$$\text{or,}\quad \lambda_o = \frac{12{,}400}{W_o}$$

$$= \frac{12{,}400}{4.5}$$

$$= 2755\ \mathring{A}.$$

Example 12.4. The photo-electric threshold of copper is 3200 Å. If ultra-violet light of wavelength 2500 Å falls on it, find (a) the maximum kinetic energy of the photo-electrons ejected, (b) maximum velocity of the photo-electrons and (c) the value of the work function.

Soln.

(a) Maximum kinetic energy of photo-electrons is

$$T_{max} = h\nu - h\nu_o$$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)$$

$$= \frac{6.625\times10^{-34}\times3\times10^{8}}{10^{-10}}\left(\frac{1}{2500} - \frac{1}{3200}\right)$$

$$= \frac{6.625\times3\times7}{25\times32}\times 10^{-18}\ \text{joules}$$

$$= \frac{6.625\times3\times7\times10^{-18}}{25\times32\times1.6\times10^{-19}}\ \text{eV}$$

$$= 1.087\ \text{eV}.$$

Alternately

$$T_{max} = 12{,}400\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)\text{eV}$$ where λ and λ_o are in A.U.

$$= 12{,}400\left(\frac{1}{2500} - \frac{1}{3200}\right)\text{eV}$$

$= 1.087$ eV.

(b) Maximum velocity of the photo-electrons is given by

$$\frac{1}{2}mv_{max}^2 = T_{max}$$

$$\therefore \quad v_{max} = \sqrt{\frac{2T_{max}}{m}}$$

$$= \sqrt{\frac{2\times6.625\times3\times7\times10^{-18}}{25\times32\times9.1\times10^{-31}}}$$

$= 6.18 \times 10^5$ m/sec.

(c) Work function $w_o = h\nu_o = \frac{hc}{\lambda_o}$

$$= \frac{6.625\times10^{-34}\times3\times10^8}{3200\times10^{-10}} \text{ joules}$$

$$= \frac{6.625\times10^{-34}\times3\times10^8}{3200\times10^{-10}\times1.6\times10^{-19}} \text{ eV}$$

$= 3.88$ eV.

Alternately, $\quad w_o = \frac{12,400}{\lambda_o}$ eV λ_o in A.U.

$$= \frac{12,400}{3,200} \text{ eV}$$

$= 3.88$ eV.

Example 12.5. *A photo-electric surface has a work function of 4 eV. What is the maximum velocity of the photoelectrons emitted by light of frequency 10^{15} Hertz incident on the surface. $h = 6.6 \times 10^{-34}$ joule-sec.; $e = 1.6 \times 10^{-19}$ coulomb; $m = 9 \times 10^{-31}$ kg.*

Soln.

$w_o = 4$ eV $= 4 \times 1.6 \times 10^{-19}$ joules

$$= 6.4 \times 10^{-19} \text{ joules}$$

$$\frac{1}{2}mv^2 = h\nu - w_o$$

$$= 6.6 \times 10^{-34} \times 10^{15} - 6.4 \times 10^{-19}$$

$$= 0.2 \times 10^{-19} \text{ joules}$$

$$\therefore \quad v = \sqrt{\frac{2\times0.2\times10^{-19}}{m}}$$

$$= \sqrt{\frac{0.4\times10^{-19}}{9\times10^{-31}}}$$

$$= 2.107 \times 10^5 \text{ m/sec.}$$

***Example 12.6.** Calculate the threshold frequency corresponding wavelength of radiation incident on a cert whose work function is 3.31×10^{-19} J. Given, Planck's c 6.62×10^{-34} J-s.*

Soln.

Work function, $w_o = h\nu_o$

or, $\nu_o = \dfrac{w_0}{h}$

Example 12.7. *Photo-electrons are emitted with a maximum speed of 7 x 10^5 m/sec from a metal surface when light of frequency 8 x 10^{11} kHz falls on it. What is the threshold frequency of the metal.*

Soln.

$$T_{max} = h\nu - h\nu_o$$

$$\frac{1}{2}mv^2 = h(\nu - \nu_o)$$

or, $$\frac{1}{2} \times 9.1 \times 10^{-31} \times (7 \times 10^5)^2 = 6.6 \times 10^{-34} (8 \times 10^{14} - \nu_o)$$

or, $$6.6 \times 10^{-34} \nu_o = (52.8 - 22.3) \times 10^{-20}$$

or, $$\nu_o = \frac{30.5 \times 10^{-20}}{6.6 \times 10^{-34}} = 4.62 \times 10^{14} \text{ Hz.}$$

Example 12.8. *A tungsten cathode whose threshold wavelength is 2300 Å is irra...*

Cal...

Example 13.6. Calculate the glancing angle on the cube (110) of a rock salt crystal ($a = 2.81\,\mathring{A}$) corresponding to second order diffraction maximum for the X-rays of wavelength $0.71\,\mathring{A}$.

Soln.

From the Bragg's relation

$2d \sin\theta = n\lambda$ we have

$$\sin\theta = \frac{n\lambda}{2d}$$

here $n = 2$

$\lambda = 0.71\,\mathring{A}$

and $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$

$$= \frac{2.81}{\sqrt{1^2 + 1^2}} = \frac{2.81}{\sqrt{2}}\,\mathring{A}$$

$$= 1.987\,\mathring{A}$$

$$= \frac{2 \times 0.71\,\mathring{A}}{2 \times 1.9871\,\mathring{A}}$$

$$= 0.3573$$

$$\therefore \quad \theta = \sin^{-1}(0.3573)$$

$$= 20^\circ\, 54'.$$

***Example 13.7.** X-rays of wavelength $0.36\,\mathring{A}$ are diffracted in first order at an angle of 4.80 in Bragg's crystal spectrometer. Find the effectiv...*

$$= 0.3573$$

$$\therefore\quad \theta = \sin^{-1}(0.3573)$$

$$= 20^\circ\ 54'.$$

$$= \frac{2.81}{\sqrt{1^2+1^2}} = \frac{2.81}{\sqrt{2}}\ \mathring{A}$$

$$= 1.987\ \mathring{A}$$

Example 13.7. *X-rays of wavelength 0.36 Å are diffracted in first order at an angle of 4.80 in Bragg's crystal spectrometer. Find the effective spacing of atomic layers in the crystal.*

Soln.

From the Bragg's relation, we have

$$2d \sin\theta = n\lambda$$

$$\text{or,}\quad d = \frac{n\lambda}{2\sin\theta}$$

Here $\theta = 4.80^\circ$, or, $\sin\theta = 0.084$

$\lambda = 0.36\ \mathring{A}$ $n = 1$

$$\therefore\quad d = \frac{n\lambda}{2\sin\theta} = \frac{1\times 0.36\ \mathring{A}}{2\times 0.084} = 2.15\ \mathring{A}.$$

Example 13.8. *Calculate glancing angle at which the first and second order diffraction maxima will be observed when X-rays of 1.92 Å wavelength are reflected from a cleavage plane of calcite for which d = 3.029 Å.*

Soln.

Bragg's relation for X-ray diffraction is given by

$$2d \sin\theta = n\lambda$$

where d = inter-planer separation of lattice planes

n = order of diffraction maximum

λ = wavelength of incident X-rays

and θ = glancing angle at which diffraction maximum is observed.

In the first order, we have n = 1

$$\therefore \sin\theta = \frac{n\lambda}{2d} = \frac{1\times 1.92\,\text{Å}}{2\times 3.029\,\text{Å}} = 0.3169$$

$$\therefore \theta = \sin^{-1}(0.3169) = 18°29'$$

For the second order, n = 2

$$\therefore \sin\theta = \frac{n\lambda}{2d} = \frac{1\times 1.92\,\text{Å}}{2\times 3.029\,\text{Å}} = 0.6338$$

$$\therefore \theta = \sin^{-1}(0.6338) = 39°20'.$$

Example 13.9. *The radiation from an X-ray tube operated at 50KV are diffracted by a cubic KCl crystal of molecular mass 74.6 and density 1.99 x 10^3 kg/m^3. Calculate (i) the short wavelength limit of the spectrum from the tube and (ii) glancing angle for first-order reflection from the principal planes of the crystal for that wavelength.*

Example 13.10. In a Bragg's spectrometer, the glancing angle for first order spectrum was observed to be 8°. Calculate the wavelength of X-rays if d = 2.82 x 10^{-10}m. At what angle will the second maximum occur?

Soln.

$$2d \sin \theta = n\lambda$$

$$\lambda = \frac{2d \sin \theta}{n}$$

(i) Here $n = 1$, $\theta = 8°$ $\therefore \sin \theta = 0.1392$

$$d = 2.82 \times 10^{-10} m$$

$$\therefore \lambda = \frac{2 \times 2.82 \times 10^{-10} \times 0.1392}{1}$$

$$= 0.785 \times 10^{-10} m.$$

(ii) For the second order $n = 2$,

$$\therefore \sin \theta = \frac{n\lambda}{2d} = \frac{2 \times 0.785 \times 10^{-10}}{2 \times 2.82 \times 10^{-10}} = 0.2783$$

$$\therefore \theta = \sin^{-1}(0.2783) = 16.15°.$$

Example 13.11. A set of lattice planes reflects X-rays of wavelength 1.32 Å at a glancing angle of 9°30′. Deduce the possible spacing for this set of planes.

Soln.

we have

$$2d \sin \theta = n\lambda$$

$\therefore \quad \theta = \sin^{-1}(0.2783) = 16.15°.$

Example 13.11. *A set of lattice planes reflects X-rays of wavelength 1.32 Å at a glancing angle of 9°30′. Deduce the possible spacing for this set of planes.*

Soln.

we have

$$2d \sin\theta = n\lambda$$

$$\text{or,} \quad d = \frac{n\lambda}{2\sin\theta}$$

Here $\theta = 9° \, 30'$ $\qquad \therefore \quad \sin\theta = 0.1650$

$$\lambda = 1.32 \, \text{Å} = 1.32 \times 10^{-10} \text{m}$$

$$\therefore \quad d = \frac{n \times 1.32 \times 10^{-10}}{2 \times 0.1650}$$

$$= n \times 4 \times 10^{-10} \text{m.}$$

where n = 1, 2, 3, 4, ...

Hence the possible spacings are

4×10^{-10}m, 8×10^{-10}m, 12×10^{-10}m and so on *i.e.*, 4 Å, 8 Å, 12 Å or integral multiples of 4 Å.

Example 13.12. *Compute the wavelength of the most energetic photons emitted by an X-ray tube operated at a steady potential of 80,000 volts. At what glancing angle would these photons be reflected, in the first order from the 100 planes of sodium?*

Soln.

is identical with the classical formula for the momentum. The last term is therefore called the *relativistic-correction term.*

Example 11.21 *A particle is moving with a speed of 0.5c. Calculate the ratio of its rest mass and the mass while in motion.*

Sole.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

or,
$$\frac{m_0}{m} = \sqrt{1-\frac{v^2}{c^2}}$$

Here $v = 0.5c$

$$\therefore \quad \frac{m_0}{m} = \sqrt{1-\frac{(0.5c)^2}{c^2}}$$

$$= \sqrt{1-(0.5c)^2}$$

$$= 0.866.$$

***Example 11.22** Calculate the velocity that one atomic mass unit will have if it has a kinetic energy equal to twice the rest mass energy.*

Soln.

We have

$$E = mc^2 = m_0c^2 + T$$

Here $T = 2m_0c^2$

$\therefore \quad mc^2 = m_0c^2 + 2m_0c^2$

or, $3m_0c^2 = mc^2$

or, $m = 3m_0$

But $\quad m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$\therefore \quad 3m_0 = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

or, $\sqrt{1-\dfrac{v^2}{c^2}} = \dfrac{m_0}{3m_0} = \dfrac{1}{3}$

or, $1-\dfrac{v^2}{c^2} = \dfrac{1}{9}$

or, $\dfrac{v^2}{c^2} = 1-\dfrac{1}{9} = \dfrac{8}{9}$

or, $v^2 = \left(\dfrac{8}{9}\right)c^2$

or, $v = \sqrt{\dfrac{8}{9}}\cdot c$

***Example 11.24** What is the length of a metre stick moving parallel to its length when its mass is 3/2 of its rest mass?*

Soln.

We have $m = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$ .... (i)

or, $\dfrac{m}{m_0} = \dfrac{1}{\sqrt{1-v^2/c^2}}$; or $\dfrac{m_0}{m} = \sqrt{1-v^2/c^2}$

and $L = L_0\sqrt{1-v^2/c^2}$ .... (ii)

Dividing (ii) by (i)

$$\frac{L}{m} = \frac{L_0}{m_0}\left(1-v^2/c^2\right)$$

$$\therefore \quad L = \frac{m}{m_0} . L_0\,(1-v^2/c^2) = \left(\frac{m}{m_0}\right) L_0 \left(\frac{m_0}{m}\right)^2 = \frac{m_0}{m} . L_0$$

Here $\dfrac{m_0}{m} = \dfrac{2}{3}$ and $L = 1$ metre

$$\therefore L = \frac{2}{3} \times 1 = 0.667\text{m}.$$

Exam...

$$\left(\frac{\quad}{m_0}\right) L_0 \left(\frac{m_0}{m}\right) = \frac{m_0}{m} \cdot L_0$$

Here $\frac{m_0}{m} = \frac{2}{3}$ and $L = 1$ metre

$$\therefore \; L = \frac{2}{3} \times 1 = 0.667\text{m}.$$

***Example 11.25** A particle of mass 10^{-24} kg is moving with a speed of 1.8×10^8 m/s. Calculate its mass when it is motion.*

Soln.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Here $m_0 = 10^{-24}$ kg

$v = 1.8 \times 10^8$ m/sec

$c = 3 \times 10^8$ m/sec

$\frac{v}{c} = 0.6$

$$\therefore \quad m = \frac{10^{-24}}{\sqrt{1-(0.6)^2}}$$

$$= 1.25 \times 10^{-24} \text{ kg.}$$

Example 11.26 *4.18×10^{-3} kg of a substance is fully converted to heat energy. Calculate the amount of heat produced.*

Soln.

Here $m = 4.18 \times 10^{-3}$ kg

$C = 3 \times 10^{8}$ m/sec.

$$E = mc^2$$

$$= 4.18 \times 10^{-3} \times (3 \times 10^{8})^2 \text{ J}$$

$$\therefore \quad \text{Heat produced} = \frac{E}{4.18} \text{ calories}$$

$$= \frac{4.18 \times 10^{-3} \times (3 \times 10^{8})^2}{4.18}$$

$$= 9 \times 10^{13} \text{ calories.}$$

Example 11.27 *Two particles each of rest mass 3×10^{-25} kg approach each other in head on collision. If each ...*

$$\sqrt{1-\left(\frac{12}{13}\right)}$$

$= 7.8 \times 10^{-25}$ kg.

Example 11.28 *Electrons are accelerated upto a kinetic energy of 10^9 eV. Find (i) the ratio of their mass to the rest mass (ii) the ratio of their velocity to the velocity of light and (iii) the ratio of their energy to the rest mass energy.*

Soln.

$$U = 10^9 \text{ eV} = 10^9 \times 1.6 \times 10^{-19} \text{ J}$$

Also $U = mc^2$

$$m = \frac{U}{c^2} = \frac{10^9 \times 1.6 \times 10^{-19} \text{J}}{(3\times10^8)^2}$$

$= 1.77 \times 10^{-27}$ kg.

Rest mass of electron,

$m_0 = 9 \times 10^{-31}$ kg.

(i) $$\frac{m}{m_0} = \frac{1.77\times10^{-27}}{9\times10^{-31}} = 1.95 \times 10^3$$

(ii) $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

or, $$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1.95 \times 10^3 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \left[\frac{1}{1.95\times10^3}\right]^2$$

$$\frac{v^2}{c^2} = 1 - \left[\frac{1}{1.95\times10^3}\right]^2$$

$$\frac{v}{c} = \left[1-\left\{\frac{1}{1.95\times10^3}\right\}^2\right]^{1/2}$$

$$= \left[1-\frac{1}{2\times(1.95)^2\times10^6}\right]^2$$

$$= \left[1-1.315\times10^{-7}\right]$$

(iii) Rest mass energy,

$$U_0 = m_0c^2 = 9 \times 10^{-31} \times (3 \times 10^8)^2$$

$$= 8.1 \times 10^{-14} \text{ J}$$

$$\frac{U}{U_0} = \frac{10^9 \times1.6\times10^{-19}}{8.1\times10^{-14}} = 1.975 \times 10^3.$$

***Example 11.29** At what speed will an electron move in order*

Example 11.32 *Calculate the kinetic energy of an electron with a velocity of 0.98 c times the velocity of light in the laboratory system.*

Soln.

When the electron moves with a velocity 0.98 c, its mass becomes

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$ where m_0 is the rest mass of the electron.

$$= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = 5.02\ m_0$$

Relativistic formula for kinetic energy

$$T = (m - m_0)\, c^2 = (5.02\ m_0 - m_0)\, c^2$$

$$= 4.02\ m_0 \times c^2 = 4.02 \times 9.11 \times 10^{-31} \times (3 \times 10^8)^2\ \text{J}$$

$$= 3.396 \times 10^{-13}\ \text{J}.$$

Example 11.33 *Show that the momentum of a particle of rest mass m_0 and kinetic energy K_E is given by the expression.*

$$\sqrt{K_E^2 \ldots$$

Example 11.34 Calculate the mass of the electron when it is moving with a K.E. of 10 MeV.

Soln.

$$K.E. = (m - m_0)\, c^2 = 10 \text{ MeV} = 10 \times 1.6 \times 10^{-13} \text{ J}$$

$$\therefore \quad m = m_0 + \frac{10\times1.6\times10^{-13}\text{ J}}{\left(3\times10^{8}\right)^2}$$

$$= 9.1 \times 10^{-31} + 176 \times 10^{-31}$$

$$= 185.1 \times 10^{-31} \text{ Kg.}$$

Example 11.35 Does the mass of a substance increase on melting? Why?

Soln.

Yes. Because an amount of energy equal to the specific latent heat of fusion has been added to the substance.

Thus the mean life τ turns out to be the reciprocal of λ.

From $\boxed{T_{1/2} = \frac{0.693}{\lambda}}$, we get

$$\tau = \frac{T_{1/2}}{0.693} \cong 1.44 T_{1/2}$$

or $$T_{1/2} = 0.693\tau$$

The mean life of radium is therefore equal to 1620/0.693 = 2337.66 years and that of radon 3.8/0.693 = 5.5 days.

Example 16.1 The half-life of a radioactive substance is 30 days. Calculate (i) the radioactive decay constant, (ii) the mean life (iii) the time taken for ¾ of the original number of atoms to disintegrate and (iv) the time for 1/8 of the original number of atoms to remain unchanged.

Soln:

(i) $T_{1/2} = 30$ days

$$\therefore \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30\,d} = 0.0231 \text{ per day}$$

(ii) Mean life, $\tau = \frac{1}{\lambda} = \frac{1}{0.0231 d^{-1}} = 43.29$ days

(iii) From $N = N_0 e^{-\lambda t}$, we have

$$\frac{1}{4} N_0 = N_0 e^{-\lambda t} \quad \text{where} \quad N = N_0 - \frac{3N_0}{4} = \frac{1}{4} N_0$$

or $$\frac{\frac{1}{4} N_0}{N_0} = e^{-\lambda t} \;; \quad \text{or } e^{-\lambda t} = \frac{1}{4}$$

or $\quad e^{\lambda t} = 4 \ ; \quad \lambda t = \log_e 4$

$$\therefore \quad t = \frac{\log_e 4}{\lambda} = \frac{}{0.0231} = 60 \text{ days}$$

(iv) The number of atoms left, $N = \frac{1}{8} N_0$

$$\therefore \quad \frac{N}{N_0} = \frac{1}{8} = e^{-\lambda t}$$

or $\quad e^{\lambda t} = 8$

$$\therefore \quad \lambda t = \log_e 8 \ ; \text{ or } \ t = \frac{\log_e 8}{0.0231} = \frac{}{0.0231} = 90 \text{ days}$$

Example 16.2 The half-life of radium is 1620 years. In how many years will one gram of pure element (i) lose one centigram and (ii) be reduced to one centigram?

Soln.

The decay constant of radium is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1620 \text{ years}} = 4.28 \times 10^{-4} \ y^{-1}$$

(i) Let t be the time during which one centigram of radium is lost due to disintegration. The amount remaining is $(1 - \frac{1}{100}) = 0.99$ gm

From $\quad N = N_0 e^{-\lambda t}$, we have

$$\frac{N}{N_0} = e^{-\lambda t} \ ; \text{ or } \ \frac{0.99 N_0}{N_0} = e^{-\lambda t}$$

or $\quad e^{-\lambda t} = 0.99 = \frac{99}{100}$

$$e^{\lambda t} = \frac{100}{99} \ ; \text{ or } \ \lambda t = \log_e(\frac{100}{99})$$

$$\therefore \quad t = \frac{\log_e(100/99)}{\lambda} = \frac{}{4.28\times10^{-4}\text{ years}} = 23.68 \text{ years}$$

(ii) Now N = 0.01 gm

$$\therefore \quad \frac{N}{N_0} = 0.01 = \frac{1}{100} = e^{-\lambda t}$$

or $\quad e^{\lambda t} = 100 \qquad \therefore \lambda t = \log_e 100$

$$\therefore \quad t = \frac{\log_e 100}{\lambda} = \frac{}{4.28\times10^{-4}\text{ years}} = 10{,}760 \text{ years}$$

Example 16.3 *1gram of radium is reduced by 2.1 mg in 5 years by α-decay. Calculate the half-life of radium.*

Soln.

The amount of radium left at the end of 5 years is, $N = 1 - 2.1\times10^{-3}$ $= 1 - 0.0021 = 0.9979$ gm

From $\frac{N}{N_0} = e^{-\lambda t}$ we have

$$\frac{0.9979}{1.0} = e^{-\lambda t} = 0.9979$$

Now $t = 5$ years

$$e^{-5\lambda} = 0.9979 \;;\quad \text{or}\quad e^{5\lambda} = \frac{1}{0.9979}$$

or $\quad 5\lambda = \log_e(\frac{1}{0.9979})$

$$\therefore \quad \lambda = \frac{\log_e(1/0.9979)}{5\text{ y}} = \frac{}{5\text{ y}} = 41.4468\times10^{-5}\text{ y}$$

$$\therefore \quad T_{1/2} = \frac{0.6931}{41.4468\times10^{-5}\text{ y}} = 1672 \text{ years.}$$

...gadro's number. ...radioactive substance and

$$\text{Activity} = \lambda N = 8.8\times10^{-14}\times\frac{3.364\times10^{-3}\times6.02\times10^{23}}{234}$$

$$= 7.69\times10^{5} \text{ disintegrations per seconds.}$$

***Example 16.6** A counter rate meter is used to measure the activity of a radioactive sample. At a certain instant, the count rate was recorded as4750 counts per minute. Five minutes later, the count rate recorded was 2700 counts per minute. Compute (i) the decay constant and (ii) the half-life of the sample.*

Soln.

(a) $N = N_0 e^{-\lambda t}$

or $\dfrac{N}{N_0} = e^{-\lambda t}$; $e^{\lambda t} = \dfrac{N_0}{N}$

Now $N_0 = dN_1/dt = 4750$

and $\quad N = dN_2/dt = 2700$

$$\therefore \quad e^{5\lambda} = \frac{4750}{2700}$$

or $$5\lambda = \text{loge}\left(\frac{4750}{2700}\right)$$

or $$\lambda = \frac{\log_e(1.76)}{5} = 0.113 \text{ per minute.}$$

(b) $$T_{1/2} = \frac{0.693}{0.113} = 6.1 \text{ minutes.}$$

16.8 Units of Radioactivity

In radioactivity the numb...

disint...

$\therefore \quad m = 80.9 \text{ gm}$

Example 16.7 Find the activity of 1mg (10^{-3}gm) of radon (R_n^{222}). The half-life of radon is 3.8 days.

Soln.

$$\lambda = \frac{0.693}{3.8 \times 24 \times 3600} = 2.1 \times 10^{-6} \text{ s}^{-1}$$

Number of atoms in 10^{-3} gm,

$$N = \frac{10^{-3} \times 6.02 \times 10^{23}}{222}$$

So activity, $A = \dfrac{dN}{dt}$ (ignoring the minus sign)

$$= \lambda N$$

$$= \frac{2.1 \times 10^{-6} \times 10^{-3} \times 6.02 \times 10^{23}}{222}$$

$$= 5.7 \times 10^{12} \text{ disintegration per second}$$

$$= \frac{5.7\times10^{12}}{3.7\times10^{10}} = 153 \text{ Ci}$$

$$= \frac{5.7\times10^{12}}{10^{6}} = 5.7 \times 10^{6} \text{ rd}$$

$$= 5.7 \times 10^{3} \text{ GBq.}$$

16.9 Isotopes, Isobars, Isotones, Isodiapheres and Isomers

Isotopes: Atoms which have the same atomic number Z (hence have similar chemical properties) but different mass number A (atomic weight) are called *isotopes* (meaning the same place in the periodic table). Obviously, the different isotopes of an element contain different number of neutrons in their nuclei.

Exam

***Example 16.8** Some amount of a radio-active substance of half-life 30 days is spread inside a room. Consequently the level of radiation inside the room became 50 times the permissible level for normal occupancy of the room. After how many days the room would be safe for occupation?*

Soln.

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = \frac{1}{50}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\lambda = \frac{0.693}{30} d^{-1} = 0.0231 d^{-1}$$

$$e^{\lambda t} = \frac{N_0}{N} = 50$$

$$\lambda t = \ln(50) = 3.912$$

$$t = \frac{3.912}{0.0231} = 169.35 \text{ days}$$

***Example 16.9** A sample of pitchblende has a lead-uranium weight ratio of 9/40. Calculate the age of the mineral. (Given: half-life of uranium = 4.1 × 10⁹ y and the atomic weights of lead and ur...*

Example 16.10 *A piece of an ancient wood boat shows an activity of C^{14} of 3.9 disintegrations per minute per gram of carbon. Estimate the age of the boat if the half-life of C^{14} is 5568 years. Assume that the activity of fresh C^{14} is 15.6 disintegrations per minute per gram.*

Soln.

Let the age of the boat be t years.

From $N = N_0 e^{-\lambda t}$, we have

Here

activity $\lambda N = 3.9$

and $\lambda N_0 = 15.6$

$$\therefore \quad \frac{\lambda N_0}{\lambda N} = \frac{N_0}{N} = \frac{15.6}{3.9}$$

$$\lambda = \frac{0.693}{5568} y^{-1} = 1.24 \times 10^{-4} y^{-1}.$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

or $$e^{\lambda t} = \frac{N_0}{N} = \frac{15.6}{3.9}$$

or $$\lambda t = \ln\left(\frac{15.6}{3.9}\right)$$

or $$t = \frac{\ln\left(\frac{15.6}{3.9}\right)}{\lambda}$$

$$= \frac{\ln\left(\frac{15.6}{3.9}\right)}{1.24 \times 10^{-4} y^{-1}} = 1.118 \times 10^4 \text{ yrs.}$$

Example 16.11 *Calculate the activity of K^{40} in 100 kg man assuming that 0.35% of the body weight is potassium. The abund*

$$a = \frac{\;}{2} = \frac{\;}{2} A = 3.15 A$$

Example 19.3 Calculate the lattice constant of potassium bromide from the following data:

Atomic weight of potassium $= 39.1$

Atomic weight of bromine $= 79.9$

Density of potassium bromide $= 2.7 \times 10^3\ kg/m^3$

Avogadro's number $= 6.02 \times 10^{26}$ *per kg-mode.*

Potassium bromide has fcc crystal structure.

Soln.

Let "a" be the lattice constant and ρ the density of potassium bromide. Then

mass in each unit cell $= a^3 \cdot \rho$

Also mass in each unit cell $= \dfrac{nM}{N}$

where n = No. of molecules per unit cell of potassium bromide = 4 (for fcc structure).

M = molecular weight of potassium bromide

= (39.1 + 79.9) = 119.

N = Avogadro's number $= 6.02 \times 10^{26}$ per kg-mole.

$$\therefore \quad a^3\rho = \frac{nM}{N}$$

$$\text{or,} \quad a^3 = \frac{nM}{\rho N} = \frac{4 \times 119}{2.7 \times 10^3 \times 6.02 \times 10^{26}}$$

$$\text{or,} \quad a = \left[\frac{476}{16.254 \times 10^{29}}\right]^{1/3}$$

$$= 6.57 \times 10^{-10}\ \text{m}$$

$$= 6.57\ \mathring{A}\ .$$

Example 19.4 Calculate the lattice constant of cesium chloride which has a simple cubic lattice. The density of CsCl is $4 \times 10^3\ kg/m^3$. *Atomic weight of cesium* $= 132.9$ *and atomic weight of Cl* $= 35.5$. *Avogado's number* $= 6.02 \times 10^{26}$ *per kg-mole.*

Soln.

As in the earlier examples, the lattice constant is given by

$$a = \left[\frac{n\,M}{N\rho}\right]^{1/3}$$

n for a simple cubic system is 1.

$$\therefore \quad a = \left[\frac{1 \times 168.4}{6.02 \times 10^{26} \times 4 \times 10^{3}}\right]^{1/3} \quad [M = 132.9 + 35.5 = 168.4]$$

$$= 4.12 \times 10^{-10}\ \text{m}$$

$$= 4.12\ \mathring{A}.$$

Example 19.5 Sodium is a bcc crystal. Its density is 9.6 x 10² kg/m³ and atomic weight is 23. Calculate the lattice constant for a sodium crystal.

Soln.

The lattice constant is given by

$$a = \left[\frac{n\,M}{N\rho}\right]^{1/3}$$

For a bcc crystal n = 2.

$$N = 6.02 \times 10^{26} \text{ per kg-mode.}$$

$$a = \left[\frac{2 \times 23}{6.02 \times 10^{26} \times 9.6 \times 10^{2}}\right]^{1/3}$$

$$= \left[0.8 \times 10^{-28}\right]^{1/3}$$

$$= 4.3 \times 10^{-10}\,\text{m}$$

$$= 4.3\ \mathring{A}.$$

Example 19.6 If density of NaCl is 2.163 gm/cm³ and its molecular weight 58.45, find the spacing between planes parallel to the cubic lattice faces of the NaCl crystal. Given that Avogadro's number is 6.02 x 10²³ molecules /g-mole.

Soln.

From the definition of Avogadro's number (number of molecules in 1 gm-molecular weight of the substance), NaCl has 6.02×10^{23} molecules in 58.45gm. Hence

$$\text{No. of molecules/gm} = \frac{6.02 \times 10^{23}}{58.45} = 1.03 \times 10^{22}$$

$$\text{No. of molecules/cm}^3 = 1.03 \times 10^{22} \times 2.163$$
$$= 2.23 \times 10^{22}$$

Since NaCl has two atoms per molecule

$$\text{No. of atoms/cm}^3 = 2 \times 2.23 \times 10^{22} = 4.46 \times 10^{22}.$$

Crystal lattice of NaCl consists of a multitude of cubes with atoms at the corners, there will be $[4.46 \times 10^{22}]^{1/3} = 3.5 \times 10^7$ atoms in a row of 1 cm length. The spacing between atoms and hence between rows or planes is

$$d = \frac{1}{3.54 \times 10^7} = 2.83 \times 10^{-8}\,\text{cm} = 2.83\ \text{Å}$$

Example 19.7 Copper has fcc structure and its atomic radius is 1.278 A.U. Calculate its density. Atomic weight of copper = 63.54.

Soln.

As seen from Art. 19.10, the lattice constant for fcc structure is given by

$$a = 4r/\sqrt{2} = 4 \times 1.278/\sqrt{2} = 3.61\ \text{A.U.}$$

$$\text{Volume of unit cell} = a^3 = (3.61 \times 10^{-8})\text{cm}^3$$
$$= 47 \times 10^{-24}\ \text{cm}^3$$

If its density is ρ, then mass of unit cell

$$= 47 \times 10^{-24} \times \rho\ \text{gm}$$

Now let us find the mass of unit cell of copper in a differer way. Each unit cell of fcc structure has 4 atoms. The mass of this atoms obviously represent the mass of the cell. Since 6.02 x 10 atoms of copper weigh 63.54 gm, the mass of 4 atoms

$$= \frac{4 \times 63.54}{6.02 \times 10^{23}} = 42.2 \times 10^{-23}\,\text{gm}$$

Equating the two masses we have,

$$47 \times 10^{-24} \times \rho = 4.22 \times 10^{-23}$$

$$\text{or,} \quad \rho = \frac{42.2 \times 10^{-23}}{47 \times 10^{-24}} = 0.898 \times 10^{-23} \times 10^{24}$$

$$= 8.98\,\text{gm/cc}$$

Alternately, we know

$$a^3\rho = \frac{n\,A}{N}$$

$$\text{or,} \quad \left(3.61 \times 10^{-8}\right)^3 \rho = \frac{4 \times 63.54}{6.02 \times 10^{23}}, \quad \text{or,} \quad \rho = 8.98\ \text{gm/cc}$$

Example 19.8 For a simple cubic lattice calculate

(i) the next neighbour distance, and

(ii) ratio of nearest neighbour distance to the next nearest neighbour distance

Soln.

(i) In a simple cubic lattice, the nearest neighbour distance $d_1 = a$ where a is the side of the unit cell. The next nearest neighbour lies at diagonally opposite end of the face of the unit cell. Therefore, the next neighbour distance

$$d_2 = \sqrt{a^2 + a^2} = a\sqrt{2}$$

(ii) The ratio of the nearest neighbour distance to the next nearest neighbour distance

$$d_1 : d_2 = a : a\sqrt{2}$$

$$= 1 : \sqrt{2}$$

respectively.

Example 19.15 The lattice constants of a simple lattice is a. Find the lattice spacings between (111), (112) and (113) lattice planes.

Soln.

The lattice spacing for a given set of parallel planes (hk$l$) is given by

$$d_{hkl} = \frac{1}{\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{1/2}}$$

For (111) lattice planes

$h = 1$, $k = 1$ and $l = 1$ and since the crystal is simple cubic, $a = b = c$

$$d_{111} = \frac{1}{\left[\frac{1^2}{a^2}+\frac{1^2}{a^2}+\frac{1^2}{a^2}\right]^{1/2}} = \frac{1}{(3/a^2)^{1/2}} = \frac{a}{\sqrt{3}}$$

Similarly for the (112) lattice planes, $h = 1$, $k = 1$, $l = 2$

or, $$d_{112} = \frac{1}{\left[\frac{1^2}{a^2}+\frac{1^2}{a^2}+\frac{2^2}{a^2}\right]^{1/2}} = \frac{1}{(6/a^2)^{1/2}} = \frac{a}{\sqrt{6}}$$

and for the (113) planes

$$d_{113} = \frac{1}{\left[\frac{1^2}{a^2}+\frac{1^2}{a^2}+\frac{3^2}{a^2}\right]^{1/2}} = \frac{1}{(11/a^2)^{1/2}} = \frac{a}{\sqrt{11}}$$

Example 19.16 For a simple cubic lattice of lattice parameter 2.04 Å, calculate the spacing of lattice planes (2 1 2).

Soln.

$$d_{hkl} = \frac{1}{\left[\frac{h^2}{a^2}+\frac{k^2}{a^2}+\frac{l^2}{a^2}\right]^{1/2}}$$

Here $h = 2$, $k = 1$, $l = 2$

and $a = b = c = 2.04$ Å

$$\therefore\ d_{212} = \frac{1}{\left[\frac{2^2}{(2.04)^2}+\frac{1^2}{(2.04)^2}+\frac{2^2}{(2.04)^2}\right]^{1/2}}$$

$$= \frac{1}{\left[\frac{4}{4.16} + \frac{1}{4.16} + \frac{4}{4.16}\right]^{1/2}}$$

$$= \frac{1}{(9/4.16)^{1/2}} = \frac{2.04}{3} = 0.68 \text{ Å}$$

Example 19.17 In a tetragonal lattice a = b = 2.12 Å, c = 1.81 Å. Find (i) the lattice separation between (110) planes, (ii) the density of lattice points in (110) planes.

Soln.

(i) The lattice separation is given by

$$d_{hkl} = \frac{1}{\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{1/2}}$$

Here h = 1, k = 1 and l = 0

and a = b = 2.12 Å and c = 1.81 Å

$$\therefore\ d_{110} = \frac{1}{\left[\frac{1^2}{(2.12)^2} + \frac{1^2}{(2.12)^2} + \frac{0}{(1.81)^2}\right]^{1/2}}$$

$$= \frac{1}{\left[\frac{1^2}{4.49} + \frac{1^2}{4.49} + 0\right]^{1/2}} = \frac{1}{(2/4.49)^2} = \frac{1}{(0.445)^{1/2}}$$

$$= 1.5 \text{ Å}$$

(ii) The density of lattice points is given by

$$\rho = \frac{nd}{v}$$

where n = number of lattice points per unit cell

d = lattice separation

v = volume of unit cell

For a tetragonal system, n = 1 and v = a.b.c

$$\therefore\ \rho = \frac{1.d}{v} = \frac{1.5\overset{o}{A}}{\left(2.12\overset{o}{A}\right) x \left(2.12\overset{o}{A}\right) x \left(1.81\overset{o}{A}\right)}$$

$$= \frac{1.51 x 10^{-10}}{8.1348 x 10^{-30}} = 0.184 x 10^{20} = 1.84 x 10^{19}\,\text{atoms}/m^3$$

Example 19.18 *Show that in a simple cubic lattice the separation between the successive lattice planes (100), (110) and (111) are in the ratio 1: 0.71 : 0.58.*

Soln.

For simple cubic lattice a = b = c

$$d_{100} = \frac{1}{\left[\frac{1^2}{a^2} + \frac{0}{a^2} + \frac{0}{a^2}\right]^{1/2}} = \frac{1}{(1/a^2)^{1/2}} = a$$

$$d_{110} = \frac{1}{\left[\frac{1^2}{a^2} + \frac{1}{a^2} + \frac{0}{a^2}\right]^{1/2}} = \frac{1}{(2/a^2)^{1/2}} = \frac{a}{\sqrt{2}}$$

$$d_{111} = \frac{1}{\left[\frac{1^2}{a^2} + \frac{1^2}{a^2} + \frac{1^2}{a^2}\right]^{1/2}} = \frac{1}{(3/a^2)^{1/2}} = \frac{a}{\sqrt{3}}$$

$$\therefore\ d_{100} : d_{110} : d_{111} = a : \frac{a}{\sqrt{2}} : \frac{a}{\sqrt{3}} = 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{3}}$$

$$= 1 : 070 : 058.$$

Example 19.19 Deduce the density of lattice points in (100), (110) and (111) planes in NaCl cubic lattice which belongs to fcc lattice. Given: lattice parameter = 5.63, number of lattice points per unit cell = 4.

Soln.

The density of lattice points is given by

$$\boxed{\rho = \frac{nd}{v}}$$

where n = no. of lattice points per unit cell

d = separation between lattice planes

v = volume of each unit cell (a^3)

for fcc crystal structure n = 4

$$d_{hkl} = \frac{1}{\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{1/2}}$$

For (100) plane, $d_{100} = \frac{a}{2}$

$$\therefore \rho_{100} = \frac{n \cdot d_{100}}{v} = \frac{4 \times a}{2 \times a^3}$$

$$= \frac{4 \times 5.63 \mathring{A}}{2 \times \left(5.63 \mathring{A}\right)^3} = \frac{4 \times 5.63 \times 10^{-10} \text{ m}}{2 \times (5.63)^3 \times 10^{-30} \text{ m}^3}$$

$= 0.0631 \times 10^{20}$ atom/m^3

$= 6.31 \times 10^{18}$ atoms/m^3

For (110) plane, $d_{110} = \frac{a}{2\sqrt{2}}$

$$\therefore 110 = \frac{nd_{110}}{a^3} = \frac{4 \times 5.63 \times 10^{-10}}{2\sqrt{2} \times (5.63)^3 \times 10^{-30}}$$

$= 0.446 \times 10^{20}$

$= 4.46 \times 10^{18}$ atoms / m^3

For (111) plane, $d_{111} = \frac{a}{\sqrt{3}}$

$$\therefore \quad 111 = \frac{nd_{111}}{a^3} = \frac{4 \times a}{\sqrt{3} \times a^3}$$

$$= \frac{4 \times 5.63 \times 10^{-10}}{1.732 \times (5.63)^3 \times 10^{-30}}$$

$$= 0.0728 \times 10^{20}$$

$$= 7.28 \times 10^{18} \text{ atoms /m}^3$$

Example 19.20 Fi