

Time Series Analysis of Monthly 1-Year Treasury Bill Rates

Group 3

Shirley Yuan, Gwen Li, and Maisie Tang

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Outline

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Introduction

Treasury Bills (T-Bills) are government issued short-term investment vehicles.

- Considered liquid and risk-free asset
- An important measure of interest rate

Why care about interest rates?

- A critical indicator of the overall macroeconomic condition
- Reflect both Fed policy and economic events that affect the market

How does the Fed regulate the economy by adjusting interest rates?

- During economic recession, the Fed lowers interest rates to stimulate economic growth but low interest rates can result in high inflation
- The Fed hikes the interest rates as a way of controlling inflation

Introduction Cont'd



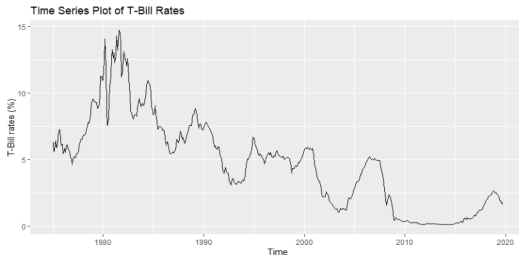
How can interest rates inform investment decisions?

- For individual investors, anticipated low interest rates signal opportunities in shorting the bond market to invest in the equity market
- For private equity (PE) investors, low interest rates mean lower cost of capital, easier access to funds and higher returns

In this study, we focus on forecasting 1Q 2020 T-Bill rates to inform investment decisions for both individual and PE fund investors.

Data

- The monthly 1-year Treasury Bill rates data is sourced from the Federal Reserve Bank of St. Louis website (FRED)
- Spans from Jan 1975 to Oct 2019, containing 538 data points (516 training data points and 22 test data points)



- Covers some of the most notable economic booms and busts, including the 1970s economic crash, the "dot com" bubble in the 1990s, and the 2008-2009 recession

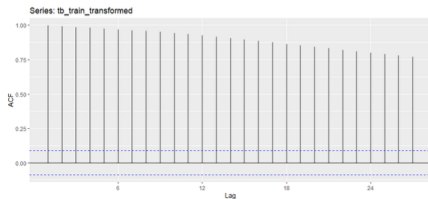
Content

We tried the following time series models to come up with the best model for T-Bill rates forecast.

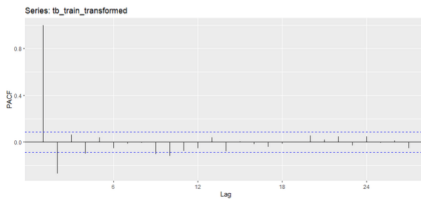
- ETS (Additive Damped Trend Method with Additive Errors)
- ARIMA(4,1,5)
- ARMA(4,5)+Garch(1,1)
- Neural Network
- SETAR(1,4)
- LSTAR
- Time Series Regression



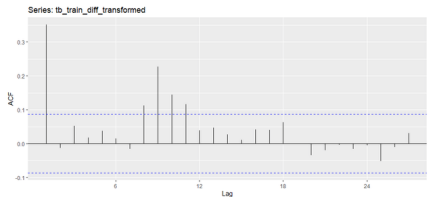
Data Analysis



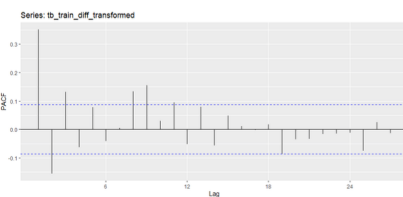
(1) ACF: transformed train set



(2) PACF: transformed train set



(3) ACF: transformed differenced train set



(4) PACF: transformed differenced train set

ETS(A,A_d,N)

Additive Damped Trend Method with Additive Errors

- Since the last recession, the T-Bill rates have remained low, therefore we use ETS model to weigh recent data more highly.
- **The fitted model is:**

$$\begin{aligned}y_t &= \ell_{t-1} + \phi b_{t-1} + \varepsilon_t \\ \ell_t &= \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t \\ b_t &= \phi b_{t-1} + \beta \varepsilon_t\end{aligned}$$

- **Model Checking:** Ljung-Box test of residuals shows that residuals cannot be considered as white noise.
- **AIC=914.2691 MAE=0.176938 MAPE=21.42893 RMSE=0.194894**

ARIMA

We anticipate ARIMA to be sufficient because:

- The ACF of original data suggests autocorrelation, implying that future data can be predicted by lagged values
- The data is non-stationary and first-order differencing needed

Starting from `auto-arma()` and running a for loop through all neighbor models, the one with the smallest AIC is: **ARIMA(4,1,5)**

- **The fitted model is:**

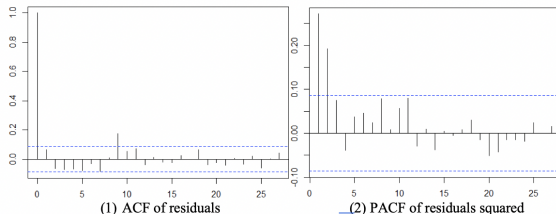
$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \phi_4 B^4)(1 - B)y_t = c \\ + (1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3 + \theta_4 B^4 + \theta_5 B^5)\varepsilon_t$$

- **Model Checking:** the result of residuals diagnosis indicates that the residuals can be considered white noise.
- **AIC=-919.7686 MAE=0.176316 MAPE=21.70815 RMSE=0.190120**

ARMA + GARCH

ARMA(4,5)+GARCH(1,1) with Gaussian innovation

- examine the ARCH effect:



- The fitted model is:

$$\begin{aligned}
 r_t &= 0.0016a_t + 0.1344r_{t-1} - 0.3039r_{t-2} + 0.4485r_{t-3} + 0.3968r_{t-4} \\
 &\quad + 0.2482a_{t-1} + 0.2636a_{t-2} - 0.1897a_{t-3} - 0.0115a_{t-4} \\
 \sigma_t^2 &= 0.0013 + 0.3926a_{t-1}^2 + 0.4985\sigma_{t-1}^2 \\
 y_t &= \sigma_t * \varepsilon_t
 \end{aligned}$$

ARMA + GARCH Cont'd

ARMA(4,5)+GARCH(1,1) with Gaussian innovation

- **Model Checking:** the result of residuals test indicates that the fitted GARCH model is sufficient.
- **AIC**=-2.077399 **MAE**=0.052220 **MAPE**=108.2037 **RMSE**=0.072367

Neural Network

1-1-1 neural network with 14 lags

(coefficients are in the Appendix of report)

- **Model Checking:** Ljung-Box test of residuals shows that residuals cannot be considered as white noise.
- **AIC=N/A MAE=0.499548 MAPE=69.30398 RMSE=0.686494**

SETAR

SETAR(1,4).

- **The fitted model is:**

$$x_t = 0.0340 + 1.1198x_{t-1} - 0.3970x_{t-2} + 0.4970x_{t-3} - 0.1988r_{t-4} + a_t$$

if $x_{t-2} < 0.2215$

$$x_t = 0.0103 + 1.5660x_{t-1} - 0.7509x_{t-2} + 0.2613x_{t-3} - 0.0744r_{t-4} + a_t$$

if $x_{t-2} \geq 0.2215$

- **Model Checking:** The p-value of the Ljung-Box test of the residuals is 0.01163, indicating that the residuals can be considered as white noise.
- **AIC**=-2402.722 **MAE**=0.068262 **MAPE**=8.312612 **RMSE**=0.095785

LSTAR

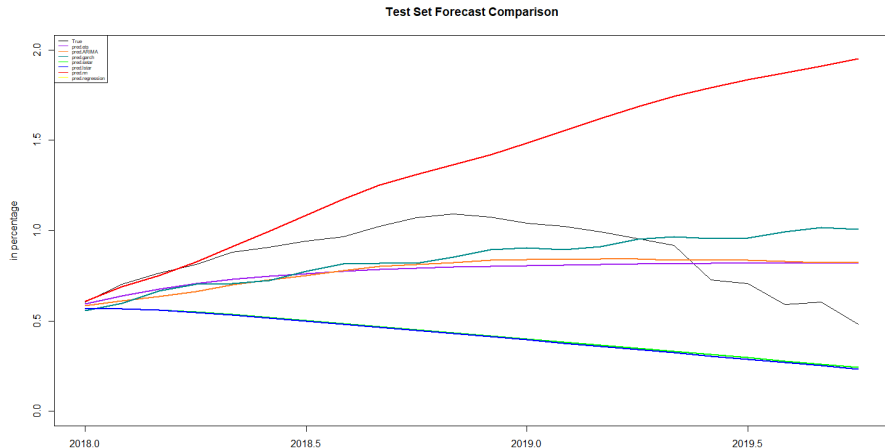
- The resulting forecast is similar to the forecast from SETAR
- **Model Checking:** Ljung-Box test of residuals shows that residuals cannot be considered as white noise.
- **AIC**=-2400.388 **MAE**=0.441605 **MAPE**=49.96487 **RMSE**=0.478271

Time Series Regression

T-Bill rates = $6.002 - 0.428 \text{ Unemployment Rate} - 2.512 \times 10^{-4} \text{ Stock Return}$

- Use unemployment rate and monthly return of Dow Jones Index to fit the model.
- Fed will lower interest rate to stimulate the economy when unemployment rate is high
- More investment flow into bond market when the stock market is not doing well.
- **Adjusted R squared** = 0.7929
- **Model Checking:** Ljung-Box test of residuals shows that residuals cannot be considered as white noise.
- **AIC**=1087.65 **MAE**=2.90366 **MAPE**=354.622 **RMSE**=2.91154

Discussion

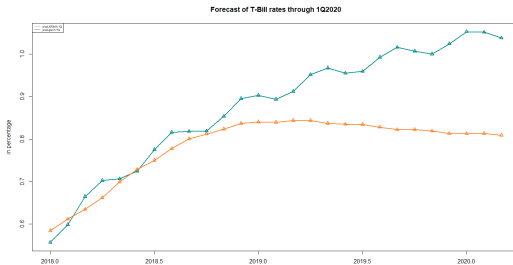


Discussion Cont'd

	ETS	ARIMA	GARCH	SETAR	LSTAR	NN	REG
AIC	914.2691	-919.7686	-2.077399	-2402.722	-2400.388	N/A	1087.65
MAE	0.176938	0.176316	0.052220	0.068262	0.441605	0.499548	2.90366
MAPE	21.42893	21.70815	108.2037	8.312612	49.96487	69.30398	354.622
RMSE	0.194894	0.190120	0.072367	0.095785	0.478271	0.686494	2.91154

- Overall, ARIMA and GARCH provide the most outstanding forecast accuracy among the 7 models applied
- ARIMA has much smaller AIC and MAPE
- Garch beats ARIMA in terms of MAE and RMSE
- ARIMA provides much closer predictions to true data towards the end of the forecast period and well captures the slightly downward trend starting in 2019, which GARCH fails to capture
- Thus, selecting ARIMA as the most sufficient model

Discussion Cont'd



- The one-year T-Bill rates in 1Q2020 stay in the low range of 0.5
- Lower interest rates signal easier access to funds and stimulates economic activities
- Bodes well for the overall economy and thus the investment activities of private equities

Recommendations

Despite suspicion of another economic recession down the road, we are bullish on the macroeconomic condition in 1Q 2020. **Thus, we recommend:**

- For private equity funds, 1Q 2020 could be an ideal time for fund raising and transaction activities due to the anticipated lower interest rates;
- For individual investors, consider shorting the bond market to make long positions in the equity market.

Thank you!

Questions?