Modern Physics

Bhor Atom Model:

Bhor assumed basically Rutherford Nuclear model of the atom and tried to overcome the defects of the model. He proposed the following two postulates.

(1) The atom has a massive positively –charged nucleus and

(2) The electrons revolve round their nucleus in circular orbits- the centrifugal force being balanced, as before, by the electrostatic pull between the nucleus and electrons. However, he extended this model further by utilizing Planck's Quantum theory. He made the following three assumptions:

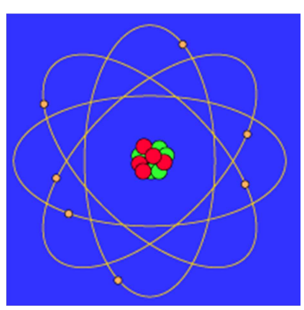
(3) An electron cannot revolve round the nucleus in any arbitrary orbit but in just certain definite and discrete orbits. Only those orbits are possible (or permitted) for which the orbital angular momentum of the electron is equal to an integral multiple of $\frac{h}{2\pi}$,where n is an integer and h is Planck's constant.Such orbits are also known as stationary orbits.

(4) While revolving in these permitted stationary (or stable) orbits, the electron does not radiate out any electro-magnetic energy .In other words, the permissible orbits are non-radiating paths of the electron.

(5) The radiates out energy only when an electron jumps from one orbit to another. If E_2 and E_1 are the energies corresponding to two orbits before and after the jump , the frequency of the emitted photon is given by the relation,

$$E_2 - E_1 = hf \quad or \quad \Delta E = hf$$

where f is the frequency of the emitted radiations.



The Bohr formulae:

Based on these postulates, Bohr derived the formulae for
(i) the radii of the stationary orbits and
(ii) the total energy of the electron in the orbit.

Calculation of the radii:

Consider an atom whose nucleus has a positive charge ***Ze*** and mass ***M***. For hydrogen, Z=1. Let an electron of charge (-e) and mass m move round the nucleus in an orbit of radius *r*. Since $M \gg m$, the nucleus is stationary. Hence the mass of the nucleus does not come into the calculations.

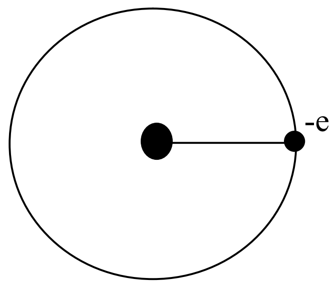


Fig.-01

$$\left.\begin{matrix} The\ electrostatic\ force\ of\ attraction \\ between\ the\ nucleus\ and\ the\ electron \end{matrix}\right\} = \frac{1}{4\pi\varepsilon_o}\frac{(Ze)(-e)}{r^2} \text{ ....} (1)$$

The centrifugal force on the electron = $\frac{mv^2}{r}$ .... (2)

The system will be stable if $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{Ze^2}{r^2}$ .... (3)

According to Bohr's first postulate,

$$mvr = \frac{nh}{2\pi} \text{ or } v = \frac{nh}{2\pi rm}$$

$$\text{or, } v^2 = \frac{n^2h^2}{4\pi^2r^2m^2} \text{ ....} (4)$$

Substituting this value of v^2 in (3),

$$\frac{m}{r}\left(\frac{n^2h^2}{4\pi^2r^2m^2}\right) = \frac{1}{4\pi\varepsilon_o}\frac{Ze^2}{r^2}$$

$$\text{or, } r = \frac{n^2h^2\varepsilon_o}{\pi Ze^2m} \text{ ....} (5)$$

$\therefore$ *Radius of the* n^{th} *permissible orbit for hydrogen,* $r_n = \frac{n^2h^2\varepsilon_o}{\pi e^2m}$ $(\because Z = 1)$.

From equation (5) we find that $r_n \propto n^2$.
The radii of the orbits are in the ratio of 1: 4: 9: 16: 25 etc.

The radius of the first orbit for hydrogen atom

$$r_1 = \frac{1^2 \times (6.625 \times 10^{-34})^2 (8.854 \times 10^{-12})}{\pi (1.6 \times 10^{-19})^2 (9.11 \times 10^{-31})} m = 0.053\, nm$$

This is called the Bohr radius.

$\therefore \quad r_1 = 0.053\, nm\, ;\ r_2 = 2^2\, r_1 \ldots \ldots \ldots\, and\ r_{n=} n^2 r_1.$

Calculation of total energy:

The total energy of the electron in any orbit is the sum of its kinetic and potential energies. The potential energy of the electron is considered to be zero when it is at an infinite distance from the nucleus. P. E. of an electron in an orbit is given by the work done in bringing the electron from infinity to that orbit. This amount of work is obtained by integrating the electrostatic force of attraction between the nucleus and the electron from the limits ∞ to r.

P.E. of the electron = $\int_{\infty}^{r} \frac{Ze^2}{4\pi\varepsilon_o r^2}\, dr = \frac{-Ze^2}{4\pi\varepsilon_o r}$ .... (6)

K.E. of the electron = $\frac{1}{2} mv^2 = \frac{Ze^2}{8\pi\varepsilon_o r}$ .... (7)

$$\left[\therefore from\ (3), mv^2 = \frac{Ze^2}{4\pi\varepsilon_o r}\right]$$

∴ Total energy of the electron in the n^{th} orbit = E_n = P.E. + K.E.

$$= \frac{-Ze^2}{4\pi\varepsilon_o r} + \frac{Ze^2}{8\pi\varepsilon_o r} = \frac{-Ze^2}{8\pi\varepsilon_o r}$$

Substituting the value of r from (5),

$E_n = \frac{-me^4 Z^2}{8\varepsilon_o^2 n^2 h^2}$ .... (8)

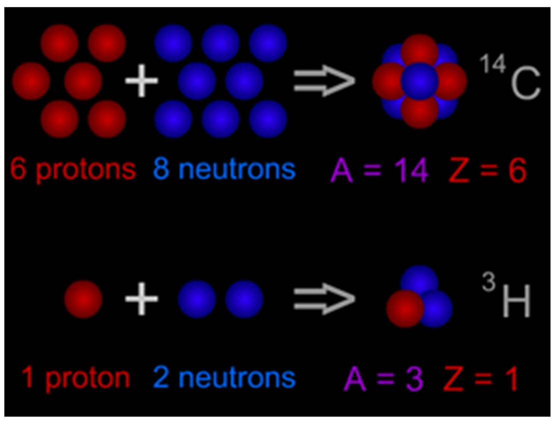
As the value of n increases, E_n increases. Hence, the outer orbits have greater energies than the inner orbits.

The Atomic Nucleus:

The nucleus (atomic nucleus) is the center of an atom. Atomic nucleus is made up of elementary particle called protons and neutrons.

A proton has a positive charge of the same magnitude as that of an electron. A neutron is electrically neutral. The proton and the neutron are considered to be two different charge states of the same particle which is called a nucleon.

A species of nucleus, known as a nuclide, is represented schematically by $_{Z}X^{A}$, where Z, the atomic number indicates the number of protons, A, the mass number , indicates the total number of protons plus neutrons and X is the chemical symbol of the species.
N=Number of neutrons=A-Z.



As an example, the carbon nucleus $_{6}C^{14}$ has Z=6 protons, A=14 nucleons and N=14-6=8 neutrons.

General Properties of a Nucleus:

Some of the important properties of an atomic nucleus are discussed below:

i) **Nuclear mass:** We know that nucleus consist of protons and neutrons. Then the mass of the nucleus is equal to the sum of the masses of neutrons and protons. Therefore, we can write

Nuclear mass=Zm_p+Nm_n.

Where, m_p and m_n are the respective proton and neutron masses and N is the neutron number. For example, the uranium nucleus U^{238} has a mass of 238 a.m.u.

ii) **Nuclear Size:** Rutherford's work on the scattering of α- particles showed that the mean radius of an atomic nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10} m. Thus the nucleus is about 10000 times smaller in radius than the atom.
The empirical formula for the nuclear radius is

$$R = r_0 A^{1/3}$$

where A is the mass number and $r_0 = 1.3 \times 10^{-1}$ m=1.3fm. Nuclei are so small that the Fermi (fm) is an appropriate unit of length .1 fm =10^{-15}m .From this formula we find that radius of the $_6C^{12}$ nucleus is $R \approx (1.3)(12)^{1/3} = 3fm.$

iii) **Nuclear Density:** The nuclear density is defined as the nuclear mass per unit volume. Since the nuclear mass is the mass of proton and neutron of an atom.

The nuclear density ρ_N can be calculated from

$$\rho_N = \frac{Nuclear\ mass}{Nuclear\ volume}.$$

$$\rho_N = \frac{Zm_p + Nm_n}{\frac{4}{3}\pi R^3}$$

In the case of $_6C^{12}$ whose nuclear density is,

$$\therefore\ \rho_N = \frac{12.0 \times 1.66 \times 10^{-27} Kg}{\frac{4}{3} \times 3.1416 \times (3 \times 10^{-15} m)^3}$$

$$= 1.8 \times \frac{10^{17} Kg}{m^3}.$$

iv) **Nuclear Charge:**
The charge of the nucleus is due to the protons contained in it. Each proton has a positive charge of $1.6 \times 10^{-19}\ C$. The nuclear charge is Ze where Z is the atomic number of the nucleus.

Mass Defect:

The difference between the measured mass M and the mass number A of a nuclide is called the mass defect.

$$\therefore \text{Mass defect, } \Delta m = M - A$$

Binding Energy:

When the Z protons and N neutrons combine to make a nucleus, some of the mass (Δm) disappears because it is converted into an amount of energy $\Delta E = (\Delta m)c^2$. This energy is called the binding energy (B.E.) of the nucleus.

If M is the experimentally determined mass of a nuclide having Z protons and N neutrons,

$$B.E. = \{(Zm_p + Nm_n) - A\}\, c^2.$$

Let us illustrate the calculation of B.E. by taking the example of the deuteron, ${}_1H^2$. A deuteron is formed by a proton and a neutron.

Mass of proton =1.007276 amu

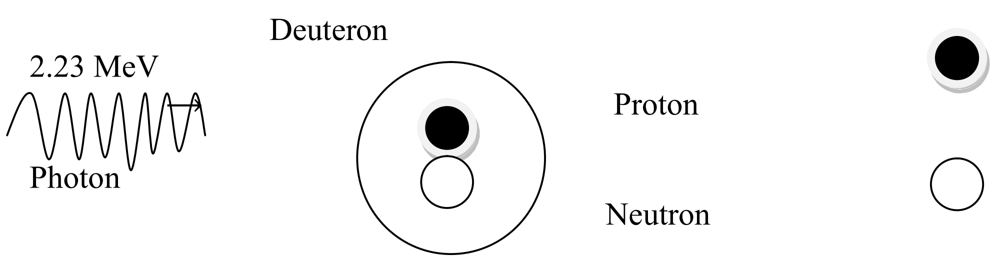
Mass of neutron =1.008665 amu

∴ Mass of (proton + neutron) in free State = 2.015941 amu

Mass of deuteron nucleus=2.013553 amu

∴ Mass defect= Δm=0.002388 amu

B.E. =0.002388×931=2.23 MeV ($\because 1\ amu = 931\ MeV$)



Nuclear Binding Energy Curve:

The binding energy curve is obtained by dividing the total binding energy of a nucleus by the number of nucleons it contains.

$$Binding\ energy\ per\ nucleon = \frac{Total\ B.E.of\ a\ nucleus}{The\ number\ of\ nucleons\ it\ contains}$$

The observed variation in the Binding Energy per nucleon is plotted as a function of mass number, A in Fig.-01.

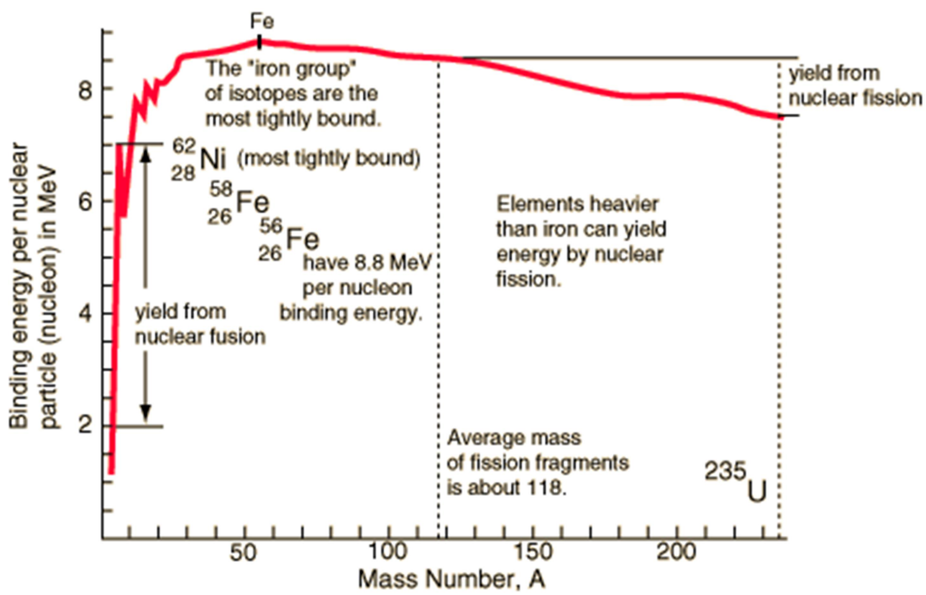


Fig.-01

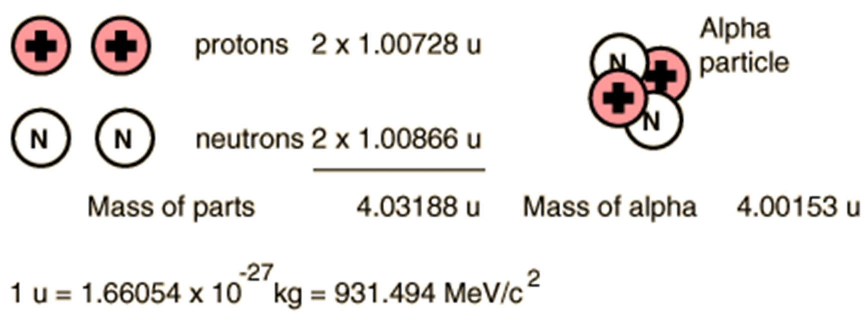
The curve rises abruptly at first and then more gradually until it reaches a maximum of 8.79 MeV at A=56, corresponding to the iron nucleus, $_{26}Fe^{56}$. The curve then drops slowly to about 7.6 MeV at the highest mass numbers. Evidently, nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate each of their nucleons. This fact suggests that a large amount of energy will be liberated if heavier nuclei can somehow be spilt into lighter ones or if light nuclei can somehow be joined to form heavier ones. The former process is known as nuclear fission and the latter as nuclear fusion. Both the processes indeed occur under proper circumstances and do evolve energy as predicted.

Nuclear Binding Energy:

Nuclei are made up of protons and neutron, but the mass of a nucleus is always less than the sum of the individual masses of the protons and neutrons which constitute it. The difference is a measure of the nuclear binding energy which holds the nucleus together. This binding energy can be calculated from the Einstein relationship:

$$\text{Nuclear binding energy} = \Delta mc^2$$

For the alpha particle Δm= 0.0304 a.m.u. which gives a binding energy of 28.3 MeV.



Example 1. **Calculate the binding energy of an α-particle and express the result both in MeV and Joules.**

Ans: B.E.=28.29 MeV, $=45.32\times10^{-13}$J

Example 2. : **The mass of $_{17}Cl^{35}$ is 34.9800 a.m.u. Calculate its binding energy. What is the binding energy per nucleon?**

Mass of $_0n^1 = 1.008665\ a.m.u.$ Mass of $_1H^1 = 1.007825\ a.m.u.$

Solution:

The given chlorine atom has 17 protons and (35-17) =18 neutrons.

Mass Mass of $_1H^1 = 1.007825\ a.m.u.\times 17 = 17.133025$

Mass of $_0n^1 = 1.008665\ a.m.u.\times 18 = 18.155970$

The mass of separate constituents=35.288995

Mass Defect $\Delta m = 35.288995 - 34.9800$

=.308995 a.m.u.

Now, 1 a.m.u. =931 MeV

∴ Binding energy of the nucleus $=931 \times 0.308995 = 288\ MeV$

No. of Nucleons =35

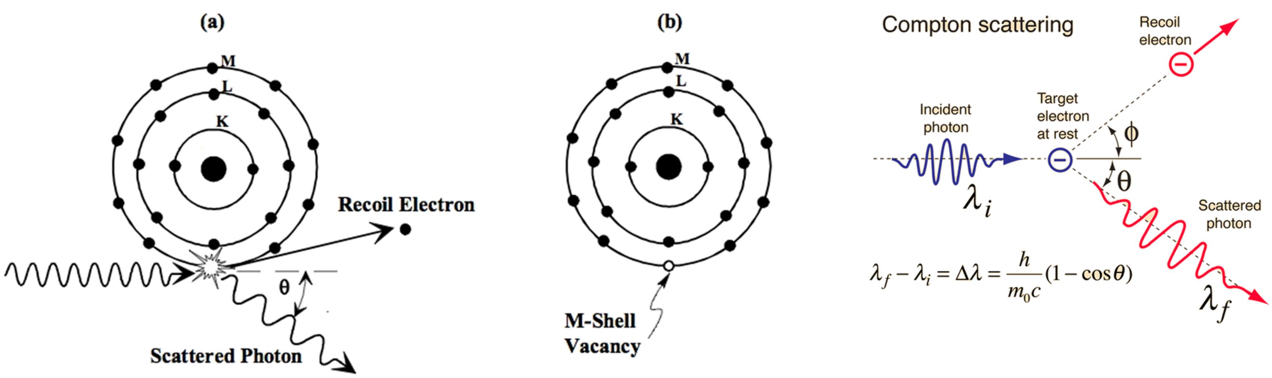
Binding energy per nucleon=288/35=8.22 MeV.

Compton Effect:

On the basis of quantum theory of radiation, Compton in 1925 proposed an adequate explation, treating the phenomenon of scattering as due to an elastic collision of two particles, the photon and the electron of the scattered.

This Process is predominant for medium energy photons with energies between 150 KeV and 25 MeV.

In this process an incident photon of energy hν interacts with a loosely bound orbital or "free" electron, transfers some of its energy to the electron. The electron (Compton electron) is ejected with kinetic energy E_k in a direction making an angle φ between 0 and 90^0 with the direction of the incident photon .The incident photon is also scattered in a direction making an angle θ between 0 and 180^0 with its initial direction with a reduced energy $h\nu'$, where ν and ν' are the frequencies of incident and scattered gamma photon respectively.
The Compton effect is shown in Fig.-01.



The kinetic energy of the Compton electron and the energy of the scattered photon are a function the energy of the incident photon and of the angles φ and θ.

The energy E_k of the Compton electron is

$$E_k = 2\alpha h\nu cos^2\varphi/\{(1+\alpha)^2 - \alpha^2 cos^2\varphi\}$$

Photo-Electric Effect:

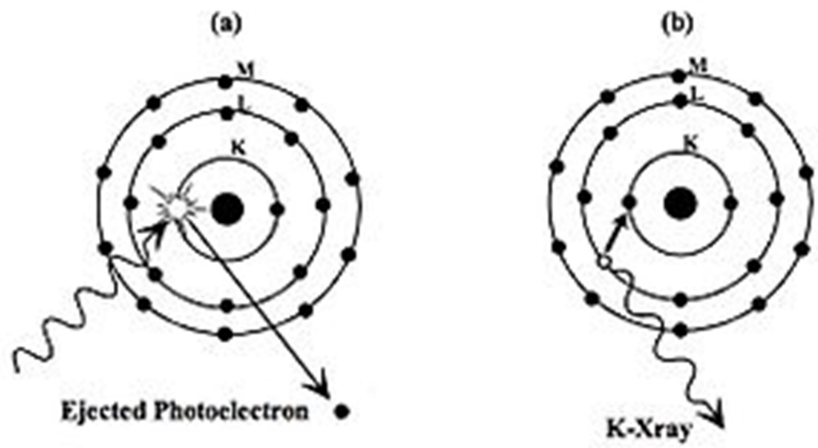
Hertz in 1887 and Hallwachs in 1888 discovered the features of photo-electric effect.

The electrons ejected out of the metal under the action of light(photon) are known as photo-electrons and this phenomenon is known as "**photo-electric effect**".

This process is predominant for photon in the low energy range, below 115 KeV ,which makes it particularly important for diagnostic X –ray or γ radiation. In this process ,a photon imparts all of its energy E to an inner shell's bound electron ; as a result ,the electron is energized and ejected from the atom with a kinetic energy E_k ,so that

$$E_k=E-W_i$$

where W_i is the binding energy of the electron in that particular orbit and $E> W_i$ .The photo electric effect is shown in Fig.-01.



If the energy E of the photon is less than W_i for an electron in the K shell ,the photoelectric effect takes place with an electron in the L shell ,and so on. An electron from a more peripheral shell (say,L shell) ,fills the vacancy created in an inner shell (say, K shell) by an ejected photoelectron .

The energy W_k-W_L released by the transition may be-

(1) emitted as a photon of fluorescence, characteristic of the element ,having an energy E given by

$$E= W_k-W_L$$

further reorganization of the electronic shell system results in the emission of characteristic X-ray .

(2) Cause Augur electrons to be ejected. These electrons will possess a kinetic energy E given by

$$E_{aug}= (W_K-W_L)-W_i$$

where W_i represents the binding energy of the Augur electron in its shell ; in this case

$$W_i= W_L \text{ or } W_L$$

Einstein's equation for photo-electric effect:

In 1905 A. Einstein made use of quantum theory of radiation to explain the basic facts and laws of photo-electric emission. According to quantum theory,

i. A light of frequency ν consist of quanta of energy or photon. Each photon has an energy $h\nu$ and travels with the velocity of light.

ii. In photon-electric effect, one photon is completely absorbed by one electron in the photo-cathode.

Consider a photon of energy $h\nu$ which ejects a photo-electron from a material, with a velocity v, then

$$\text{The kinetic energy of the electron} =\frac{1}{2}\mathrm{mv}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{i})$$

Therefore, from the conservation of energy,

$$h\nu =\frac{1}{2}\mathrm{mv}^2 + \mathrm{w}_0$$

where, w_0 =energy needed to make an electron free also called work function.
Equation (i) is the photo-electric equation of Einstein.

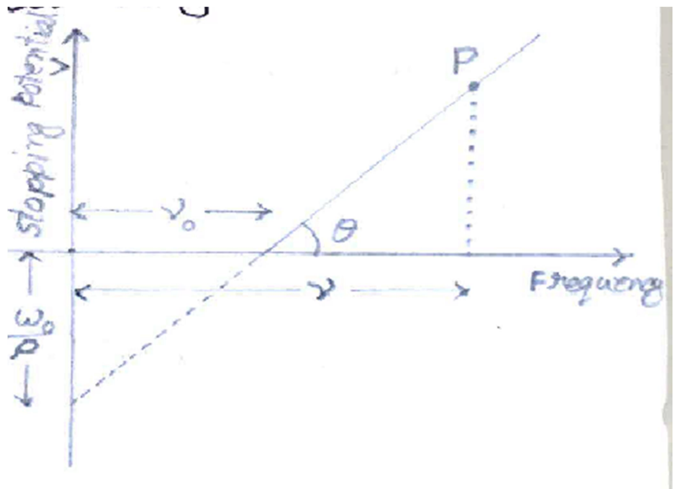
<u>Derivation of equation</u>:

Let the charge of the electron ejecting with maximum velocity from the metal plate is e and the stopping potential is v_0. If the maximum velocity of the ejecting electron is v_m, then

Maximum K.E. of the electron $=\frac{1}{2}\mathrm{mv}_\mathrm{m}^2$

Therefore, $\frac{1}{2}\mathrm{mv}_\mathrm{m}^2 = \mathrm{ev}_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

Now plotting the frequency of the radiation along X-axix and ev_0- along Y-axis a straight line is obtained which will intersect x-axis at ν_0. this frequency ν_0 is called the threshold frequency. A point p is considered on the straight line. Let the straight line is inclined with the x-axis at angle θ.



Now we get

$tan\theta = \frac{ev_0}{\nu - \nu_0}$............................(3)

Where, $tan\theta = slope\ of\ the\ straight\ line$

= h , a constant

Therefore, $h = \frac{ev_0}{\nu - \nu_0}$

$\Rightarrow ev_0 = h(\nu - \nu_0)$....................(4)

Putting this value into equation (2),

$$\frac{1}{2}mv_m^2 = h(\nu - \nu_0)$$

$$\Rightarrow \frac{1}{2}mv_m^2 = h\nu - h\nu_0$$

$$\Rightarrow \frac{1}{2}mv_m^2 = h\nu - w_0 \ldots\ldots\ldots\ldots\ldots(5)$$

Where, $h\nu_0 = w_0 = photo - electric\ work\ function.$

Equation (5) is the **Einstein's equation for photo-electric effect and** $h\nu_0 = \phi$ is the relation between work function and threshold frequency.

X-ray:

X-rays are high energy photons (1-100keV) or electromagnetic radiation, having a very short wave-length of the order of 1Å.

X-rays was discovered by Willium Rontgen in 1985 while he was studying with a discharge tube. It was discovered when the Crooke's dark space filled the whole tube inside a discharge tube.

Properties of X-rays:

1) They are propagated in straight lines with the velocity of light .i.e., 3×10^8 m/s.

2) They are not deflected by magnetic or electric fields and therefore they do not possess any charge.

3) They are highly penetrating and can pass through many solids which are opaque to visible light .e.g., wood, flesh, paper, cardboard, thin sheets of metals ,ebonite but not penetrate through heavy metals and bones.

4) They produce fluorescence in barium-platinocyanide, Zincsulphide and cadmium tungstate.

5) They have destructive effect on living tissues. Long exposure of the skin to X-rays produce reddening of the skin and kill white corpuscles of the blood. X-rays are used for destroying and burning the ulcers and tumours in the body.

Uses of X-ray:

i) X-rays are especially useful in the detection of pathology of the skeletal system, but are also useful for detecting some disease processes in soft tissue. Some notable examples are the very common chest X-ray, which can be used to identify lung diseases such as pneumonia, lung cancer or pulmonary edema, and the abdominal X-ray, which can detect intestinal obstruction, free air (from visceral perforations) and free fluid (in ascites). X-rays may also be used to detect pathology such as gallstones (which are rarely radiopaque) or kidney stones which are often (but not always) visible. Traditional plain X-rays are less useful in the imaging of soft tissues such as the brain or muscle.

ii) X-ray crystallography in which the pattern produced by the diffraction of X-rays through the closely spaced lattice of atoms in a crystal is recorded and then analysed to reveal the nature of that lattice.

iii) X-ray astronomy, which is an observational branch of astronomy, which deals with the study of X-ray emission from celestial objects.

iv) Industrial radiography uses X-rays for inspection of industrial parts, particularly welds.

v) Airport security luggage scanners use X-rays for inspecting the interior of luggage for security threats before loading on aircraft.

vi) Border security truck scanners use X-rays for inspecting the interior of trucks for at country borders.

vii) X-ray photoelectron spectroscopy is a chemical analysis technique relying on the photoelectric effect, usually employed in surface science.

Bragg's Law:

Statement:

The intensity of the reflected beam at certain angles will be maximum, where the two reflected wave from two different planes have a path difference equal to an integral multiple of the wave-length of X-rays, while at some other angles, the intensity will be minimum.

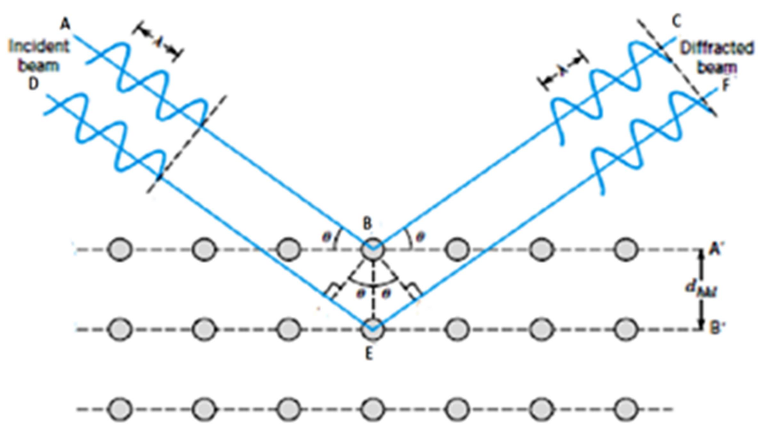
Derivation:

Let us consider parallel lattice planes equidistant from one another in a crystal structure, separated by a distance d.

Let us consider a mono-chromatic beam of X-rays AB of wave –length λ is incident on a layer of atoms in one cleavage plane Y of a crystal at a glancing angle θ at B and after reflection goes along BC. The ray DE incident on cleavage plane Z and after reflection goes along EF. BP and BQ perpendicular is drawn on DE and EF respectively.

Therefore, the path difference between the reflected waves along BC and EF

=PE+EQ.



Now,

In the ΔPBE,

$$Sin\theta = \frac{PE}{BE}$$
$$\therefore PE = BESin\theta = dSin\theta$$

Similarly,

In the ΔQBE,

$$EQ = BESin\theta = dSin\theta$$

$$\therefore \text{Path difference, } PE + EQ = dSin\theta + dSin\theta$$
$$= 2dSin\theta$$

If the path difference is an integral multiple of wave-length λ, then constructive interference will occur between the reflected beams and they will reinforce with each other .Therefore, for the reflected beam to be of maximum intensity,

$$\boldsymbol{2dSin\theta = n\lambda} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (1) ;$$
$$\text{where, } n = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, etc$$

This equation is known as Bragg's equation and represents Bragg's law for X-ray diffraction.

De Broglie's Wave:

In 1924 a young physicist, de Broglie, speculated that nature did not single out light as being the only matter which exhibits a wave-particle duality. He proposed that ordinary "particles" such as electrons, protons, or bowling balls could also exhibit wave characteristics in certain circumstances.

A photon of energy

$$E = h\upsilon$$

must carry a momentum

$$P = \frac{h\upsilon}{c} = \frac{h}{\lambda}$$

where λ is the wavelength . De Broglie suggested that whenever there are particles of momentum P, their motion is associated with a wave of wavelength.

This hypothesis is called De Broglie's hypothesis.