

# Maine-Québec Number Theory Conference

University of Maine  
October 4<sup>th</sup> and 5<sup>th</sup>, 2025

## Organizers

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| Saturday                |  |  |  |   |
|-------------------------|--|--|--|---|
|                         | DPC 105  | DPC 107  | DPC 115  | DPC 117   |
| 8:50-9:00<br>DPC 100    | Opening Remarks<br><b>Emily Haddad</b><br>Dean of CLAS<br><b>Joan Ferrini-Mundy</b><br>President of UMaine   |  |  |   |
| 9:00-9:50<br>DPC 100    | <b>Drew Sutherland</b><br>(MIT)<br><i>L-functions from nothing</i>   |  |  |   |
| 9:50-10:00              | Break  |  |  |   |
| 10:00-10:20             | <b>Mike Cerchia</b><br>(University of Maine)<br><i>Quadratic points on ell-adic modular curves</i>   | <b>Félix Baril Boudreau</b><br>(U Luxembourg/U Concordia)<br><i>Abelian varieties with homotheties</i>                                   | <b>Lucile Devin</b><br>(CRM-CNRS & U Littoral)<br><i>Vertical distribution of zeros of L functions extending the uncond. supp.</i> |   |
| 10:20-10:30             | Break  |  |  |   |
| 10:30-10:50             | <b>Sheela Devadas</b><br>(University of Maine)<br><i>Ceresa cycles, covers of curves, and unlikely intersections</i>                                     | <b>Hugo Chapdelaine</b><br>(Laval University)<br><i>Equivariant eta invariants for cusp ends of Hilbert modular varieties</i>            | <b>Doyon Kim</b><br>(University of Bonn)<br><i>Mellin-Barnes integrals of Bessel functions on <math>GL(n)</math></i>               |   |
| 10:50-11:20             | Coffee   |  |  |   |
| 11:20-11:40             | <b>Rusiru Gambheera</b><br>(UC, Santa Barbara)<br><i>Structure of fine/signed Mordell–Weil groups</i>  | <b>Louis Gaudet</b><br>(UMass Amherst)<br><i>Counting biquadratic number fields that admit quaternionic and dihedral extensions</i>      | <b>Paul Kinlaw</b><br>(U Maine Presque Isle)<br><i>On Even Pseudoprimes</i>  |   |
| 11:40-11:50             | Break  |  |  |   |
| 11:50-12:10             | <b>Sung Min Lee</b><br>(Wake Forest University)<br><i>Cyclicity and Koblitz Conj's in Arithmetic Progressions: Bias Across Cong. Classes</i>             | <b>Ashwin Iyengar</b><br>(Unaffiliated)<br><i>The Casselman–Shalika Formula</i>  | <b>Berend Ringeling</b><br>(U. de Montréal (CRM))<br><i>Zeros of Modular Forms</i>   |   |
| 12:10-1:50              | Lunch  |  |  |   |
| 1:50-2:40<br>DPC 100    | <b>Wei Zhang</b><br>(MIT)<br><i>Gross–Zagier formula in high dimensions</i>  |  |  |   |
| 2:40-2:50               | Break  |  |  |   |
| 2:50-3:05<br>(students) | <b>Arindam Bhattacharyya</b><br>(UMass Amherst)<br><i>Class Groups of Kummer Extensions associated to CM Elliptic Curves</i>                             | <b>Paresh Arora</b><br>(Louisiana State University)<br><i>Well-Poised Hypergeometric Functions</i>                                       | <b>Steven Creech</b><br>(Brown University)<br><i>Modular Murmurations and Trace Formulas</i>                                       | <b>Haochen Wu</b><br>(Dartmouth College)<br><i>Hilbert modular forms from orthogonal modular forms on binary lattices</i> |
| 3:05-3:35               | Coffee   |  |  |   |
| 3:35-3:50<br>(students) | <b>Vittoria Cristante</b><br>(Tufts University)<br><i>Torsion Subgroups Over the Compositum of (Potentially) Generalized <math>S_4</math> Extensions</i> | <b>João Campos-Vargas</b><br>(Stanford University)<br><i>Primes of the form <math>x^2 + dy^2</math> and interactions of class groups</i> | <b>Elias Dubno</b><br>(U Zurich / McGill)<br><i>A Central Limit Thm for the Winding Number of Low-Lying Closed Geodesics</i>       | <b>Zecheng Yi</b><br>(Boston University)<br><i>Continuous group cohomology of Steinberg representations</i>               |
| 3:50-4:00               | Break  |  |  |   |
| 4:00-4:20               | <b>Jack Petok</b><br>(Colby College)<br><i>Zeta functions of <math>K3</math> categories over finite fields.</i>  | <b>Andrew Schultz</b><br>(Wellesley College)<br><i>Bloch-Kato profinite groups</i>   | <b>Jakob Streipel</b><br>(University at Buffalo)<br><i>Sub-Weyl bound for <math>GL(2)</math> via trivial delta</i>                 | <b>Max Weinreich</b><br>(Harvard University)<br><i>Height growth in pentagram maps</i>                                    |
| 4:20-4:30               | Break  |  |  |   |
| 4:30-4:50               | <b>Tristan Phillips</b><br>(Dartmouth College)<br><i>Unbounded average selmer ranks in torsion families.</i>   | <b>Karen Taylor</b><br>(Bronx CC, CUNY)<br><i>A Sturm Bound for Quaternionic Modular Forms</i>   | <b>Jesse Thorner</b><br>(U Illinois U-C)<br><i>Remarks on Landau–Siegel zeros</i>  |   |

| Sunday                    |   |  |  |   |
|---------------------------|---|--|--|---|
|                           | DPC 105   | DPC 107  | DPC 115  | DPC 117   |
| 9:00-9:15<br>(students)   | <b>Dillon et. al.</b><br><i>On Second Moment<br/>Distributions Associated<br/>to Elliptic Surfaces</i>            | <b>Hazem Hassan</b><br>(McGill University)<br><i>p-adic higher<br/>Green's Functions</i>                           | <b>Mohammad Hamdar</b><br>(Concordia University)<br><i>Hecke L-functions<br/>away from the central line</i>                                  |   |
| 9:15-9:25                 | Break   |  |  |   |
| 9:25-9:40<br>(students)   | <b>Emir Eray Karabiyik</b><br>(Cornell University)<br><i>A classification of low genus<br/>modular curves</i>     | <b>Seong Eun Jung</b><br>(UMass Amherst)<br><i>Modular symbols over<br/>function fields of<br/>elliptic curves</i> | <b>Kwon et. al.</b><br><i>Breaking Universality<br/>in n-level Densities<br/>of Cuspidal Newforms</i>  |   |
| 9:40-10:10                | Coffee  |  |  |   |
| 10:10-10:25<br>(students) | <b>Pankaj Singh</b><br>(University of South Carolina)<br><i>Classifying Torsors of Tori<br/>via Brauer Groups</i> | <b>Swati</b><br>(U S Carolina, Columbia)<br><i>Explicit images for the<br/>Shimura Correspondence</i>              | <b>Siddharth R. Cherukara</b><br>(University of Oklahoma)<br><i>Atkin-Lehner decompositions<br/>for quaternionic<br/>modular forms</i>       | <b>Erick Ross</b><br>(Clemson University)<br><i>What is the asymptotic<br/>behavior of the Hecke<br/>polynomial coefficients?</i> |
| 10:25-10:35               | Break   |  |  |   |
| 10:35-10:50<br>(students) |   | <b>Nikita Lvov</b><br>(Unaffiliated)<br><i>Universality Results for<br/>Random Matrices<br/>over Local Rings</i>   | <b>Muthuvel et. al.</b><br><i>Centered Moments of<br/>Weighted One-Level Densities<br/>of GL(2) L-Functions</i>                              | <b>Iana Vranesko</b><br>(Williams College)<br><i>Using Gowers Norms in<br/>Improving Signal<br/>Recovery Conditions</i>           |
| 10:50-11:00               | Break   |  |  |   |
| 11:00-11:20               | <b>Rakvi</b><br>(The University of Maine)<br><i>Elliptic Curves, Isogenies,<br/>and Adelic Indices</i>            | <b>Tian Wang</b><br>(Concordia University)<br><i>Lower bounds for isogenies<br/>of CM elliptic curves</i>          | <b>Zhining Wei</b><br>(Brown University)<br><i>The weighted density conj.<br/>of standard L-functions ass'd<br/>to Hilbert modular forms</i> | <b>Gary Walsh</b><br>(University of Ottawa)<br><i>Revisiting an Equation<br/>of Richard Bumby</i>                                 |
| 11:20-11:30               | Break   |  |  |   |
| 11:30-12:20<br>DPC 100    | <b>Brooke Feigon</b><br>(CUNY City College of NY)<br><i>Ramanujan graphs<br/>and bigraphs</i>                     |  |  |   |

# Abstracts

## **Rakvi, The University of Maine**

### *Elliptic Curves, Isogenies, and Adelic Indices*

Let  $E$  be an elliptic curve without complex multiplication, defined over the rationals. A well-known theorem of Serre bounds the largest prime  $\ell$  for which the mod  $\ell$  Galois representation of  $E$  is nonsurjective. After proving the theorem, Serre asked whether a universal bound on the largest nonsurjective prime might exist. Although this question remains open, significant partial progress has been made. In particular, Lemos proved that the question has an affirmative answer for all  $E$  admitting a rational cyclic isogeny. In light of the considerable recent progress in understanding Galois representations of elliptic curves, Zywinia proposed a more ambitious conjecture about the possible adelic indices that can occur as  $E$  varies. In this talk, we will give an overview of some of the work in this area and discuss an ongoing project (joint with Kate Finnerty, Tyler Genao and Jacob Mayle) that aims to extend Lemos's result to prove Zywinia's conjecture for certain families of elliptic curves.

## **Swati, University of South Carolina, Columbia (student)**

### *Explicit images for the Shimura Correspondence*

For  $(r, 6) = 1$  with  $1 \leq r \leq 23$ , and a non-negative integer  $s$ , we define

$$\mathcal{A}_{r,s,N,\chi} = \{\eta(z)^r f(z) : f(z) \in M_s(N, \chi)\}.$$

In 2014, Yang showed that for  $F \in \mathcal{A}_{r,s,1,1_N}$ , the  $r$ -th Shimura image associated to the theta-multiplier  $\text{Sh}_r(F | V_{24}) = G \otimes \chi_{12}$  where  $G \in S_{r+2s-1}^{\text{new}}(6, -(\frac{8}{r}), -(\frac{12}{r}))$ . He proved a similar result for  $(r, 6) = 3$ . His proofs rely on trace computations in integral and half-integral weights.

In this talk, we provide a constructive proof of Yang's result. We obtain explicit formulas for  $\mathcal{S}_r(F)$ , the  $r$ -th Shimura lift associated to the eta-multiplier defined by Ahlgren, Andersen, and Dicks, when  $1 \leq r \leq 23$  is odd and  $N = 1$ . We also obtain formulas for lifts of Hecke eigenforms multiplied by theta-function eta-quotients and lifts of Rankin-Cohen brackets of Hecke eigenforms with theta-function eta-quotients.

## **Paresh Arora, Louisiana State University (student)**

### *Well-Poised Hypergeometric Functions*

Hypergeometric functions can be viewed as traces of Frobenius for certain Galois representations. In this talk, I will discuss how finite-field hypergeometric functions decompose into lower-rank pieces when the underlying data satisfy a special symmetry known as the "well-poised" condition. In particular, I will present reductions for the well-poised  ${}_3F_2(-1)$ ,  ${}_4F_3(1)$ , and  ${}_5F_4(-1)$  cases.

**Arindam Bhattacharyya, University of Massachusetts Amherst (student)**

*Class Groups of Kummer Extensions associated to CM Elliptic Curves*

Let  $F$  be an imaginary quadratic field and let  $E$  be an elliptic curve defined over its Hilbert class field  $H_F$  with CM by the maximal order of  $F$ . Let  $p \geq 5$  be a prime that splits in  $F$  as  $(p) = \mathfrak{p}\bar{\mathfrak{p}}$ . We use Galois cohomology to bound the  $p$ -rank of the class group of  $H_F(Q)$  where  $Q$  is a  $\mathfrak{p}^{th}$  root of a non-torsion point  $P$ . This gives us a CM elliptic curve of Schaefer-Stubbley's results on the  $p$ -rank of  $\mathbb{Q}(N^{1/p})$ .

**Félix Baril Boudreau, Université du Luxembourg/Université de Concordia**

*Abelian varieties with homotheties*

Let  $A$  be an Abelian variety defined over a number field  $K$ . The celebrated Bogomolov-Serre theorem states that, for any prime  $\ell$ , the  $\ell$ -adic representation of the absolute Galois group of  $K$  contains all  $c$ -th power homotheties, where  $c$  is a positive constant depending only on the dimension of  $A$ . If  $K$  is a global function field, the analogous statement fails in general, since Zahrin has shown the existence of ordinary Abelian varieties of positive dimensions defined over  $K$ , for which  $G_\ell$  only contains finitely many homotheties.

In this talk, I will discuss my ongoing joint work with Sebastian Petersen (University of Kassel), in which we prove, under suitable additional assumptions, an analogue of Bogomolov-Serre Theorem when  $K$  is a finitely generated field of positive characteristic.

**João Campos-Vargas, Stanford University (student)**

*Primes of the form  $x^2+dy^2$  and interactions of class groups*

Inspired by a theorem of Kaplansky, we show that any prime represented by two of the forms  $x^2+17y^2$ ,  $x^2+65y^2$ ,  $x^2+1105y^2$  is actually represented by all three. The same is true for  $x^2+34y^2$ ,  $x^2+66y^2$ ,  $x^2+1122y^2$ . We establish these results by investigating how class groups of different quadratic fields interact.

**Mike Cerchia, University of Maine**

*Quadratic points on  $\ell$ -adic modular curves*

A subgroup  $H$  of  $GL_2(\mathbb{Z}_\ell)$  corresponds to modular curve  $X_H$ , which roughly parametrizes elliptic curves whose associated  $\ell$ -adic Galois representation lands inside of  $H$ . The first task when trying to determine all possible images of Galois of  $E/K$  as  $K$  varies over all quadratic extensions is to determine which of these modular curves have an infinite number of quadratic points, which is what we do in this talk. This is joint work with Rakvi.

**Hugo Chapdelaine, Laval University**

*Equivariant eta invariants for cusp ends of Hilbert modular varieties*

Let  $X$  be a cusp end of a Hilbert modular variety  $Y$  associated to a totally real number field  $K$  of degree  $g$  over  $\mathbb{Q}$ . In particular,  $X$  is a manifold of real dimension  $2g - 1$  and it comes with a natural torus fibration  $F = \mathbb{R}^g/M \rightarrow X \rightarrow B$  where  $B$  itself is a real torus of dimension  $g - 1$  and  $M \subseteq K$  is a lattice. Choose an ordered  $\mathbb{Z}$ -basis  $\mathfrak{B} = (\epsilon_1, \dots, \epsilon_{g-1})$  of  $\pi_1(B) \simeq \mathbb{Z}^{g-1}$ , where the  $\epsilon_i$ 's are totally positive units of  $K$ . To the space  $X$  one can associate a Hilbert space  $V$  on which a certain self-adjoint Dirac operator  $A$  acts. In this talk we shall present a generalization of a theorem due to Atiyah-Donnelly-Singer which involves  $X$  and the pair  $(V, A)$ . As a first step, ADS proved that the value  $\eta_A(0)$ , where  $\eta_A(s)$  is the spectral eta function associated to the Dirac operator  $A$ , is equal to the special value  $L_X(0)$  where  $L_X(s)$  is a Shimizu L-function associated to  $X$ . In a second step, they prove that  $\eta_A(0)$  is actually equal to the topological signature defect  $\sigma(X)$  of  $X$ . Our main goal in this talk is to present a generalization of this result to a “non-standard equivariant setting” which we now explain. We start with a unitary character  $\chi : B \rightarrow S^1$  which, thanks to the choice of the ordered  $\mathbb{Z}$ -basis  $\mathfrak{B}$  of  $\pi_1(B)$ , corresponds to a specific loop  $\alpha_\chi : S^1 \rightarrow B$  which thus provides a class in  $\pi_1(B)$ . We then choose two commuting isometries  $\sigma$  and  $\tau$  of  $X$  of finite order so that our construction will depend on the choice of a triple  $(\alpha_\chi, \sigma, \tau)$ . In that way, the original setting considered by ADS corresponds to the trivial triple  $(e_0, \text{Id}_X, \text{Id}_X)$  where  $e_0$  is the class of the trivial loop on  $B$ . For the talk, we intend to explain the analogues of  $L_X(s)$  and  $\eta_A(s)$  in this larger setting.

**Siddharth Ramakrishnan Cherukara, University of Oklahoma (student)**

*Atkin-Lehner decompositions for quaternionic modular forms*

In this talk, I will discuss Atkin-Lehner decompositions for quaternionic modular forms. Building on Casselman's work, we use representation-theoretic results for local quaternionic division algebras to study these forms and describe their newspaces. Finally, I will outline applications of these results, including connections to classical modular forms and Eichler's basis problem.

**Steven Creech, Brown University (student)**

*Modular Murmurations and Trace Formulas*

Given a family  $\mathcal{F}$  of modular forms, a murmuration phenomenon is a correlation between the corresponding root numbers appearing in the functional equations of the associated  $L$ -functions of the modular forms and the Fourier coefficients of the modular forms. In this talk, I will outline how trace formulas can be used as a tool to study murmurations of modular forms in the weight aspect.

**Vittoria Cristante, Tufts University (student)***Torsion Subgroups Over the Compositum of (Potentially) Generalized  $S_4$  Extensions*

Let  $E/\mathbb{Q}$  be an elliptic curve and let  $K/\mathbb{Q}$  be a (possibly infinite) extension. An interesting problem is to describe the torsion subgroup  $E(K)_{\text{tors}}$ . This problem has been resolved for various types of extensions, with cases of particular interest being when  $K$  is the compositum of all quadratic or all cubic fields. In this talk, we present ongoing work toward understanding the torsion structure for the compositum of quartic extensions.

**Sheela Devadas, University of Maine***Ceresa cycles, covers of curves, and unlikely intersections*

The Ceresa cycle, first studied by Ceresa in the 1980s, is an example of a homologically trivial but algebraically nontrivial cycle on the Jacobian of a smooth curve. While the Ceresa cycle of a hyperelliptic curve is always zero, less is known about the set of genus  $g$  curves for which the Ceresa cycle vanishes or is torsion. In joint work with Padma Srinivasan, Toren D’Nelly-Warady, and Tejasi Bhatnagar we consider Ceresa cycles of families of curves with more than one map to an elliptic curve. We leverage dimension reduction techniques from classical algebraic geometry (such as intersection theory) to compute when the Ceresa cycle is torsion via points on an elliptic curve, rather than 1-dimensional subvarieties of the Jacobian.

**Lucile Devin, CRM-CNRS & Université du Littoral Côte d’Opale***Vertical distribution of zeros of  $L$  functions – extending the unconditional support*

Joint with Martin Čech, Daniel Fiorilli, Kaisa Matomäki and Anders Södergren. I will discuss the distribution of low-lying zeros of  $L$ -functions in families of degree two, for which, thanks to good trace formulas, we are able to extend the unconditional support in the Katz–Sarnak prediction.

**Lawrence Dillon, University of Washington (student)****Pramana Saldin, University of Wisconsin (student)***On Second Moment Distributions Associated to Elliptic Surfaces*

Define an elliptic surface fibered over projective space  $\pi: \mathcal{E}_T \rightarrow \mathbb{P}^1$  which can be thought of as a one-parameter family of smoothly varying elliptic curves. The fibers here can be written in short Weierstrass form,  $\pi^{-1}(t): y^2 = x^3 + A(t)x + B(t)$ . In 2024, Cheek et. al., in an effort to find candidates to disprove Steven J. Miller’s bias conjecture, estimated numerically the average value of  $B_2(p) = (A_2(p) - p^2)/p^{3/2}$  over many families, where  $A_2(p) = \sum_{t(p)} a_t(p)^2$  is the second moment of the trace of Frobenius. They plotted the distribution of  $B_2(p)$  (as  $p$  varies) and conjectured that the variance of that distribution is always an integer. We verify this conjecture in the case of pencils of cubics and, assuming a form of the general Sato-Tate conjecture for motives, provide evidence that this should hold generically.

There is also a way to construct a sequence of families from a base family. Simply take the polynomials  $A(T)$  and  $B(T)$  and replace them with  $A(T^n)$  and  $B(T^n)$ , yielding the  $n$ -cover surface  $\mathcal{E}_{T^n}$ . These surfaces have many interesting properties that we are able to investigate numerically. We give a few interesting examples and prove that for a certain class of families, the second moment over  $\mathcal{E}_{T^{p^n}}$  has variance  $n$  for any prime  $p$ .

**Elias Dubno, University of Zurich / McGill (student)**

*A Central Limit Theorem for the Winding Number of Low-Lying Closed Geodesics*

Sarnak showed that for all closed geodesics on the modular surface, the winding divided by the geometric length has a Cauchy limiting distribution. We will discuss why this Cauchy law arises in that setting, and why the situation changes if one restricts to “low-lying” geodesics. In this case, we show that the winding numbers have a Gaussian limiting distribution when normalized by any natural notion of length. The proof uses the continued fraction coding of geodesics, which allows us to take a probabilistic approach.

**Brooke Feigon, CUNY City College of New York**

*Ramanujan graphs and bigraphs*

Expander graphs are sparse, highly connected graphs. They have applications to Computer Science, coding theory, neural networks and other areas. Ramanujan graphs are optimal expander graphs. Lubotzky, Phillips and Sarnak (LPS) first constructed infinite families of regular Ramanujan graphs of fixed degree in the late 1980s using the Ramanujan Conjecture for  $\mathrm{PGL}(2)$ . I will discuss the LPS construction as well as recent work of mine with S. Evra, K. Maurischat and O. Parzanchevski constructing Ramanujan (and non-Ramanujan) biregular bipartite graphs via automorphic representations of  $\mathrm{U}(3)$ .

**Rusiru Gambheera, University of California, Santa Barbara**

*Structure of fine/signed Mordell-Weil groups*

I will discuss some results on the algebraic structure of fine Mordell–Weil groups and plus/minus Mordell–Weil groups of an elliptic curve in the cyclotomic  $\mathbb{Z}_p$ -extensions of certain abelian number fields. Our results generalize (and strengthen) the work of Lei. Moreover, we prove a result on the structure of the (plus/minus) Selmer groups in the cyclotomic  $\mathbb{Z}_p$ -extensions of  $\mathbb{Q}$ . As a consequence, we give evidence towards an affirmative answer for the Kurihara–Pollack problem. This is joint work with Debanjana Kundu.



**Louis Gaudet, University of Massachusetts Amherst**

*Counting biquadratic number fields that admit quaternionic and dihedral extensions*

Many interesting problems in arithmetic statistics involve counting number fields (ordered by their discriminants, say) with certain properties. In joint work with Siman Wong (UMass Amherst), we establish asymptotic formulae for the number of biquadratic extensions of  $\mathbb{Q}$  that admit a degree-2 extension with Galois group  $G$ , where  $G$  is either the quaternion group or the dihedral group (of order 8). We will discuss these results and how they are proved, and we will discuss their significance with regard to a theorem of Tate on lifts of projective Galois representations.

**Mohammad Hamdar, Concordia University (student)**

*Hecke  $L$ -functions away from the central line*

We will talk about new phenomena observed when studying the first moment of cubic Hecke  $L$ -functions at any  $s$  in the critical strip. We show that there is a phase transition in the moment at  $s = \frac{1}{3}$ ; and an interesting symmetry in the moment happens when  $s > \frac{1}{3}$  and  $s < \frac{1}{3}$ . In particular, at  $s = \frac{1}{2}$ , we improve the known asymptotic and prove the existence of a new secondary term. We also investigate this phenomena for general  $\ell^{th}$  order Hecke  $L$ -functions, and look at it from a Random Matrix Theory perspective.

**Hazem Hassan, McGill University (student)**

*$p$ -adic higher Green's Functions*

Stark-Heegner cycles are conjectural real-quadratic analogues of Heegner cycles. The Archimedean contribution of the height pairing of Heegner cycles is equal to the higher Green's functions. I will discuss the generalization of Rigid Meromorphic cocycles to higher weight and use them to construct a  $p$ -adic version of the higher Green's functions which. Values of the  $p$ -adic higher Green's functions were numerically verified to lie in compositum of abelian extensions of real-quadratic fields.

**Ashwin Iyengar, Unaffiliated**

*The Casselman-Shalika Formula*

The Langlands program is in large part concerned with understanding  $L$ -functions of modular forms, and more generally of automorphic representations. These representations are global in nature, but have local pieces given by representations of  $p$ -adic groups. Of those representations, there is a distinguished class called the unramified ones, which are in some sense the simplest, but must be thoroughly understood in order to understand automorphic representations fully. The Casselman-Shalika formula gives an explicit description of a certain class of unramified representations, called the generic ones, in terms of a natural algebra of commuting linear operators acting on a certain space of functions. I will explain

this formula in a motivated way, talk about its connection to L-functions, and then discuss joint work with Milton Lin and Konrad Zou to geometrize this formula.

**Seong Eun Jung, UMass Amherst (student)**

*Modular symbols over function fields of elliptic curves*

Developed by Manin, modular symbols are classes of paths on  $\mathbb{P}^1(\mathbb{Q})$ : elements of the relative homology group  $H_1(X, \text{cusps})$  where  $X$  is the modular curve, the quotient of a congruence subgroup of  $\text{SL}(2, \mathbb{Z})$  on the complex upper half plane. He was able to find the explicit generators for this group as well as a complete set of relations. Later, Teitelbaum constructed modular symbols over the rational function field  $\mathbb{F}_q(T)$ . He was able to define modular symbols and the complete set of relations for this case. Building off of Teitelbaum's work, we discuss what happens over function fields of elliptic curves. We first construct the analogs of the complex upper half plane,  $\text{SL}(2, \mathbb{Z})$ , and the quotient space. Then we define modular symbols and the generators. Finally, we present an algorithm that allows us to write any symbol as the sum of the generators when the field has class number one.

**Emir Eray Karabiyik, Cornell University (student)**

*A classification of low genus modular curves*

Let  $G$  be an open subgroup of  $\text{GL}_2 \hat{\mathbb{Z}}$  satisfying  $\det(G) = \hat{\mathbb{Z}}^\times$  and  $-I \in G$ . Associated to  $G$ , there is a modular curve  $X_G$  defined over  $\mathbb{Q}$ , which weakly parametrizes elliptic curves whose image of the Galois representation lies in  $G$ . Fixing a genus  $g$ , we give a classification of modular curves of genus  $g$ . In particular, we show that modular curves of genus  $g$  lie in finitely many families of  $\mathbb{Q}^{\text{ab}}$ -twists of modular curves. We also describe an algorithm for computing all families of modular curves of a fixed genus  $g$  and for computing projective models for modular curves of genus  $g$ .

**Doyon Kim, University of Bonn**

*Mellin–Barnes integrals of Bessel functions on  $\text{GL}(n)$*

I will discuss a Mellin–Barnes approach to certain conjectural integral representations of Bessel functions on  $\text{GL}(n)$  proposed by Buttcane. A change of variables simplifies the oscillatory factors and produces Mellin–Barnes representations of the type appearing in his recent work. I will present explicit computations for Weyl elements with simple combinatorial structure in arbitrary rank, including  $w_{1,n-1}$  and  $w_{2,n-2}$ .

**Paul Kinlaw, University of Maine, Presque Isle**

*On Even Pseudoprimes*

By Fermat's little theorem, if  $p$  is a prime number then  $2^p \equiv 2 \pmod{p}$ . One might use this congruence as a primality test. Indeed, if the congruence fails then we know that  $p$  is definitely not a prime. However, this primality test has limitations. If the congruence holds, we can only say that  $p$  is a probable prime, as there are rare counterexamples which are false positives. A *pseudoprime* is a composite number  $n$  such that  $2^n \equiv 2 \pmod{n}$ . We will discuss recent work on numerical bounds for the sum of reciprocals of pseudoprimes, including even pseudoprimes, which appear to be much more rare than odd pseudoprimes. Time permitting, we will also discuss explicit bounds for the counting function of even pseudoprimes. This includes joint work with Duc Nguyen, William Cheng, and Jonathan Bayless

**Say-Yeon Kwon, Princeton University (student)**

**Meiling Laurence, Yale University / SMALL REU (student)**

**Steven Zanetti, University of Michigan (student)**

*Breaking Universality in  $n$ -level Densities of Cuspidal Newforms*

The Katz-Sarnak density conjecture states that, as the analytic conductor  $R \rightarrow \infty$ , the distribution of the normalized low-lying zeros converges to the scaling limits of eigenvalues clustered near 1 of subgroups of  $U(N)$ . Evidence shows that this conjecture holds in many families, including the family of holomorphic cusp newforms. In 2009, Miller computed lower-order terms for the 1-level density of some  $GL(2)$  families. We extend Miller's work by identifying family-dependent lower-order correction terms in the weighted 1-level and 2-level densities of holomorphic cusp newforms up to  $O(1/\log^4 R)$  error, sharpening Miller's  $O(1/\log^3 R)$  error. We consider cases where the level is prime or when the level is a product of two, not necessarily distinct, primes. We show that the rates at which the prime factors of the level tend to infinity lead to different lower-order terms, breaking the universality of the main behavior.

**Sung Min Lee, Wake Forest University**

*Cyclicity and Koblitz Conjectures in Arithmetic Progressions: Bias Across Congruence Classes*

The cyclicity and Koblitz conjectures concern the distribution of primes for which the reduction of an elliptic curve over  $\mathbb{Q}$  is cyclic or of prime order, respectively. Serre gave a conditional proof of the cyclicity conjecture in 1976, while the Koblitz conjecture remains open. In this talk, we investigate these problems in the setting of arithmetic progressions. Building on Zywin's approach, we formulate the Koblitz conjecture for primes in arithmetic progressions. We then refine a theorem of Jones to obtain results on the moments of the associated constants in both conjectures. Finally, we highlight a somewhat surprising phenomenon: the two constants exhibit opposite biases across congruence classes. This is joint work with Jacob Mayle and Tian Wang.

**Nikita Lvov, Unaffiliated**

*Universality Results for Random Matrices over Local Rings*

The cokernel of a random  $p$ -adic square matrix is a random abelian  $p$ -group. When the entries are i.i.d random variables, the distribution of this group asymptotically converges to a limiting distribution. It was shown by Wood and others that this limiting distribution, called the Cohen-Lenstra distribution, does not depend on the distribution of the individual entries. This is known as the universality phenomenon for random  $p$ -adic matrices. We extend this result in several ways. Firstly, we prove an analogue for matrices with coefficients in more general local rings. This has potential applications for understanding characteristic polynomials of random matrices. Secondly, we prove a quantitative version of universality, by exhibiting an exponential convergence rate. Thirdly, we show that our results yield a dynamic version of universality: almost surely, given an infinite random matrix with i.i.d. entries, the cokernels of the top left minors are distributed according to the same universal distribution. All three results follow from a single general theorem, which we call the "column-swapping estimate", an analogue of Lindeberg replacement.

**Vishal Muthuvel, Columbia University (student)**

**Lawrence Dillon, University of Washington (student)**

**Pramana Saldin, University of Wisconsin (student)**

*Centered Moments of Weighted One-Level Densities of  $GL(2)$   $L$ -Functions*

The Katz-Sarnak philosophy conjectures that the low-lying zeros (near the central point) of families of  $L$ -functions are well modeled by eigenvalues (near 1) of certain random matrix ensembles. In 2018, Knightly and Reno proved that the exact nature of this symmetry can depend on how the  $L$ -functions are weighted. They observed both orthogonal and symplectic symmetry in the one-level density of cuspidal newforms for different choices of weights. We observe the same dependence of symmetry on weights in the  $n^{\text{th}}$  centered moments of this one-level density, for smooth test functions whose Fourier transforms are supported in  $(-1/2n, 1/2n)$ . The proof proceeds by generalizing Knightly and Reno's weighted trace formula from prime powers to all positive integers, then mirroring the analysis of Hughes and Miller to compute the main and error terms of the  $n^{\text{th}}$  centered moment.

**Jack Petok, Colby College**

*Zeta functions of K3 categories over finite fields.*

We define the zeta function of a noncommutative K3 surface over a finite field, an invariant under Fourier-Mukai equivalence that can be used to define point counts in this noncommutative setting. These point counts can be negative, and can be used as an obstruction to geometricity. In particular, we study the K3 category associated to a cubic fourfold over a finite field, and show that point counts can also fail to detect nongeometricity. We also study an analogue of Honda-Tate for K3 surfaces and for K3 categories, and provide a nontrivial restriction on the possible Weil polynomials of the K3 category of a cubic fourfold.

**Tristan Phillips, Dartmouth College***Unbounded average selmer ranks in torsion families.*

In their groundbreaking work, Bhargava and Shankar established the first unconditional bounds on the average rank of elliptic curves over the rational numbers by bounding the average sizes of their 2-Selmer groups—and subsequently the 3-, 4-, and 5-Selmer groups. In contrast, Klagsbrun and Lemke Oliver demonstrated that the average size of the 2-Selmer group becomes unbounded when restricting to elliptic curves over  $\mathbb{Q}$  with a rational 2-torsion point. This talk presents a generalization of their result to broader settings, including elliptic curves with more general torsion subgroups and defined over arbitrary number fields.

**Berend Ringeling, Université de Montréal (CRM)***Zeros of Modular Forms*

Typically, the results on zeros of modular forms can be put into two categories: The zeros of Eisenstein series and the zeros of Hecke eigenforms. In the first case, all zeros are confined to an arc of the unit circle. In the second case, the zeros become equidistributed as the weight increases. In this talk we focus on the first case. We discuss the beautiful and elementary proof of Rankin and Swinnerton-Dyer and its generalization to arbitrary congruence groups. This is joint work with Gunther Cornelissen and Sebastián Carrillo Santana.

**Erick Ross, Clemson University (student)***What is the asymptotic behavior of the Hecke polynomial coefficients?*

In this presentation, we determine the asymptotic behavior of the coefficients of Hecke polynomials. In particular, this allows us to determine the signs of these coefficients when the level or the weight is sufficiently large. In all but finitely many cases, this also verifies a conjecture on the nonvanishing of the coefficients of Hecke polynomials.

**Andrew Schultz, Wellesley College***Bloch-Kato profinite groups*

A famous conjecture of Bloch and Kato, resolved by Rost and Voevodsky in 2011, says that the cohomology ring of the absolute Galois group of a field  $F$  with  $\mathbb{F}_p$ -coefficients is generated in degree one, and with relations generated in degree two. In this talk we discuss a recent paper where we characterize groups that satisfy related cohomological criteria. We also investigate superpythagorean and  $p$ -rigid fields to see how certain qualitative features implied by the Bloch-Kato conjecture manifest in these special cases.

**Pankaj Singh, University of South Carolina (student)**

*Classifying Torsors of Tori via Brauer Groups*

Using Mackey functors, we develop a general framework for classifying torsors of algebraic tori in terms of Brauer groups of finite field extensions of the base field. This generalizes Blunk’s description of tori associated with degree 6 del Pezzo surfaces to all retract rational tori—essentially the largest class for which such a classification is possible. I will also provide a classification of generalized del Pezzo varieties in terms of the Brauer group, serving as an example of this situation.

**Jakob Streipel, University at Buffalo**

*Sub-Weyl bound for  $GL(2)$  via trivial delta*

Using a refinement of the recent “trivial delta method” by means of Diophantine approximation, we obtain a sub-Weyl bound for  $GL(2)$ , thereby crossing the Weyl barrier for the first time beyond Riemann Zeta function.

**Drew Sutherland, Massachusetts Institute of Technology**

*L-functions from nothing*

The proof of the modularity theorem relating elliptic curves to modular forms via their  $L$ -functions is widely regarded as one of the crowning achievements of 20th century number theory. Long before the proof of the modularity theorem, evidence for it was amassed in the 1972 “Antwerp IV” tables comparing elliptic curves and modular forms of small conductor; D.J. Tingley’s calculation of spaces of modular forms was essential for filling some gaps in the elliptic curve table and gave a de facto complete list of all elliptic curves of conductor up to 200. In the decades that followed, this list was extended by John Cremona and became part of the  $L$ -functions and Modular Forms Database (LMFDB).

In recent decades similar efforts have been undertaken in dimension 2 (abelian surfaces), but we lack a rigorous analog of Tingley’s table of modular forms and with it any proof of completeness. Computing the relevant spaces of modular forms is prohibitively difficult, even for small conductors. In this talk I will describe joint work with Andrew Booker that makes it possible to compute these spaces indirectly via their  $L$ -functions. The result is a provably complete tabulation of  $L$ -functions of modular forms that the Langlands program predicts should arise for abelian surfaces over  $\mathbb{Q}$  of conductor up to 1500. This table reveals several gaps in our tabulations of abelian surfaces, and I will describe ongoing work to fill those gaps.

**Karen Taylor, Bronx Community College, CUNY**

*A Sturm Bound for Quaternionic Modular Forms*

In this talk we give a Sturm bound for quaternionic modular forms. Quaternionic modular forms live on lie groups  $G_J$  constructed from a cubic norm structure  $J$ . Details will be given in the  $SO(4, 3)$  case. This is joint work, with Aaron Pollack, Ilesanmi Adeboye and Charles

Burnette, initiated at the ADJOINT program held at SLMATH.

**Jesse Thorner, University of Illinois Urbana-Champaign**

*Remarks on Landau–Siegel zeros*

(Joint work with Alexandru Zaharescu and Debmalya Basak) For certain families of  $L$ -functions, we prove that if each  $L$ -function in the family has only real zeros in a fixed yet arbitrarily small neighborhood of  $s = 1$ , then one may considerably improve upon the known results on Landau–Siegel zeros. Sarnak and Zaharescu proved a similar result under much more restrictive hypotheses.

**Iana Vranesko, Williams College (Student)**

**June Duvivier, Reed College (student)**

*Using Gowers Norms in Improving Signal Recovery Conditions*

When transmitting signals, we often send the Fourier transform instead of the raw signal, as it's usually more convenient and efficient. However, often the channel is noisy, causing some of the frequencies to be unobservable and preventing accurate reconstruction. Recently Iosevich and Mayeli analyzed the sparse, finite, binary case with signal  $f : \mathbb{Z}_N^d \rightarrow \{0, 1\}$ ,  $\hat{f}$  suitably normalized, and  $S \subset \mathbb{Z}_N^d$  the set of unobserved frequencies. They prove many recovery estimates that ensure exact recovery after applying simple rounding algorithms. Specifically, they use restriction theory to show that if the additive energy

$$|\{(x, y, x', y') \in U^4 : x + y = x' + y'\}|$$

is sufficiently small for every  $U \subset S$ , then  $f$  can be recovered exactly.

We extend the use of additive energy to higher-order Gowers norms, a powerful framework from additive combinatorics that captures increasingly subtle arithmetic structure. Notably, the second Gowers norm coincides with additive energy, while higher norms generalize this by detecting longer arithmetic patterns. Using this approach, we significantly strengthen the additive uncertainty principle in the finite setting, yielding sharper recovery guarantees even in the presence of more adversarial frequency loss. Our new conditions ensure exact reconstruction under broader circumstances than previously known. Ongoing work focuses on fully characterizing the role of higher Gowers norms in signal recovery, with the goal of developing robust, structure-sensitive criteria for signal reconstruction in noisy or incomplete frequency regimes.

**Gary Walsh, University of Ottawa**

*Revisiting an Equation of Richard Bumby*

Bumby determined the set of integral solutions to  $3x^4 - 2y^2 = 1$  using an extremely clever construction, along with insightful observations concerning elements in certain quadratic and quartic number fields. Bennett and the speaker generalized Bumby's result to an infinite

family of similar equations by way of the so-called hypergeometric method. In this talk, we shamelessly beat a dead horse by revisiting the original problem, and show how it can be solved entirely over the integers.

**Tian Wang, Concordia University**

*Lower bounds for isogenies of CM elliptic curves*

Given two CM elliptic curves over a number field and a natural number  $m$ , we establish a polynomial lower bound in terms of  $m$  for the number of rational primes  $p$  such that these elliptic curves become  $m$ -isogenous modulo a prime above  $p$ . The proof builds on a version of the Gross–Kohnen–Zagier formula, with a key insight that the Fourier coefficients of the incoherent Eisenstein series can be approximated by those of coherent Eisenstein series of increasing level. Hence we can use the equidistribution of Hecke correspondences, the non-negativity of higher Green functions, and an explicit upper bound for the Petersson norm of classical modular forms in terms of finitely many Fourier coefficients. This is joint work with Edgar Assing, Yingkun Li, and Jiacheng Xi.

**Zhining Wei, Brown University**

*The weighted density conjecture of standard L-functions associated to Hilbert modular forms*

In this talk, I propose the weighted density conjecture of standard L-functions associated with Hilbert modular forms. I will also discuss the proof of low degree cases. This is joint work with Liyang Yang and Shifan Zhao.

**Max Weinreich, Harvard University**

*Height growth in pentagram maps*

In this talk, we will see how arithmetic methods have become useful in the study of dynamical systems. Pentagram maps are simple geometric operations in which one replaces a polygon by a new polygon obtained by intersecting some diagonals. If the input polygon has only rational coordinates, the Weil height of the polygon grows slowly over time, a phenomenon called Diophantine integrability. But computer experiments have shown that in some generalized pentagram maps, the Weil height appears to grow exponentially. We prove this for an infinite family of generalized pentagram maps using methods from algebraic geometry.

**Haochen Wu, Dartmouth College (student)**

*Hilbert modular forms from orthogonal modular forms on binary lattices*

We show the explicit connection between Hilbert modular forms and orthogonal modular forms arising from positive definite binary lattices over the ring of integers of a totally real



number field. Our work uses the Clifford algebra to generalize Gauss composition, and this allows us to classify the class sets of genera in terms of the class groups of the associated quadratic algebras. This is joint work with John Voight.

**Zecheng Yi, Boston University (student)**

*Continuous group cohomology of Steinberg representations*

Geometrically, Steinberg representations of  $GL_n(\mathbb{Q}_p)$  arise as cohomology groups of Drinfeld spaces. The extensions between such representations are well-understood when the coefficient ring is a characteristic 0 field equipped with the discrete topology. I will discuss the analogue of such results when the coefficient is  $\mathbb{Q}_p$  equipped with the  $p$ -adic topology.

**Wei Zhang, Massachusetts Institute of Technology**

*Gross–Zagier formula in high dimensions*

In their 1986 paper, Gross and Zagier proved a formula relating the height of Heegner point on an elliptic curve to the first derivatives of the L-function of that elliptic curve. Since then, the problem of generalizing this fundamental result to higher dimensional algebraic varieties has been of great interest. In this talk we will present some of the generalizations with relatively recent results, with an emphasis on Kudla’s program and the arithmetic Gan–Gross–Prasad conjecture.