Solar, Exoplanet and a Customised Planet Systems -A Draft Article

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The solar system is the planetary system that includes the Sun and the eight major planets, as well as their moons, asteroids, comets, and other small bodies. Exoplanet systems are planetary systems that orbit other stars, and they can have different numbers, types, and properties of planets.

In this article, we present the solar system and exoplanet systems in terms of simulating orbits with the available data and algorithms. We show **1**) Kepler's third law verification, 2) Orbits sung accurate modelling and Verlet iterative Method, 3) effect of polar angle in their speed and travel along the orbits 4) Spirographs and 6) Relative speed diagrams between various planets in a solar system and exoplanet Systems. The results show an interesting comparison between our solar system and exoplanet systems. Finally, we also show a custom planet-star formation using Kepler's third law and the custom-planet GUI-APP is formed with the help of Pygame and Tkinter packages in Python. The results show many interesting orbit formations around the sun-like planet but up to 10 times larger in size.

I. Introduction to Solar and Exoplanet Systems

The solar system formed about 4.6 billion years ago. The Sun originated from the core of a gas and dust nebula. The planets formed from the residual matter that orbited the Sun in elliptical trajectories. The Sun and any other star with planetary systems lies at one of the foci of each planet's ellipse. Sun and any exoplanet star usually sit in one of the focies of the ellipse of every one of the planets. During the past 30 years, NASA and other space researchers have identified more than 5000 exoplanet systems. Exoplanet systems are thought to form in a similar way but with variations depending on the initial conditions and the properties of the host star. For example, some exoplanet systems may form around binary or multiple stars, or around stars with different masses and temperatures than the Sun. However, in this paper, we consider only single-star systems for simulations. Eight out of nine planets in our solar (sun) system orbit the Sun in nearly circular orbits with near zero eccentricity that lies in roughly the same plane except for the planet Pluto which has a higher eccentricity of 0.25 and has an elliptical orbit which is inclined in the 3D plane. We show the 2D disk of the solar system in Figure 3. The orbits are stable and follow predictable patterns according to Newton's laws of motion and gravity. Exoplanet systems can have similar or very different orbital characteristics than the solar system.

Some exoplanets may orbit very close to their host star, very far away, or in highly eccentric or inclined orbits. Some exoplanets may also experience strong gravitational interactions with other planets or stars in their system, leading to orbital changes or instability. In this paper, I have considered the 648 exoplanets provided on the Cambridge Computational Challenge website and along with some extra information from NASA and Wikipedia to analyse some of the exoplanet orbits in detail by simulations. I considered the Trappist1 star, Kepler11 star, Uma47 star, Gliese star and Kepler649 in this paper. Figure 2 shows the 3-D Orbits of the exoplanet systems. Table 1 provides the data available about the chosen exoplanet systems compared to our solar system and Earth.

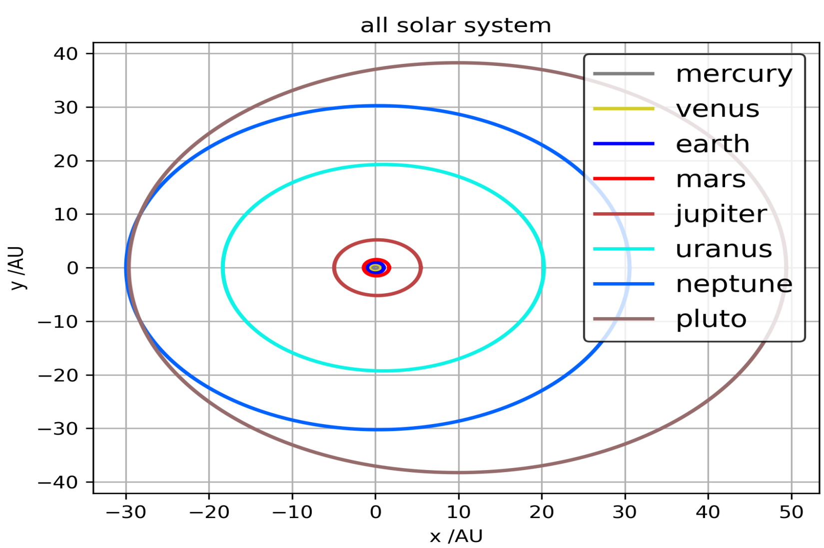


Fig. 1: All Solar System in 2D]

II. Task 1: Kepler III Law Verification of Solar and Exoplanet Systems

The time it takes for a planet to complete one orbit around the Sun is related to how far away it is from the Sun. The farther the planet is, the longer its orbital period will be. This relationship can be expressed mathematically by saying that the orbital period squared is proportional to the average distance cubed, which is known as Kepler's third law of planetary motion. This law applies to all planets in the solar system and can be used to calculate their orbital properties. Exoplanet systems also obey Kepler's third law (Equations 1 and 2). The mass of the host star also affects the orbital period and distance of an exoplanet, as well as its gravitational influence on other planets in its system which is excluded in this simple approximated formulation.

Kepler's Third Law T/Yr versus semi-major axis a/AU with be an exponential curve

$$\frac{T}{Yr} = \left(\frac{a}{AU}\right)^{3/2} \tag{1}$$

Kepler's Third Law given in (T/Yr) versus $(a/AU)^{3/2}$ will be a line with Gradient 1

$$\left(\frac{T}{Yr}\right) = \left(\frac{R}{AU}\right)^{3/2} \tag{2}$$

Kepler's Third Law plotted Log-base values $\log(T/Yr)$ versus $log\,(a/AU)$ will be a line with gradient 1.5

$$\log\left(\frac{T}{Yr}\right) = 1.5\left(\log\frac{R}{AU}\right) \tag{3}$$

Figures 2 (a) (b) show the verified Kepler III formulation shown in Equations 1 2 3 for Solar system parameters and Figures 2 (c) (d) show the Kepler III verification for 648 exoplanets parameters provided reference to Cambridge Computational Physics Challenge Data.

For the solar system, it is verified that the square of the orbital period is proportional to the cube of the orbital semi-major axis when units of years for time in (Yr) and Astronomical Units (AU) for distance are used. The gradient obtained for the linear regression line fitting is 1.0. However for the Exoplanet systems, the measurements have relatively higher errors in the measurement/model, the verification (the line fitted using linear regression for the measured data taking account of the error bars) deviated from the expected gradient 1.

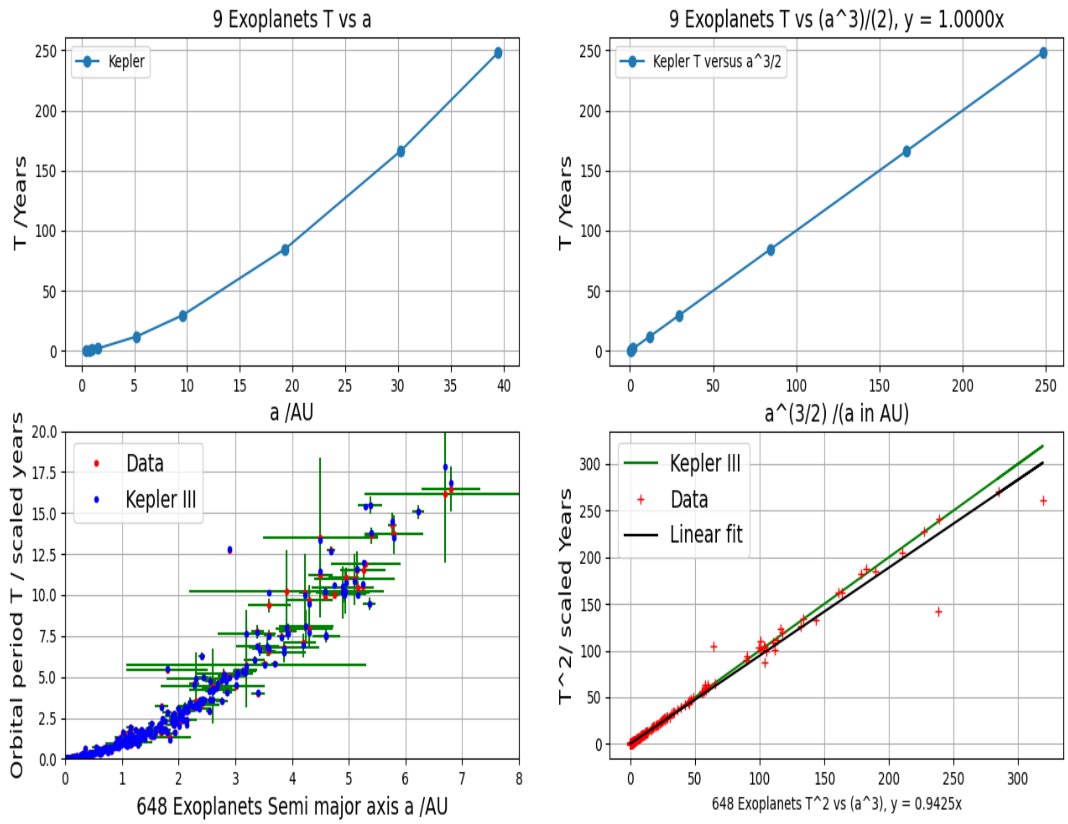
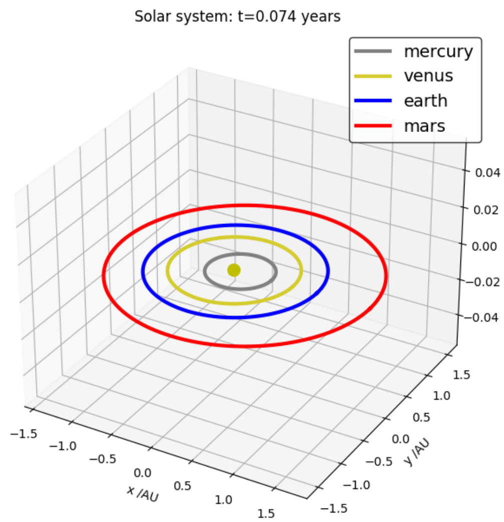


Fig. 2: Kepler's Third Law Verification for Solar and Exoplanet Systems with 648 Planets [Cambridge Physics Computational Challenge data]

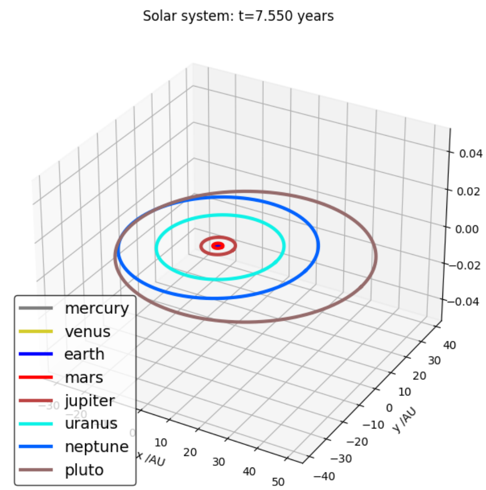
III. Task2/Task3/Task4:Solar and Exoplanet Systems Modeling using accurate and Verlet Models in 2D, 3D and animation (speed of planets related to polar angles)

- Accurate plots orbit using parameters and geometry but with approximate Kepler equation.
- Verlet Method which is an iterative orbit prediction method

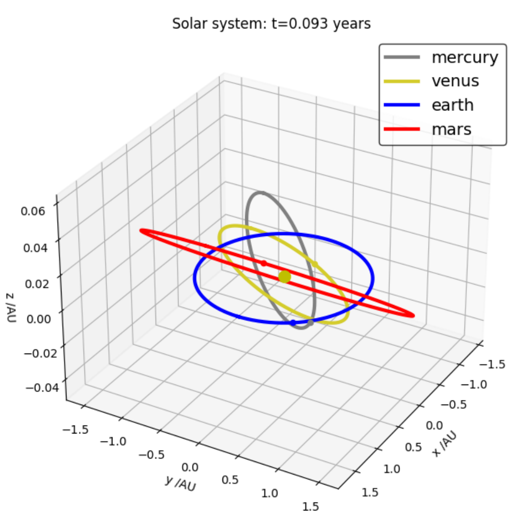
First, I considered accurate modelling of the Solar, and then in the next section the same is presented for selected Exoplanet systems. We also show the elliptical orbits and (animation on youtube) in 2D and 3D (Figure 3-5). Figure 3, subfigures show the Accurate plots of the Solar planet system's "inner orbits" and "all orbits" in 2D and 3D views.



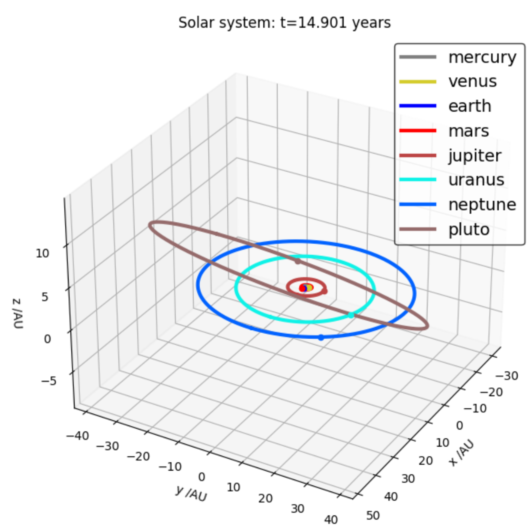
(a) Solar System Orbits 2D Inner Planets



(b) Solar System Orbits 2D All Planets



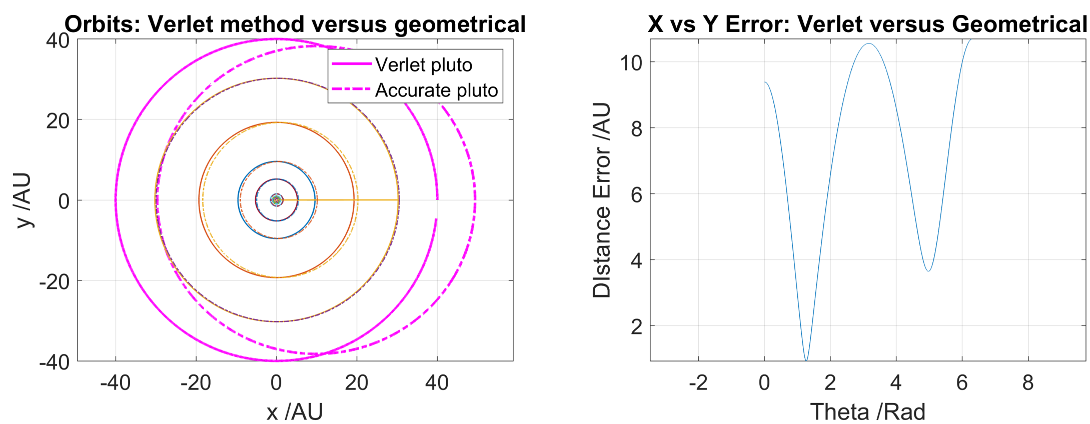
(c) Solar System Orbits 3D Inner Planets



(d) Solar System Orbits 3D All Planets

Fig. 3: Solar System Accurate Orbits in 2D and 3D using parameters [Cambridge Physics Computational Challenge Data

Figures 4 (a) (b) show the Verlet method of iterative prediction of orbits to draw the approximated orbits using semi-major axis lengths as a parameter. Figure 4 (b) shows the differences in the calculated orbits using accurate and Verlet methods; particularly for Pluto. As can be seen from the figure, the orbit error of the Verlet method for Pluto is very large compared to other orbits that are nearly circular. We show the error between the accurate and Verlet method for the orbit of Pluto in Figure 4(b).



(a) Orbits of Solar system using Verlet and Accurate modeling

(b) The distance error between both methods for Pluto

Fig. 4: 2D solar orbits for Verlet and Accurate modeling

A. Orbits of Selected Exoplanet Systems

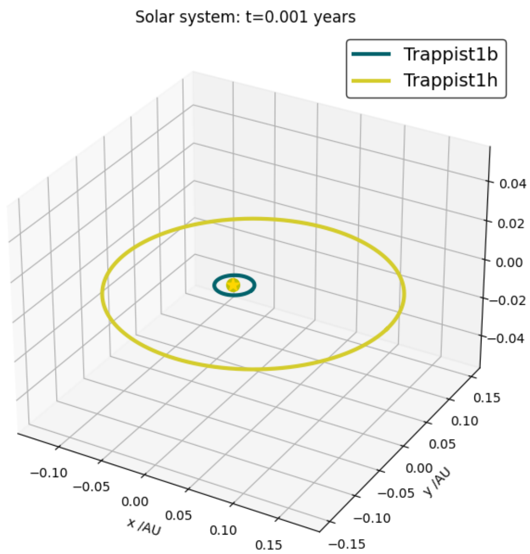
1) Orbits or Trappist I: Trappist I System Orbits with 7 planets using "Accurate Modeling" with data obtained from NASA and Wikipedia are shown in Figure 5. Trappist System is known to be in one plane (or the inclination is not known), therefore, we do not show the 3D plot.

2) Orbits or Kepler 11 with 5 planets: Kepler 11 System Orbits using "Accurate Modeling" with data obtained from NASA and Wikipedia are shown in Figure 6.

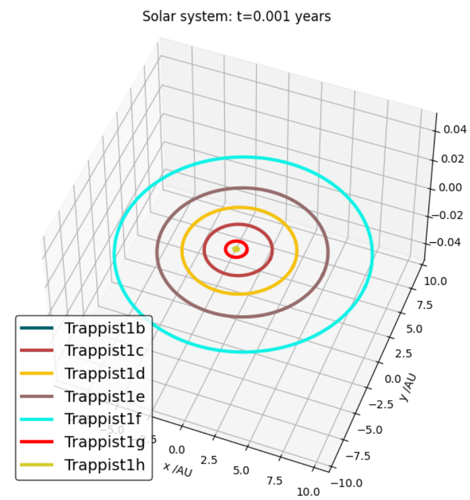
3) Orbits or Uma47 with 3 planets: Uma 47 System Orbits using "Accurate Modeling" with data obtained from NASA and Wikipedia are shown in Figure 7.

IV. Task 5: Simpson Integration and Eccentricity

Finding exact solutions to the orbital equations has no closed-form solution when considering the effects of other planets and external forces. Therefore, numerical methods are needed to



(a) Trappist I System Orbits 2D Inner Planets



(b) Trappist I System Orbits 2D All Planets

Fig. 5: Trappist 1 System Accurate Orbits in 2D and 3D using parameters [Cambridge Physics Computational Challenge Data, NASA, Wikipeda

approximate the solutions and obtain realistic values of the orbital parameters. One of these methods is Simpson integration, which is a technique for estimating the area under a curve by dividing it into segments and fitting parabolas to each segment that is required to calculate the time spent by the planets around the orbit at various locations. Simpson integration can be applied to calculate the orbital motion of planets by integrating the equations of motion over time. This paper presents the calculation of polar angle versus time around the orbit. Figure 8 shows the polar angle versus time for Pluto and also for a very high eccentricity where we can see at various locations of the polar angle the time travel is really fast whereas at other places it is the opposite. We also demonstrate this fact in our youtube animations.

V. Task 6: Spirographs

A Spirograph of planets is a graphical representation of the orbital motion of the planets around the Sun. The following are understood from Spirographs:

- shape

- size
- orientation
- eccentricity of a planet's orbit
- speed and position at different times
- gravitational interactions between the planets

A Spirograph can help us visualize and compare the orbital characteristics of different planets. I have shown various solar system Spirographs in Figure 9. I also plotted various Spirographs for exoplanet systems Kepler11 and Trappist. We can see the similarities between the exoplanet systems very clearly from Spirographs. For example, Kepler d and f are similar to Jupiter and Neptune; and Neptune-Pluto Spirograph is similar to the Trappist d-Trappiste Spirograph.

VI. Task 7: Relative Distances of Planets

Relative distances of planets are the distances between the planets and the Sun or the Earth measured in terms of astronomical units (AU) which is a very useful way to understand the relative position between two planets or the sun that cannot be easily understood otherwise due to large distances between them.

Relative distances of planets can help us understand how far apart the planets are from each other and us at a particular time even if the distances are high.

In Figures 11 and 12, I show the 2D and 3D relative distance plots for the inner planets (around the earth) and outer planets around Saturn) for the Solar system, inner planets for Kepler11 (around Kepler b) and outer planets for Trappist (around Trappist b).

VII. Custom Orbits

Finally, I aimed to explore the formation of the solar system by creating a custom orbit-planet simulator in Python. The simulator uses Tkinter and PyGame packages to develop a graphical user interface (GUI) app that allows the user to input various parameters and observe the resulting planetary orbits. The simulator solves the Kepler's formula iteratively to a tolerance level of 10^{10} and applies Kepler's third law to ensure that the orbits are proportional to the mass of the star, the gravitational force, the eccentricity, and the initial distance of the planets. The simulator can generate multiple planets with random masses and distances within the range of the solar system planets, as well as a star with a mass relative to the sun (from 0.5 to 10 times). The initial eccentricity is randomly assigned between 0 and 0.3 for each planet. The simulator demonstrates how different factors affect the shape and stability of the orbits and provides a visual and interactive way to learn about solar system formation.

Figure 13 shows four different settlements in this custom planet orbit generator attempt where the distance from the star is also shown for each of the successful planets. Successfully formed stars are in the range of 1 million km (range of Mercury and Venus). Our earth is formed around 150 million km and the simulation shows the challenge of creating stars at a distance from Earth as they mostly lose the gravitational hold with the sun. I plan to investigate and develop a much better custom orbit simulator in the future.

VIII. Conclusion

Planetary systems are diverse and complex phenomena that can be studied by comparing different examples in our galaxy and beyond. In this paper, I examined the solar system and three exoplanet systems: Trappist-1, Kepler-11, and UMa 47. I compared their characteristics such as the number, size, mass, composition, Spirograph, Relative Distances patterns and orbital features of the planets. Trappist-1 has multiple Earth-sized planets in the habitable zone and has very regular orbits and Kepler-11 has tightly packed planets with low densities. These comparisons helped me to understand the diversity and evolution of planets in our galaxy and beyond. With this knowledge, I developed a custom orbit simulator app in Python that can generate random planetary systems inspired by our solar system. The app allows the user to input various parameters and observe the resulting planetary orbits. The app is a useful tool for learning about solar system formation and dynamics, as well as for exploring the possible outcomes of different scenarios. I plan to improve the app and its functionality and the theories behind the orbit formulation in the future and make it available as a phone app orbit simulator game.

References

R1 https://exoplanets.nasa.gov/

R2 https://www.astro.phy.cam.ac.uk/research/research-activities/exoplanets

R3 https://en.wikipedia.org/wiki/Kepler-11

R4 https://en.wikipedia.org/wiki/TRAPPIST-1

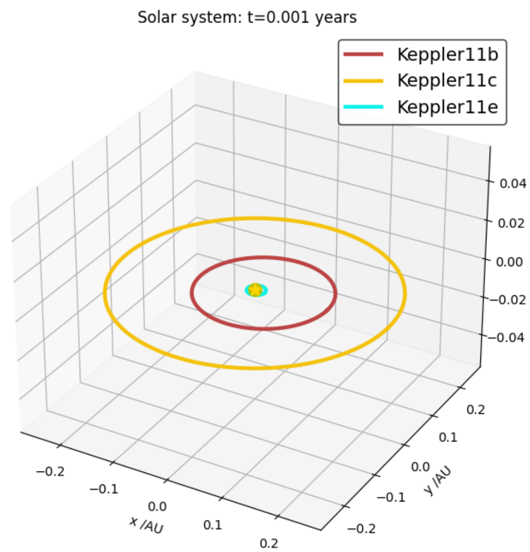
R5 https://exoplanets.nasa.gov/resources/199/kepler-186-and-the-solar-system/.

R6 https://www.nasa.gov/ames/kepler/kepler-186-and-the-solar-system/.

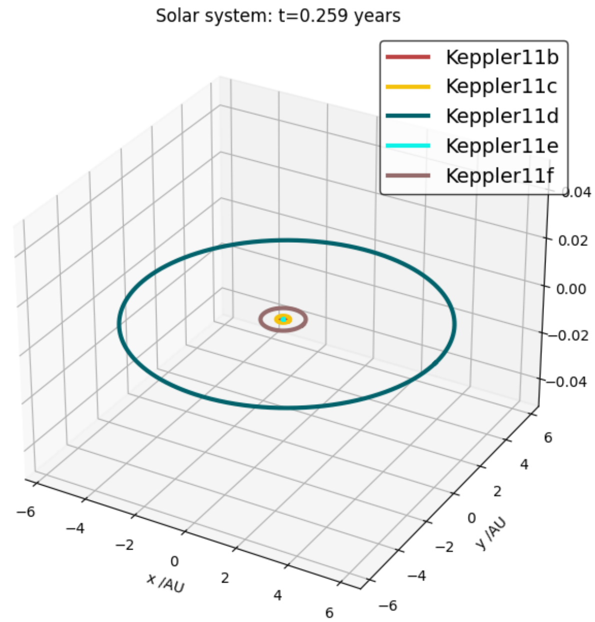
R7 https://www.nasa.gov/feature/ames/kepler/astronomers-confirm-orbital -details-of-trappist1-least-understood-planet/.

R8 https://exoplanets.nasa.gov/resources/198/kepler -186f-the-first-earth-size-planet-in-the-habitable-zone-artists-concept/.

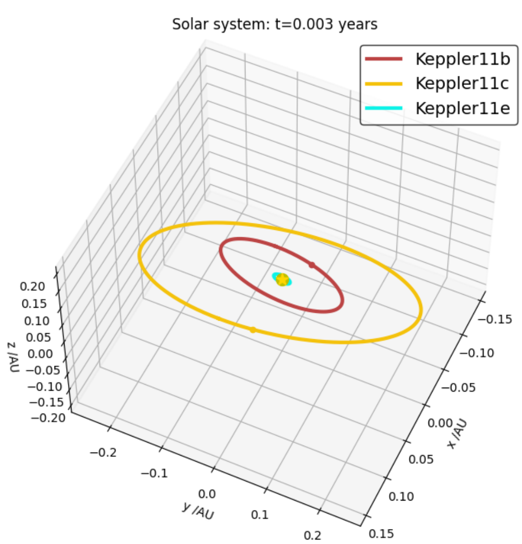
- I also acknowledge many other websites and youtube channels that helped me understand the programming aspects of the solar and exoplanet systems.



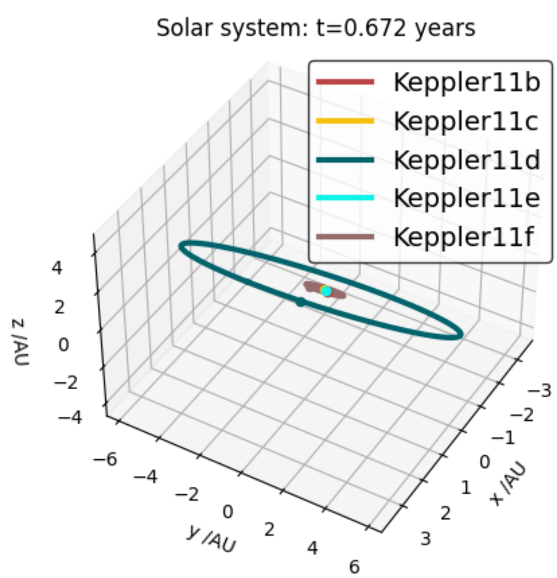
(a) Kepler 11 System Orbits 2D Inner Planets



(b) Kepler 11 System Orbits 2D All Planets

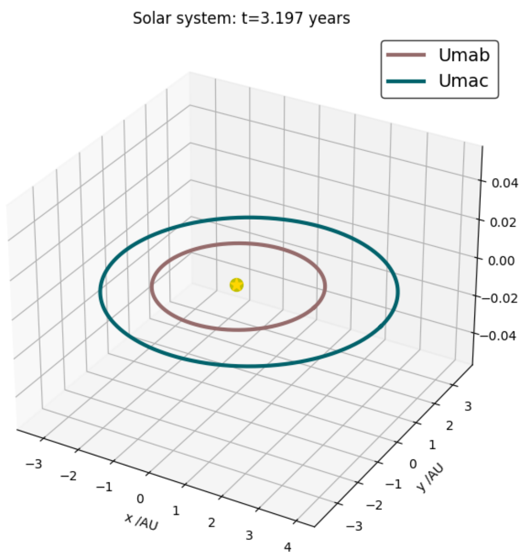


(c) Kepler 11 System Orbits 3D Inner Planets

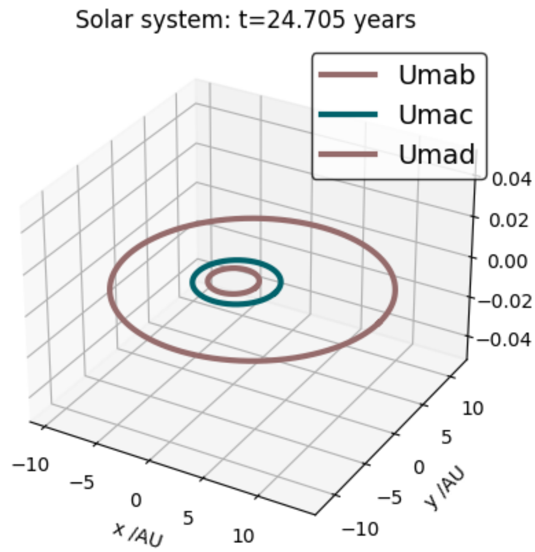


(d) Kepler 11 System Orbits 3D All Planets

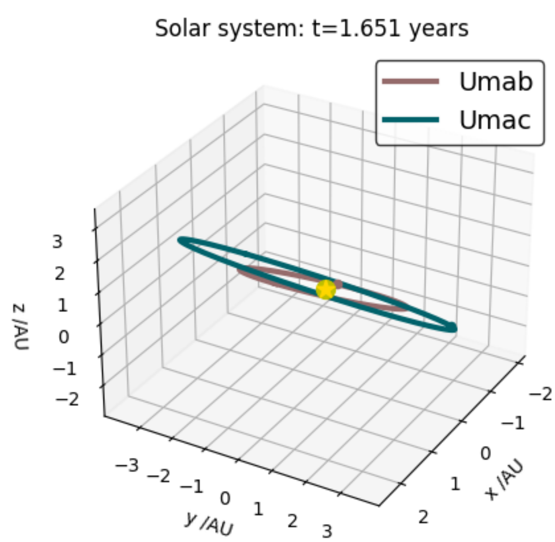
Fig. 6: Kepler 11 System Accurate Orbits in 2D and 3D using parameters [Cambridge Physics Computational Challenge Data, NASA and Wikipedia



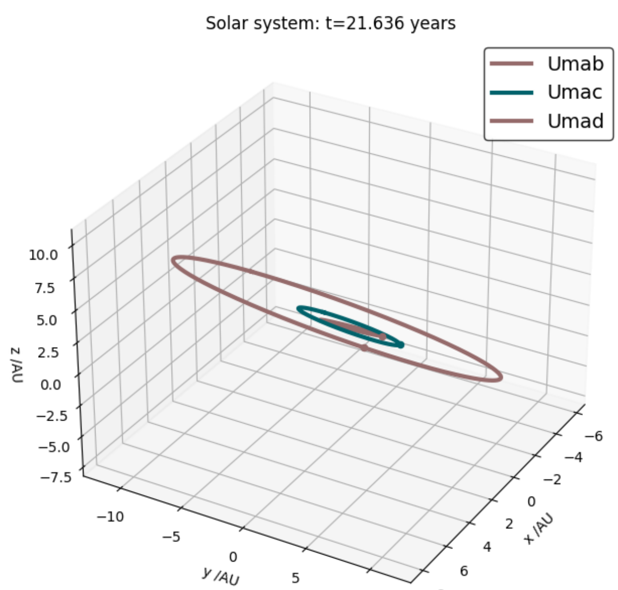
(a) Uma47 System Orbits 2D Inner Planets



(b) Uma47 System Orbits 2D All Planets

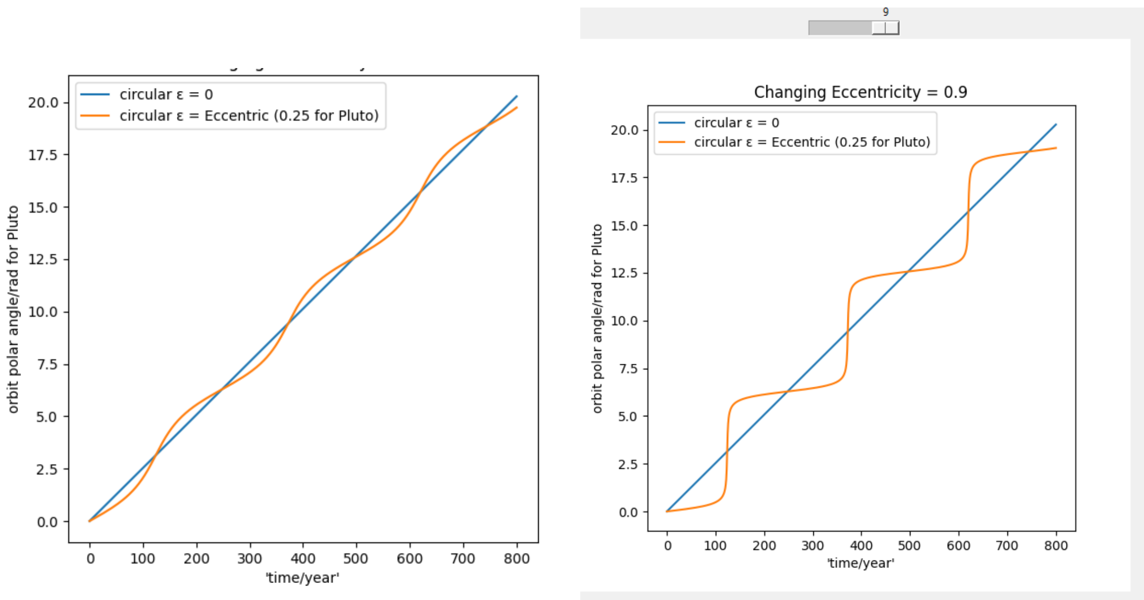


(c) Uma47 System Orbits 3D Inner Planets



(d) Uma47 System Orbits 3D All Planets

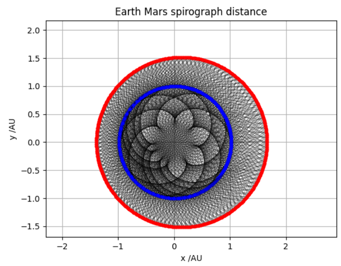
Fig. 7: Uma 47 System Accurate Orbits in 2D and 3D using parameters [Cambridge Physics Computational Challenge Data, NASA and Wikipedia



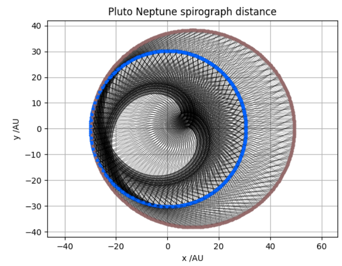
(a) Simpson Integration for Polar angle versus time with actual eccentricity = 0.25 and eccentricity =0

(b) Simpson Integration for Polar angle versus time with an imaginary eccentricity = 0, 0.25, 0.9

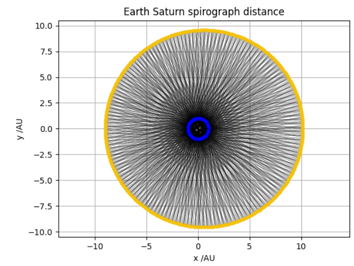
Fig. 8: Simpson Integration for Polar angle versus time with an imaginary eccentricity = 0, 0.25, 0.9



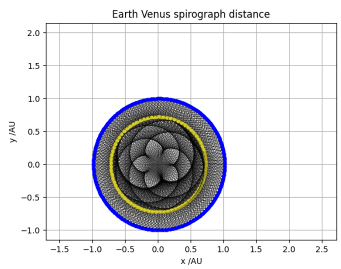
(a) Earth-Mars



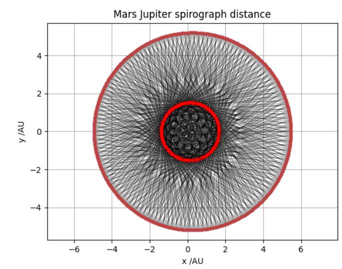
(b) Pluto-Neptune



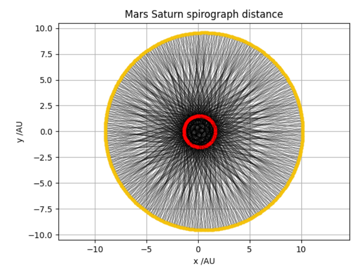
(c) Earth-Sun



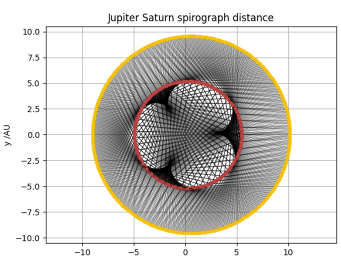
(d) Earth-Venus



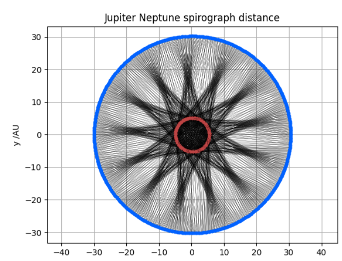
(e) Mars-Jupiter



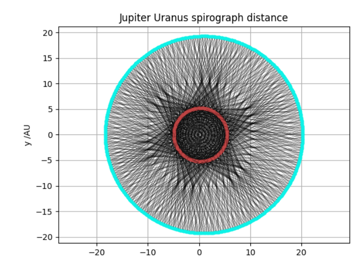
(f) Mars-Saturn



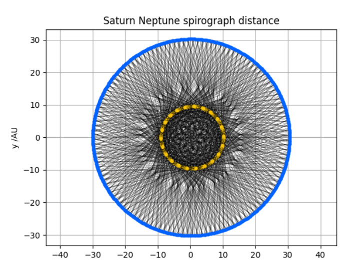
(g) Jupiter-Saturn



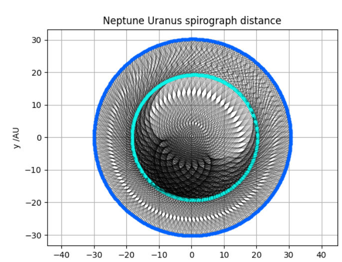
(h) Jupiter-Neptune



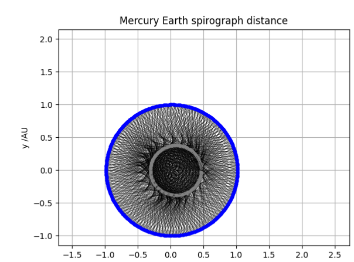
(i) Jupiter-Uranus



(j) Saturn-Neptune

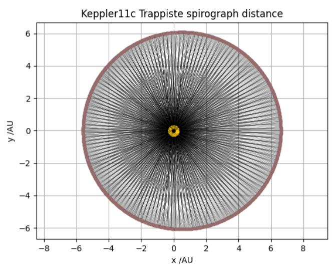


(k) Uranus-Neptune

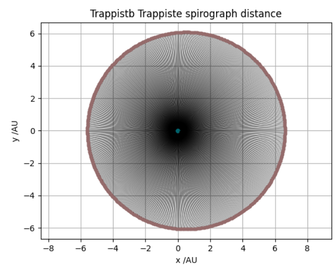


(l) Pluto-Neptune

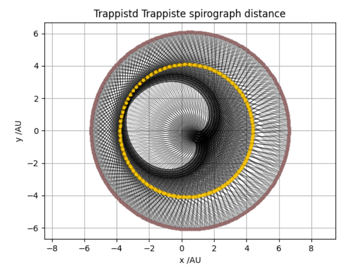
Fig. 9: Spirographs of Solar System



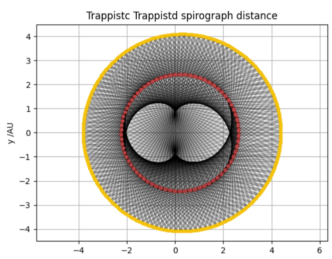
(a) Trappist c - Trappist e



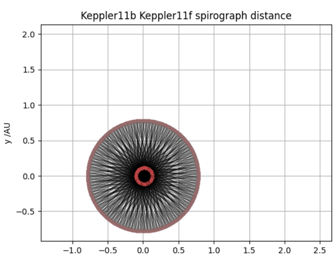
(b) Trappist b - Trappist e



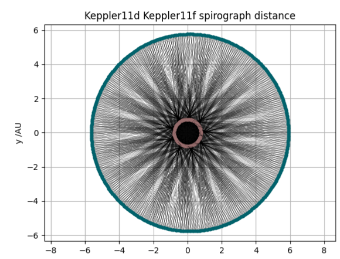
(c) Trappist d - Trappist e



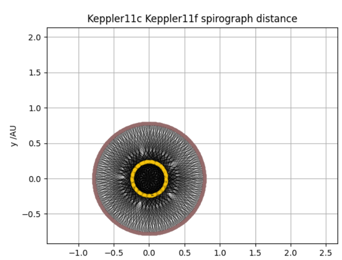
(d) Trappist b - Trappist e



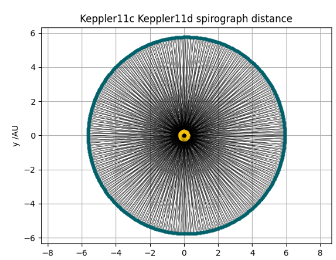
(e) Kepler b - Kepler f



(f) Kepler d - Kepler f

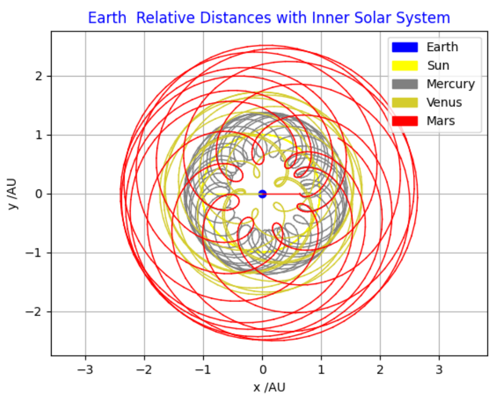


(g) Kepler c - Kepler f

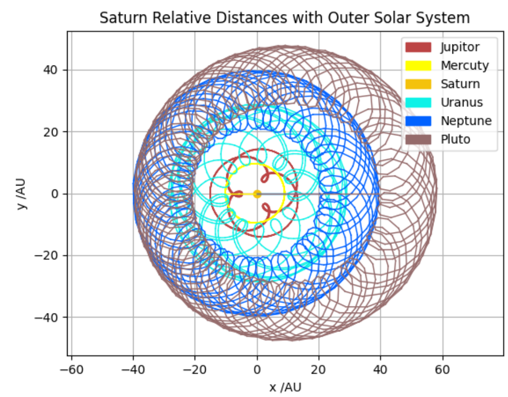


(h) Kepler d - Kepler c

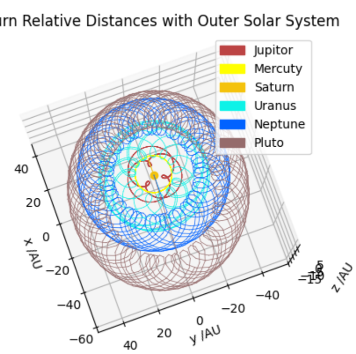
Fig. 10: Spirographs of Exoplanet Systems Kepler11 and Trappist 1



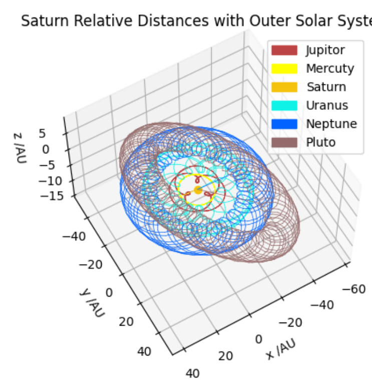
(a) Solar Inner Planets Relative Distance With Earth



(b) Solar Outer Planets Relative Distanced to Saturn

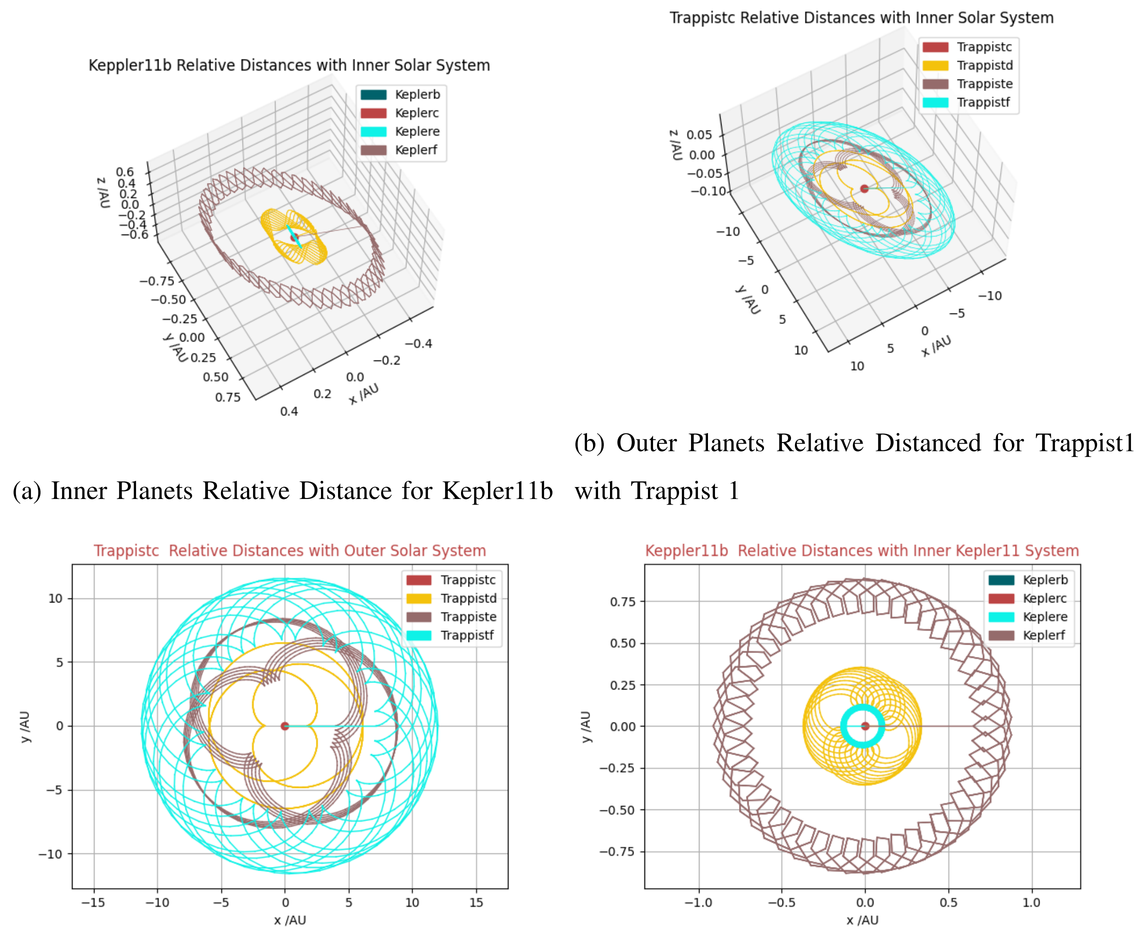


(c) Solar Outer Planets Relative Distanced to Saturn in 3D



(d) Solar Outer Planets Relative Distanced to Saturn in 3D in different angle of view

Fig. 11: Relative Distances of Solar System



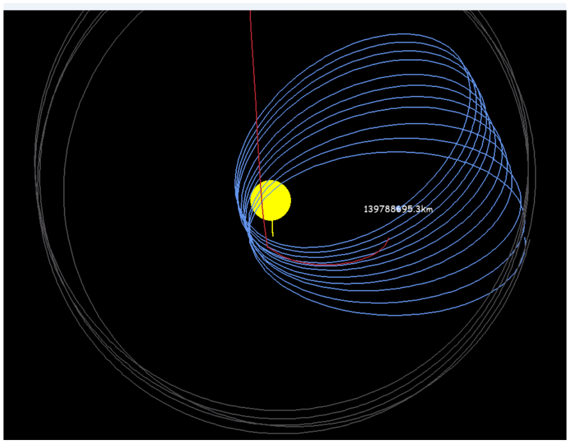
(a) Inner Planets Relative Distance for Kepler11b

(b) Outer Planets Relative Distanced for Trappist1 with Trappist 1

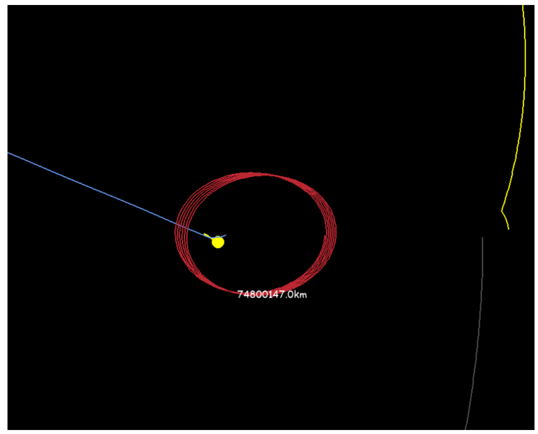
(c) Outer Planets Relative Distanced for Trappist1b in 2D view

(d) Inner Planets Relative Distance for Kepler11b in 2D view

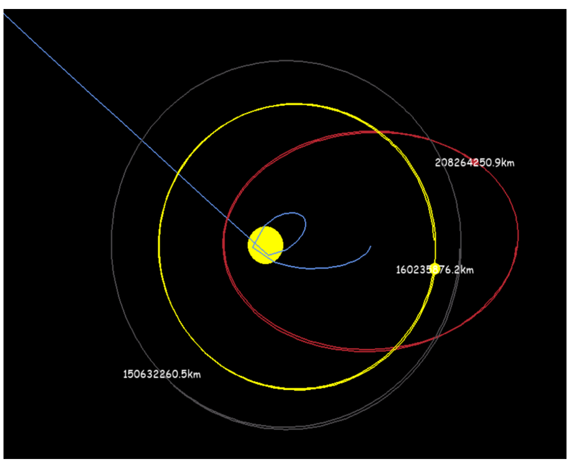
Fig. 12: Relative Distances of Exoplanet Systems



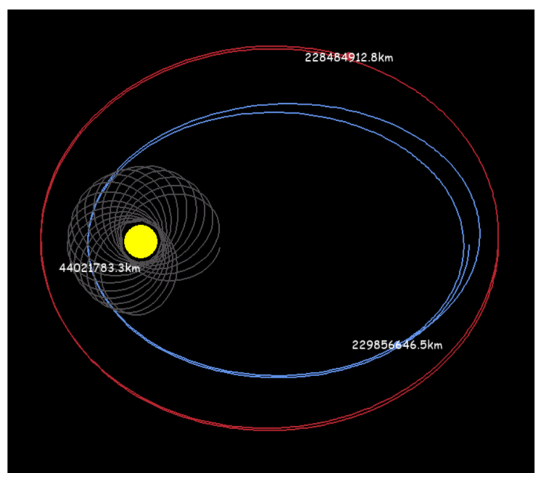
(a) Custom Orbits 1: Forms one circular orbit and one elliptical orbit with inclination



(b) Custom Orbits 2: Single elliptical orbit formed



(c) Custom Orbits 3: 2 Circular and one elliptical orbits formed



(d) Custom Orbits 4: 2 Elliptical and one Strange Behaviour and possibly the planet will be destroyed by the star

Fig. 13: Custom Planet Orbit Formation Using Kepler's Third Law and Pygame