Homework Problems on Markov Chain and Queuing Theory

2014097081 CHO YOONSANG

1. A machine can work in two method. It works in same method or change to another method of operation every one hour according to the transition probability matrix P

a) Compute the transition probability matrix.

0.4	0.6
0.6	0.4

b) b) If the system is in run in Methods one at 5:30 pm, what is the probability that on the same day it will be in same method one at 8:30 pm?

3hours have passed

$P^3 =$

0.5008	0.4992
0.4992	0.5008

Probability that on the same day it will be in same method one at 8:30 is **0.5008**

2. In Daegu during spring the weather was quite strange of its has both sunny and rainy days and follows a Markov chain with two states. A sunny day is followed by a sunny day with probability 0.8. A rainy day is by a rainy day with probability 0.6. a) Daegu has sunny weather Today. What is the chance of rain the day after tomorrow?

0.8	0.2
0.4	0.6

→

0.72	0.28
0.56	0.44

→ The chance of rian the day after tomorrow is **0.28**

b) Compute the probability that April 1 next year is rainy in Daegu (Bonus)

Probability that April 1 next year is rainy in Deageu is **0.33333333**

0.8	0.2
0.4	0.6

→ stable (converge)

0.66666667	0.33333333
0.66666667	0.33333333

3. A technician is to be hired to repair machines which breakdown at an average rate of 3 per hour. The breakdown follow Poisson distribution. Non-Productive time of a machine is considered to cost 16$ per hour. Two technicians have been interviewed. One is slow but cheap while other is fast but expensive. The slow technician charges 8$ per hour and he services break-down machines at the rate of 4 per hour. The fast technician demands 10$ per hour and he services at an average rate of 6 per hour. Which technician should be hired?

Slow technician	Fast technician
λ = 3/hr, μ = 4/hr, charge = 8$/hr	λ = 3/hr, μ = 6/hr, charge = 10$/hr

Suppose) Two technicioans repair for 8 hours.

- **Slow technician**

  $= \lambda \times 8 \times W \times 16 + 8 \times 8$

Since the average time is $W = \frac{1}{\mu - \lambda} = \frac{1}{4-3} = 1$

$= 3 \times 8 \times 1 \times 16 + 8 \times 8 = 384 + 64 = 448$

- **Fast technician**

  $= \lambda \times 8 \times W \times 16 + 10 \times 8$

Since the average time is $W = \frac{1}{\mu - \lambda} = \frac{1}{6-3} = \frac{1}{3}$

$= 3 \times 8 \times \frac{1}{3} \times 16 + 10 \times 8$

$= 128 + 80 = 208$

$\therefore$ Fast technician should be hired