortest route that will enable him to deliver the letters as well as save on fuel, then it becomes a linear programming problem. Thus, LP will be used to get the optimal solution which will be the shortest route in this example [2].

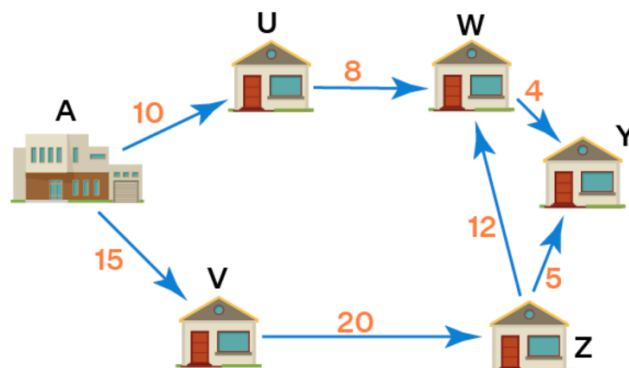


Figure 1 – Distance between houses.

1.3 Problem statement

Organizations face the problem of properly allocating resources to achieve great results while adhering to limits in many real-world circumstances. Such optimization challenges may be systematically addressed with linear programming which creates an objective function and limitations before using mathematical methods of optimization to find the best possible solutions.

2. Numerical Methods for Linear Programming

2.1 Introduction

2.2 Linear Programming formula

Firstly, the basic structure of LP problem is being depicted in terms of a canonical form when optimizing objective functions under constraints, this is shown as [3].

$$\begin{aligned} \max f &= c_1x_1 + c_2x_2 + \cdots + c_px_p \quad \text{s.t.} \\ a_{11}x_1 + a_{12}x_2 + \cdots + a_{1p}x_p &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2p}x_p &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mp}x_p &\leq b_m \\ x_1, x_2, \dots, x_p &\geq 0. \end{aligned}$$

Equation 1 – canonical form of linear programming problem

Where:

f – objective function (a hyperplane)

m – the first m inequalities are the structural constraints

p – the last p inequalities are the nonnegativity conditions

The solution to equation 1 is found within the p -dimensional feasible region k , formed by the constraint system and nonnegativity conditions [3].

A linear programming problem involves decision variables (x , y), an objective function (Z), constraints, and non-negative restrictions. The objective function maximizes or minimizes the solution, while constraints limit the variables' value, ensuring they always have non-negative values [2].

Linear Programming general formula can be expressed as.

Objective Function: $Z = ax + by$

Constraints: $cx + dy \leq e, fx + gy \leq h$. The inequalities can also be " \geq "

Non-negative restrictions: $x \geq 0, y \geq 0$

Equation 2 – General formula of linear programming problem

To solve LP problems.

1. Identify the decision variables.
2. Write up the goal function. Verify if the function should be maximized or decreased.
3. Write down the constraints.
4. Make sure that the variables used for the choice are larger than or equal to 0. (Non-detrimental constraint)
5. Solve the LP problem using the preferred method.

2.3 Simplex Method

The simplex method is a fast and valuable tool for solving complex linear programming problems. However, its slowness in certain problems has puzzled theoreticians. The ellipsoid method, a new algorithm, has shown little prospect of outperforming the simplex method in practice [4]. It involves repeatedly shifting from one variable to another along the borders of the feasible region.

2.4 Interior point Method

Interior Point Methods are optimization algorithms used to find the optimal solution of mathematical problems by moving from one point on the objective function to another in the feasible region. They are commonly used for linear and nonlinear programming problems. The primal-dual interior-point method is the most theoretically elegant and successful computationally, developed by researchers like Kojima, Monteiro, McShane, Choi, and Lustig [5].

Interior point methods and, simplex method are two examples of optimization techniques that are made possible by linear programming, which implies linear objective function and constraints. Larger issues are difficult for the simplex approach to solve because of the exponential rise of possible solutions. However, it is effective for small to moderate difficulties. Although they can handle non-linear constraints more effectively and are appropriate for bigger problems, interior point approaches may demand more time and processing resources [6]. Hence for the purpose of this report, simplex method is being used to solve problems.

3. Simplex method workings

There are 3 main steps to simplex method. Assuming there is a company with two products A and B, it has limited resources in terms of labor materials. Each unit of A requires 2 hours of labor and 3 units of material, as B requires an hour and 2 units. The profit per units is 5 for A and 4 for B. we will use simplex method to show how the company can maximize it's profits given the source constraints.

Formulation of the objective function and constraints

x_1 : Number of units of A

x_2 : Number of units of B

Objective function:

$$Z = 5x_1 + 4x_2$$

Constraints:

Labor constraint : $2x_1 + x_2 \leq 40$ (total available labor hours)

Material constraint : $3x_1 + 2x_2 \leq 60$ (total available material units)

Non – negativity constraints : $x_1, x_2 \geq 0$ (total available labor hours)

2.5 Initial simplex matrix

$$2x_1 + x_2 + s_1 = 40$$

$$2x_1 + 2x_2 + s_2 = 60$$

x	y	s_1	s_2
5	4	0	0
2	1	1	0
3	2	0	1

Where:

- Row 1 represents the coefficients of the objective function (unit profit).
- Row 2 represents the labor constraint with slack variable S1.
- Row 3 represents the material constraint with slack variable S2.

2.6 Determine the pivot column and exiting variable.

Identify the column with the highest negative entry. This is called the pivot column.

As seen product A is has the highest negative entry of 5. And the lowest ratio is the labor constraint which is the exiting variable.

Row 2 is being divided by 2 to make the pivot element 1 and row 1 is being subtracted 2 times row 2 from row 1 to make other elements 0.

$$\begin{array}{cccc|c}
 x & y & s_1 & s_2 & \\
 1 & 0 & 2 & -1 & 30 \\
 0 & 1 & -1 & 2 &
 \end{array}$$

2.7

2.8 The step is being repeated till the bottom-most row has no negative entries.

$$\begin{array}{cccc|c}
 x & y & s_1 & s_2 & \\
 1 & 0 & \frac{2}{3} & -\frac{1}{3} & 10 \\
 0 & 1 & -\frac{1}{3} & \frac{2}{3} &
 \end{array}$$

2.9 Optimal values

$$x_1 = 10 \text{ units of product A}$$

$$x_2 = 10 \text{ units of product B}$$

$$\text{Maximum profit : 90}$$

By following these iterative steps and updating the tableau accordingly, we have determined the optimal production quantities of products A and B that maximize the company's profits within the given resource constraints.

4. Discussion and conclusion

Linear Programming has several advantages in with regards to problem optimizations as it enables effective resolution of issues in which the design variables' goal function is linear producing fast calculation times and unique solutions. It's necessity for the objective function and constraints to be linear that may not adequately capture real world problems is its limitation [7].

The simplicity and efficiency of the simplex method makes it a popular solution for small to moderate sized linear programming problems. However, it struggles with large scale problems due to its exponential increase in possible solutions and its tendency to get stuck at local optima. This emphasizes the need for an alternative optimization algorithm such as interior point, which uses a primal dual approach to navigate complex solution spaces efficiently, improving convergence properties and scalability for larger problem instances [8].

5. References

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