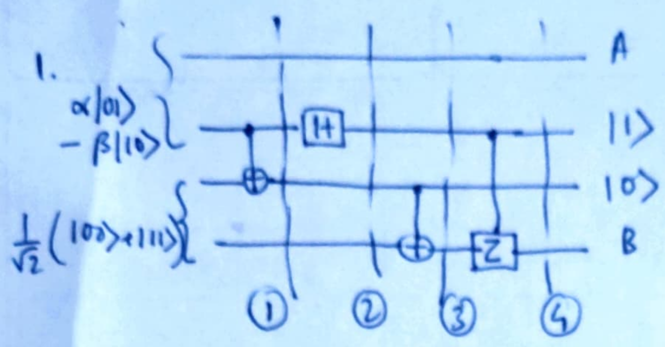
Mainak Nandi Assignment - 1 23N0325

1.



$$\left(\alpha|01\rangle - \beta|10\rangle\right)\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(\alpha|0100\rangle + \alpha|0111\rangle - \beta|1000\rangle - \beta|1011\rangle\right)$$

After ①, $|\psi\rangle = \frac{1}{\sqrt{2}}\left(\alpha|0110\rangle + \alpha|0101\rangle - \beta|1000\rangle - \beta|1011\rangle\right)$

~~② $|\psi\rangle = \frac{1}{\sqrt{2}}\alpha|0-10\rangle$~~

② $$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)|10\rangle + \alpha|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)|10\rangle \right.$$
$$\left. - \beta|1\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle|0\rangle - \beta|1\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|11\rangle\right]$$

$$= \frac{1}{2}\left[\alpha\left\{|0010\rangle - |0110\rangle + |0010\rangle - |0110\rangle\right\}\right.$$
$$\left. - \beta\left\{|1000\rangle + |1100\rangle + |1011\rangle + |1111\rangle\right\}\right]$$

After ③ $$= \frac{1}{2}\left[\alpha\left\{|0011\rangle - |0111\rangle + |0011\rangle - |0111\rangle\right\}\right.$$
$$\left. - \beta\left\{|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right\}\right]$$

$$= \alpha\left\{|0011\rangle - |0111\rangle\right\} - \frac{\beta}{2}\left\{|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right\}$$

④ $$= \alpha\left\{|0011\rangle + |0111\rangle\right\} - \frac{\beta}{2}\left\{|1000\rangle + |1100\rangle + |1010\rangle + |1110\rangle\right\}$$

Now, 2nd qubit is $|1\rangle$ and ~~1st~~ 3rd qubit is $|0\rangle$

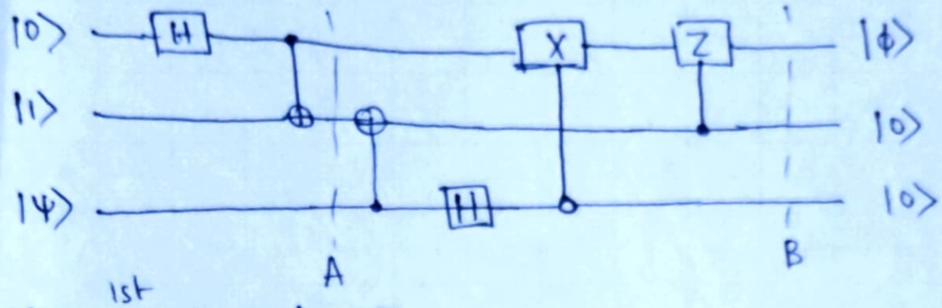
$\therefore$ The state is ~~α~~ $-\frac{\beta}{2}|1100\rangle$

$\therefore$ $A = -\frac{\beta}{2}|1\rangle$

$B = |0\rangle$

2. $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\Psi\rangle = |0\rangle \otimes |1\rangle \otimes |\psi\rangle$



After 1st Hadamard gate,

$$|\Psi\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |1\rangle \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\left(|0\rangle \otimes |1\rangle \otimes |\psi\rangle\right) + \left(|1\rangle \otimes |1\rangle \otimes |\psi\rangle\right)\right]$$

$$= \frac{1}{\sqrt{2}}\left[\left(|01\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)\right) + \left(|11\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)\right)\right]$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|010\rangle + \beta|011\rangle + \alpha|110\rangle + \beta|111\rangle\right]$$

At A, $|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\alpha|010\rangle + \alpha|100\rangle + \beta|011\rangle + \beta|101\rangle\right]$

Then; $|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\alpha|010\rangle + \alpha|100\rangle + \beta|001\rangle + \beta|110\rangle\right]$

After 2nd Hadamard,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\alpha|01\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + \alpha|10\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + \beta|00\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) + \beta|11\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{2}\left[\alpha|010\rangle + \alpha|011\rangle + \alpha|100\rangle + \alpha|101\rangle + \beta|000\rangle - \beta|001\rangle + \beta|110\rangle + \beta|111\rangle\right]$$

Then, $|\Psi\rangle = \frac{1}{2}\left[\alpha|110\rangle + \alpha|011\rangle + \alpha|000\rangle + \alpha|101\rangle + \beta|100\rangle - \beta|001\rangle + \beta|010\rangle + \beta|111\rangle\right]$

Finally, $|\Psi\rangle = \frac{1}{2}\left[-\alpha|110\rangle + \alpha|011\rangle + \alpha|000\rangle + \alpha|101\rangle + \beta|100\rangle - \beta|001\rangle + \beta|010\rangle - \beta|111\rangle\right]$

Now, last state is $|\phi\rangle \otimes |00\rangle$

$$\therefore |\Psi\rangle = \frac{1}{2}\left[\alpha|000\rangle + \beta|100\rangle\right]$$

$$\therefore |\phi\rangle = \alpha|0\rangle + \beta|1\rangle = |\psi\rangle$$

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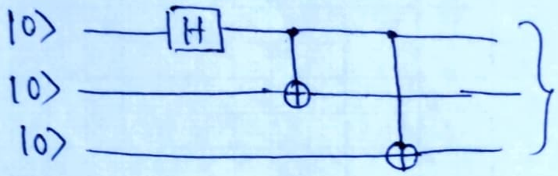
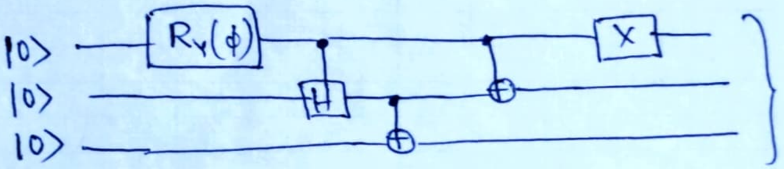
3. $$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$|\psi\rangle_{W} = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$

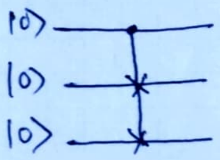


$|\psi\rangle_{GHZ}$



$|\psi\rangle_{W}$

4.



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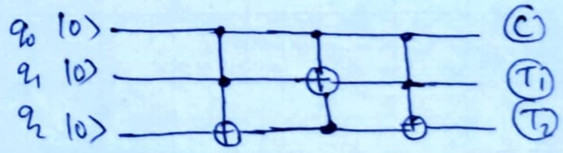

Toffoli : ~~too~~ controlled by q_0 & q_1 on q_2

Toffoli : controlled by q_0 & q_2 on q_1

Toffoli : controlled by q_0 & q_1 on q_2

Controlled swap gate

Truth table ($C = 1$)

C	T_1	T_2	T_1' (Output)	T_2' (Output)
1	0	0	0	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	1