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Cellular Automata with Memory

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Abstract. Standard Cellular Automata (CA) are ahistoric (memoryless): i.e., the new state of a cell depends on its neighbourhood configuration only at the preceding time step. Historic memory of all past iterations can be incorporated into CA by featuring each cell by a summary of all its past states. CA with memory turn out to be a natural and interesting generalization of the CA paradigm

INTRODUCTION

Standard Cellular Automata (CA) are ahistoric (Markovian): i.e., no memory of previous iterations except the last one is taken into account to decide the next one. Thus, if $\sigma_i^{(T)}$ is taken to denote the value of cell i at time step T , the site values evolve by iteration of the mapping: $\sigma_i^{(T+1)} = \phi(\sigma_j^{(T)} \in N_i)$, where ϕ is a rule operating on the neighborhood (N) of the cell i . The subject has been reviewed recently by Ilachinski [1] and Wolfram [2].

A natural variation of the basic paradigm is given when the unaltered rules (ϕ) actuate over cells featured by a summary of their past states (s): $\sigma_i^{(T+1)} = \phi(s_j^{(T)} \in N_i)$. We refer to these automata by considering historic memory as *historic* and to the standard ones as *ahistoric*.

The easiest way of taking past states into account is to feature every cell by means of its most frequent past state. In case of a tie, the cell will be featured by its last state. We will exemplify the effect of this kind of memory using an easy rule: Rule 1022, a two dimensional, two states ($k = 2$) rule operating on nearest neighbors ($r = 1$) which assigns a live state to any cell in whose neighborhood there would be at least one live cell. In the ahistoric model, a single site live cell would grow as fast as it can (i.e., at the speed of light) generating squares of live cells with their side size increasing two units with every time step. Things happen much more slowly in the historic model (see Fig. 1): the outer live cells, the perimeter of every square of live cells, are not featured as live cells until the number of times they *live* ($\sigma=1$) is equal to the number of times they were *dead* ($\sigma=0$). Then the automaton fires a new outer perimeter of live cells. Thus at most time steps there is a phase shift between the last configuration and the most frequent state configuration, which only coincide at the powers of two time steps.

Since the local rules of both *historic* and *ahistoric* automata remain unaltered, and the latest and most frequent states coincide after the first two iterations, the *historic* and *ahistoric* evolution patterns are the same till $T=3$. But after the third time step, the last and the most frequent states often differ, and consequently the patterns for the historic and ahistoric automata typically diverge at $T=4$. Most frequent memory causes

extinction starting from a single seed in a small number of rules, but the typical effect of this kind of memory is either the *punctuated equilibrium* growth described in Rule 1002 or the generation of short period oscillators which tend to appear fairly soon (see [3] and [4]).

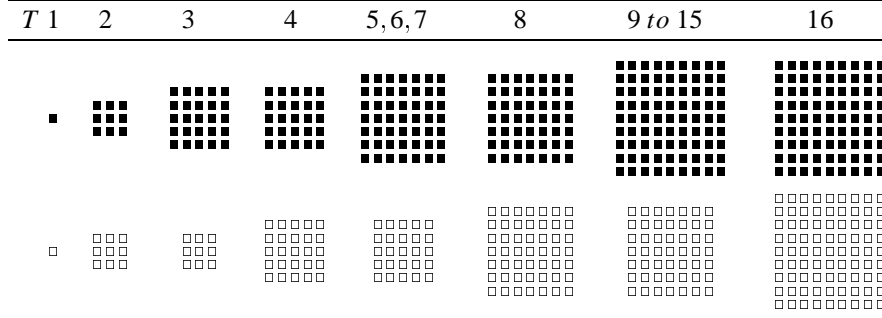


FIGURE 1. Evolving patterns of Rule 1022 starting from a single site live cell in the most frequent memory model. Evolution up to $T = 16$. Live cells: ■ last ($\sigma^{(T)} = 1$), □ most frequent ($s^{(T)} = 1$).

WEIGHTING MEMORIES

Historic memory can be weighted by applying a geometric discounting process in which the state obtained τ time steps before the last round, so $\sigma^{(T-\tau)}$ is actualized to the value: $\alpha^\tau \sigma^{(T-\tau)}$, α being the *memory factor* ($0 \leq \alpha \leq 1$). A given cell will be featured by the rounded weighted mean of all its past states. This well known mechanism fully ponders the last round ($\alpha^0 = 1$), and tends to *forget* the older rounds. Thus, after time step T , the unrounded weighted mean (m) of the states of a given cell will be given by (1), and in the $k=2$ scenario, the rounded weighted mean state (s) will be obtained by comparing m to 0.5 as in (2).

$$m_i^{(T)}(\sigma_i^{(1)}, \sigma_i^{(2)}, \dots, \sigma_i^{(T)}) = \frac{\sum_{t=1}^T \alpha^{T-t} \sigma_i^{(t)}}{\sum_{t=1}^T \alpha^{T-t}} \quad (1) \quad s_i^{(T)} = \begin{cases} 1 & \text{if } m_i^{(T)} > 0.5 \\ \sigma_i^{(T)} & \text{if } m_i^{(T)} = 0.5 \\ 0 & \text{if } m_i^{(T)} < 0.5 \end{cases} \quad (2)$$

The effect of keeping different degrees of memory by examining the transition from $\alpha = 1.0$ (fully historic) to $\alpha=0.0$ (ahistoric) was reported in Refs. [3] and [4] when

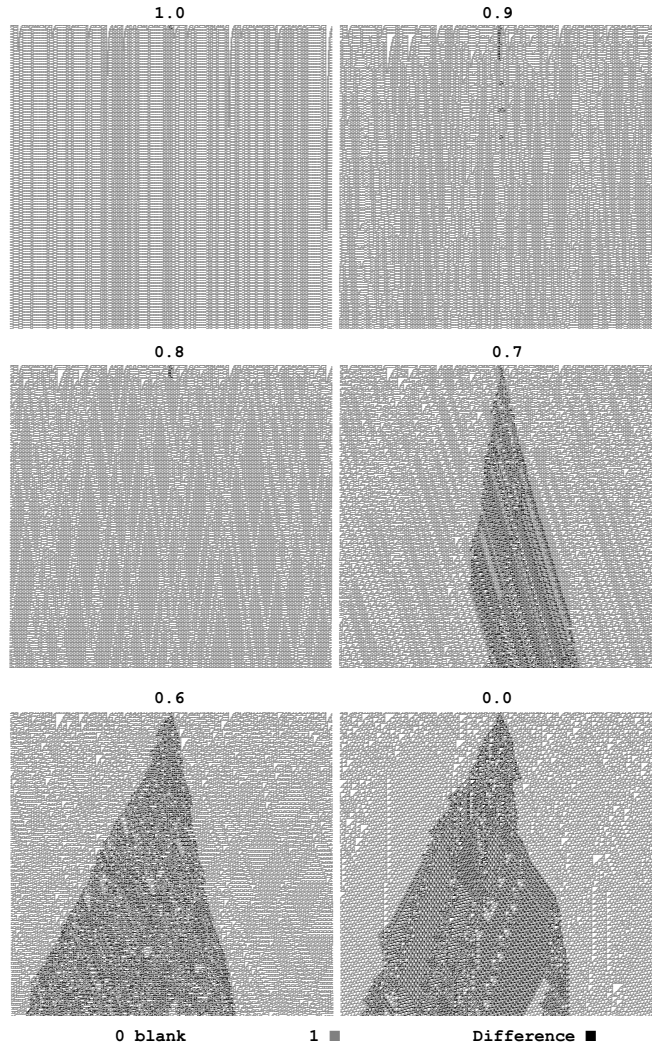


FIGURE 2. Evolving patterns of the *elementary* Rule 110 from an initial configuration assigned at random. Historic memory is geometrically discounted with a memory factor (α) indicated over every spatio-temporal pattern. Evolution up to $T=377$ in a lattice with 377 sites. *Dead* cells are coded as blank, *live* cells in grey. Darker cells show the damage induced when reversing the state of the central site.

starting with a single site live cell in the $k=2, r=1$ scenario ¹. As a rule, discounting, as implying less historic information retained, implies an approach to the ahistoric model; but the transition which decreases the memory factor α , from $\alpha = 1.0$ (fully historic) to $\alpha = 0.0$ (ahistoric model), is not always regular. Deviations from the expected evolution have been traced. Figure 2 shows the effect of memory on one of the most important *elementary* rules (Rule 110) starting from a random initial configuration. Memory gradually transforms the *chaotic* spatio-temporal pattern in the ahistoric scenario into the structured (periodic Wolfram's Class II) pattern in the fully historic model. Darker cells indicate differences in patterns resulting from change in the value of its initial center site. The perturbations in the ahistoric model propagate to the right and left at great velocity at any time (*butterfly effect*). As an overall rule, historic memory produces a preserving effect (that Fig. 2 illustrates) regarding the damage induced by the reversal of a single site value.

OTHER MEMORIES

A number of average-like memory mechanisms can readily be proposed by writing (1) as : $m_i^{(T)} = \frac{\Omega_i^{(T)}}{\Delta(T)}$, with $\Omega_i^{(T)} = \sum_{t=1}^T \delta(t) \sigma_i^{(t)}$ and $\Delta(T) = \sum_{t=1}^T \delta(t)$. Thus, the decay in the weighting factor can be implemented in an exponential way instead of in the geometric one: $\delta(t) = e^{-\beta(T-t)}$ (equivalent to $\alpha = e^{-\beta}$, β ranging in \mathbb{R}^+); or designed in an inverse way so that the older rounds are remembered more than the more recent ones by choosing: $\delta(t) = \alpha^{t-1}$.

All the weighting memory mechanisms described are *holistic* in their information demands: for evaluating the weighted mean of a cell, it is necessary to know its whole states series in time. In order to avoid this demand, previous states can be pondered with $\delta(t) = t^c$ or $\delta(t) = c^t$. This mechanism is not *holistic* but *accumulative* in its demand of knowledge of past history : for evaluating $\Omega_i^{(T)}$ it is not necessary to know the whole $\{\sigma_i^{(t)}\}$ series because it is determined by the *accumulation* of the contribution of the last state ($\delta(T)\sigma_i^{(T)}$) to the already accumulated $\Omega_i^{(T-1)}$. The larger the c , the more heavily the recent past is taken into account and consequently the closer the scenario to the ahistoric one. Choosing integer c values allows working only with integers (*à la* CA), which is a clear computational advantage of this model over the geometric discount memory. But this memory mechanism has a serious drawback : t^c *explodes*, even for $c=2$, as T grows (see [7]). The mere unrounded mean states can be used to embed memory in continuous valued CA (such as *coupled map lattices* or *fuzzy CA*): $\sigma_i^{(T+1)} = \phi(m_j^{(T)} \in N_i)$. Departing from the weighted mean memory scheme, cells can be featured by the parity

¹ In previous works we analyzed the effect of memory in the historic formulation of Conway's Game of Life [5] and in the spatial formulation of the Prisoner's Dilemma [6].

of the sum of previous states ($s_i^{(T)} = \sum_{t=1}^T \sigma_i^{(t)} \bmod 2$) or, in the *interest* memory scenario, a cell will be featured as dead if all its previous states are equal (zero *interest* means boring), and as alive if any of them are different: $s_i^{(T)} = 0$ iff $\sigma_i^{(1)} = \sigma_i^{(2)} = \dots = \sigma_i^{(T)}$. The standard models of CA and CA with memory can be combined by considering two types of cell characterization of the neighborhood N . Thus for a subset of N , let it be S , the cells can be featured by their last state and the remaining \bar{S} by their historic states. Thus, $\sigma_i^{(T+1)} = \phi(\sigma_i^{(T)} \in S, s_i^{(T)} \in \bar{S})$.

It should be emphasized that the memory mechanism just described is different from that of other CA with memory reported in the literature. Typically, they determine the configuration at time $T+1$ in terms of the configurations at both time T and time $T-1$. Particularly interesting is the second order in time (memory of capacity two) mechanism of the form: $\sigma_i^{(T+1)} = \phi(\sigma_j^{(T)} \in N_i) \ominus \sigma_i^{(T-1)}$. This mechanism guarantees the invertibility and the time-reversal invariance, i.e. reversibility, on account of the algebraic properties of the subtraction module k (noted \ominus). Thus, defining $\sigma_i^{(T-1)}$ in the above equation: $\sigma_i^{(T-1)} = \phi(\sigma_j^{(T)} \in N_i) \ominus \sigma_i^{(T+1)}$. This reversibility device operates independently of the first component of the second member, so incorporating memory into it, the mechanism: $\sigma_i^{(T+1)} = \phi(s_j^{(T)} \in N_i) \ominus \sigma_i^{(T-1)}$ remains reversible (see [8]).

CHALLENGES

We have introduced the type of CA with Memory (**CAM**) and surveyed its properties in the simplest scenario: a single site seed. A more complete analysis of **CAM** is left for future work. Starting with randomly assigned live cells will allow the study of the statistical and algebraic properties of **CAM**, a study to be extended to rules with range $r = 2$ and $k=3$. It would also be interesting to investigate how memory could be used in the Margolus neighborhood as well as Morita's Partitioned CA, which are used for reversible automata.

Embedding memory into CA implies additional computing time and memory demands, especially when considering geometric discount, as it is necessary to preserve the whole states series in time, so to study asymptotic properties in large lattices, with a significant number of different simulations, it is expected to resort to parallel computing.

CAM structurally exhibit a growth inhibition and a slowdown in error propagation features found exceptionally in standard CA. The extent of these effects can be modulated by varying the memory factor (α). This could mean a potential advantage of **CAM** over standard CA as a new tool for modelling slow diffusive growth from small regions (a common phenomenon in nature) and in the design of fault tolerant devices: resilience in the face of errors is an important issue when considering systems with a large number of components, in which faults are highly probable. The robust **CAM** could play a role in this (nanotechnologic) scenario. Reversible **CAM** applications in cryptography are also possible.

In the mean-time, the introduction of CAM contributes to the study of CA "Zoology", i.e., the study of CA for its own sake. Thus, CAM increase CA-(*bio*)diversity.

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