Transformada de Laplace

PARTE 1) $$H(s) = \frac{50\,(s+5)}{(s+5)(s+1)}$$

ⓐ Transformada inversa $\rightsquigarrow$ raízes $[-5, -1]$ $\rightsquigarrow$ $\frac{1}{s-\lambda}$ (*1)

$$\frac{50.(s+5)}{(s+5)(s+1)} = \frac{K1}{(s+5)} + \frac{K2}{(s+1)}$$ (FUNÇÕES PARCIAIS)

$$\frac{50\,\cancel{(s+5)}}{\cancel{(s+5)}(s+1)}(s+5) = \frac{K_1\cancel{(s+5)}}{\cancel{(s+5)}} + \frac{K2\,(s+5)}{(s+1)}$$

onde $\boxed{s=-5}$

$$\frac{50\,(s+5)}{(s+1)} = K1 + \frac{K2\,(s+5)}{(s+1)}$$

$$\frac{50.(0)}{-4} = K1 + 0 \rightsquigarrow \boxed{K1 = 0}$$

$$\frac{50\cancel{(s+5)}}{\cancel{(s+5)}\cancel{(s+1)}}.\cancel{(s+1)} = \frac{K1(s+1)}{(s+5)} + \frac{K2\,\cancel{(s+1)}}{\cancel{(s+1)}}$$

onde $\boxed{s=-1}$

$$50 = 0 + K2 \rightsquigarrow \boxed{K2 = 50}$$

$$H(s) = \frac{0}{(s+5)} + \frac{50}{(s+1)} \iff \boxed{h(t) = 50\,e^{-t}\,u(t)}$$

ⓒ emcomtrar $X(s)$ com

TABELA

$$x(t) = \sin(t)\, u(t) \rightsquigarrow \frac{1}{s^2+1}$$

ou seja, pela tabela $\sin bt\, u(t) \iff \frac{b}{s^2+b^2}$.

$$X(s) = \frac{1}{s^2+1} \quad e \quad H(s) = \frac{50(s+5)}{(s+5)(s+1)}$$

através do denominador característico extraído através da operação $X(s) \cdot H(s)$ foi possível calcular os raízes

roots([$s^4 + 6s^3 + 6s^2 + 6s + 5$])

$$r_1 = -5 \ ; \ r_2 = -1 \ ; \ r_3 = +j \ ; \ r_4 = -j$$

$$\frac{50s+250}{(s+1)(s+5)(s-j)(s+j)} = \frac{K1}{(s+1)} + \frac{K2}{(s+5)} + \frac{K3}{(s-j)} + \frac{K4}{(s+j)}$$

$$\frac{(50s+250)\cancel{(s+1)}}{\cancel{(s+1)}(s+5)(s-j)(s+j)} = \frac{K1\cancel{(s+1)}}{\cancel{(s+1)}} + \frac{K2(s+1)}{(s+5)} + \frac{K3(s+1)}{(s-j)} + \frac{K4(s+1)}{(s+j)}$$

onde $\boxed{s=-1}$

$$\frac{50(-1)+250}{(-1+5)(-1-j)(-1+j)} = \boxed{K1 = 25}$$

tilibra

$$\frac{(50s+250)\cancel{(s+5)}}{(s+1)\cancel{(s+5)}(s-j)(s+j)} = \frac{K1\,(s+5)}{(s+1)} + \frac{K2\cancel{(s+5)}}{\cancel{(s+5)}} + \frac{K3(s+5)}{(s-j)} + \frac{K4(s+5)}{(s+j)}$$

onde $\boxed{s=-5}$ $\quad 50.(-5) = -250 + 250 = 0$

$$\boxed{K2=0}$$

$$\frac{(50s+250)\cancel{(s-j)}}{(s+1)(s+5)\cancel{(s-j)}(s+j)} = \frac{K1(s-j)}{(s+1)} + \frac{K2(s-j)}{(s+5)} + \frac{K_3\cancel{(s-j)}}{\cancel{(s-j)}} + \frac{K4(s-j)}{(s+j)}$$

onde $\boxed{s=+j}$

$$\frac{j50+250}{(j+1)(j+5)(j2)} = \boxed{K3 = -12{,}5 - j12{,}5}$$

$$\frac{(50s+250)\cancel{(s+j)}}{(s+1)(s+5)(s-j)\cancel{(s+j)}} = \frac{K1(s+j)}{(s+1)} + \frac{K2(s+j)}{(s+5)} + \frac{K_3(s+j)}{(s-j)} + \frac{K4\cancel{(s+j)}}{\cancel{(s+j)}}$$

onde $\boxed{s=-j}$

$$\frac{-j50+250\ \cancel{(s+j)}}{(s+1)(s+5)(s-j)\cancel{(s+j)}} = \boxed{K4 = -12{,}5 + j12{,}5}$$

$$y(s) = \frac{25}{(s+1)} + \frac{0}{(s+5)} + \frac{(-12{,}5 - j12{,}5)}{(s-j)} - \frac{12{,}5 + j12{,}5}{(s+j)}$$

$\downarrow$ Passando p/ domínio tempo

$$y(t) = (25e^{-t} + 17{,}677e^{-j2{,}35}e^{jt} + 17{,}677e^{j2{,}35}e^{-jt})\,u(t)$$