# When Inequality Meets Finance: Household Portfolios and the Corporate Bond Boom \*

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#### Abstract

How does rising income inequality reshape financial markets? This paper demonstrates that shifts in the income distribution can influence corporate debt structure by altering household saving behavior. When labor income risk rises and increases income inequality, households accumulate more liquid wealth, held in deposits and corporate bonds, as a precautionary motive, but the composition of that wealth varies across the distribution due to portfolio adjustment frictions. In a general equilibrium model with heterogeneous households and firms facing financial frictions, this reallocation reduces the liquidity premium, the extra return paid by bonds relative to deposits, and pushes firms toward greater reliance on bond financing. The model explains nearly 59.5% of the observed increase in bond reliance and 57.9% of the decline in the liquidity premium in U.S. data since 1989. These findings reveal a new transmission channel from inequality to credit markets, one that operates through the composition, rather than just the level, of household savings.

JEL Codes: D31, E21, E44, G32

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# 1 Introduction

A growing body of research (Piketty et al. (2018); Kuhn et al. (2020); Kopczuk et al. (2010)) has documented the persistent rise in income inequality in the United States (U.S.) and has begun to examine its macroeconomic consequences, particularly its role in shaping natural interest rates and output (Mian et al. (2021); Auclert et al. (2018)). A related but unexplored question that arises is: How has the long-term increase in income inequality influenced the shift toward corporate bonds over traditional bank lending?

The existing literature (Attanasio et al. (2012); Aguiar and Bils (2015); Heathcote et al. (2010a); Meyer and Sullivan (2023)) has primarily examined the empirical relationship between income and consumption inequality, as it sheds light on households' ability to buffer income shocks and thus provides a rationale for government intervention. However, little is known about the pass-through from rising income inequality to the interest rates on deposits and corporate bonds. Understanding the drivers of this pass-through is relevant for both economists and policymakers because it determines households' portfolio choices and, ultimately, their ability to smooth consumption.

This paper contributes by proposing a novel mechanism that links income inequality, corporate debt structure, and real interest rates. By emphasizing the interaction between household portfolio choices and firm-level financing decisions, it highlights an underexplored channel through which inequality affects corporate leverage, the liquidity premium, and the distribution of liquid wealth (deposits and bond savings) across households. The core contribution of this paper is to demonstrate theoretically that shifts in the income distribution affect the composition of household savings across assets with varying degrees of liquidity, such as deposits and corporate bonds, which in turn influence firms' optimal debt structure and equilibrium interest rates.

The paper begins by motivating the mechanism with three stylized facts from U.S. data. First, there is a positive comovement between income inequality and the corporate debt structure, both at the aggregate and firm levels. Second, the deposit shares, measured as the share of deposits in total liquid wealth, have evolved heterogeneously across the income distribution. This pattern highlights two salient features. On the one hand, households in the lower part of the income distribution tend to hold a greater share of deposits relative to those at the top. On the other hand, this suggests the presence of frictions or inertia in portfolio rebalancing, even among assets typically classified as liquid.<sup>1</sup> Third, the spread between the return on corporate bonds and short-term commercial paper, a proxy for the liquidity premium in corporate credit markets, has declined

<sup>&</sup>lt;sup>1</sup>Bayer et al. (2019); Kaplan et al. (2018) treat both deposits and corporate bonds as liquid assets.

over time. This downward trend mirrors the decreasing cost differential for firms choosing between bond and bank financing

Based on these facts, I then develop a model with heterogeneous households that face portfolio adjustment frictions and liquidity needs, allocating wealth differently depending on their labor income and initial liquid wealth. To capture the rigidity of portfolio rebalancing, the model assumes that households face stochastic adjustment costs, drawn from a common distribution, when changing their bond holdings. Consequently, households optimally weigh the marginal benefits against the costs of adjustment, resulting in infrequent and lumpy rebalancing of their bond portfolios. These adjustment costs can be viewed as representations of behavioral biases or informational frictions that discourage households from rebalancing when they misperceive the risks associated with bonds.

Thereby, in the model, although both deposits and bonds mature in one period, deposits are modeled as more liquid because households can access them without frictions at any time with perfect flexibility. In contrast, bond holdings can only be adjusted with some probability, creating an endogenous friction. The liquidity premium in equilibrium thus reflects the value households place on the ability to reallocate funds flexibly, rather than differences in maturity.

Moreover, firms are subject to idiosyncratic shocks that affect both their production and the value of assets pledged as collateral. These shocks generate repayment risk and exacerbate moral hazard, thereby creating financial frictions in the contracting relationship between firms and intermediaries. Financial intermediaries (traditional banks and bond markets) channel household savings into firm credit, but they have notable distinctions. Banks possess monitoring technologies that mitigate informational asymmetries, allowing them to pool risk across firms and ensure repayment to households. Firms, in turn, endogenously adjust the mix of bank loans and bond issuance in response to shifts in credit supply. This feedback loop generates a novel transmission channel from inequality to real interest rates, with testable implications for household deposit shares, corporate leverage, and the liquidity premium.

The baseline estimation is disciplined by a set of targeted moments that capture both household portfolio patterns and firm financing behavior in the U.S. economy around 1990. On the household side, the model matches the relative size of deposits to bonds, the distribution of deposit holdings across the income distribution, and the share of borrowers. On the firm side, it aligns with the external funding relative to net worth, the use of bank loans and collateral, and the ratio of assets to sales. At the aggregate level, the calibration also matches with observed returns on bonds, the liquidity premium, and overall corporate debt structure. Together, these moments ensure that the

steady state provides a realistic benchmark for analyzing the effects of rising income inequality.

In my main exercise, I increase the dispersion of idiosyncratic labor productivity shocks to generate the observed rise in income inequality. Such an increase can be interpreted as the outcome of several forces documented in the literature, including a widening skill premium from skill-biased technological change, shifts in labor market composition that concentrate income among business owners, technological change and automation that polarize wages between high and low-skill workers, globalization and trade shocks that depress manufacturing wages, and institutional or policy changes such as declining unionization and reduced tax progressivity.

After increasing labor income risk to reflect higher income inequality, the model delivers a strong quantitative fit to key macro-financial patterns observed in the U.S. economy between 1989 and 2019. It reproduces the link between rising income inequality, measured by the P90/P10 income ratio, and firms' increasing reliance on bond financing, accounting for 59.5% of the observed change in firms' financing. The model also captures the heterogeneous reallocation of household savings across the income distribution, including the decline in deposit intensity among top households and the relative shift toward deposit holdings at the bottom. Moreover, it partially matches the empirical reduction in the liquidity premium, accounting for 57.9% of the observed drop. The model also offers a structural explanation for changes in household financial intermediation and the evolution of the corporate debt structure. These results underscore the connection between household portfolios and aggregate credit market outcomes. Even under simplified assumptions, they provide a foundation for analyzing structural transformations in financial markets.

Another important lesson from the model is that rising income inequality can generate redistributive effects on household portfolios, resulting in a compression of liquid wealth inequality even as income and consumption inequality increase. While standard intuition suggests that greater labor income dispersion should amplify disparities in financial asset holdings, the model instead predicts a decline in the Gini coefficient of liquid wealth, from 0.42 to 0.40. This outcome stems from a decrease in the liquidity premium, which raises the marginal value of bond holdings for households in the upper-middle of the distribution, enabling them to rebalance their portfolios more heavily toward bonds. By contrast, households at the bottom remain concentrated in deposits, while those at the top, already heavily invested in bonds, face a low marginal value of additional bond holdings and increase their positions only marginally. These offsetting responses result in a more balanced accumulation of liquid wealth across the distribution. This mechanism highlights a new transmission channel from inequality to financial markets, one that narrows gaps in liquid asset holdings while simultaneously compressing liquidity premia.

The model highlights distinct but interrelated distributional consequences of rising income inequality. First, liquid wealth inequality widens across most of the distribution, as upper-middle households accumulate bonds at a faster pace than lower-income groups. Second, consumption inequality follows a hump-shaped pattern where lower and middle-income households experience modest gains, upper-middle households benefit the most, while the top 1% see a sharp decline in consumption due to their disproportionate exposure to falling bond returns. Third, income inequality displays a similar profile, rising between the upper-middle and the bottom but narrowing between the top 1% and the rest, reflecting how shifts in financial returns redistribute resources away from the very top. Taken together, these results underscore that the surge in inequality is not uniform across dimensions as upper-middle households emerge as the relative winners, while the top 1% lose ground in liquid wealth, income, and especially consumption.

Contribution to the literature: This paper contributes to four main strands of the literature. First, this paper complements and extends the growing literature on income and wealth inequality (e.g., Mian et al. (2020); Auclert et al. (2021); Kuhn et al. (2020); Kopczuk et al. (2010); Smith et al. (2023); Gomez (2025); Fernandez-Villaverde et al. (2024); Cao and Luo (2017)) by linking distributional dynamics to the structure of firm financing. It also complements empirical evidence Aguiar and Bils (2015); Attanasio et al. (2012), who show that consumption inequality has closely mirrored the rise in income inequality in the U.S. over recent decades. Moreover, recent studies have emphasized the redistributive effects of macroeconomic shocks. For instance, Coibion et al. (2017) shows that contractionary monetary policy disproportionately affects low-income households, thereby exacerbating income and consumption inequality. Similarly, Heathcote et al. (2010a,b) document a persistent rise in income inequality and labor income risk since the 1980s. These findings underscore the importance of understanding how rising income risk shapes household saving behavior and its macroeconomic consequences, an interaction that is central to the mechanism explored in this paper.

While these studies document and explain the evolution of inequality and its implications for aggregate demand, savings, and wealth distribution on the top end, this paper focuses on how shifts in the joint distribution of household income and liquid asset holdings affect the composition of corporate debt, specifically, the relative reliance on bank loans versus bond financing. By embedding heterogeneous households and firms in a general equilibrium framework, the model uncovers a novel transmission channel. Rising inequality reallocates savings toward upper-middle-wealth households with stronger preferences for less liquid, higher-yield assets (e.g., corporate bonds), thereby reshaping the supply of intermediary capital and altering firm debt structure. This perspective adds a new dimension to the inequality debate by showing how financial structure, not

just macroeconomic aggregates, responds endogenously to changes in the income distribution.

Second, it builds on heterogeneous-agent models with two-asset structures. While recent research (Doerr et al. (2024); Bayer et al. (2019); Kaplan et al. (2014, 2018)) has examined how heterogeneity in household saving behavior shapes macroeconomic outcomes, such as employment, investment, and consumption, this paper focuses on a complementary but previously underexplored mechanism. I develop a general equilibrium model in which households, differentiated by income and initial liquid wealth face portfolio adjustment frictions and choose how to allocate their savings across assets with different liquidity properties. These heterogeneous frictions shape the aggregate composition of household savings, particularly the balance between deposits and corporate bonds, which in turn affects firms' relative reliance on bank loans versus bond financing.

Unlike much of the existing literature, which abstracts from within-firm debt composition, this paper explicitly models how household heterogeneity gives rise to endogenous corporate debt structure decisions.<sup>3</sup> This interaction introduces a novel mechanism through which income inequality affects the liquidity premium, the cost of capital, and ultimately the equilibrium real interest rate.

Third, it contributes to the literature on financial frictions and financial heterogeneity. Holmstrom and Tirole (1997), Rampini and Viswanathan (2019), and Villacorta (2018) emphasize the role of intermediary capital in shaping firm leverage and financial dynamics. While the latter two papers focus on the joint dynamics of firm and bank net worth, this paper abstracts from banks' net worth to concentrate on long-run general equilibrium effects driven by changes in household savings composition. Furthermore, it contributes to the literature on financial accelerator proposed by Bernanke et al. (1998) that emphasizes the importance of the external finance premium, which is the difference between the cost of external funds (borrowing) and the opportunity cost of internal funds (retained earnings). In this paper, the comparison between the respective sources of external financing (bank loans and bonds) and the return on capital is also presented, and it is key for firms to determine their optimal leverage and debt structure. However, this external finance premium depends on the endogenous decision of households to balance their portfolios of deposits and bonds.

Fourth, this paper contributes to the literature on liquidity premia, interest rates, and financial disruptions. Hall (2016) and Mian et al. (2021) emphasize heterogeneity in risk preferences and household saving behavior as key drivers of the long-run decline in global real interest rates.

<sup>&</sup>lt;sup>2</sup>Kaplan et al. (2018) and Bayer et al. (2019) explore household portfolio choices between illiquid assets (e.g., housing or equity) and liquid assets (e.g., deposits or bonds). In contrast, this paper focuses on the macroeconomic implications of changes in the composition of liquid household assets, specifically, deposits and corporate bonds.

<sup>&</sup>lt;sup>3</sup>Doerr et al. (2024) also studies firm-level debt composition, but focuses on differences across firm types (e.g., public vs. private), whereas this paper examines variation within firms driven by household-side heterogeneity.

Fontaine and Garcia (2012), Gromb and Vayanos (2018), and He and Krishnamurthy (2013) argue that financial disruptions stem from common shocks to the wealth of financial intermediaries or speculators, which impair their ability to engage in arbitrage or provide liquidity. This paper complements those theories of intermediary capital constraints by highlighting how households' savings decisions, shaped by rising inequality, can endogenously influence the supply of intermediary capital. In doing so, it introduces a structural source of liquidity stress that affects corporate debt composition through general equilibrium channels.

The remainder of the paper is organized as follows. Section 2 presents the motivating empirical evidence. Section 3 develops the quantitative model and derives closed-form solutions for key credit variables. Section 4 discusses the numerical implementation and describes the model estimation, Section 5 analyzes the quantitative results, and Section 6 provides the conclusion. The appendices offer additional technical details and supporting material for the sections throughout the paper.

# 2 Motivating evidence

The quantitative model is motivated by three empirical patterns in the U.S. economy. First, there is a positive co-movement between income inequality and the corporate debt structure. Second, the heterogeneous composition of liquid wealth across households, particularly in the allocation between deposits and bonds. Third, the downward trend in the corporate liquidity premium.

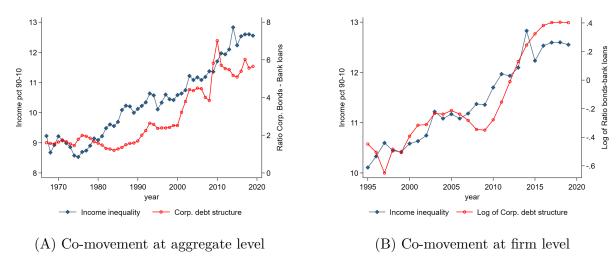
# 2.1 Positive co-movement between income inequality and corporate debt structure

Over the past five decades, income inequality in the U.S. has increased steadily. During this same period, firms have gradually shifted away from traditional bank lending and increasingly relied on bond markets for external financing, a trend reflected in the rising ratio of corporate bonds to bank loans using the Current Population Survey (CPS), the Z.1 Financial Accounts, and the Capital IQ databases, as shown in Figure 1.

This transformation in corporate debt structure is observable not only in aggregate data but also at the firm level, indicating a broad and persistent change in how firms, particularly larger ones, access credit.<sup>4</sup> Notably, the increasing reliance on bond financing occurred during a period of

<sup>&</sup>lt;sup>4</sup>The rising log-ratio of corporate bonds to bank loans at the firm level reflects a growing reliance on bond

Figure 1: Income inequality and corporate debt structure.



Notes: Figure A plots the evolution of income inequality (blue line), together with the ratio of corporate bonds to bank loans in the liabilities of non-financial corporations (red line), using aggregate data from the Current Population Survey (CPS) and the Z.1 Financial Accounts. Figure B presents the firm-level counterpart, illustrating the co-movement between income inequality and the 3-year moving average of the log-ratio of corporate bonds to bank loans, constructed from the CPS and Capital IQ databases. In both cases, income inequality is measured as the ratio of the 90th-to-10th percentile of the income distribution, allowing a direct comparison between aggregate and firm-level patterns.

historically low interest rates, when both bank loans and bonds became cheaper. This suggests that when interest rates are very low, the shift in debt structure reflects differing elasticities of demand across the two debt instruments.

Using U.S. time-series regressions for the period 1967-2021 from the Z1. Financial Accounts and the Current Population Survey (CPS), I estimate the relationship between various measures of income inequality and the aggregate ratio of corporate bonds to bank loans, controlling for a linear time trend t:

$$y_t = \beta_0 + \beta_1 Ineq_t + \beta_2 t + \epsilon_t \tag{1}$$

where the dependent variable  $y_t$  is the ratio of corporate bonds to bank loans for non-financial corporate businesses, and  $Ineq_t$  denotes alternative time-varying measures of income inequality, specifically, the income ratios of the 80th to 10th percentile, 90th to 10th percentile, and 95th to 10th percentile.

financing relative to bank credit.

The results in Table 1 indicate a strong and statistically significant positive association between income inequality and the relative importance of bond financing. Specifically, in column (1), a one-unit increase in the 80th-to-10th percentile income ratio is associated with a 0.937 increase in the corporate bond-to-bank loan ratio, controlling for the linear time trend. The corresponding estimates for the 90th-to-10th and 95th-to-10th ratios are 0.679 and 0.381, respectively. All three specifications capture over 80% of the variation in the dependent variable, as reflected in the adjusted  $R^2$ .

Table 1: Relation between the ratio of corporate bonds-to-bank loans and income inequality

	(1)	(2)	(3)
	Corp.Bonds-Banks	Corp.Bonds-Banks	Corp.Bonds-Banks
Time-trend	0.0565***	0.0419*	0.0487**
	(0.0178)	(0.0215)	(0.0241)
Disposable income ratio 80:10	$0.937^{**}$		
	(0.365)		
Disposable income ratio 90:10		$0.679^{***}$	
		(0.243)	
Disposable income ratio 95:10			0.381**
			(0.169)
Observations	55	55	55
$\mathbb{R}^2$ adj.	0.807	0.809	0.803

Standard errors in parentheses

Notes: Each specification is estimated separately for the three inequality measures, following the model in Equation 1. The standard errors are adjusted for heteroskedasticity, allowing for valid statistical inference even when the variability of the error terms changes over time.

Importantly, this association is not confined to the U.S. Cross-country evidence also reveals a positive relationship between disposable income inequality and the structure of credit markets. Using annual data from the Global Financial Development Database and the World Inequality Database, I estimate pooled OLS regressions to explore this relationship:

$$y_{it} = \beta_0 + \beta_1 Ineq_{it} + \beta_3' X_{it} + u_t + v_i + \epsilon_{it}$$
(2)

where the dependent variable  $y_{it}$  denotes alternative measures of bond market structure for country i in year t, including the share of domestic non-financial corporate bonds in total domestic bonds outstanding (Corp. bonds %) and the annual change in public bond market capitalization as a

<sup>\*</sup> p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

share of GDP ( $\Delta$  Bond Mkt Cap). The variable  $Ineq_{it}$  captures different measures of income inequality, such as the S80/S20 ratio (the share of disposable income held by the top 20% relative to the bottom 20%) for the total population and for individuals aged 65 and above, and the ratio of disposable income in the 50th to the 10th percentile (P50/P10) and 90th to the 10th percentile (P90/P10) for the total population. To control for income levels and the size of the credit market, I include lagged GDP per capita and the lagged ratio of total liabilities to GDP, and  $u_t$  and  $v_i$  are time and country fixed effects, respectively.

One limitation of the datasets used in Table 2 is the absence of a consistent and well-defined variable capturing the volume of traditional bank loans, which restricts a direct analysis of bond vs. bank financing. Despite this constraint, Table 2 offers strong empirical support for the hypothesis that rising income inequality is associated with a greater presence of non-financial corporate bonds in domestic bond markets. Columns (1)-(3) show that higher inequality, measured by the ratio of the share of disposable income in the 80th to the 20th percentile (S80/S20), and the ratios of disposable income in the 50th to the 10th percentile (P50/P10) and 90th to the 10th percentile (P90/P10), is significantly and positively correlated with the share of outstanding bonds issued by non-financial corporations.<sup>5</sup>

In Column (1), a one-unit increase in the S80/S20 ratio, capturing a more top-heavy income distribution, is associated with a 0.69 percentage point rise in the corporate share of domestic bonds, significant at the 10% level. Alternative measures of inequality yield consistent results. Columns (2) and (3) show that the P50/P10 and P90/P10 ratios are associated with increases of 4.74 and 0.69 percentage points, respectively, in the corporate bond share for each one-unit rise in the ratio. Both effects are statistically significant at the 10% level. Taken together, these results suggest that more unequal economies tend to exhibit shifts in their portfolios toward private, non-financial debt instruments. This finding is consistent with a supply-side mechanism in credit markets, as wealthier households capture a larger share of disposable income, they channel their savings into relatively safe yet higher-yielding corporate bonds, thereby reshaping the composition of domestic bond markets.

<sup>&</sup>lt;sup>5</sup>Based on data availability, the sample covers annual observations from 2000 to 2021 for the following 18 countries: Australia, Brazil, Canada, China, Croatia, Denmark, Hungary, Iceland, Israel, Japan, South Korea, Mexico, Norway, Russia, South Africa, Sweden, Switzerland, and Turkey.

Table 2: Relation between non-financial corporate bonds and measures of income inequality

	(1)	(2)	(3)	(4)
	Corp. bonds $\%$	Corp. bonds $\%$	Corp. bonds $\%$	$\Delta$ Bond Mkt Cap
S80/S20 Total pop.	0.685*			
	(0.332)			
P50/P10 Total pop.		$4.741^{*}$		
		(2.662)		
P90/P10 Total pop.			0.690*	1.209*
			(0.337)	(0.609)
Lag GDP per capita	0.0282	0.0247	0.0280	0.150***
	(0.0597)	(0.0580)	(0.0595)	(0.0425)
Lag Liabilities to GDP	-0.0214***	-0.0212***	-0.0217***	0.00667
	(0.00336)	(0.00333)	(0.00327)	(0.0133)
Lag Inflation	0.0732	0.0835	0.0775	$0.769^{**}$
	(0.136)	(0.133)	(0.136)	(0.358)
Observations	173	173	173	246
Country FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Clustered SE by country	Yes	Yes	Yes	Yes

Standard errors in parentheses

Notes: Columns (1)-(3) use the ratio of the total amount of domestic non-financial corporate bonds and notes outstanding to the total amount of domestic bonds and notes outstanding as the dependent variable. The measure of liabilities to GDP includes all debt liabilities, such as bonds, debentures, notes, and money market or other negotiable debt instruments. All these variables are sourced from the Global Financial Development Database. Column (4) uses the annual change in public bond market capitalization as a share of GDP (in %), taken from the Financial Development and Structure Dataset (revised September 2019), as the dependent variable. Measures of income inequality are obtained from the World Inequality Database (WID), and information for the control variables is obtained from the World Bank. All regressions include country and time fixed effects, and standard errors are clustered at the country level.

Column (4) shifts the focus from bond market composition to bond market growth, using the annual change in public bond market capitalization relative to GDP as the dependent variable.<sup>6</sup> The results show that higher income inequality, measured by the P90/P10 ratio, is positively and significantly associated with the expansion of public bond markets. A one-unit increase in the

<sup>\*</sup> p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

<sup>&</sup>lt;sup>6</sup>Based on data availability, the sample covers annual data from 2000 to 2017 for the following 54 countries: Argentina, Australia, Australia, Belgium, Brazil, Canada, Chile, China, Colombia, Croatia, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, South Korea, Luxembourg, Malaysia, Malta, Mexico, Netherlands, New Zealand, Norway, Pakistan, Peru, Philippines, Poland, Portugal, Russia, Saudi Arabia, Singapore, Slovak Republic, Slovenia, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom and United States.

P90/P10 ratio is associated with a 1.21 percentage points increase in bond market capitalization growth, significant at the 10% level. This suggests that greater income concentration at the top may contribute not only to shifts in the composition of financial markets but also to their overall size. One possible channel is that top-income households have a stronger marginal propensity to save, thereby increasing the aggregate demand for fixed-income securities, including public bonds. Together, these findings highlight that both structural inequality and macroeconomic fundamentals shape the dynamics of bond market growth.

Although these estimates are not intended to be interpreted as causal effects, they suggest a consistent and robust association between income inequality and the development of corporate bond markets. To address the possibility that this relationship reflects broader macro-financial conditions rather than inequality itself, the regressions control for several additional covariates. GDP per capita is included to capture the role of economic development, since richer economies typically exhibit more sophisticated financial systems and deeper capital markets. Liabilities-to-GDP proxies for the overall degree of financial depth and indebtedness, which could affect the relative space for corporate versus other forms of debt. Inflation is added as a measure of macroeconomic stability, as high or volatile inflation tends to erode investor demand for long-term fixed-income instruments and may hinder bond market development. Together, these controls help isolate the relationship between inequality and corporate bond markets from other structural or macroeconomic drivers.

This raises a key question: Is income inequality more closely associated with firms' debt structure, the composition of borrowing between banks and bond markets, rather than their capital structure, traditionally defined by the trade-off between debt and equity? While classical corporate finance emphasizes capital structure decisions, recent trends suggest a shift in the relevant margins of adjustment. In particular, U.S. initial public offerings (IPOs) activity has declined sharply, with many firms delaying or avoiding equity issuance. For instance, Doidge et al. (2013) documents that between the 1990s and 2000s, IPO activity in the U.S. fell by 8%, even as IPO capital raised outside the U.S. grew by 65%. Also, Gao et al. (2013) shows that from 1980 to 2000, an average of 310 firms went public each year in the U.S., whereas since 2000, the average has dropped to just 99 IPOs annually, with the decline particularly pronounced among smaller firms.

In contrast, there has been a marked increase in corporate bond issuance. The average issue size in the U.S. rose from approximately USD 479 million during the 2000-2007 period to USD 837 million between 2008 and 2018. Over the same time frame, the total volume of debt raised through corporate bond markets doubled, from roughly USD 300 billion in 2000 to USD 600 billion in 2018

OECD (2019). These trends suggest that the composition of debt, between bank loans and bonds, has become an increasingly important margin of firm financing, potentially influenced by shifts in household savings behavior and broader macro-financial dynamics.

## 2.2 Heterogeneous composition of liquid wealth across households

According to Buchak et al. (2024), the period from the late 1970s to the mid-1990s witnessed significant technological advances in the issuance of debt securities, including the rise of securitization and the development of private debt markets. These innovations contributed to a marked reduction in informationally sensitive lending. In parallel, financial innovation also reached retail markets, expanding household access to a broader range of financial products.<sup>7</sup>

Despite these advances, households continue to exhibit considerable inertia in rebalancing their portfolios, even between highly liquid assets such as deposits and bonds. In a frictionless environment without portfolio adjustment costs, all households would shift entirely toward the asset offering the highest yield. In reality, however, households across the income distribution hold a mix of assets, reflecting frictions, preferences, and constraints that limit such reallocation. Moreover, in the specific case of deposits and bonds, the data show that the relative importance of deposits varies systematically not only across the income distribution (Figure 2) but also across wealth groups (Figure A2).

Figure 2 reveals heterogeneity in the composition of household liquid assets across income percentiles. While lower-income households (bottom 20%) allocate nearly 80% of their liquid assets to deposits, the share of deposits declines sharply with income. In contrast, households in the top 1% allocate only nearly one-third of their liquid wealth to deposits, with the remaining two-thirds held in debt securities such as corporate bonds. This systematic shift indicates that higher-income earners exhibit a stronger preference for higher yields but less cash-like instruments, while lower-income earners remain disproportionately reliant on deposits. These differences in portfolio composition underscore the relevance of modeling household heterogeneity in liquidity demand when analyzing credit market dynamics.

On the other hand, while life-cycle motives affect total wealth accumulation, survey evidence shows relatively uniform liquid saving behavior across age cohorts. Figure A5 shows that the ratio of deposits to bond savings varies modestly across age groups, particularly among older cohorts (40 years old and above). While younger households consistently exhibit higher deposit

<sup>&</sup>lt;sup>7</sup>For instance, the rise of online brokerage platforms, bond ETFs, and reduced trading costs.

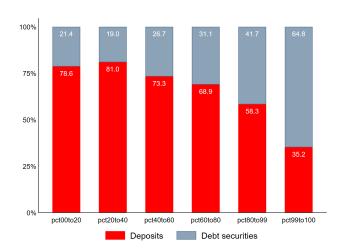


Figure 2: Composition of liquid wealth across income percentiles

Notes: Liquid wealth is defined as the sum of deposits and bond holdings (excluding U.S. and municipal securities). The red and blue bars display the average share of deposits and bonds as a percentage of total liquid wealth, respectively, across income percentiles from 1989 to 2019 based on data from the Distributional Finances Accounts (DFA).

reliance, the overall cross-age variation is small compared to the inequality observed across the income distribution. This suggests that age-cohort dynamics alone cannot explain the substantial heterogeneity in liquidity holdings, motivating a focus on inequality and portfolio sorting across the income and wealth dimensions. Nonetheless, cross-country evidence indicates that income inequality within the 65-and-over population is strongly and consistently associated with a higher corporate share of domestic bonds (see Appendix B for details).

This pattern motivates the model's central assumption that households differ not only in their level of wealth but also in how they allocate liquidity; the stark variation in deposit intensity across the income distribution points to underlying frictions that influence portfolio choices. Thereby, the model captures this rigidity by assuming that households face stochastic adjustment costs when changing their bond holdings, with these costs drawn from a common distribution. As a result, households optimally trade off the marginal benefit and cost of adjusting their bond portfolios, leading to infrequent and lumpy rebalancing behavior. These adjustment costs can reflect a broad range of real-world frictions, such as transaction fees, informational barriers, behavioral biases, inattention, or regulatory constraints, that discourage frequent portfolio shifts. Modeling them stochastically allows the framework to capture gradual portfolio dynamics observed in the data while maintaining tractability and realistic microfoundations.

### 2.3 The Downward Trend of the Corporate Liquidity Premium

In this paper, the corporate liquidity premium is defined as the spread between the interest rate on corporate bonds and that on bank loans of comparable maturity and credit quality. Figure 3 presents two spreads related to the real rates of corporate bonds and bank loans. The red line shows the return differential between high-rated corporate bonds with maturities of one to two years and 3-month commercial paper issued by AA-rated firms.<sup>8</sup> Although commercial papers are not bank loans, Allen et al. (2023) describes that commercial papers offer large firms the ability to raise short-term debt as an alternative to bank loans. Complementing this, the blue line uses firm-level data to show the average cost differential between bond and bank financing for comparable maturities. Both series display a downward trend over time.

I interpret the red line as a proxy for the liquidity premium in corporate credit markets. Because firms issue the bonds in question with minimal credit risk, and despite having longer maturities than commercial papers, they are highly liquid and actively traded in secondary markets. Thus, the observed spread is more likely to reflect compensation for lower immediacy or liquidity rather than for default risk.

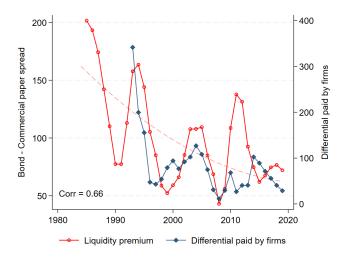
While the bond-commercial paper spread does not directly measure household returns, it reflects equilibrium conditions in financial markets that influence the pricing of liquidity across the entire economy. In particular, a decline in this spread suggests that the compensation required to hold less liquid assets has fallen, which may lower the effective return differential between instruments available to households, such as deposits and bonds. This supports the idea that improvements in corporate bond market liquidity and intermediation efficiency can indirectly shape household portfolio choices by compressing liquidity premium and altering the relative attractiveness of financial assets.

Despite the long-run trend, both spreads also exhibit strong cyclical dynamics, co-moving during major macro-financial episodes such as the early 2000s downturn, the 2008 financial crisis, and the post-2012 recovery. This procyclicality of credit is well established in the literature (Holmstrom and Tirole, 1997; Diamond and Dybvig, 1983; Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). Disentangling supply and demand remains empirically challenging, but Becker and Ivashina (2014) show that firms substitute away from bank loans and toward bond financing when credit conditions tighten or banks face stress. Similarly, Zhu (2021) find that bondholder inflows predict lower

<sup>&</sup>lt;sup>8</sup>The spot rate for a given maturity is the yield on a zero-coupon bond that pays a single amount at that maturity.

likelihoods of equity issuance and bank loan initiation, further underscoring the substitutability and interconnectedness of credit instruments.

Figure 3: Spread of corporate bonds - commercial papers and differential cost paid by firms



Notes: The solid red line plots the three-year moving average of the difference between the annual average yield on corporate bonds rated AAA, AA, or A with maturities of one to two years, and the interest rate on 3-month commercial papers offered to AA-rated firms, both adjusted by inflation, based on FRED data. The red dashed line plots the quadratic trend of this spread. The blue line displays the three-year moving average of the difference between the average cost of funds from corporate bond issuance and bank loans, constructed using firm-level maturity data from Capital IQ.

However, the raw spread in Figure 3 includes the Treasury term premium, which embeds expectations about interest rates and macroeconomic risk. Figure A6 adjusts for this by subtracting the Treasury yield curve slope (e.g., the difference between one-year and three-month Treasury rates), isolating the component of the spread that more directly reflects credit and liquidity risk. The adjusted liquidity premium becomes noticeably flatter but still trends downward and shows strong cyclical patterns. Key episodes, such as the compression of spreads in the early 1990s, the early 2000s, and the post-2010 period, remain visible and align with model predictions. These cyclical shifts are consistent with a flight to liquidity and quality, driven by precautionary savings motives and a rising preference for liquid assets, central features of the theoretical mechanism in the model.

In line with this evidence, my model features an endogenous link between the two debt instruments: the collateral value of each affects firms' financing decisions. Changes in household saving behavior affect the composition of firms' liabilities, which, in turn, impact their asset holdings and net worth, key determinants of the collateral value required by banks and bond investors. This mechanism captures how the household side of the economy can shape firms' access to credit through both

channels simultaneously.

Therefore, the empirical patterns documented in this section support the core mechanism of the paper. Rising income inequality alters the composition of household savings, concentrating bond demand among higher-income households while leaving lower-income households more reliant on deposits. Since financial intermediaries depend on household savings, this shift reshapes the aggregate supply of intermediary capital. In the model, changes in deposit and bond holdings affect the lending capacity of banks and bond markets, thereby influencing the relative cost of financing across credit channels and, ultimately, firms' debt structure and the equilibrium liquidity premium.

#### Mechanisms behind the permanent rise in income inequality

In the next section, I develop a quantitative model that rationalizes the connection between income inequality and the corporate bond boom discussed above. Before doing so, it is useful to briefly review the main explanations for the persistent increase in U.S. income inequality over the past several decades:

- 1. Rising skill premium: Wage inequality has widened as the return to education and skills has increased, particularly since the 1980s. The literature on skill-biased technological change indicates that the demand for highly educated workers has outpaced the supply, thereby widening the wage gap between college-educated and non-college-educated workers. However, Heathcote et al. (2010a) document that wage differentials by education, age, and race have stabilized since 2000, while the gender gap has continued to narrow.
- 2. Changing labor market composition: The rise of self-employment, entrepreneurship, and the concentration of income among business owners has contributed to top-end inequality. Much of the top business income stems from pass-through entities and private equity, which are taxed at the individual rather than entity level, rather than from wages. Smith et al. (2019) ask whether the typical top earner is primarily a human capitalist or a financial capitalist, and conclude that most top earners are human-capital rich, not financial-capital rich.
- 3. Technological change and automation: Advances in information technology and automation have displaced routine jobs, compressing middle-skill wages while raising demand for both high-skill workers, who complement technology, and low-skill service workers (Autor et al., 2006). Complementing this, Heathcote et al. (2010a) document that the most significant occupational shifts in recent decades have been the relative rise in wages for men in non-routine

<sup>&</sup>lt;sup>9</sup>Their definition of labor income includes wages plus three-quarters of pass-through income.

manual occupations compared to those in routine (manual and non-manual) occupations.

- 4. Globalization and trade integration: Increased import competition, particularly from China since the early 2000s, has depressed wages in manufacturing and other tradable sectors, widening income dispersion across regions and occupations. Autor et al. (2013) find that import competition accounts for about one-quarter of the aggregate decline in U.S. manufacturing employment. At the same time, transfer payments for unemployment, disability, retirement, and healthcare rose sharply in more trade-exposed labor markets.
- 5. Institutional and policy changes: Declining unionization, lower real minimum wages, and changes in top marginal tax rates have also amplified wage and after-tax income inequality. Using data from the mid-1970s and early 1990s, Card (1998) shows that unionization among men fell substantially, and that the decline in union coverage explains 10% 20 % of the rise in male wage inequality. Piketty and Saez (2003) document a U-shaped pattern of top income inequality over the 20th century, with a dramatic shift in the composition of top incomes from capital to labor. They argue that technological change alone cannot account for this pattern, and that institutional factors, fiscal policy, and social norms have played critical roles in shaping the wage structure.

These mechanisms help explain the persistent rise in U.S. wage dispersion in recent decades. In the next section, however, my analysis abstracts from these multiple channels and interprets the increase in income inequality as arising exclusively from higher dispersion in idiosyncratic labor productivity shocks.

# 3 Quantitative model

In this section, I develop a dynamic general equilibrium model with long-lived heterogeneous households operating in incomplete markets and heterogeneous firms that face a constant probability of exit each period. Firms endogenously choose their optimal corporate debt structure. The economy consists of three types of agents: households, financial intermediaries, including a representative traditional bank and bond markets, and productive firms.

#### 3.1 Households

Time is discrete and infinite. Each period, a unit mass of households is indexed by  $i \in [0, 1]$ . Households live infinitely, have time-separable preferences, and have a discount factor  $\beta$ . They obtain income from supplying labor inelastically composed of wages  $W_t$  and its idiosyncratic labor productivity  $z_{it}$ , from saving in illiquid assets (corporate bonds)  $b_{it}$ , and from interest on liquid assets (deposits)  $d_{it}$ . The government collects labor income taxes at a rate  $\tau^g$ , so the after-tax labor income is  $(1 - \tau^g) W_t z_{it}$ .

Whenever a household adjusts its holdings of illiquid assets, it needs to pay a stochastic adjustment cost  $\chi_{it}$  that follows a logistic distribution with mean  $\mu_{\chi}$  and scale  $\sigma_{\chi}$ . Following Bayer et al. (2019), this formulation ensures that the first-order conditions in a model with fixed adjustment probabilities are equivalent to those in a model with state-dependent adjustment probabilities. The presence of these adjustment costs induces households to optimally trade off the marginal benefit and cost of rebalancing their bond portfolios, resulting in infrequent and lumpy adjustment behavior. These costs can capture a wide range of real-world frictions, such as transaction fees, information processing constraints, inattention, or regulatory barriers, that discourage frequent portfolio reallocations.

Holdings of liquid assets have a lower bound  $-\underline{d}$ , and holdings of illiquid assets have to be non-negative. Then, households will maximize:

$$\max_{\{c_{it}, b_{it+1}, d_{it}\}_{t=0}^{\infty}} \mathbb{E}_{0} \left[ \sum_{t=0}^{\infty} \beta^{t} u(c_{it}) - \mathbb{I}_{\Delta b_{it} \neq 0} \chi_{it} \right]$$
s.t  $c_{it} + d_{it+1} + b_{it+1} = (1 - \tau^{g}) W_{t} z_{it} + (1 + r_{t}^{d}) d_{it} + (1 + r_{t}^{b}) b_{it} + \Pi_{t}$ 

$$d_{it} \geq -\underline{d}, \quad b_{it} \geq 0$$

$$z_{it} = \frac{\tilde{z}_{it}}{\int \tilde{z}_{it}} \quad \text{where} \quad \tilde{z}_{it} = \exp\left(\rho \log \tilde{z}_{it-1} + \varepsilon_{it}\right), \quad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_{t}\right)$$
(3)

where instantaneous utility is  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$  with  $\gamma \ge 0$ . Labor income follows a log-AR(1) process, and each household receives a fixed share of firms' dividends,  $\Pi_t$ .<sup>10</sup>

To capture the empirical fact that households at the bottom of the wealth distribution still hold a positive amount of bond assets, I introduce liquidity constraints. Specifically, households are allowed to access unsecured credit to finance consumption and/or increase bond holdings when

 $<sup>^{10}</sup>$ This share is proportional to the household's labor productivity and cannot be traded as an asset.

portfolio adjustment is permitted. This credit incurs an additional cost  $\hat{r} > 0$ , which generates a mass of households with zero unsecured borrowing but the potential to take on debt.<sup>11</sup>

$$r_t^d = \begin{cases} r_t^d + \hat{r} &, d_{it} < 0 \\ r_t^d &, d_{it} \ge 0 \end{cases}$$

$$\tag{4}$$

Households maximize their lifetime utility taking as given equilibrium paths for the real wage  $\{W_t\}_{t\geq 0}$ , real return of liquid  $\{r_t^d\}_{t\geq 0}$  and illiquid assets  $\{r_t^b\}_{t\geq 0}$  where  $\{W_t, r_t^d, r_t^b\}_{t\geq 0}$  will be determined by market clearing conditions for liquid and illiquid assets. Furthermore, the policy functions of the household's problem will be nonlinear functions of the state variables  $(d_t, b_t, z_t)$  and joint distribution  $\Theta_t := G(d_t, b_t, z_t)$ .

Finally, three equations characterize the household's problem (recursive form):

$$V_{a}(d,b,z|\Theta) = \max_{d'_{a},b'_{a}} u\left(c\left(d,d'_{a},b,b',z\right)\right) + \beta \mathbb{E}V\left(d',b',z'|\Theta'\right)$$

$$V_{n}(d,b,z|\Theta) = \max_{d'_{n}} u\left(c\left(d,d'_{n},b,b,z\right)\right) + \beta \mathbb{E}V\left(d',b,z'|\Theta'\right)$$

$$\mathbb{E}V\left(d',b',z|\Theta\right) = \mathbb{E}_{\chi',z'}\left[\max\left\{V_{a}\left(d',b',z'|\Theta\right) - \chi',V_{n}\left(d',b',z'|\Theta\right)\right\}\right]$$
(5)

A household will choose to pay the fixed cost to adjust its portfolio if and only if the value function conditional on adjusting  $(V_a)$ , net of the adjustment cost  $(\chi')$ , is greater than or equal to the value function conditional on not adjusting  $(V_n)$ :  $V_a(d',b',z'|\Theta) - \chi' \geq V_n(d',b',z'|\Theta)$ . The adjustment cost  $\chi$  is drawn from a cumulative distribution function  $F_{\chi}$  with mean  $\mu_{\chi}$  and scale  $\sigma_{\chi}$ . The probability of adjustment is therefore given by:

$$\chi' \leq V_a(d', b', z'|\Theta) - V_n(d', b', z'|\Theta)$$

$$\nu^*(d', b', z'|\Theta) \coloneqq F_{\chi}(\chi') = F_{\chi}(V_a(d', b', z'|\Theta) - V_n(d', b', z'|\Theta))$$

$$\nu^*(d', b', z'|\Theta) \coloneqq F_{\chi}[V_a(d', b', z'|\Theta) - V_n(d', b', z'|\Theta)]$$
(6)

Conditional on paying the adjustment cost, a household chooses a portfolio that balances the higher liquidity of deposits - useful for smoothing consumption - against the higher return offered by bond savings in equilibrium.

Equation 6 implies that even for states where  $\Delta V \equiv V_a - V_n$  is small, there is still a nonzero

<sup>&</sup>lt;sup>11</sup>This cost represents an intermediation fee for banks, reflecting the need to monitor unsecured borrowers to prevent default.

probability of adjustment, and for states with large  $\Delta V$ , adjustment is highly likely but not certain. This stochastic formulation replaces the sharp "inaction regions" typical of models with deterministic fixed adjustment costs.<sup>12</sup> Here, the cutoff cost  $\chi^*(d,b,z) = \Delta V(d,b,z)$  defines the threshold at which a household is indifferent between adjusting and not. The CDF of  $\chi$  gives the adjustment probability, so it rises smoothly as  $\Delta V$  increases. Because the model integrates over all possible realizations of  $\chi$  when solving for policies, the policy and value functions are smooth in (d,b,z).

In steady state, all households in the same state (d, b, z) have identical policy functions and value functions, and thus make the same ex-ante optimal decision given their adjustment probability  $\nu^*$ . The stochastic cost affects behavior only through this probability, not by creating cross-sectional dispersion in choices among households in the same state (d, b, z). In aggregate, the economy exhibits state-dependent adjustment probabilities rather than sharp, permanent separation between adjusting and non-adjusting states.

#### 3.1.1 Euler Equations

The policies of consumption, deposit, and bond holdings are denoted by  $c_i^*$ ,  $d_i^*$ , and  $b_i^*$ , respectively, where  $i \in \{a, n\}$ . The first-order conditions for an interior solution of the households' problem are:

$$[b^*] \frac{\partial u\left(c_a^*\right)}{\partial c} = \beta \mathbb{E} \left[ \nu \frac{\partial u\left(c_a^*\left(d_a^*, b^*, z'\right)\right)}{\partial c} \left(1 + r^b\right) + (1 - \nu) \frac{\partial V_n\left(d_a^*, b^*, z'\right)}{\partial b} \right]$$
(7)

$$[d_a^*] \frac{\partial u\left(c_a^*\right)}{\partial c} = \beta \mathbb{E}\left(\left(1 + r^d\right) \left[\nu \frac{\partial u\left(c_a^*\left(d_a^*, b^*, z'\right)\right)}{\partial c} + (1 - \nu) \frac{\partial u\left(c_n^*\left(d_a^*, b^*, z'\right)\right)}{\partial c}\right]\right) \tag{8}$$

$$[d_n^*] \frac{\partial u\left(c_n^*\right)}{\partial c} = \beta \mathbb{E}\left(\left(1 + r^d\right) \left[\nu \frac{\partial u\left(c_a^*\left(d_n^*, b, z'\right)\right)}{\partial c} + (1 - \nu) \frac{\partial u\left(c_n^*\left(d_n^*, b, z'\right)\right)}{\partial c}\right]\right) \tag{9}$$

with derivations provided in Appendix C.1. These conditions imply that when a household decides between holding deposits and bond savings, it must compare three margins. First, the one-period return differential  $r^b - r^d$ , weighted by the marginal utility of consumption when ad-

 $<sup>^{12}</sup>$ In models with deterministic fixed adjustment costs, optimal household behavior often exhibits steady-state inaction regions ("bands"). Specifically, if  $\chi$  is deterministic, there exists a set of states such that households do not adjust their portfolio when the value gain from adjusting does not exceed the cost:  $V_a(d',b',z'|\Theta) - V_n(d',b',z'|\Theta) < \chi'$ .

justment is allowed. Second, the return on deposits when adjustment is not possible, given by  $(1 + r^d) \frac{\partial u(c_n^*(d_a^*,b^*,z'))}{\partial c}$ . Third, the marginal value of bond holdings in the no-adjustment state,  $\frac{\partial V_n(d_a^*,b^*,z')}{\partial b}$ , which captures both the utility from the bond's return stream and the option value of selling bonds in future periods when adjustment becomes available.

#### 3.2 Firms

Departing from the standard RBC literature, I introduce idiosyncratic risk to the firms' stock of assets that directly affects production. This approach, as in Villacorta (2018), allows me to decompose firms' collateral into two components: (i) a riskless portion, whose value is preserved regardless of whether an adverse shock hits the firm, and (ii) a risky portion, whose value is directly affected when the firm experiences such a shock.

#### 3.2.1 Investment block

Each period, a unit mass of productive firms is indexed by  $j \in (1,2]$ . Firms share a common technology  $A_t$ . They are heterogeneous along two dimensions: initial net worth  $n_{jt}$  and the realization of an idiosyncratic productivity shock  $\omega_{jt} \in \{0,1\}$  that follows a Bernoulli process with probability  $p_H$  that represents the fraction of firms that succeed in their projects. Also, whenever there is an adverse shock  $\omega_{jt} = 0$ , there is a partial loss of capital  $\kappa > 0$ , and the firm can't produce in period t. Each firm produces an undifferentiated good  $y_{jt}$  using a production function  $f(A_t, k_{jt+1}, l_{jt}, \omega_{jt}) = \omega_{jt} A_t k_{jt+1}^{\alpha} l_{jt}^{1-\alpha}$  and depreciation of assets is denoted by  $\delta$ .<sup>13</sup>

Workers supply labor to a representative firm under wage contracts that are contingent on firm productivity, with no reallocation of labor across firms. Firms hire workers before the realization of  $\omega_{jt}$ , but wage payments occur after. I assume that wages are contingent on firm productivity to prevent firms from operating at a loss when  $\omega_{jt} = 0$ . Also, there is no reallocation of workers after the realization of  $\omega_{jt}$ , preventing any labor adjustments that could distort the return on capital. The labor market is competitive, with firms taking  $w_t$  as given, and wages falling to zero if the firm fails.

Thus, for individual firms that take wages as given, the gross profit depends on wages and is linear in their own capital. However, since wages are determined by aggregate capital, a firm's marginal profit on capital is ultimately influenced by aggregate capital, exhibiting decreasing

<sup>&</sup>lt;sup>13</sup>In this section, the terms "capital" and "assets" are used interchangeably.

returns. Finally, each firm's marginal return on capital is defined by:

$$R_{jt}^{K} = \begin{cases} \alpha A K_{t+1}^{\alpha - 1} + 1 - \delta & \text{with } p_{H} \\ (1 - \delta)(1 - \kappa) & \text{with } 1 - p_{H} \end{cases}$$
 (10)

**Assumption 1:**  $\mathbb{E}_t R_{jt}^K \equiv R_t^K > 1$ . Return on capital is high enough to incentivize households to save.

#### 3.2.2 Financing block

In the absence of financial frictions, firms would finance themselves using the cheapest available instrument, and any cost differential between bonds and bank loans would be eliminated by competition in the financial sector. Consequently, a positive liquidity premium, defined as the wedge between less liquid bonds and more liquid deposits, would not persist in equilibrium, as arbitrage by firms and financial intermediaries would equalize marginal costs of funding. To capture realistic debt market dynamics, my model introduces financial frictions due to moral hazard in the spirit of Holmstrom and Tirole (1997). Firms have access to both bank loans and bond financing, and choose their debt composition to maximize expected net worth, subject to contractual arrangements that mitigate these frictions. Firms raising external funds can pledge their entire stock of assets as collateral, allowing both banks and bondholders to hold claims on the firm's assets. However, the enforceability of financial contracts determines the priority structure of these claims, ensuring that bondholders are repaid first while banks bear the riskier portion of debt.

Also, firms report the realization of  $\omega_{jt}$  independently to banks and bondholders. Since bondholders lack monitoring capabilities, they cannot verify the firm's true state, creating a moral hazard problem that restricts bond financing to risk-free debt. Hence, the amount of risk-free bonds that firms can issue is limited by the safe part of their capital, which is determined by the marginal return of capital  $1 - \kappa$ . Then, the riskless borrowing constraint of bond issuance is defined by:

$$R_t^B L_{jt}^B \le (1 - \delta)(1 - \kappa)k_{jt+1}$$
 (11)

In contrast, banks can monitor firms, allowing them to accept risky debt while enforcing incentive constraints to deter misreporting. This separation of reporting justifies the different financial contract structures for bondholders and banks. As a result, firms have incentives to misreport the

true realization of  $\omega_{jt}$  and divert resources for private benefits.

Furthermore, banks accept risky debt limited by their monitoring effectiveness  $\phi \in (0, 1)$ . Unlike bondholders, banks can monitor firms, allowing them to extend risky debt that is not fully collateralized. To prevent misreporting, banks impose an incentive compatibility constraint, ensuring that firms truthfully report their outcomes rather than diverting funds. Appendix E describes the constraints on financial contracts assumed in the model.

Without enforcement mechanisms, firms may divert resources for private benefits. Introducing banks into the model expands firms' access to external financing while limiting their ability to expropriate funds. Banks mitigate misreporting through monitoring, which acts as an enforcement device, restricting firms from diverting a fraction  $\phi$  of the risky value of capital. Since banks assume the riskier part of the firm's liabilities, they only accept risky debt up to a fraction  $\phi$  of the risky capital that can be effectively monitored. The stochastic loan rate promised to banks at t is denoted by:

$$R_{jt}^{D} = \begin{cases} R_t^D & \omega_{jt} = 1\\ 0 & \omega_{jt} = 0 \end{cases}$$
 (12)

where  $\omega_{jt}$  denotes the realization of the idiosyncratic productivity shock. The zero recovery value for the bank loan is a normalization: given the binomial nature of the idiosyncratic shock, any loan with a positive recovery value can be decomposed into a risk-free component and a risky recovery debt instrument.

Given the loan rate  $R_t^D$ , the total promise from the risky bank loans issued by firms is limited by:

$$R_t^D L_{it}^D \le \phi \kappa (1 - \delta) k_{jt+1} \tag{13}$$

Firms begin period t with initial net worth  $n_{jt}$  and can raise additional financing from both households and banks by issuing either risk-free bonds or risky bank loans. I denote with  $L_{jt}^{B}$  the amount of funds borrowed through risk-free bonds and  $L_{jt}^{D}$  the funds obtained via risky bank loans. Total capital invested by firm j at period t is:

$$k_{jt+1} = n_{jt} + L_{jt}^B + L_{jt}^D (14)$$

and combining Equations 11, 13 and 14, I get

$$k_{jt+1} \le \frac{1}{\left(1 - \frac{(1-\delta)(1-\kappa)}{R_t^B} - \frac{\phi\kappa(1-\delta)}{R_t^D}\right)} n_{jt}$$
 (15)

where  $R_t^B$  and  $R_t^D$  are endogenous objects that clear the market of risk-free bonds and risky bank loans, respectively.

Firm's leverage: It is important to notice that Equation 15 imposes a maximum leverage of firms that depends on their initial net worth and the relative repayment (per unit) to financial intermediaries. Moreover, lower interest rates and greater monitoring effectiveness ( $\phi$ ) enhance firms' ability to expand their capital stock by increasing their borrowing capacity.

#### 3.2.3 Firm's problem

Firms will maximize their net worth and only will pay dividends to entrepreneurs when they exit the market which occurs each period with probability  $\tau$ . Firms have a discount factor equivalent to the discount factor of entrepreneurs denoted by  $\Lambda$ . Conditional on existing, firms start period t with an initial net worth  $n_{jt}$ , and they borrow from households and banks implying that the law of motion of net worth and capital after the realization of the idiosyncratic productivity shock are defined by:

$$n_{jt+1} = R_{jt}^K k_{jt+1} - R_t^B L_{jt}^B - R_{jt}^D L_{jt}^D$$
(16)

Thus, firms will choose optimally  $L_{jt}^B$  and  $L_{jt}^D$  to maximize their respective net worth as follows:

$$V_t^F(n_{jt}) = \max_{L_{it}^B, L_{jt}^D} \tau \Lambda \mathbb{E}_t[n_{jt+1}] + (1 - \tau) \Lambda \mathbb{E}_t \left[ V_{t+1}^F(n_{jt+1}) \right]$$
(17)

subject to 11, 13, 14, and 16.

#### 3.3 Financial intermediaries

There are two types of financial intermediaries: (i) bond markets composed of uninformed investors who invest in secured firm loans, and (ii) traditional banks, which are deposit-taking institutions

that invest all household deposits in firm loans that are not necessarily secured. When idiosyncratic shocks hit firms and some become unable to repay, banks have the ability to pool risk across firms and ensure repayment to households. This risk-sharing capacity allows banks to offer deposit contracts that are safe from the perspective of savers, despite the underlying exposure to firm default risk.

#### 3.3.1 Bond markets

Bond markets consist of uninformed investors, treated here as a single entity with no specialized technology for monitoring or capital production. These investors pool household savings, offering a return of  $1 + r_t^b$ . Then, profits in the bond market are defined by  $\pi_t^B = R_t^B L_t^B - (1 + r_t^b) B_t = R_t^B \int_j L_{jt}^B d_j - (1 + r_t^b) B_t$ . Since the bond market is perfectly competitive, equilibrium implies  $\pi_t^B = 0$ . Moreover, as total bond issuance equals the resources collected from households,  $L_t^B = B_t = \int_i b_{it} d_i$  leading to:

$$R_t^B = 1 + r_t^b \tag{18}$$

#### 3.3.2 Banks

There is a continuum of competitive banks, treated as a single party that collects deposits from households at a cost of funding  $1 + r^d$ . Banks possess a specialized monitoring technology, which incurs a per-unit cost  $\hat{c}$  for each unit of firm credit. This monitoring mitigates the moral hazard problem by improving borrower discipline. Banks are short-lived: they operate for one period, after which they are replaced by new banks with identical technology.

In the absence of unsecured consumer credit, bank profits are given by:  $\pi_t^D = \tilde{R}_t^D L_t^D - \left(1 + r^d\right) D_t - \hat{c} L_t^D \ge 0$  and the average return on banks' portfolio loans is given by  $\tilde{R}_t^D = \frac{\int_j R_{jt}^D L_{jt}^D d_j}{L_t^D}$ . Since risk diversification is costless and by LLN then the average return per unit of capital is:

$$\tilde{R_t^D} = p_H R_t^D \frac{\int_j L_{jt}^D}{L_t^D} = p_H R_t^D$$

then  $p_H R_t^D L_t^D \ge (1 + r^d) D_t + \hat{c} L_t^D$  and since banks have no networth  $L_t^D = D_t$  then the aggregate supply of bank loans is defined by:

$$p_H R_t^D = 1 + r_t^d + \hat{c} (19)$$

Introducing unsecured consumer credit: Households can access unsecured consumer credit  $(d_{it} < 0)$ , subject to an intermediation cost  $\hat{r}$  per unit of credit and a monitoring cost  $\epsilon$  incurred by banks for each unit of household debt. The monitoring technology is intended to mitigate default risk. Under this setup, the profits of banks from consumer lending are given by:

$$\pi_t^D = \int_j R_{jt}^D L_{jt}^D d_j - \left(1 + r_t^d\right) \int_i \mathbb{I}_{d_{it} \ge 0} |d_{it}| d_i + \left(1 + r^d + \hat{r} - \epsilon\right) \int_i \mathbb{I}_{d_{it} < 0} |d_{it}| d_i - \hat{c} L_t^D \ge 0 \tag{20}$$

I define the composition of total deposits based on the sign of individual household positions. Specifically, the share of total deposits that are non-negative and negative are denoted by  $S_t^+ = \frac{\int_i \mathbb{I}_{d_{it} \geq 0} |d_{it}| d_i}{\int_i |d_{it}| d_i}$  and  $S_t^- = \frac{\int_i \mathbb{I}_{d_{it} < 0} |d_{it}| d_i}{\int_i |d_{it}| d_i}$ , respectively. By construction, these shares satisfy  $S_t^+ + S_t^- = 1$ . In a competitive market, where bank profits from deposit intermediation satisfy  $\pi_t^D = 0$ , the following condition must hold:

$$\pi_t^D = p_H R_t^D L_t^D - \left( \left( 1 + r_t^d \right) S_t^+ - \left( 1 + r^d + \hat{r} - \epsilon \right) S_t^- \right) \int_i |d_{it}| d_i - \hat{c} L_t^D \ge 0$$

where  $L_t^D = D_t$  and  $D_t = S_t^+ \int_i |d_{it}| d_i - S_t^- \int_i |d_{it}| d_i$ 

$$(p_H R_t^D - \hat{c}) (S_t^+ - S_t^-) = (1 + r_t^d) S_t^+ - (1 + r^d + \hat{r} - \epsilon) S_t^-$$
 (21)

For simplicity, I assume that the extra interest rate charged on household borrowing  $\hat{r}$ , fully covers the monitoring cost  $\epsilon$  associated with unsecured consumer credit. Moreover, given that monitoring firms is also costly, the supply of risky bank loans in the presence of unsecured consumer credit is defined by:

$$p_H R_t^D = 1 + r_t^d + \hat{c} (22)$$

Equation 22 indicates that, all else equal, a rise in  $\hat{c}$  requires a lower deposit rate  $r_t^d$  to restore balance.

# 3.4 Aggregation

The aggregate output and the stock of assets are defined by:

$$Y_t = p_H A K_t^{\alpha} \tag{23}$$

$$K_{t+1} = N_t + L_t^B + L_t^D (24)$$

As firms exit over time with probability  $\tau$ , I assume that every period a new set of firms enters and replaces them so that the set of firms remains  $\mathbb{J}=(1;2]$ . Every period, a measure  $\tau$  of firms enters the market. Each new firm is endowed with a random initial capital with a mean equal to the fraction  $\zeta$  of the average net worth of existing firms. By aggregating Equation 16 and considering the entry and exit of firms, the initial aggregate networth in t+1 is defined by:

$$N_{t+1} = (1 - \tau + \tau \zeta) \left( R_t^K K_{t+1} - R_t^B L_t^B - p_H R_t^D L_t^D \right)$$
 (25)

where the return on capital (per unit)  $R_t^K$  and the average bank loan repayment rate (per unit):

$$R_t^K = p_H \alpha A K_{t+1}^{\alpha - 1} + p_H (1 - \delta) \kappa + (1 - \delta) (1 - \kappa)$$
(26)

# 3.5 General equilibrium

A recursive equilibrium is a set of households' policy functions  $\{c_i^*, d_i^*, b_i^*, \nu_i^*\}, \forall i \in \mathbb{I}$ , firms' policy functions  $\{l_j^*, k_j^*, L_j^{*B}, L_j^{*D}, n_j^*\}, \forall j \in \mathbb{J}$ , set of value functions of households  $\{V_i^a, V_i^n\}, \forall i \in \mathbb{I}$ , firms  $\{V_j^F\}, \forall j \in \mathbb{J}$ , and banks and investors policy functions  $\{R^D, R^B\}$  that depends on prices  $\{r^d, r^b, w, R^K\}$ , distributions  $\Theta$  over individual asset holdings such that:

- Given prices  $\{r^d, r^b, w\}$ ,  $\{V_i^a, V_i^n\}$ , distributions  $\Theta$ , policy functions  $\{c_i^*, d_i^*, b_i^*, \nu_i^*\}$  solve the households' problem.
- Given  $\{R^D, R^B, w, R^K\}$  and initial networth  $n_j$ , policy functions  $\{l_j^*, k_j^*, L_j^{*B}, L_j^{*D}\}$  solve the firms' problem.

- Given  $\{r^d, r^b, D_t, B_t\}$ , policy function  $\{\tilde{R}^D, R^B\}$  solves the banks and bond markets problem.
- The market of goods and labor clears

$$\int_{i} c_i^* d_i = \int_{j} y_j^* d_j \tag{27}$$

$$\int_{j} l_j^* d_j = 1 \tag{28}$$

• The risk-free bond market and risky bank loans clears

$$\int_{i} b_i^* d_i = \int_{j} L_j^{*B} d_j \tag{29}$$

$$\int_i d_i^* d_i = \int_j L_j^{*D} d_j \tag{30}$$

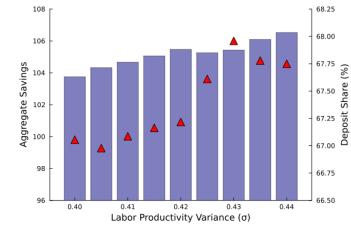
• Aggregate output, the law of motion of capital and networth are defined by Equations 23, 24 and 25, respectively.

## 3.6 Equilibrium characterization

#### 3.6.1 Aggregate savings and income inequality

Using the numerical methods described in Section 4, this section examines how aggregate savings and their composition evolve as income inequality increases, holding equilibrium prices constant. Figure 4 shows how rising idiosyncratic labor productivity risk affects both the level and composition of household savings in partial equilibrium. As the variance  $\sigma$  increases and prices (i.e.,  $r^d, r^b, W$ ) are fixed, aggregate savings rise steadily, consistent with households engaging in precautionary saving to buffer higher income uncertainty. Interestingly, the deposit share, shown by the red dots, also increases with income risk, indicating a portfolio reallocation toward the more liquid assets. This shift reflects the heightened value of flexibility when income becomes riskier, even at the cost of lower returns on deposits relative to illiquid assets (the liquidity premium is  $r^b - r^d = 0.0565$ ). Moreover, a higher income risk not only amplifies saving behavior but also reshapes the liquidity structure of household portfolios, with implications for credit markets.

Figure 4: Total savings and their composition under income inequality



Notes: Bars (left axis) display aggregate household savings as the variance of labor productivity,  $\sigma$ , increases uniformly from 0.4 to 0.45. Red dots (right axis) show the share of deposits in total savings, capturing the composition of liquid wealth. The exercise uses the household parameters reported in Table 3, holding prices fixed at  $r^b = 0.075$ ,  $r^d = 0.0185$ , and W = 17.5.

#### 3.6.2 Labor market

Since wage contracts are contingent on firm productivity, firms pay wages  $w_t$  only if they generate revenue:

$$w_t = \begin{cases} w_t^* & \text{if } \omega_{jt} > 0\\ 0 & \text{if } \omega_{jt} = 0 \end{cases}$$

The labor market is competitive, with firms taking  $w_t$  as given, and wages falling to zero if the firm fails. Thus, for a given  $k_{jt+1}$ , firms maximize cash flows by choosing labor to solve:

$$y_{jt} = \max_{l_{jt}} \mathbb{E}_t \left[ \omega_{jt} \right] \left( A k_{jt+1}^{\alpha} l_{jt}^{1-\alpha} - w_t l_{jt} \right)$$

where the optimal labor decision is linear in capital:

$$l_{jt}^{*}(k_{jt+1}) = \left(\frac{A(1-\alpha)}{w_t}\right)^{\frac{1}{\alpha}} k_{jt+1}$$

and each firm's gross profits are defined by:

$$\pi\left(k_{jt+1}, \omega_{jt}\right) = \omega_{jt} \alpha A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} k_{jt+1} \tag{31}$$

Therefore, the marginal return of a firm's capital will be influenced by idiosyncratic productivity  $\omega_{jt}$  and also depends on wages and the common technology factor. As a result, the marginal product of capital is identical across all firms, given by  $\frac{\partial \pi_{jt}}{\partial k_{jt+1}} = \omega_{jt} \alpha A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$ . Moreover, in the labor market, the aggregate demand for labor is:

$$\int_{j \in \mathbb{J}} l_{jt}^*(k_{jt+1}) \, dj = \left(\frac{A(1-\alpha)}{w_t}\right)^{\frac{1}{a}} K_{t+1}$$

and the wage  $w_t$  clears the market of labor. Also, given the aggregate fixed labor supply from households, the equilibrium wage solves:

$$\left(\frac{A\left(1-\alpha\right)}{w_t}\right)^{\frac{1}{a}}K_{t+1} = 1
\tag{32}$$

then wages depend on aggregate capital, and by substituting Equation 32 into Equation 31, I obtain:

$$\frac{\partial \pi_{jt}}{\partial k_{jt+1}} = \begin{cases} \alpha A K_{t+1}^{\alpha - 1} & \text{with } p_H \\ 0 & \text{with } 1 - p_H \end{cases}$$

#### 3.6.3 Credit regimes

The firm's state space over bank loans and bonds  $(L_t^D, L_t^B)$  can be divided into two regions: (i) an unconstrained credit regime, where neither borrowing constraint binds, and (ii) a constrained credit regime, where one or both borrowing constraints are binding. Owing to the linearity of the firm's borrowing policies, the distribution of firms' net worth is irrelevant; hence, the analysis in the following sections abstracts from this distribution. Details of the firm's dynamic problem and the derivation of the conditions that characterize these regimes are provided in Appendix D.1 and D.2.

Unconstrained credit regime: If borrowing constraints are not binding, the returns across all funding sources are equalized - that is, the return on capital equals the cost on corporate bonds, which also equals cost of risky bank loans:  $R_t^K = R_t^B = p_H R_t^D$ . Also, in the steady state, firms' net worth satisfies  $(1 - \tau)R^K = 1$ , which ensures that firms do not accumulate net worth indefinitely and remain near the steady state.

Corollary 1. Under Assumption 3.2.1, the condition  $(1 - \tau)R_t^K = 1$ , and given that the return on capital in 26 holds, the equilibrium stock of capital lies within the interval  $K_t \in [\underline{K}, \overline{K})$  where the lower and upper bounds are defined as follows:

$$\underline{K} \equiv \left(\frac{\alpha A p_H}{\frac{1}{1-\tau} - p_H (1-\delta)\kappa + (1-\delta)(1-\kappa)}\right)^{\frac{1}{1-\alpha}} \tag{33}$$

$$\bar{K} \equiv \left(\frac{\alpha A p_H}{1 - p_H (1 - \delta)\kappa + (1 - \delta)(1 - \kappa)}\right)^{\frac{1}{1 - \alpha}} \tag{34}$$

Constrained credit regime: When either or both borrowing constraints bind, the firm's debt composition becomes endogenous. Specifically, when the constraint on risky bank loans or on bonds is binding, the firm's first-order conditions yield the following expressions, respectively.

**Lemma 1.** Kink point on risk-free bond return under a tight risky bank loan constraint. Suppose that the bank constraint binds. Then, the kink point on risk-free bond return,  $\bar{R}_t^B$ , is defined by:

$$\bar{R}_{t}^{B} \equiv \frac{p_{H}(\alpha A K_{t+1}^{\alpha-1} + (1-\delta)\kappa) + (1-\delta)(1-\kappa-\phi\kappa)}{1 - \frac{\phi\kappa(1-\delta)}{R_{t}^{D}}} \equiv \frac{R_{t}^{K} - \phi\kappa(1-\delta)}{1 - \frac{\phi\kappa(1-\delta)}{R_{t}^{D}}}$$
(35)

Furthermore, if  $R_t^B < \bar{R}_t^B$  implies that the marginal value of borrowing through risk-free bonds is positive, then the bond constraint binds.

**Lemma 2.** Kink point on risky bank loan return under a tight risk-free bond constraint. Suppose that the bond constraint binds. Then, the kink point on risky bank loan return,  $R_t^D$ , is defined by:

$$p_H \bar{R}_t^D \equiv \frac{p_H(\alpha A K_{t+1}^{\alpha - 1} + (1 - \delta)\kappa)}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_t^B}} \equiv \frac{p_H(R_t^K - (1 - \delta)(1 - \kappa))}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_t^B}}$$
(36)

Furthermore, if  $R_t^D < \bar{R}_t^D$  implies that the marginal value of borrowing through risky bank loans is positive, then the bank loan constraint binds.

#### 3.6.4 Optimal borrowing decisions

The following propositions can thus characterize the firm's demand for risk-free bonds and risky bank loans.

**Proposition 3.** The firm's demand for risk-free bonds is defined by:

$$L_{jt}^{B} = \begin{cases} \bar{L}_{jt}^{B} & \text{if } R_{t}^{B} < \bar{R}_{t}^{B} \\ [0, \bar{L}_{jt}^{B}] & \text{if } R_{t}^{B} = \bar{R}_{t}^{B} \\ 0 & \text{if } R_{t}^{B} > \bar{R}_{t}^{B} \end{cases}$$
(37)

where 
$$\bar{L}_{jt}^{B} = \frac{(1-\kappa)(1-\delta)}{R_{t}^{B}} \frac{1}{\left(1 - \frac{(1-\kappa)(1-\delta)}{R_{t}^{B}} - \frac{\phi\kappa(1-\delta)}{p_{H}R_{t}^{D}}\right)} n_{jt}$$
 and  $\bar{R}_{t}^{B} = \frac{R_{t}^{K} - \phi\kappa(1-\delta)}{1 - \frac{\phi\kappa(1-\delta)}{R_{t}^{D}}}$ .

**Proposition 4.** The firm's demand for risky bank loans is defined by:

$$L_{jt}^{D} = \begin{cases} \bar{L_{jt}^{D}} & \text{if } R_{t}^{D} < \bar{R_{t}^{D}} \\ [0, \bar{L_{jt}^{D}}] & \text{if } R_{t}^{D} = \bar{R_{t}^{D}} \\ 0 & \text{if } R_{t}^{D} > \bar{R_{t}^{D}} \end{cases}$$
(38)

where 
$$\bar{L_{jt}^D} = \frac{\phi \kappa (1-\delta)}{R_t^D} \frac{1}{\left(1 - \frac{(1-\kappa)(1-\delta)}{R_t^B} - \frac{\phi \kappa (1-\delta)}{p_H R_t^D}\right)} n_{jt}$$
 and  $\bar{R_t^D} = \frac{R_t^K - (1-\delta)(1-\kappa)}{1 - \frac{(1-\delta)(1-\kappa)}{R_t^B}}$ 

#### 3.6.5 Market for risk-free bonds and risky bank loans

The aggregate demand for risk-free bonds is defined by:

$$L_{t}^{B} = \begin{cases} \frac{(1-\kappa)(1-\delta)}{R_{t}^{B}} \frac{1}{\left(1 - \frac{(1-\kappa)(1-\delta)}{R_{t}^{B}} - \frac{\phi\kappa(1-\delta)}{p_{H}R_{t}^{D}}\right)} N_{t} & \text{if } R_{t}^{B} < \bar{R}_{t}^{B} \\ \left[0, \frac{(1-\kappa)(1-\delta)}{R_{t}^{B}} \frac{1}{\left(\frac{1}{1-\delta} - \frac{1-\kappa}{R_{t}^{B}} - \frac{\phi\kappa}{p_{H}R_{t}^{D}}\right)} N_{t}\right] & \text{if } R_{t}^{B} = \bar{R}_{t}^{B} \\ 0 & \text{if } R_{t}^{B} > \bar{R}_{t}^{B} \end{cases}$$

$$(39)$$

whereas the supply of risk-free bonds is defined by:

$$R_t^B = 1 + r_t^b \tag{40}$$

Furthermore, the aggregate demand for risky bank loans is defined by:

$$L_{t}^{D} = \begin{cases} \frac{\phi\kappa(1-\delta)}{R_{t}^{D}} \frac{1}{\left(\frac{1}{1-\delta} - \frac{1-\kappa}{R_{t}^{B}} - \frac{\phi\kappa}{p_{H}R_{t}^{D}}\right)} N_{t} & \text{if } R_{t}^{D} < \bar{R}_{t}^{D} \\ \left[0, \frac{\phi\kappa(1-\delta)}{R_{t}^{D}} \frac{1}{\left(\frac{1}{1-\delta} - \frac{1-\kappa}{R_{t}^{B}} - \frac{\phi\kappa}{p_{H}R_{t}^{D}}\right)} N_{t}\right] & \text{if } R_{t}^{D} = \bar{R}_{t}^{D} \\ 0 & \text{if } R_{t}^{D} > \bar{R}_{t}^{D} \end{cases}$$
(41)

whereas the supply of risky bank loans is defined by:

$$p_H R_t^D = 1 + r_t^d + \hat{c} (42)$$

Therefore, the demand for risky bank loans and risk-free bonds is jointly determined, as changes in the interest rate of one instrument affect the firm's leverage decisions and, consequently, the demand for both types of financing. Figure 5 illustrates the relationship between aggregate deposits, aggregate bonds, and two key firm-side outcomes: (A) the aggregate net worth of firms, and (B) the return on capital  $R_t^K$ . Panel A reveals that net worth is a non-monotonic function of the financial asset composition. At low levels of deposits and bonds, firms are largely unconstrained and indifferent between bond and loan financing, enabling them to rely more on their internal funding (net worth). However, as the economy accumulates liquid wealth beyond a certain threshold, especially when concentrated in deposits, financial frictions begin to bind. In equilibrium, firms substitute away from bank loans toward bond financing as the relative cost of borrowing through banks rises.

1.076 50 50 -1.075 1.074 40 40 100 Bonds Bonds ® 1.073 1.072 Agg. Agg. 1.071 1.070 10 10 1.069 1.068 0 0 Agg. Deposits Agg. Deposits (A) Firm net worth (B) Return on capital

Figure 5: Net worth and return on capital

Notes: Figures (A) and (B) present heatmaps of firm net worth and the return on capital, respectively, as functions of aggregate deposits (x-axis) and aggregate bonds (y-axis). Parameters as in Table 3.

The shift in financing composition, driven by the tightening of borrowing constraints, leads firms to accumulate more internal funds, thereby shaping the equilibrium path of the return on capital. Panel B exhibits the reaction of the return on capital to these dynamics. In constrained regions, the marginal return on capital increases due to restricted firm financing, whereas in the unconstrained region, capital markets clear at a lower return, reflecting the lower cost of external funding. Notably, the point labeled "SS2" (the new steady state with higher productivity risk) is located in a region with lower capital returns and higher reliance on external funding than "SS1" (the original steady state), highlighting the nontrivial impact of income risk on firm-side outcomes.

Thus, Figure 6 maps the economy's feasible allocations into constrained and unconstrained regimes. The blue dots denote combinations of aggregate deposits and bonds for which financial constraints (either on bonds or deposits) bind. The region labeled "Constrained Regime" includes allocations with relatively high bond supply, reflecting firms' increased reliance on bond financing as bond interest rate declines. However, despite lower borrowing costs, the economy enters a constrained regime because firms hit an upper limit on how much they can borrow through risk-free bonds, due to the collateral constraint. This forces firms to adjust their debt structure.

In contrast, the "Unconstrained Regime" appears when bonds and deposits are scarce (to the left of the blue region) or when there is not enough bond supply to lower borrowing costs (to the right of blue region). Finally, the combinations of deposits and bonds inside the gray region

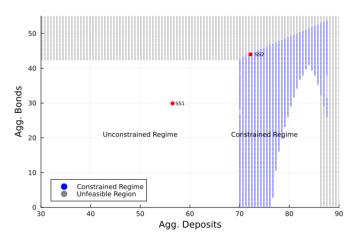


Figure 6: Unconstrained and constrained regimes

Notes: The figure distinguishes between feasible credit regimes and unfeasible allocations. White areas correspond to the unconstrained regime, while blue areas denote the constrained regime. Unfeasible allocations arise when either bond or bank collateral falls short of the required credit, in which case financial intermediaries are unwilling to lend. The exercise is based on the firm and intermediary parameters reported in Table 3.

reflect the unfeasible area where the value of collateral of risk-free bonds or risky bank loans are not high enough to cover the resources collected by bond markets and banks. Importantly, the equilibrium in SS2 is located inside the constrained region, reinforcing the idea that higher income risk leads households to demand more deposits and bonds, indirectly tightening constraints on firm financing. This endogenous shift in regime has important implications for both capital allocation and macroeconomic outcomes.

These results are closely related to Bernanke and Gertler (1989), where firm net worth governs the strength of borrowing constraints and amplifies real fluctuations. In this model, however, financial frictions arise endogenously from the interaction between household saving behavior and firm financing decisions.

Moreover, Bernanke and Gertler (1989) define the external finance premium as the difference between the cost of external funds and the opportunity cost of internal funds (retained earnings). This premium arises because lower net worth increases the risk of borrower default, raising the monitoring and agency costs borne by lenders. As a result, the external finance premium is countercyclical, widening in downturns when borrower balance sheets weaken.

In contrast, in my model, a similar wedge emerges between two forms of external financing, bond financing and bank loans. Unlike in Bernanke and Gertler (1989), where the premium reflects the gap between internal and external funding costs, here it captures the relative cost of these

two external sources. This wedge depends on the credit regime in which firms operate: in the unconstrained regime, it is zero because firms are indifferent between bonds and bank loans, whereas in the constrained regime, it becomes non-zero as firms substitute away from the relatively more expensive instrument. The wedge is determined endogenously by household saving behavior and financial frictions in credit markets.

Thus, while in Bernanke and Gertler (1989) the external finance premium is driven by borrower net worth and affects access to all external funds, in my framework, the relative cost of external funds is shaped by shifts in the supply of liquid savings, which feed back into firm financing choices. Both mechanisms link financial frictions to real outcomes, but they operate through different margins, Bernanke and Gertler (1989) through the internal-external margin, and this paper through the bank-bond margin.

#### 3.6.6 Liquidity premium

In aggregate, the total holdings of bonds and deposits will be positive if and only if the liquidity premium, as  $LP_t \equiv r_t^b - r_t^d$ , is positive.

**Proposition 5.** The liquidity premium is given by:

- Under the unconstrained credit regime:  $LP_t = \hat{c} > 0$
- Under the constrained credit regime:
  - If risk-free bond constraint binds:  $LP_t = (1 \kappa) \frac{K_{t+1}}{L_t^B} (p_H \bar{R}_t^D \hat{c})$
  - If risky bank loans constraint binds:  $LP_t = \bar{R}_t^B (p_H \phi \kappa \frac{K_{t+1}}{L_t^D} \hat{c})$
  - If both constraints bind:  $LP_t = (1 \kappa) \frac{K_{t+1}}{L_t^B} (p_H \phi \kappa \frac{K_{t+1}}{L_t^D} \hat{c})$

Importantly, for sufficiently high monitoring costs  $\hat{c}$ , the liquidity premium remains positive regardless of the regime. This guarantees interior solutions in equilibrium, where firms use a mix of safe and risky debt. Moreover, the liquidity premium acts as a key pricing signal that shapes the allocation of funding across instruments and affects macro-financial transmission channels.

Besides, in equilibrium, when the bond constraint binds, an increase in aggregate bonds  $(L_t^B)$  generates two effects on the liquidity premium. First, the effective assets-to-bonds ratio,  $(1-\kappa)\frac{K_{t+1}}{L_t^B}$ , declines as the denominator rises. Second, the net maximum cost of bank loans,  $p_H \bar{R}_t^D - \hat{c}$ , also

falls because the upper bound on risky bank loans  $(\bar{R}_t^D)$  increases. Taken together, these two forces imply a negative overall effect on the liquidity premium.

Conversely, when the risky bank loans constraint binds, an increase in aggregate deposits  $(L_t^D)$  generates two effects on the liquidity premium. First, maximum cost of risk-free bonds,  $\bar{R}_t^B$ , increases. Second, the effective assets-to-bank loans ratio,  $(p_H\phi\kappa\frac{K_{t+1}}{L_t^D})$ , declines as the denominator rises. In combination, these two forces imply a positive overall effect on the liquidity premium.

#### 4 Estimation

The dynamic problem 5 is solved numerically by discretizing the joint distribution  $\Theta(d, b, z)$  over household states. In contrast, the dynamic problem 17 is solved at the representative firm level, since the linearity of firm policies implies that the distribution of networth across firms does not affect aggregate outcomes.

I approximate the labor productivity process  $(e_i \equiv \log z_i)$  using a symmetric grid of  $n_e$ . Specifically, the grid points are:

$$e_i = \alpha \left( i - \frac{n_e - 1}{2} \right), i = 0, 1, 2, ..., n_e$$
 (43)

where the scaling parameter  $\alpha = \frac{2\sigma_1}{\sqrt{n_e-1}}$  is tied to the innovation standard deviation  $\sigma_1$  in the initial steady state. Importantly, this grid is held fixed when computing the second steady state as well. Moreover, I exponentiate the grid points to obtain  $\tilde{z}_i = \exp e_i$ . Note that these  $\tilde{z}_i$  are not normalized to have unit mean since normalization arises endogenously once the stationary distribution of the Markov chain is computed.

The transition probability matrix,  $\mathbf{P_z}$ , is approximated using a Monte Carlo method. For each state, I simulate shocks from the AR(1) process of log income and assign probabilities based on the simulated frequencies. Starting from a uniform distribution over states, the stationary distribution  $\pi$  is obtained iterating from  $\pi^{k+1} = \pi^k \mathbf{P_z}$ , until convergence. The resulting stationary distribution  $\pi$  is normalized to sum to one and used to determine the effective normalization of income levels.

The asset grid is log-spaced, with  $n_d = n_b = 36$  grid points for deposits and bonds, respectively, resulting in a total of  $n_d \times n_b \times n_z = 20{,}736$  grid points. To solve the household problem, I implement the endogenous grid method (EGM), following the approach of Bayer et al. (2019).

Furthermore, since the distribution of net worth across firms is irrelevant for aggregation, the firm side of the model is solved in representative terms. Consequently, the relevant demand functions for credit, defined in Equations 39 and 41, depend only on aggregate asset stocks and interest rates. Further details of the algorithm are provided in the Appendix G.

Table 3 reports the parameters estimated for the period 1989-1995. I jointly estimate ten parameters; four related to households, two to banks, and four to firms, using the one-step Generalized Method of Moments (GMM) with the identity matrix as the weighting matrix. The parameters are chosen to minimize the distance between a set of model-implied moments (e.g., household deposit shares, share of borrowers, firm's leverage ratio, liquidity premium, etc.) and their empirical counterparts.

To achieve identification, some parameters are fixed following values commonly used in the literature. Specifically, the initial standard deviation of the innovation to log-income ( $\sigma = 0.4$ ), the persistence of the income process ( $\rho = 0.9$ ), the coefficient of relative risk aversion ( $\gamma = 2.45$ )<sup>14</sup>, the capital share ( $\alpha = 0.4$ )<sup>15</sup>, and the labor income tax ( $\tau^g = 0.3$ ) are held fixed at values commonly used in the literature (e.g., Aiyagari (1994); Bergholt et al. (2022); Hall (2016); Bayer et al. (2019)). After estimating the model, I adjust  $\sigma$  in a manner consistent with the observed rise in income inequality, explicitly targeting the increase in the P90/P10 income ratio over the last 30 years. In this exercise, I assume a depreciation rate of zero to identify the monitoring effectiveness parameter ( $\phi$ ) and the partial loss of asset stock parameter ( $\kappa$ ). Under this assumption, the firm's asset stock should be interpreted broadly to encompass any factor of production that must be financed in advance, such as intangible, physical, or organizational capital. This generalization allows the model to capture a wide range of investment needs while focusing on how financial frictions shape firm behavior.

Table 4 reports the eleven targeted moments that discipline the model using key household and corporate financial variables. Since the objective of the estimation is to evaluate how rising income inequality shapes the corporate debt structure, I combine information from two complementary data sources. The average deposits-to-bonds ratio is taken from the 1989 SCF, which provides a detailed benchmark of the wealth distribution across households and thus captures how balance sheets are allocated between liquid and illiquid assets at the start of the period. In contrast, the cross-sectional deposit shares are derived from the 1989-1995 DFA, which provides consistent annual information on the distribution of disposable income, allowing me to track how income

<sup>&</sup>lt;sup>14</sup>Hall (2016) implements the model using  $\gamma \in \{2.0, 2.5\}$ .

<sup>&</sup>lt;sup>15</sup>Bergholt et al. (2022) document a structural decline in the U.S. labor share over the past two decades. For their Monte Carlo exercises, they adopt a median labor share of 0.6, which corresponds to a capital share of 0.4.

Table 3: Estimated Parameters

Parameter	Value	Description	Target
Households			
$\sigma$	0.4	Innovation of productivity	Aiyagari (1994)
ho	0.9	Persistence of productivity	Aiyagari (1994)
$\gamma$	2.45	Relative risk aversion	Hall (2016)
$\beta$	0.945	Discount factor	Table 4
$\hat{r}$	5.7%	Penalty rate	Table 4
$\mu_\chi$	562.63	Mean adjustment cost	Table 4
$\sigma_\chi$	194.13	Scale adjustment cost	Table 4
Banks			
$\hat{c}$	0.06	Monitoring cost	Table 4
$\phi$	0.88	Monitoring effectiveness	Table 4
Firms			
$\alpha$	0.4	Capital share	Bergholt et al. (2022)
$p_H$	0.996	Probability partial loss	Table 4
$\kappa$	0.70	Partial loss	Table 4
A	4.04	Technology	Table 4
au	7.11%	Exit probability	Table 4
Government			
$ au^g$	0.3	Labor income tax	Bayer et al. (2019)

inequality maps into savings composition in the early 1990s. Using both sources ensures that the estimation is anchored in the wealth heterogeneity observed in the SCF while also capturing the income-based reallocation patterns documented in the DFA.

In addition to the deposits-to-bonds ratio and the cross-sectional deposit shares, I also target the share of borrowers. This moment is crucial for identification, as it helps pin down the scale parameter of adjustment costs ( $\sigma_{\chi}$ ) and prevents the model from allocating an unrealistically large share of aggregate resources to deposits that would otherwise be transformed into risky bank loans in general equilibrium. By disciplining the extent of borrowing activity, it ensures that the debt market is not matched solely through savings behavior. Incorporating the borrower share thus anchors the lower tail of the wealth distribution, sharpens the identification of borrowing constraints, and improves the joint fit of household portfolio choices and the aggregate debt structure. The remaining moments are based on aggregate data, specifically the leverage of non-financial corporations and the returns on aggregate financial assets.

The first important result is that while the model overstates the aggregate deposit-to-bond ratio

Table 4: Targeted moments

Targets	Model	Data	Source	Parameter
Deposits / Bonds $\frac{D}{B}$	2.13	1.02	SCF (1989)	Discount factor
Deposit share (1st quintile)	88.4	86.6	DFA (1989-1995)	Scale adj.costs
Deposit share (3rd quintile)	74.2	80.1	DFA (1989-1995)	Scale adj.costs
Deposit share (Top 1%)	30.5	28.3	DFA (1989-1995)	Mean adj.costs
Share of borrowers $(\%)$	4.6	10.0	SCF (1989)	Penalty rate
Ext. funding / Networth $\frac{(D+B)}{N}$	2.18	1.97	Z1 FA (1990-1993)	Exit probability
Bank loans / Assets $\frac{D}{K}$	0.47	0.21	Z1 FA (1990-1993)	Prob. partial loss
Collateral for banks	0.57	0.59	Gupta et al. (2022)	Monit. effectiveness
Assets / Sales $\frac{K}{V}$	5.04	4.54	FRED (1992-1993)	Technology
Bond return	7.65%	7.27%	FRED (1989-1993)	Partial loss
Liquidity premium	5.88%	5.46%	FRED (1989-1993)	Monitoring cost

relative to the data (Model: 2.13 vs. Data: 1.02), this reflects a trade-off in the estimation that prioritizes matching the cross-sectional deposit shares and the liquidity premium. Regarding the deposit share across the income distribution, the model predicts slightly higher values, particularly at the bottom and top, but it nevertheless reproduces the monotonic pattern observed in the DFA. The model also accounts for nearly half of the total share of borrowers. Because all household savings are assumed to be intermediated into firm credit, the aggregate ratio of deposits to bonds directly maps into the firm-level corporate bond structure, defined as  $\frac{B}{B+D}$ . The model produces a bond share of 32.0%, which corresponds to 47% of the empirical corporate bond structure recorded in the data.<sup>16</sup>

Importantly, the model does not incorporate international investors or foreign capital inflows, which in reality provide a significant share of corporate bond financing. Despite this limitation, the model captures a substantial portion of the observed corporate bond structure, underscoring its ability to replicate key financial ratios using only domestic household savings and intermediation mechanisms.

A second key result is that the model predicts a higher reliance on external funding than reported in the Z1 Financial Accounts for the period 1990-1995 (Model: 2.18 vs. Data: 1.97). This gap reflects the model's simplifying assumption that all household savings, particularly deposits, are fully channeled into firms' credit. In reality, banks hold reserves, and by abstracting from these reserves, the model overstates the supply of credit available to firms. Despite these simplifying

 $<sup>^{16}</sup>$ According to the Z1 Financial Accounts, U.S. corporations reported an average bond share of 67.8% between 1990 and 1993. The model reproduces a value of 32.0%, capturing approximately 47% of the observed structure.

assumptions, the model closely matches several other corporate firm financing ratios, including the ratio of bank loans to asset stock, the share of collateralizable assets held by banks measured by the upper-bound of bank loans-to-assets  $(\frac{\phi\kappa}{R^D})$ , and the asset-to-sales ratio. Finally, the model almost matches the real return of high-quality corporate bonds<sup>17</sup> and the liquidity premium implied by the data<sup>18</sup> for the period 1989-1993, as reported by FRED.

Figure 7 presents the average deposit share by income percentile using DFA data from 1989 to 1995, along with the model-implied estimates. Income is defined as the sum of labor and capital income. The model closely replicates the downward-sloping pattern observed in the data, capturing the fact that lower-income households hold a higher share of deposits relative to bonds compared to those at the top of the distribution.

Finally, Figure A7 presents two Lorenz curves: the blue line reflects the distribution implied by the model, while the red line represents the Lorenz curve constructed from 1989 SCF data on liquid wealth (defined as deposits and bond holdings). Consistent with the visual comparison, the Gini coefficient implied by the model indicates lower liquid wealth inequality than the empirical Gini coefficient.

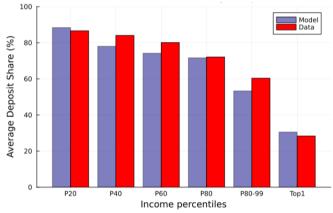


Figure 7: Deposit shares across income percentiles (Model vs. Data)

Notes: This figure displays the estimated shares of deposits across income percentiles compared to data from the Distributional Financial Accounts for 1989-1995. Income in the model is computed as the sum of labor income, capital income (returns on deposits and bonds, annualized), and dividends. The estimated relationships are obtained using a local linear regression with a Gaussian kernel and a bandwidth of 0.025.

<sup>&</sup>lt;sup>17</sup>The real return is calculated as the average of the HQMCB1YR and HQMCB2YR series, adjusted for inflation using the CPIAUCNS series. All data are sourced from the FRED portal.

<sup>&</sup>lt;sup>18</sup>The liquidity premium is computed as the difference between the real return on corporate bonds and the 3-month commercial paper rate for AA-rated firms, using the series CP3M and CPN3M. Since CP3M was discontinued in 1997, I have complemented it with CPN3M from that point onward. All data was adjusted by inflation using the CPIAUCNS series.

#### 5 Quantitative results

In this section, I examine the effect of the rise in income inequality over the past 30 years in the U.S. on the model's key outcome variables. Based on the CPS data during the periods 1985-1990 and 2015-2019, income inequality, measured by the P90/P10 income ratio, increased by 24.3%, from 10.06 to 12.50, which I interpret as reflecting an 8.4% rise in labor income risk. As households face greater income risk, they adjust their savings behavior accordingly. Figure 8B shows that households consistently accumulate more savings, while reducing the share held in deposits across the income distribution. The decline in deposit shares is most pronounced for households in the bottom and middle of the distribution (Figure 8A).

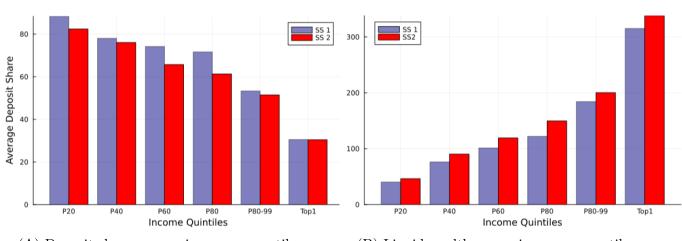


Figure 8: Estimated deposit shares and liquid wealth (Period 1 vs. Period 2)

(A) Deposit shares across income percentiles (B) Liquid wealth across income percentiles

Notes: Figure A displays the estimated shares of deposits across income percentiles before and after the increase in labor income risk. Figure B displays the estimated liquid wealth (deposits plus bond holdings) across income percentiles before and after the increase in labor income risk.

Consequently, the aggregate deposits-to-bonds ratio declines by 0.41 points in levels (from 2.13 to 1.72), driven mainly by households in the middle of the income distribution. Deposit shares fall across the distribution, with the sharpest reductions observed in the middle quintile (down 8.5 percentage points, from 74.2% to 65.7%) and the bottom quintile (down 6.0 percentage points, from 88.4% to 82.4%). Households in the top 1% reduce their deposit share only marginally, from 30.5% to 30.4%, reflecting their already solid reliance on bonds. Overall, the model implies a consistent shift toward bond holdings across the distribution, with wealthier households continuing

<sup>&</sup>lt;sup>19</sup>I assume that the increase in the P90/P10 ratio is exclusively driven by rising labor income dispersion.

Table 5: Selected moments: Initial and counterfactual steady state

Moments	Steady State 1 SS 1	Counterfactual SS 2	Change
Std. dev. of productivity	0.40	0.43	8.4%
Implied corp. bond structure $\frac{(B)}{B+D}$	32.0%	36.8%	15.0%
Deposits / Bonds $\frac{D}{B}$	2.13	1.72	-0.41
Deposit share (1st quintile)	88.4	82.4	-5.98
Deposit share (3rd quintile)	74.2	65.7	-8.47
Deposit share (Top 1)	30.5	30.4	-0.1
Gini coefficient (Liquid wealth)	0.42	0.40	-0.02
Share of borrowers (%)	4.6	4.2	-0.4
Ext. funding / Networth $\frac{(D+B)}{N}$	2.18	3.30	1.12
Assets / Sales $\frac{K}{V}$	5.04	5.12	0.08
Bond return	7.65%	6.29%	-1.37
Liquidity premium	5.88%	4.16%	-1.73

to hold a substantial share in bonds. On the firm side, the model predicts a 15.0% increase in the corporate bond share, from 32.0% to 36.8%, as firms rely more heavily on bond financing. This response is qualitatively consistent with the first empirical pattern documented in the motivating evidence of this paper. Quantitatively, it corresponds to an elasticity of 1.79, meaning that a 1% increase in labor income risk generates a 1.79% rise in bond reliance.

Regarding liquid wealth inequality, the model predicts a modest decline in the Gini coefficient, by 0.02 points (from 0.42 to 0.40). This reduction is driven by changes in saving behavior: households in the middle and lower parts of the income distribution substantially reduce their reliance on deposits and increase their holdings of bonds, thereby raising their liquid wealth. Nevertheless, wealthier households continue to accumulate a disproportionate share of total liquid wealth, so the overall decline in inequality remains limited. Consistent with this result, the model also predicts a fall in the share of borrowers, from 4.6% to 4.2%, indicating that fewer households are at their borrowing constraint, and relatively more can self-finance through savings in the face of higher income risk.

Additionally, the model predicts an increase in both corporate leverage and the assets-to-sales ratio. Specifically, the ratio of external funding to firms' net worth rises by 1.12 points, from 2.18 to 3.30, reflecting greater reliance on external finance. At the same time, the assets-to-sales ratio increases by 0.08 points, from 5.04 to 5.12. These changes are driven by a higher level of liquid savings among households, particularly as bonds become increasingly attractive relative to bank

loans, thereby reducing their cost and encouraging firms to tilt their capital structure toward bond financing.

Regarding the liquidity premium, the model predicts a decline of 1.73 percentage points (from 5.88% to 4.16%), which is qualitatively consistent with the third empirical pattern highlighted in the motivating evidence of this paper. A regime shift on the firm side of the model drives this change. As aggregate bond holdings increase in response to higher income risk, the supply of funds in the bond market expands to the point where the risk-free bond constraint becomes binding. This transition leads to a decline in the bond interest rate as credit demand becomes inelastic in the constrained regime, unlike in the unconstrained regime, where the demand curve is perfectly elastic.

Table 6: Returns and wages in equilibrium

Factor return	Steady State 1	Counterfactual	Change %
Bonds return $(r^b)$	7.65%	6.29%	-17.8%
Deposits return $(r^d)$	1.77%	2.13%	20.3%
Wages $(W)$	18.05	18.25	1.1%

Table 6 reports the equilibrium outcomes under two steady states: a baseline and a counterfactual with higher income inequality. The increase in inequality is reflected in a rise in the standard deviation of idiosyncratic labor productivity, from 0.40 to 0.43. This shock amplifies labor income risk and deepens the heterogeneity across households' after-tax labor income and wealth accumulation paths.

The general equilibrium response includes adjustments in key prices. The return on bonds declines from 7.65% to 6.29% (a 17.8% decrease), reflecting a greater demand for illiquid bonds. Meanwhile, the deposit rate increases from 1.77% to 2.13% (a 20.3% rise), driven by the firms' higher cost of bank loans compared to bonds. Also, wages experience a modest 1.1% increase, due to a rise in firms' stock of assets.

Table 7: Aggregates in equilibrium

Aggregate variables	Steady State 1	Counterfactual	Change %
Consumption	20.01	19.97	-0.2%
Deposits	70.47	73.34	6.9%
Bonds	33.15	43.84	32.3%
Total savings	103.62	119.18	15.0%
Output	29.97	30.29	1.1%
Firms' networth	47.50	36.10	-24.0%
Total assets	151.11	155.21	2.7%

Table 7 reports the equilibrium aggregates across the two steady states. Aggregate consumption remains essentially unchanged, falling by only 0.2%. In contrast, total savings rise by 15.0%, driven primarily by a 32.3% increase in bond holdings alongside a 6.9% rise in deposits. On the firm side, net worth declines sharply by 24.0%, reflecting firms' substitution away from internal financing toward cheaper external debt as bond returns fall (Table ??). This shift lowers the cost of market borrowing and reduces reliance on retained earnings. At the macroeconomic level, total assets expand by 2.7% and output rises by 1.1%, consistent with higher capital accumulation facilitated by more favorable external financing conditions.

#### Effect of Financial Asset Returns, Wages, and Income Inequality

Figure 9 provides a decomposition of the total change in households' asset holdings across income percentiles. Figure 9A displays changes in bond holdings, while Figure 9B shows changes in deposit holdings. Each bar separates the contribution of four channels: the pure effect of higher inequality (blue), the joint effect of higher inequality and bond returns (red), the joint effect of higher inequality and deposit returns (green), and the joint effect of higher inequality and wages (orange). The black line plots the total joint effect, summarizing the combined impact across channels.

Figure 9A shows that bond holdings rise most strongly in the upper tail, particularly for households in the fourth quintile and the top 1%. Although the sharp decline in bond returns (from 7.65% to 6.29%) dampens the incentive to hold bonds, top-earners households continue to expand their bond positions. This reflects their relatively smaller portfolio adjustment frictions, which make reallocating precautionary savings into bonds more attractive than into deposits.

Figure 9B highlights a different pattern for deposits. The largest increase in deposits is concentrated among households in the second quintile, while the top 1% also expand their deposit

holdings, though mainly as a byproduct of the interaction between higher inequality and falling bond returns. By contrast, the households in the third and fourth quintiles reduce their deposit exposure, substituting into bonds when returns are favorable (red bars).

The contribution of wage changes is limited. Average wages rise slightly by 1.1% between steady states, but their effect on portfolio reallocation is negligible for most households. The only visible impact appears at the very top of the distribution, where higher wages, in interaction with greater inequality, induce a modest reallocation away from bonds and toward deposits.

Taken together, these results underscore a heterogeneous portfolio response to rising inequality. Lower and middle-income households accumulate deposits as a buffer against income risk, whereas households at the top shift their savings composition toward bonds. This divergence generates the aggregate patterns reported in Tables 7 and 6, namely a substantial rise in bond holdings relative to deposit holdings.

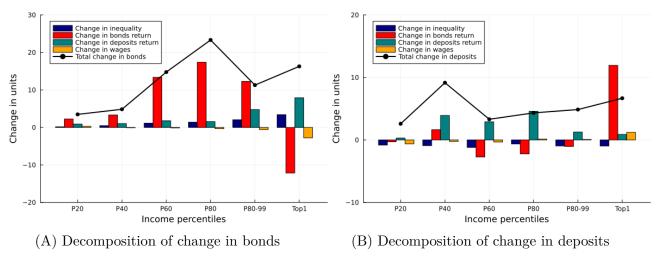


Figure 9: Decomposition of the total effect of higher income inequality

Notes: Figures A and B illustrate how changes in bond and deposit holdings vary across wealth percentiles in response to higher income inequality. The blue bars isolate the effect of higher inequality, the red bars reflect the joint effect of higher inequality and changes in bond returns, the green bars reflect the joint effect of higher inequality and changes in deposit returns, and the orange bars capture the joint effect of higher inequality and changes in wages. The black line represents the joint effect of all three channels.

#### Distributional Effects on Liquid Wealth Inequality

Figure 10 illustrates the decomposition of changes in total liquid wealth across income percentiles. This decomposition highlights the heterogeneous impact of inequality and price adjustments across the income distribution. The figure reveals that all groups experience an increase in liquid wealth

in response to rising income inequality, but the relative contributions vary. Households in the upper tail of the distribution see the most significant absolute gains, primarily driven by the joint effects of higher income inequality and changes in financial asset returns. While the top 1% continues to accumulate substantial liquid wealth, the increase is less disproportionate compared to upper-middle-income groups.

This more evenly distributed growth in liquid assets helps explain the model's prediction of a decline in the Gini coefficient for liquid wealth, from 0.42 to 0.40. In particular, the stronger response among middle-income households reflects the lower portfolio adjustment frictions relative to households at the bottom. As these households reallocate toward bonds, inequality in liquid wealth holdings compresses, despite rising income inequality, highlighting the redistributive role of return dynamics in general equilibrium.

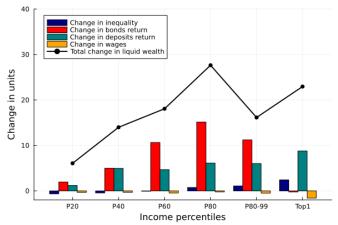


Figure 10: Decomposition of the total effect on liquid wealth

Notes: This figure illustrates how changes in liquid wealth vary across wealth percentiles in response to rising income inequality. The blue bars isolate the direct effect of increased inequality on liquid wealth. The red, green, and orange bars capture how this effect interacts with changes in bond returns, deposit returns, and wages, respectively. The black line represents the total general equilibrium effect, combining all four channels simultaneously.

However, a more specific comparison of liquid wealth across the income distribution reveals that inequality is still increasing when comparing households in the fourth quintile with lower income groups. For example, the P80/P20, P80/P40, and P80/P60 ratios rise by 0.20 (from 3.03 to 3.23), 0.06 (from 1.60 to 1.66), and 0.05 (from 1.21 to 1.26) points, respectively, signaling an increase in top-to-bottom wealth inequality. However, inequality declines within the top and the lower income groups. For instance, the top 1%/P20 and top 1%/P80 ratios decline by 0.53 (from 7.82 to 7.29) and 0.32 points (from 2.58 to 2.26), respectively. This pattern suggests that while most

of the income distribution experiences a surge in liquid wealth inequality, households in the lower percentiles increase their liquid wealth proportionally more than those in the top 1%, thereby reducing the relative gap between the very top and the rest of the distribution.

#### Distributional Effects on Income Inequality

The decomposition in Figure 11 reveals that rising income inequality not only reshapes the distribution of income directly but also interacts with financial returns and general equilibrium feedbacks. For households in the lower-middle of the distribution, the overall effect is positive, mainly driven by the rise in the deposit return. Upper-middle income groups experience the largest gains, while households in the top 1% face an adverse effect primarily driven by the decline in bond returns. The hump-shaped profile of the black line, which captures the total general equilibrium effect, underscores how return dynamics redistribute income gains away from the top 1% toward the rest of the distribution.

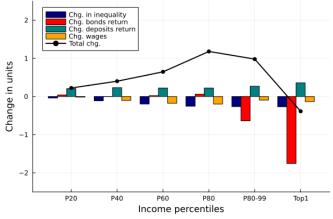


Figure 11: Decomposition of the total effect on total income

Notes: This figure illustrates how changes in total income vary across wealth percentiles in response to rising income inequality. The blue bars isolate the direct effect of increased inequality on income. The red, green, and orange bars capture how this effect interacts with changes in bond returns, deposit returns, and wages, respectively. The black line represents the total general equilibrium effect, combining all four channels simultaneously.

A closer comparison of total income across percentiles reveals that inequality still rises when contrasting the fourth quintile with lower-income groups. For example, the P80/P20, P80/P40, and P80/P60 ratios increase by 0.05 (from 4.21 to 4.26), 0.03 (from 2.27 to 2.30), and 0.02 (from 1.51 to 1.53), respectively, signaling an increase in top-to-bottom income inequality. By contrast, inequality declines within the top and bottom of the distribution: the top 1%/P20 and top 1%/P80 ratios fall by 0.62 (from 12.20 to 11.58) and 0.18 points (from 2.90 to 2.72), respectively. This

pattern suggests that while inequality widens between the upper-middle and the lower groups, households at the very top lose ground relative to both the bottom and the upper-middle, reflecting their disproportionate exposure to falling bond returns. Thus, the rise in income inequality is driven mainly by upper-middle households, while the very top experiences relative losses.

#### Distributional Effects on Consumption Inequality

Figure 12 decomposes the impact of rising inequality on household consumption across the income distribution. As in the case of income, the overall effect is positive for most households in the lower and middle percentiles, reflecting both higher deposit returns and general equilibrium feedbacks. The gains are relatively modest for the first, second, and third quintiles but peak at the fourth quintile, where consumption rises most strongly. By contrast, the top 1% experience a sharp decline in consumption, primarily driven by their exposure to falling bond returns. The red bar in Panel Top 1% highlights how the drop in  $r^b$  disproportionately reduces disposable resources for the wealthiest households, leading to a steep fall in their consumption.

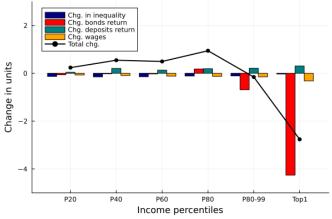


Figure 12: Decomposition of the total effect on consumption

Notes: This figure illustrates how changes in consumption vary across wealth percentiles in response to rising income inequality. The blue bars isolate the direct effect of increased inequality on consumption. The red, green, and orange bars capture how this effect interacts with changes in bond returns, deposit returns, and wages, respectively. The black line represents the total general equilibrium effect, combining all four channels simultaneously.

This pattern mirrors the hump-shaped profile observed in income but with more substantial distributional effects. While upper-middle households gain in both income and consumption, the top 1% now face a much larger contraction on the consumption side, suggesting that the fall in bond returns not only reduces their income but also amplifies the decline in their ability to consume. Hence, inequality in consumption narrows between the very top and the rest of the distribution,

even as upper-middle households pull further ahead of lower-income groups.

However, a closer comparison of consumption across percentiles reveals that inequality still rises when contrasting the fourth quintile with lower-income groups. For example, the P80/P20, P80/P40, and P80/P60 ratios increase by 0.05 (from 2.37 to 2.42), 0.01 (from 1.57 to 1.58), and 0.03 (from 1.23 to 1.26), respectively, signaling an increase in top-to-bottom consumption inequality. By contrast, inequality declines within the top and bottom of the distribution: the top 1%/P20 and top 1%/P80 ratios fall by 0.52 (from 5.63 to 5.11) and 0.26 points (from 2.37 to 2.11), respectively. This pattern suggests that while inequality widens between the upper-middle and the lower groups, households at the very top lose ground relative to both the bottom and the upper-middle, reflecting their disproportionate exposure to falling bond returns. Thus, the rise in income inequality is driven mainly by upper-middle households, while the very top experiences relative losses.

The relationship between income and consumption inequality is a topic far from reaching consensus within the profession. These results, which exclude the households in the top 1%, complement the evidence in Aguiar and Bils (2015); Attanasio et al. (2012), who show that consumption inequality closely mirrors income inequality when comparing high- and low-income households across different categories of goods. However, Meyer and Sullivan (2023) and Heathcote et al. (2010a) conclude that the rise in overall consumption inequality was small.

#### Comparing Predicted and Observed Changes

Table 8 summarizes changes in key moments from both the data and the model. The model successfully replicates the decline in the aggregate deposits-to-bonds ratio between the 1989 and 2016 SCF and predicts a substantial rise in corporate bond reliance, accounting for 59.5% of the increase observed in the data. It also captures broad distributional patterns in savings behavior. For example, it explains 70.7% of the observed decline in the deposit share of the third quintile and 36.8% for the first quintile. By contrast, the model underpredicts changes at the very top: it yields only a 0.1 percentage point decline in the deposit share of the Top 1%, whereas the data show an increase of 15.3 percentage points.

This discrepancy reflects the model's narrow focus on income risk as the sole driver of savings composition. While rising income risk generates a shift toward bonds for lower and middle-income households, through higher equilibrium wages and a lower liquidity premium, it does not capture other mechanisms that led top-income households to raise their deposit holdings.

Moreover, while the model predicts a slight decrease in liquid wealth inequality, the data show an

increase of 0.11 units in the Gini coefficient, based on SCF estimates. Finally, the model closely matches the decline in the liquidity premium, accounting for 57.8% of the drop observed in the data. Overall, these results highlight the model's ability to replicate key structural patterns in household behavior and firm financing in response to rising labor income risk.

Table 8: Change in moments (Model vs. Data)

Moments	Change-Model	Change-Data	Source
Implied corp. debt structure $\frac{(B)}{B+D}$	15.0%	25.2%	Z1 FA (1990-2019)
Deposits / Bonds $\frac{D}{B}$	-0.41	-0.40	SCF (1989-2016)
Deposit share (1st quintile)	-5.98	-16.28	DFA (1989-2016)
Deposit share (3rd quintile)	-8.47	-11.98	DFA (1989-2016)
Deposit share (Top 1)	-0.1	15.31	DFA (1989-2016)
Share of borrowers (%)	-0.39	-0.30	SCF (1989)
Gini coefficient (Liquid wealth)	-0.02	0.11	SCF (1989-2016)
Bond return	-1.37	-5.70	FRED (1989-2019)
Liquidity premium	-1.73	-2.99	FRED (1989-2019)
Ext. funding / Networth $\frac{(D+B)}{N}$	1.12	3.46	Z1 FA (1989-2016)
Assets / Sales $\frac{K}{Y}$	0.08	0.37	FRED (1992-2019)

#### 6 Conclusions

This paper formalizes a novel mechanism linking income inequality, corporate debt structure, and the liquidity premium. It delivers three main contributions.

First, this paper offers a structural explanation for changes in household financial intermediation and the evolution of the corporate debt structure. These results underscore the model's ability to link household-level risk exposure to aggregate credit market outcomes, even under simplified assumptions, providing a robust foundation for analyzing structural transformations in financial markets.

Second, the model provides a strong quantitative fit to key macro-financial patterns observed in the U.S. economy over the past three decades (1989-2019). It reproduces the link between rising income inequality, measured by the P90/P10 income ratio, and firms' increasing reliance on bond financing, accounting for 59.5% of the observed change in firms' financing. It also captures the heterogeneous reallocation of household savings across the income distribution, including the decline in deposit intensity among top households and the relative shift toward bond holdings,

especially at the middle. Moreover, it partially matches the empirical reduction in the liquidity premium, accounting for 57.9% of the observed drop.

Third, it demonstrates that rising income inequality can have redistributive effects on household portfolio composition. Higher labor income risk leads to an increase in liquid wealth, consumption, and income inequality among households in the upper income bracket and lower-income households. However, the model predicts a compression in these measures of inequality when comparing households in the top 1% and lower-income groups. While standard intuition suggests that higher labor income dispersion amplifies financial asset holdings inequality, I find that the Gini coefficient of liquid wealth slightly declines in equilibrium, from 0.42 to 0.40, due to the stronger response among middle-income households to reallocate their savings towards bonds. This reallocation is higher in the group because it reflects the lower portfolio adjustment frictions relative to households at the bottom. As these households reallocate toward bonds, inequality in liquid wealth holdings compresses, despite rising income inequality, highlighting the redistributive role of return dynamics in general equilibrium.

Moreover, the model predicts a rise in both income and consumption inequality between households at the upper and lower ends of the distribution. Specifically, the P80/P20 ratio increases from 4.21 to 4.26 for income and from 2.37 to 2.42 for consumption. These shifts are driven by higher income risk, which amplifies gains for households in the upper part of the distribution, while those at the bottom remain constrained by borrowing limits and portfolio adjustment frictions that restrict their ability to smooth consumption.

Finally, this framework offers a foundation for analyzing the distributional and macro-financial effects of fiscal and financial policies. In particular, the model can be extended to study targeted tax policies, such as progressive capital income taxation or redistributive transfers, and their impact on household saving composition, asset prices, and credit market dynamics. Given its rich heterogeneity and general equilibrium structure, it is well-suited to evaluate how taxes that alter households' disposable income or relax borrowing constraints influence both inequality and firms' financing choices. Such extensions could shed light on the trade-offs faced by policymakers aiming to reduce inequality without distorting incentives for saving or investment, and help clarify the role of fiscal tools in economies experiencing persistent income inequality.

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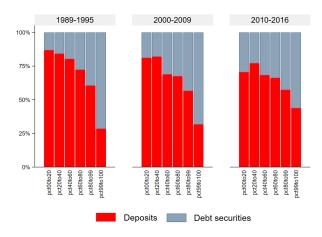
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### Appendix

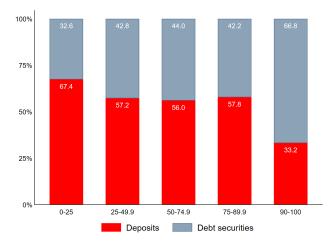
#### A Figures

Figure A1: Liquid wealth composition across income distribution



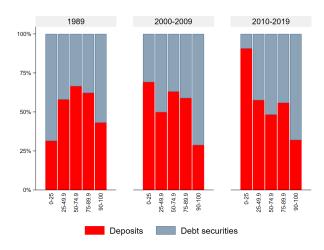
Notes: This figure shows the deposits and bond savings as a share of liquid wealth in different periods based on the Distributional Financial Accounts.

Figure A2: Average liquid wealth composition across wealth quartiles



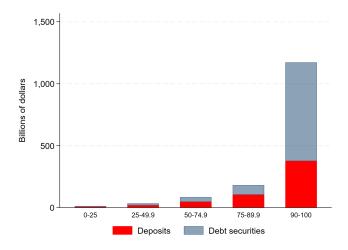
Notes: This figure shows the shares in % of deposits, such as certificates of deposit and transaction accounts, and bond savings, based on the Survey of Consumer Finances from 1989 to 2019.

Figure A3: Liquid wealth composition across wealth quartiles



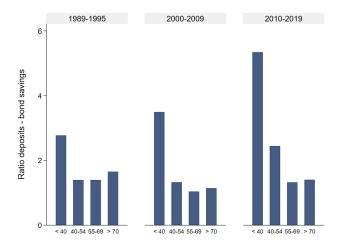
Notes: This figure shows the shares in % of deposits, such as certificates of deposit and transaction accounts, and bond savings, based on the Survey of Consumer Finances.

Figure A4: Deposit and bond holdings by wealth quartile



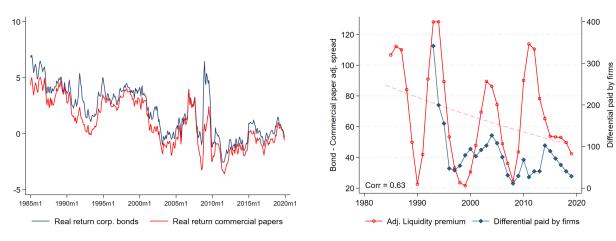
Notes: This figure shows the holdings of deposits, such as certificates of deposit and transaction accounts, and bond savings, expressed in billions of dollars, based on the Survey of Consumer Finances.

Figure A5: Deposit-to-Bonds by age-cohorts



Notes: This figure shows the relative importance of deposits in different periods based on the Distributional Finance Accounts data.

Figure A6: Monthly real returns and adjusted spreads



- (A) Real returns of corp. bonds and commercial papers
- (B) Adjusted liquidity premium and spread paid by firms

Notes: Figure A shows the returns of high-quality corporate bonds (bonds rated AAA, AA, or A with maturities of one to two years) and 3-month commercial papers offered to AA-rated firms adjusted by inflation based on monthly data published by FRED. In Figure B, the red line plots the three-year moving average of the difference between the annual average yield on high-quality corporate bonds net of the return of U.S Treasuries at 1 year of maturity, and the interest rate on 3-month commercial papers offered to AA-rated firms net of the return of U.S Treasuries at 3 months of maturity, based on FRED data. The blue line displays the three-year moving average of the difference between the average cost of funds from corporate bond issuance and bank loans, constructed using firm-level maturity data from Capital IQ. All series were adjusted by realized inflation.

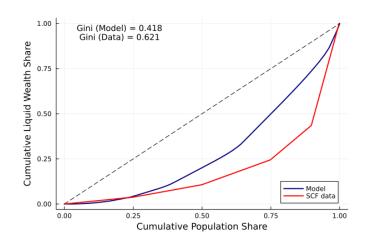


Figure A7: Lorenz curve of liquid wealth (Model vs. Data)

Notes: The figure displays the Lorenz Curve and Gini coefficient of liquid wealth. The red and blue lines display the Lorenz Curve based on the data of deposits and bond savings of households in the 1989 SCF sample and those implied by the model, respectively.

# B Discussion: Non-financial corporate bonds and income inequality within the elderly population

The cross-country effect of inequality appears particularly pronounced when focusing on the elderly population. Column (1) shows that a one-unit increase in the S80/S20 ratio among individuals aged 65 and over is associated with a 1.24 percentage point rise in the corporate share of domestic bonds, significant at the 1% level. Column (2) reveals an even stronger effect for the P90/P50 ratio within this subgroup: a one-unit increase is linked to an 8.93 percentage point increase in the corporate bond share, also significant at the 1% level. Column (3) further confirms the pattern, as the P90/P10 ratio is positively associated with a 2.57 percentage point increase in the corporate bond share, again highly significant.

These cross-sectional results suggest that income inequality among the elderly is strongly correlated with the composition of domestic bond markets. A plausible mechanism is that older, wealthier households, who both hold a growing share of disposable income and are more likely to save rather than consume, channel their resources toward relatively safe yet higher-yielding corporate bonds. In aging economies where income is increasingly concentrated among retirees, this savings behavior may indirectly reshape credit allocation, reinforcing the broader link between inequality and corporate debt structures.

	/1\	(0)	(2)
	(1)	(2)	(3)
	Corp. bonds %	Corp. bonds %	Corp. bonds $\%$
S80/S20 Pop. above 65	1.242***		
	(0.420)		
P90/P50 Pop. above 65		8.927***	
		(2.845)	
P90/P10 Pop. above 65			2.570***
			(0.639)
Lag GDP per capita	0.0498	0.0415	0.0427
	(0.0870)	(0.0912)	(0.0811)
Lag Liabilities to GPD (%)	-0.0175***	-0.0191***	-0.0182***
	(0.00547)	(0.00514)	(0.00477)
Lag Inflation	-0.0287	0.000477	-0.0259
-	(0.181)	(0.152)	(0.153)
Observations	150	150	150
Country FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes
Clustered SE by country	Yes	Yes	Yes

Standard errors in parentheses

Table A1: Relation between non-financial corporate bonds and measures of income inequality within the elderly population. Columns (1)â(3) use as the dependent variable the share of domestic non-financial corporate bonds and notes outstanding in total domestic bonds and notes outstanding. The liabilities-to-GDP measure includes all debt liabilitiesâbonds, debentures, notes, and money market or other negotiable debt instrumentsâand is sourced from the Global Financial Development Database and the World Bank. All regressions include country and time fixed effects, with standard errors clustered at the country level.

#### C Households

## C.1 Dynamic problem with two assets and logistic distribution of adjustment costs

$$\mathbb{E}V(d,b;z) = V_{n}(d,b;z) + \int_{0}^{V_{a}-V_{n}} (V_{a}(d,b;z) - V_{n}(d,b;z) - \chi) f(\chi) d\chi$$

$$\mathbb{E}V(d,b;z) = V_{n}(d,b;z) + V_{a}(d,b;z) F_{\chi} [V_{a}(d,b;z) - V_{n}(d,b;z)]$$

$$-V_{n}(d,b;z) F_{\chi} [V_{a}(d,b;z) - V_{n}(d,b;z)] - \int_{0}^{V_{a}-V_{n}} \chi f(\chi) d\chi$$
(44)

<sup>\*</sup> p < 0.10, \*\* p < 0.05, \*\*\* p < 0.01

$$\mathbb{E}V(d,b;z) = V_{n}(d,b;z) + \int_{0}^{V_{a}-V_{n}} (V_{a}(d,b;z) - V_{n}(d,b;z) - \chi) f(\chi) d\chi$$

$$\mathbb{E}V(d,b;z) = V_{n}(d,b;z) + V_{a}(d,b;z) F_{\chi} [V_{a}(d,b;z) - V_{n}(d,b;z)]$$

$$-V_{n}(d,b;z) F_{\chi} [V_{a}(d,b;z) - V_{n}(d,b;z)] - \int_{0}^{V_{a}-V_{n}} \chi f(\chi) d\chi$$
(45)

Let's compact the state space  $\omega := (d, b)$ , then:

$$\mathbb{E}V\left(\omega;z\right) = V_a\left(\omega;z\right)\nu^*\left(\omega;z\right) + \left(1 - \nu^*\left(\omega;z\right)\right)V_n\left(\omega;z\right) - \int_0^{V_a\left(\omega;z\right) - V_n\left(\omega;z\right)} \chi f(\chi)d\chi \tag{46}$$

Then, the expected of the value function has two components: the first two terms coincides with the problem when there are fixed adjustment probabilities. The last term is the expected conditional adjustment cost.

$$\frac{\partial \mathbb{E}V\left(\omega;z\right)}{\partial \omega} = \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} + \left[\frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} - \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega}\right] \left[F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]\right] + \frac{F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]}{\partial \omega} \left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right] - \frac{\partial}{\partial \omega} \int_{0}^{V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)} \chi f(\chi) d\chi$$
(47)

By Leibniz integral rule:

$$\frac{\partial \mathbb{E}V\left(\omega;z\right)}{\partial \omega} = \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} + \left[\frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} - \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega}\right] \left[F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]\right] + \frac{F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]}{\partial \omega} \left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right] - \left[\left(\left(V_{a}\left(\omega|z\right) - V_{n}\left(\omega|z\right)\right)f_{\chi}\left[V_{a}\left(\omega;z\right)\right] - V_{n}\left(\omega;z\right)\right] - \left[\left(\left(V_{a}\left(\omega|z\right) - V_{n}\left(\omega|z\right)\right)f_{\chi}\left[V_{a}\left(\omega;z\right)\right]\right) - \left(\left(V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right)f_{\chi}\left[V_{a}\left(\omega;z\right)\right] - \left(\left(V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right)f_{\chi}\left[V_{a}\left(\omega;z\right)\right]\right] + \int_{0}^{V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)} \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} + \int_{0}^{V_{a}\left(\omega;z\right)} \frac{\partial$$

$$\frac{\partial \mathbb{E}V\left(\omega;z\right)}{\partial \omega} = \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} + \left[\frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} - \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega}\right] \left[F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]\right] \\
+ f_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right] \frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} - \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} \left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right] \\
- \left[f_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right] \left(V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right) \left[\frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} - \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega}\right] \\
+ \underbrace{\int_{0}^{V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)}_{=0} \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} \chi f(\chi) d\chi}_{=0} \tag{49}$$

$$\frac{\partial \mathbb{E}V\left(\omega;z\right)}{\partial \omega} = \frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} \left[F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]\right] 
+ \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega} \left[1 - F_{\chi}\left[V_{a}\left(\omega;z\right) - V_{n}\left(\omega;z\right)\right]\right] \frac{\partial \mathbb{E}V\left(\omega;z\right)}{\partial \omega} 
= \nu^{*}\left(\omega;z\right) \frac{\partial V_{a}\left(\omega;z\right)}{\partial \omega} + \left(1 - \nu^{*}\left(\omega;z\right)\right) \frac{\partial V_{n}\left(\omega;z\right)}{\partial \omega}$$
(50)

In simple words, F.O.C of the expected value function with fixed adjustment probabilities and a model with state-dependent adjustment probabilities are the same.

Therefore, the Euler Equations (for interior solutions) around the optimal policies:  $c_i^*$ ,  $d_i^*$  and  $b^*$  are the optimal policies of consumption, deposits and bond holdings where  $i \in \{a, n\}$ , respectively.

$$[b^*] \frac{\partial u\left(c_a^*\right)}{\partial c} = \beta \mathbb{E} \left[ \nu \frac{\partial V_a\left(d_a^*, b^*, z'\right)}{\partial b} + (1 - \nu) \frac{\partial V_n\left(d_a^*, b^*, z'\right)}{\partial b} \right]$$

$$[d_a^*] \frac{\partial u\left(c_a^*\right)}{\partial c} = \beta \mathbb{E} \left[ \nu \frac{\partial V_a\left(d_a^*, b^*, z'\right)}{\partial d} + (1 - \nu) \frac{\partial V_n\left(d_a^*, b^*, z'\right)}{\partial d} \right]$$

$$[d_n^*] \frac{\partial u\left(c_n^*\right)}{\partial c} = \beta \mathbb{E} \left[ \nu \frac{\partial V_a\left(d_n^*, b, z'\right)}{\partial d} + (1 - \nu) \frac{\partial V_n\left(d_n^*, b, z'\right)}{\partial d} \right]$$

$$(51)$$

Using the Envelope Theorem conditions (EC), I get:

$$[b] \frac{\partial V_{a}(d,b,z)}{\partial b} = \frac{\partial u\left(c_{a}^{*}\left(d,b,z\right)\right)}{\partial c}\left(1+r^{b}\right)$$

$$[d] \frac{\partial V_{a}(d,b,z)}{\partial d} = \frac{\partial u\left(c_{a}^{*}\left(d,b,z\right)\right)}{\partial c}\left(1+r^{d}\right)$$

$$[b] \frac{\partial V_{n}(d,b,z)}{\partial b} = \frac{\partial u\left(c_{n}^{*}\left(d,b,z\right)\right)}{\partial c}\left(1+r^{b}\right) + \beta v\mathbb{E}\left[\frac{\partial V_{a}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)}{\partial b}\right]$$

$$+\beta\left(1-\nu\right)\mathbb{E}\left[\frac{\partial V_{n}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)}{\partial b}\right]$$

$$= \frac{\partial u\left(c_{n}^{*}\left(d,b,z\right)\right)}{\partial c}\left(1+r^{b}\right) + \beta v\left(1+r^{b}\right)\mathbb{E}\left[\frac{\partial u\left(c_{a}^{*}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)\right)}{\partial c}\right]$$

$$+\beta\left(1-\nu\right)\mathbb{E}\left[\frac{\partial V_{n}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)}{\partial b}\right]$$

$$[d] \frac{\partial V_{n}\left(d,b,z\right)}{\partial d} = \frac{\partial u\left(c_{n}^{*}\left(d,b,z\right)\right)}{\partial c}\left(1+r^{d}\right)$$

Plugging Equation 52 into Equation 51:

$$[b^*] \frac{\partial u\left(c_a^*\right)}{\partial c} = \beta \mathbb{E} \left[ \nu \frac{\partial u\left(c_a^*\left(d_a^*, b^*, z'\right)\right)}{\partial c} \left(1 + r^b\right) + \left(1 - \nu\right) \frac{\partial V_n\left(d_a^*, b^*, z'\right)}{\partial b} \right]$$

$$[d_a^*] \frac{\partial u\left(c_a^*\right)}{\partial c} = \beta \mathbb{E} \left( \left(1 + r^d\right) \left[ \nu \frac{\partial u\left(c_a^*\left(d_a^*, b^*, z'\right)\right)}{\partial c} + \left(1 - \nu\right) \frac{\partial u\left(c_n^*\left(d_a^*, b^*, z'\right)\right)}{\partial c} \right] \right)$$

$$[d_n^*] \frac{\partial u\left(c_n^*\right)}{\partial c} = \beta \mathbb{E} \left( \left(1 + r^d\right) \left[ \nu \frac{\partial u\left(c_a^*\left(d_n^*, b, z'\right)\right)}{\partial c} + \left(1 - \nu\right) \frac{\partial u\left(c_n^*\left(d_n^*, b, z'\right)\right)}{\partial c} \right] \right)$$

$$(53)$$

Finally, to compute the expected adjustment costs  $AC(v; \mu_x, \sigma_x)$ , assuming that  $F_{\chi}$  is a logistic distribution defined by:

$$F(\chi) = \frac{1}{1 + \exp\left\{-\frac{\chi - \mu_x}{\sigma_x}\right\}}$$

$$1 + \exp\left\{-\frac{\chi - \mu_x}{\sigma_x}\right\} = \frac{1}{F(\chi)} \Rightarrow -\frac{\chi - \mu_x}{\sigma_x} = \ln\left(\frac{1}{F(\chi)} - 1\right)$$

$$\Rightarrow \frac{\mu_x}{\sigma_x} = \ln\left(\frac{1}{p} - 1\right) + \frac{\chi}{\sigma_x} \Rightarrow \chi = \sigma_x \left[\frac{\mu_x}{\sigma_x} - \ln\left(\frac{1 - p}{p}\right)\right].$$
(54)

Thus, I get that  $\chi = \mu_x + \sigma_x [\ln(p) - \ln(1-p)]$ . Hence, the expected conditional adjustment cost is:

$$\underbrace{\int_{0}^{V_{a}(\omega;z)-V_{n}(\omega;z)} \chi f(\chi) d\chi}_{\equiv AC(v;\mu_{x},\sigma_{x})} = \int_{0}^{F^{-1}\left(\underbrace{V_{a}(\omega;z)-V_{n}(\omega;z)}_{\equiv v}\right) \chi \underbrace{dF(\chi)}_{=dp} 
= \int_{0}^{v} F^{-1}(p) \underbrace{dp}_{=dp} = \int_{0}^{v} \mu_{x} + \sigma_{x} \left[\ln(p) - \ln(1-p)\right] dp \tag{55}$$

Recall that  $\int \ln x dx = x (\ln x - 1) + c$ , then

$$\int_{0}^{V_{a}(\omega;z)-V_{n}(\omega;z)} \chi f(\chi) d\chi = \mu_{x} v + \left(\sigma_{x} \left[ p \left( \ln p - 1 \right) - \int_{0}^{v} \ln \left( 1 - p \right) dp \right] \right) \Big|_{0}^{v}$$
 (56)

Solve  $\int_0^v \ln(1-p) dp$  using integration by parts where  $u = \ln(1-p)$  and dv = dp. Then  $du = -\frac{1}{1-p}dp$  and v = p, therefore  $\int_0^v \ln(1-p) dp = p \ln(1-p) + \int_0^v p \frac{1}{1-p} dp$ .

Solve  $\int_0^v p \frac{1}{1-p} dp$  replacing p where u=1-p then dp=-du and  $\int_0^v p \frac{1}{1-p} dp=-\int_0^v \frac{1-u}{u} du=$ 

 $-\int_0^v \left(\frac{1}{u}-1\right) du = u - \int_0^v \frac{1}{u} du = u - \ln |u|$ . Therefore, combining the results

$$\int_{0}^{V_{a}(\omega;z)-V_{n}(\omega;z)} \chi f(\chi) d\chi = \mu_{x}v + \sigma_{x} \left( \left[ p \left( \ln p - 1 \right) - \left[ p \ln \left( 1 - p \right) + \left( 1 - p \right) - \ln \mid 1 - p \mid \right] \right) \right) \right) dy$$

$$= \mu_{x}v + \sigma_{x} \left( \left[ v \ln v - v \right] - \left[ v \ln \left( 1 - v \right) + \left( 1 - v \right) - \ln \mid 1 - v \mid - \left( 0 \ln \left( 1 - 0 \right) + \left( 1 - 0 \right) - \ln \mid 1 - 0 \mid \right) \right] \right)$$

$$= \mu_{x}v + \sigma_{x} \left( \left[ v \ln v - v \right] - \left[ v \ln \left( 1 - v \right) + \left( 1 - v \right) - \ln \mid 1 - v \mid - 1 \right] \right)$$

$$= \mu_{x}v + \sigma_{x} \left( v \ln v - v \ln \left( 1 - v \right) + \ln \mid 1 - v \mid \right)$$

$$= \mu_{x}v + \sigma_{x} \left( v \ln v - v \ln \left( 1 - v \right) + \ln \mid 1 - v \mid \right) \right)$$

$$(57)$$

For every  $v \in [0,1)$ 

$$\int_{0}^{V_{a}(\omega;z)-V_{n}(\omega;z)} \chi f(\chi) d\chi = \mu_{x} v + \sigma_{x} \left( v \ln v + (1-v) \ln (1-v) \right)$$
 (58)

#### D Firms

#### D.1 Firm's optimal borrowing decision

Solving the dynamic problem of firms:

$$V_{t}^{F}(n_{jt}) = \tau \Lambda \mathbb{E}_{t} [R_{jt}^{K} k_{jt+1} - R_{t}^{B} L_{jt}^{B} - R_{jt}^{D} L_{jt}^{D}]$$

$$+ (1 - \tau) \Lambda \mathbb{E}_{t} \left[ V_{t+1}^{F}(n_{jt+1}) \right] + \lambda_{bt} \left( (1 - \kappa) k_{jt+1} - R_{t}^{B} L_{jt}^{B} \right)$$

$$+ \lambda_{dt} \left( \phi \kappa k_{jt+1} - R_{jt}^{D} L_{jt}^{D} \right)$$

Conjecture:  $V_t^F(n_{jt}) = \theta_t n_{jt}$  and policies  $L_{jt}^B = \tilde{b} n_{jt}$ ,  $L_{jt}^D = \tilde{d} n_{jt}$  then  $k_{jt+1} = \left(1 + \tilde{b} + \tilde{d}\right) n_{jt}$ . Therefore,

$$\begin{split} V_t^F(n_{jt}) &= \theta_t n_{jt} = \tau \Lambda \mathbb{E}_t [R_{jt}^K \left( 1 + \tilde{b} + \tilde{d} \right) - R_t^B \tilde{b} - R_{jt}^D \tilde{d}] n_{jt} \\ &+ (1 - \tau) \Lambda \mathbb{E}_t \left[ \theta_{t+1} (R_{jt}^K \left( 1 + \tilde{b} + \tilde{d} \right) - R_t^B \tilde{b} - R_{jt}^D \tilde{d})) \right] n_{jt} \\ &+ \lambda_{bt} \left( (1 - \kappa) \left( 1 + \tilde{b} + \tilde{d} \right) - R_t^B \tilde{b} \right) n_{jt} + \lambda_{dt} \left( \phi \kappa \left( 1 + \tilde{b} + \tilde{d} \right) - R_{jt}^D \tilde{d} \right) n_{jt} \end{split}$$

where  $n_{jt+1} = R_{jt}^K \left(1 + \tilde{b} + \tilde{d}\right) n_{jt} - R_t^B \tilde{b} n_{jt} - R_{jt}^D \tilde{d} n_{jt}$ . Moreover,  $R_{jt}^K$  and  $R_{jt}^D$  are drawn from a

common distribution, independent across firms, then  $\mathbb{E}_t R_{jt}^K = \mathbb{E}_t R_t^K$  and  $\mathbb{E}_t R_{jt}^D = \mathbb{E}_t R_t^D$ . Suppose  $\theta_{T+1}$  is known and independent of firm j (e.g., terminal value or steady state). Then, the expectation  $\mathbb{E}_T[\theta_{T+1} \cdot X_{jT}]$  can be evaluated as:

$$\mathbb{E}_T[\theta_{T+1} \cdot X_{jT}] = \theta_{T+1} \cdot \mathbb{E}_T[X_{jT}] = \theta_{T+1} \cdot \mathbb{E}_T[X_T]$$

since  $\theta_{t+1}$  is not firm-specific, and the shocks to  $X_{jt}$  are identically distributed across firms. Thus,  $\theta_t$  computed from this recursion is also not firm-specific.

$$\theta_{t} = \tau \Lambda \mathbb{E}_{t} \left[ R_{t}^{K} \left( 1 + \tilde{b} + \tilde{d} \right) - R_{t}^{B} \tilde{b} - R_{t}^{D} \tilde{d} \right]$$

$$+ (1 - \tau) \Lambda \mathbb{E}_{t} \left[ \theta_{t+1} \left( R_{t}^{K} \left( 1 + \tilde{b} + \tilde{d} \right) - R_{t}^{B} \tilde{b} - R_{t}^{D} \tilde{d} \right) \right]$$

$$+ \lambda_{bt} \left( (1 - \kappa) \left( 1 + \tilde{b} + \tilde{d} \right) - R_{t}^{B} \tilde{b} \right) + \lambda_{dt} \left( \phi \kappa \left( 1 + \tilde{b} + \tilde{d} \right) - R_{t}^{D} \tilde{d} \right)$$

$$(59)$$

which proves the conjecture. Then, F.O.Cs are defined by:

$$[\tilde{b}] \quad \tau \Lambda(\mathbb{E}_t R_{jt}^K - R_t^B) + (1 - \tau) \Lambda \mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_t^B \right) \right]$$

$$+ \lambda_{bt} \left( (1 - \delta)(1 - \kappa) - R_t^B \right) + \lambda_{dt} \left( \phi \kappa (1 - \delta) \right) = 0$$

$$(60)$$

$$[\tilde{d}] \quad \tau \Lambda \mathbb{E}_{t}(R_{jt}^{K} - R_{jt}^{D}) + (1 - \tau) \Lambda \mathbb{E}_{t} \left[ \theta_{t+1} \left( R_{jt}^{K} - R_{jt}^{D} \right) \right]$$

$$+ \lambda_{bt} \left( (1 - \delta)(1 - \kappa) \right) + \lambda_{dt} \left( \phi \kappa (1 - \delta) - R_{jt}^{D} \right) = 0$$

$$(61)$$

both conditions are equal to zero in equilibrium, otherwise it they are greater than zero, firms will borrow infinitely amount of resources, and if it is lower than zero, firms will borrow nothing.

Case 1: Both financial constraints are not binding. Under this scenario  $\lambda_{bt}$ ,  $\lambda_{dt} = 0$ 

$$[\tilde{b}] \quad \tau \Lambda(\mathbb{E}_t R_{jt}^K - R_t^B) + (1 - \tau) \Lambda \mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_t^B \right) \right] = 0$$

$$\left[\tilde{d}\right] \quad \tau \Lambda \mathbb{E}_{t}(R_{jt}^{K} - R_{jt}^{D}) + (1 - \tau)\Lambda \mathbb{E}_{t}\left[\theta_{t+1}\left(R_{jt}^{K} - R_{jt}^{D}\right)\right] = 0$$

To satisfy both conditions, then  $\mathbb{E}_t R_{jt}^K = R_t^B$ ,  $\mathbb{E}_t (R_{jt}^K - R_{jt}^D) = 0$ ,  $\mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_{jt}^D \right) \right] = cov(\theta_{t+1}, R_{jt}^K - R_{jt}^D) + \mathbb{E}_t \left[ \theta_{t+1} \right] \mathbb{E}_t \left[ R_{jt}^K - R_{jt}^D \right] = 0$  and  $\mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_t^B \right) \right] = cov(\theta_{t+1}, R_{jt}^K - R_t^B) + \mathbb{E}_t \left[ \theta_{t+1} \right] \mathbb{E}_t \left[ R_{jt}^K - R_t^B \right] = 0$ 

Case 2: Only the bond financial constraint is binding. Under this scenario  $\lambda_{dt} = 0$  and  $\lambda_{bt} > 0$ 

$$[\tilde{b}] \quad \lambda_{bt} = \frac{\tau \Lambda(\mathbb{E}_t R_{jt}^K - R_t^B) + (1 - \tau) \Lambda \mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_t^B \right) \right]}{R_t^B - (1 - \delta)(1 - \kappa)}$$

$$\begin{split} \left[ \tilde{d} \right] \quad \tau \Lambda \mathbb{E}_t (R_{jt}^K - R_{jt}^D) + (1 - \tau) \Lambda \mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_{jt}^D \right) \right] \\ \quad + \lambda_{bt} \left( (1 - \delta)(1 - \kappa) \right) = 0 \end{split}$$

Combining these two conditions,

$$\tau \mathbb{E}_t(R_{jt}^K - R_{jt}^D) + (1 - \tau) \mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_{jt}^D \right) \right]$$
$$+ \frac{(1 - \delta)(1 - \kappa)}{R_t^B - (1 - \delta)(1 - \kappa)} \left( \tau (\mathbb{E}_t R_{jt}^K - R_t^B) + (1 - \tau) \mathbb{E}_t \left[ \theta_{t+1} \left( R_{jt}^K - R_t^B \right) \right] \right) = 0$$

$$\tau \mathbb{E}_{t}(R_{jt}^{K} - R_{jt}^{D}) + (1 - \tau) \left( cov(\theta_{t+1}, R_{jt}^{K} - R_{jt}^{D}) + \mathbb{E}_{t}(R_{jt}^{K} - R_{jt}^{D}) \mathbb{E}_{t}\theta_{t+1} \right) + \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} \left( \tau (\mathbb{E}_{t}R_{jt}^{K} - R_{t}^{B}) + (1 - \tau) \left( cov(\theta_{t+1}, R_{jt}^{K} - R_{t}^{B}) + \mathbb{E}_{t}(R_{jt}^{K} - R_{t}^{B}) \mathbb{E}_{t}\theta_{t+1} \right) \right) = 0$$

$$\mathbb{E}_{t} \left( \frac{R_{jt}^{K}}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B}}} - R_{jt}^{D} \right) (\tau + (1 - \tau)\mathbb{E}_{t}\theta_{t+1})$$

$$- \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} \left( R_{t}^{B} \left( \tau + (1 + \tau)\mathbb{E}_{t}\theta_{t+1} \right) \right)$$

$$+ (1 - \tau) \left[ cov(\theta_{t+1}, R_{jt}^{K} - R_{jt}^{D}) + \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} cov(\theta_{t+1}, R_{t}^{K} - R_{t}^{B}) \right] = 0$$

$$\mathbb{E}_{t} \left( \frac{R_{jt}^{K}}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B}}} - R_{jt}^{D} \right) (\tau + (1 - \tau)\mathbb{E}_{t}\theta_{t+1})$$

$$- \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} \left( R_{t}^{B} \left( \tau + (1 + \tau)\mathbb{E}_{t}\theta_{t+1} \right) \right)$$

$$+ (1 - \tau) \left[ cov(\theta_{t+1}, R_{t}^{K}) - cov(\theta_{t+1}, R_{t}^{D}) + \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} (cov(\theta_{t+1}, R_{t}^{K}) - cov(\theta_{t+1}, R_{t}^{B})) \right] = 0$$

$$\mathbb{E}_{t} \left( \frac{R_{jt}^{K}}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B}}} - R_{jt}^{D} \right) (\tau + (1 - \tau)\mathbb{E}_{t}\theta_{t+1})$$

$$- \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} \left( R_{t}^{B} \left( \tau + (1 + \tau)\mathbb{E}_{t}\theta_{t+1} \right) \right)$$

$$+ (1 - \tau) \left[ cov(\theta_{t+1}, R_{t}^{K}) \left[ 1 + \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} \right] - cov(\theta_{t+1}, R_{t}^{D}) \right] = 0$$

$$\mathbb{E}_{t} \left( \frac{R_{jt}^{K} - (1 - \delta)(1 - \kappa)}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B}}} - R_{jt}^{D} \right) (\tau + (1 - \tau)\mathbb{E}_{t}\theta_{t+1})$$

$$+ (1 - \tau) \left[ cov(\theta_{t+1}, R_{jt}^{K}) \left[ 1 + \frac{(1 - \delta)(1 - \kappa)}{R_{t}^{B} - (1 - \delta)(1 - \kappa)} \right] - cov(\theta_{t+1}, R_{t}^{D}) \right] = 0$$

Recall that  $R_t^K = \omega_{jt}(\alpha A_t K_{t+1}^{\alpha-1} + (1-\delta)\kappa) + (1-\delta)(1-\kappa)$ 

$$\mathbb{E}_{t} \left( \omega_{jt} \left( \frac{\alpha A_{t} K_{t+1}^{\alpha-1} + (1-\delta)\kappa}{1 - \frac{(1-\delta)(1-\kappa)}{R_{t}^{B}}} - R_{jt}^{D} \right) \right) (\tau + (1-\tau) \mathbb{E}_{t} \theta_{t+1}) 
+ (1-\tau) \left[ cov(\theta_{t+1}, R_{t}^{K}) \left[ \frac{1}{1 - \frac{(1-\delta)(1-\kappa)}{R_{t}^{B}}} \right] - cov(\theta_{t+1}, R_{t}^{D}) \right] = 0$$

$$\mathbb{E}_{t} \left( \omega_{t} \left( \frac{\alpha A_{t} K_{t+1}^{\alpha-1} + (1-\delta)\kappa}{1 - \frac{(1-\delta)(1-\kappa)}{R_{t}^{B}}} - R_{t}^{D} \right) \right) (\tau + (1-\tau) \mathbb{E}_{t} \theta_{t+1}) 
+ (1-\tau) \left[ cov(\theta_{t+1}, \omega_{jt}) \left( \frac{\alpha A_{t} K_{t+1}^{\alpha-1} + (1-\delta)\kappa}{1 - \frac{(1-\delta)(1-\kappa)}{R_{t}^{B}}} - R_{t}^{D} \right) \right] = 0$$
(62)

In the model, the covariance term  $cov(\theta_{t+1}, \omega_{jt})$  vanishes since the individual idiosyncratic shocks  $\omega_{jt}$  are independent of aggregate prices. Since  $\theta_{t+1}$  depends only on  $R_t^B, R_t^D$ , and  $R_t^K$  which are determined by aggregate states. With a continuum of firms, an individual draw has measure zero impact on equilibrium prices, ensuring conditional independence. Thus, the covariance term drops out, and the relevant expectation reduces to the first component in Equation 62.

Case 3: Only the risky bank loan financial constraint is binding. Under this scenario  $\lambda_{bt} = 0$  and  $\lambda_{dt} > 0$ 

$$[\tilde{b}] \quad \tau \Lambda \mathbb{E}_{t}(R_{jt}^{K} - R_{t}^{B}) + (1 - \tau)\Lambda \mathbb{E}_{t}\left[\theta_{t+1}\left(R_{jt}^{K} - R_{t}^{B}\right)\right] + \lambda_{dt}\left(\phi\kappa(1 - \delta)\right) = 0$$

$$[\tilde{d}] \quad \lambda_{dt} = \frac{\tau \Lambda \mathbb{E}_{t} (R_{jt}^{K} - \omega_{jt} R_{t}^{D}) + (1 - \tau) \Lambda \mathbb{E}_{t} \left[ \theta_{t+1} \left( R_{jt}^{K} - \omega_{jt} R_{t}^{D} \right) \right]}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)}$$

Combining these two conditions,

$$\tau \Lambda \mathbb{E}_{t}(R_{jt}^{K} - R_{t}^{B}) + (1 - \tau)\Lambda \mathbb{E}_{t} \left[\theta_{t+1} \left(R_{jt}^{K} - R_{t}^{B}\right)\right] + \frac{\phi \kappa (1 - \delta)}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} \left(\tau \Lambda \mathbb{E}_{t}(R_{jt}^{K} - \omega_{jt} R_{t}^{D}) + (1 - \tau)\Lambda \mathbb{E}_{t} \left[\theta_{t+1} \left(R_{jt}^{K} - \omega_{jt} R_{t}^{D}\right)\right]\right) = 0$$

$$\tau \mathbb{E}_{t}(R_{jt}^{K} \left[ \frac{\omega_{jt} R_{t}^{D}}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} \right] - R_{t}^{B}) + (1 - \tau) \mathbb{E}_{t}(\theta_{t+1}) \mathbb{E}_{t}(R_{jt}^{K} \left[ \frac{\omega_{jt} R_{t}^{D}}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} \right] - R_{t}^{B}) + (1 - \tau) cov \left( \theta_{t+1}, R_{jt}^{K} - R_{t}^{B} \right)$$

$$+\frac{\phi\kappa(1-\delta)}{\omega_{jt}R_t^D - \phi\kappa(1-\delta)} \left(-\tau \mathbb{E}_t(\omega_{jt}R_t^D) + (1-\tau) \left[cov\left(\theta_{t+1}, R_{jt}^K - \omega_{jt}R_t^D\right) - \mathbb{E}_t(\theta_{t+1})\mathbb{E}_t(\omega_{jt}R_t^D)\right]\right) = 0$$

$$\begin{split} \mathbb{E}_{t} \left( R_{jt}^{K} \left[ \frac{\omega_{jt} R_{t}^{D}}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} \right] - R_{t}^{B} \right) (\tau + (1 - \tau) \mathbb{E}_{t}(\theta_{t+1})) \\ - \frac{\phi \kappa (1 - \delta)}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} \mathbb{E}_{t}(\omega_{jt} R_{t}^{D}) \left( \tau + (1 - \tau) \mathbb{E}_{t}(\theta_{t+1}) \right) \\ + (1 - \tau) \left[ cov \left( \theta_{t+1}, R_{jt}^{K} - R_{t}^{B} \right) + \frac{\phi \kappa (1 - \delta)}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} cov \left( \theta_{t+1}, R_{jt}^{K} - \omega_{jt} R_{t}^{D} \right) \right] = 0 \end{split}$$

$$\begin{split} \left[ \mathbb{E}_t \left( (R_{jt}^K - \phi \kappa (1 - \delta)) \left[ \frac{\omega_{jt} R_t^D}{\omega_{jt} R_t^D - \phi \kappa (1 - \delta)} \right] - R_t^B \right) \right] (\tau + (1 - \tau) \mathbb{E}_t (\theta_{t+1})) \\ + (1 - \tau) \left[ cov \left( \theta_{t+1}, R_{jt}^K \right) \left[ \frac{\omega_{jt} R_t^D}{\omega_{jt} R_t^D - \phi \kappa (1 - \delta)} \right] - \frac{\phi \kappa (1 - \delta)}{\omega_{jt} R_t^D - \phi \kappa (1 - \delta)} cov \left( \theta_{t+1}, \omega_{jt} R_t^D \right) \right] = 0 \end{split}$$

$$\mathbb{E}_{t}\left(\frac{\omega_{jt}(R_{jt}^{K}-\phi\kappa(1-\delta))}{\omega_{jt}-\frac{\phi\kappa(1-\delta)}{R_{t}^{D}}}-R_{t}^{B}\right)(\tau+(1-\tau)\mathbb{E}_{t}(\theta_{t+1}))$$

$$+(1-\tau)\left[cov\left(\theta_{t+1},R_{jt}^{K}\right)\left[\frac{\omega_{jt}R_{t}^{D}}{\omega_{jt}R_{t}^{D}-\phi\kappa(1-\delta)}\right]-\frac{\phi\kappa(1-\delta)}{\omega_{jt}R_{t}^{D}-\phi\kappa(1-\delta)}cov\left(\theta_{t+1},\omega_{jt}R_{t}^{D}\right)\right]=0$$

Recall that  $R_{jt}^K = \omega_{jt}(\alpha A_t K_{t+1}^{\alpha-1} + (1-\delta)\kappa) + (1-\delta)(1-\kappa)$ 

$$\mathbb{E}_{t} \left( \frac{\omega_{jt} (R_{jt}^{K} - \phi \kappa (1 - \delta))}{\omega_{jt} - \frac{\phi \kappa (1 - \delta)}{R_{t}^{D}}} - R_{t}^{B} \right) (\tau + (1 - \tau) \mathbb{E}_{t}(\theta_{t+1}))$$

$$+ (1 - \tau) cov \left(\theta_{t+1}, \omega_{jt}\right) \left[ \frac{\omega_{jt} R_{t}^{D} (\alpha A_{t} K_{t+1}^{\alpha - 1} + (1 - \delta) \kappa) - \phi \kappa (1 - \delta) R_{t}^{D}}{\omega_{jt} R_{t}^{D} - \phi \kappa (1 - \delta)} \right] = 0$$

$$\mathbb{E}_{t} \left( \frac{R_{jt}^{K} - \phi \kappa (1 - \delta)}{1 - \frac{\phi \kappa (1 - \delta)}{\omega_{jt} R_{t}^{D}}} - R_{t}^{B} \right) (\tau + (1 - \tau) \mathbb{E}_{t}(\theta_{t+1}))$$

$$+ (1 - \tau) cov \left(\theta_{t+1}, \omega_{jt}\right) \left[ \frac{\alpha A_{t} K_{t+1}^{\alpha - 1} + (1 - \delta) \kappa - \phi \kappa (1 - \delta)}{1 - \frac{\phi \kappa (1 - \delta)}{\omega_{jt} R_{t}^{D}}} \right] = 0$$
(63)

Similarly to Case 1, the covariance term  $cov(\theta_{t+1}, \omega_{jt})$  vanishes since the individual idiosyncratic shocks  $\omega_{jt}$  are independent of aggregate prices. Thus, the covariance term drops out, and the relevant expectation reduces to the first component in Equation 63.

Case 4: Both financial constraints are binding. In this scenario  $\lambda_{bt}$ ,  $\lambda_{dt} > 0$ . This implies that the conditions 62 and 63 are strictly positive, i.e.,

$$\frac{\alpha A_t K_{t+1}^{\alpha - 1} + (1 - \delta)\kappa}{1 - \frac{(1 - \delta)(1 - \kappa)}{R_t^B}} - R_t^D > 0$$
(64)

$$\frac{R_{jt}^K - \phi \kappa (1 - \delta)}{1 - \frac{\phi \kappa (1 - \delta)}{\omega_{it} R_t^D}} - R_t^B > 0 \tag{65}$$

These inequalities indicate that the relative return to capital, measured against each alternative debt instrument, exceeds the respective borrowing rate. In other words, both bank loans and bonds

offer financing at rates below the effective marginal value of capital. As a result, firms optimally choose to hold and utilize both forms of debt simultaneously.

### D.2 Regimes

#### D.2.1 Unconstrained credit regime

This regime consists of Case 1, as explained in the previous section.

#### D.2.2 Constrained credit regime

This regime comprises Cases 2, 3, and 4. Specifically, under Case 4, where firms' financing constraints are binding, then

$$B_t = L_t^B = \frac{(1 - \delta)(1 - \kappa)}{1 + r_t^b} K_{t+1}$$
(66)

$$D_t = L_t^D = \frac{\phi \kappa (1 - \delta)}{\left(1 + r_t^d\right) \frac{S_t^+}{2S_t^+ - 1}} K_{t+1}$$
(67)

Plugging 66 and 67 in 24, I get:

$$K_{t+1} = N_t + \frac{(1-\delta)(1-\kappa)}{1+r_t^b} K_{t+1} + \frac{\phi\kappa(1-\delta)}{\left(1+r_t^d\right)\frac{S_t^+}{2S_t^+-1}} K_{t+1}$$

$$K_{t+1} = \frac{1}{1 - \frac{(1-\delta)(1-\kappa)}{1+r_t^b} - \frac{\phi\kappa(1-\delta)}{(1+r_t^d)\frac{S_t^+}{2S_t^+-1}}} N_t \equiv \bar{\theta} N_t$$

which is the law of motion of capital in equilibrium when all financial constraints are binding. Furthermore,  $\bar{\theta}$  represents the overall leverage of credit markets and firms. Finally, the law of motion of aggregate networth is defined by:

$$N_{t+1} = (1 - \tau + \tau \zeta) \left( R_t^K - R_t^B (1 - \delta) \frac{1 - \kappa}{1 + r_t^b} - p_H R_t^D (1 - \delta) \frac{\phi \kappa}{\left( 1 + r_t^d \right) \frac{S_t^+}{2S_t^+ - 1}} \right) \bar{\theta} N_t$$

$$N_{t+1} = (1 - \tau + \tau \zeta) \left( R_t^K - (1 - \delta)(1 - \kappa) - (1 - \delta)p_H \phi \kappa \right) \bar{\theta} N_t$$
(68)

**Steady state:** Using 68 and 26, I can find the steady state of capital such that  $N_{t+1} = N_t$ . Then,

$$(1 - \tau + \tau \zeta) \left( R_{SS}^{K} - (1 - \delta)(1 - \kappa) - (1 - \delta)p_{H}\phi\kappa \right) \bar{\theta} = 1$$

$$(1 - \tau + \tau \zeta) \left( p_{H}\alpha A K_{SS}^{\alpha-1} + p_{H}(1 - \delta)\kappa - (1 - \delta)p_{H}\phi\kappa \right) \bar{\theta} = 1$$

$$p_{H} (1 - \tau + \tau \zeta) \left( \alpha A K_{SS}^{\alpha-1} + \kappa (1 - \delta)(1 - \phi) \right) \bar{\theta} = 1$$

$$K_{SS} = \left[ \frac{1}{\alpha A} \left( \frac{1}{p_{H}\bar{\theta} (1 - \tau + \tau \zeta)} - \kappa (1 - \delta)(1 - \phi) \right) \right]^{\frac{1}{\alpha - 1}}$$

$$K_{SS} = \left( \frac{1 - p_{H}\bar{\theta} (1 - \tau + \tau \zeta) \kappa (1 - \delta)(1 - \phi)}{\alpha A p_{H}\bar{\theta} (1 - \tau + \tau \zeta)} \right)^{\frac{1}{\alpha - 1}}$$

$$K_{SS} = \left( \frac{\alpha A p_{H}\bar{\theta} (1 - \tau + \tau \zeta)}{1 - p_{H}\bar{\theta} (1 - \tau + \tau \zeta) \kappa (1 - \delta)(1 - \phi)} \right)^{\frac{1}{1 - \alpha}}$$
(69)

# E Moral hazard and financial contracts

This section describes the financial contracts between firms, banks, and bondholders, highlighting how monitoring and incentive constraints shape short-term financing arrangements.

The binomial nature of the firm's asset value (collateral) allows monitoring to play a central role in capital intermediation: it prevents the firm from diverting more than a fraction  $\phi$  of its risky

capital. The parameter  $\phi$  measures the effectiveness of monitoring in curbing misreporting. A higher  $\phi$  reflects stronger enforcement, making it more difficult for firms to divert resources.

By including banks in financial contracts, firms gain access to greater external financing, but at the cost of tighter discipline. Monitoring restricts the firm's ability to misreport or expropriate funds, and to ensure truthful reporting, banks impose the following incentive compatibility constraint:

$$\pi \left( k_{jt+1}, \omega_{jt} = 1 \right) + (1 - \delta) k_{jt+1} - R_t^B L_{jt}^B - R_t^D \left( \omega_{jt} = 1 \right) L_{jt}^D \ge \pi \left( k_{jt+1}, \omega_{jt} = 0 \middle| \omega_{jt} = 1 \right) + (1 - \delta) k_{jt+1} - R_t^B L_{jt}^B - R_t^D \left( \omega_{jt} = 0 \middle| \omega_{jt} = 1 \right) L_{jt}^D - \phi (1 - \delta) \kappa k_{jt+1}$$

$$\Rightarrow R_t^D(\omega_{jt} = 1) L_{jt}^D - R_t^D(\omega_{jt} = 0 | \omega_{jt} = 1) L_{jt}^D \le \phi \kappa (1 - \delta) k_{jt+1}$$
 (70)

Thus, bank monitoring imposes a constraint on firms, as they lose a fraction  $\phi \in (0,1)$  of the risky capital due to monitoring enforcement. Banks accept risky debt, but only up to the fraction of risky capital that can be effectively monitored. Additionally, since bondholders (households) can't monitor firms, they only accept risk-free contracts, requiring full collateralization. Bondholders also recognize that banks have a senior claim in failure states. To ensure bondholders' risk-free repayment, they impose the following feasibility constraint:

$$(1 - \delta)(1 - \kappa)k_{jt} \ge R_t^B L_{jt}^B + R_t^D (\omega_{jt} = 0) L_{jt}^D$$
(71)

Meanwhile, banks assume the riskier portion of the contract and must ensure they can be repaid under any reporting strategy. Therefore, banks impose their feasibility constraint, ensuring they are repaid independently of the firm's report:

$$\alpha A_t K_{t+1}^{\alpha - 1} k_{jt} + (1 - \delta) k_{jt} \ge \max \{ R_t^D \left( \omega_{jt} = 0 | \omega_{jt} = 1 \right) L_{it}^D, R_t^D \left( \omega_{jt} = 1 \right) L_{it}^D \}$$
 (72)

This framework ensures that firms can pledge their full capital stock as collateral, but contract structure dictates claim priority:

• Bondholders' repayment is fully secured by the risk-free portion of capital since bondholders only accept risk-free debt, requiring full collateral backing.

• Banks accept risky debt, constrained by monitoring effectiveness and the firm's ability to repay under all reporting states. Thus, banks lend against the total productive resources of the firm, accepting risk in exchange for monitoring oversight.

Thus, while firms can pledge their full capital stock, the feasibility constraint ensures that bond-holders' repayment is always secured first, and only the remaining risky portion is exposed to strategic misreporting and monitoring frictions. Finally, risk-free bonds and zero-recovery risky bank loans can satisfy 70, 71 and 72. These conditions are the foundation of the financial contract structure analyzed in the main part of the paper.

### F National accounts

A household's budget constraint is defined by:

$$c_{it}^a + d_{it+1}^a + b_{it+1}^a = (1 - \tau^g) p_H w_t z_{it} + (1 + r_t^d) d_{it} + (1 + r_t^b) b_{it} + \tau^f N_t^F(z_{it}) + \hat{r} d_{it} \mathbb{I}_{d_{it} < 0}$$

$$c_{it}^n + d_{it+1}^n + b_{it} = (1 - \tau^g) p_H w_t z_{it} + (1 + r_t^d) d_{it} + (1 + r_t^b) b_{it} + \tau^f N_t^F(z_{it}) + \hat{r} d_{it} \mathbb{I}_{d_{it} < 0}$$

Then, the average consumption and savings over a household's lifetime is:

$$\nu_{it}c_{it}^{a} + (1 - \nu_{it})c_{it}^{n} + \nu_{it}d_{it+1}^{a} + (1 - \nu_{it})d_{it+1}^{n} + \nu_{it}b_{it+1}^{a} + (1 - \nu_{it})b_{it}$$

$$= (1 - \tau^{g})p_{H}w_{t}z_{it} + (1 + r_{t}^{d})d_{it} + (1 + r_{t}^{b})b_{it} + \tau^{f}N_{t}^{F}(z_{it}) + \hat{r}d_{it}\mathbb{I}_{d_{it}<0}$$

where  $w_t = A (1 - \alpha) K_t^{\alpha}$  and  $\tau^f = \tau - \tau \zeta$  and  $p_H w_t = W_t$ . Then, in aggregate:

$$C_t^* + D_{t+1}^* + B_{t+1}^* = (1 - \tau^g) p_H A (1 - \alpha) K_t^{\alpha} + (1 + r_t^d) D_t^* + (1 + r_t^b) B_t^* + \tau^f N_t^F + \hat{r} D_t^-$$
 (73)

where consumption is defined by  $C_t^* = \int_i \left(\nu_{it}c_{it}^a + (1-\nu_{it})c_{it}^n\right)d_i$ , deposit holdings by  $D_{t+1}^* = \int_i \left(\nu_{it}d_{it+1}^a + (1-\nu_{it})d_{it+1}^n\right)d_i$ , bond holdings by  $B_t^* = \int_i \left(\nu_{it}b_{it+1}^a + (1-\nu_{it})b_{it}\right)d_i$  and borrowing by  $D^- = \int d_i \mathbb{I}_{d_{it}<0}$ .

Entrepreneurs' total networth is defined by:

$$N_t^E = (1 - \tau^f) N_t^F$$

Firms' networth is defined by:

$$N_{t}^{F} = R_{t}^{K} K_{t} - R_{t}^{B} L_{t}^{B} - p_{H} R_{t}^{D} L_{t}^{D}$$

Therefore, in steady state  $B^*_{t+1} = B^*_t \equiv B^*$  and  $D^*_{t+1} = D^*_t \equiv D^*$ 

$$C^* + N^E = (1 - \tau^g)p_H A (1 - \alpha) K^{\alpha} + r^d D^* + r^b B^* + \tau^f N^F + (1 - \tau^f) N^F + \hat{r} D^-$$

$$C^* + N^E = p_H A (1 - \alpha) K^{\alpha} - \tau^g p_H A (1 - \alpha) K^{\alpha} + r^d D^* + r^b B^* + N^F + \hat{r} D^-$$

where total taxes collected by government (T) satisfies the following:  $T \equiv \tau^g p_H A (1 - \alpha) K^{\alpha}$  and G = T.

$$C^* + G^* + N^E = p_H A (1 - \alpha) K^{\alpha} + r^d D^* + r^b B^* + R^K K - R^B L^B - p_H R^D L^D + \hat{r} D^-$$

$$C^* + G^* + N^E = p_H A (1 - \alpha) K^{\alpha} + r^d D^* + r^b B^* + R^K K - (1 + r^b) B^* - (1 + r^d + \hat{c}) D^* + \hat{r} D^-$$

$$C^* + G^* + N^E = p_H A (1 - \alpha) K^{\alpha} + R^K K - B^* - (1 + \hat{c}) D^* + \hat{r} D^-$$

$$C^* + G^* + N^E + B^* + D^* = p_H A (1 - \alpha) K^{\alpha} + R^K K - \hat{c}D^* + \hat{r}D^-$$

$$C^* + G^* + K^* = p_H A (1 - \alpha) K^{\alpha} + R^K K - \hat{c}D^* + \hat{r}D^-$$

where  $R^K = p_H \alpha A K^{\alpha-1} + p_H (1 - \delta) \kappa + (1 - \delta) (1 - \kappa)$ .

$$C^* + G^* + K^* = p_H A (1 - \alpha) K^{\alpha} + \left( p_H \alpha A K^{\alpha - 1} + p_H (1 - \delta) \kappa + (1 - \delta) (1 - \kappa) \right) K^{\alpha} - \hat{c} D^* + \hat{r} D^{-}$$

$$C^* + G^* + K^* = p_H A (1 - \alpha) K^{\alpha} + p_H \alpha A K^{\alpha} + p_H (1 - \delta) \kappa K + (1 - \delta) (1 - \kappa) K$$
$$-\hat{c} D^* + \hat{r} D^-$$

$$C^* + G^* + K^* (1 - p_H(1 - \delta)\kappa - (1 - \delta)(1 - \kappa)) = p_H A (1 - \alpha) K^{*\alpha} + p_H \alpha A K^{\alpha} - \hat{c}D^* + \hat{r}D^-$$

Finally, the resource constraint in steady steady is defined by:

$$C^* + G^* + K^* \left( 1 - (1 - \delta) \left( 1 + p_H \kappa - \kappa \right) \right) + \hat{c} D^* - \hat{r} D^- = p_H A K^{*\alpha}$$
 (74)

Therefore,

$$I = K^* (1 - (1 - \delta) (p_H \kappa + (1 - \kappa)))$$
(75)

# G Numerical solution: Algorithm

#### G.1 Households

- 1. Define spaced grid nodes  $\mathcal{H}_d = \{d_1, d_2, ..., d_D\}, \mathcal{H}_b = \{b_1, b_2, ..., b_B\}, \mathcal{H}_z = \{z_1, z_2, ..., z_Z\}$  and transition matrix  $\mathbf{P}^{20}$ . The state space is defined by  $S = [i_{N_Z} \otimes b, i_{N_B} \otimes d, z \otimes i_{N_D}]$  where  $N_D, N_B$  and  $N_z$  are nodes for  $\mathcal{H}_d, \mathcal{H}_b$  and  $\mathcal{H}_z$ . The number of nodes for S can be defined by  $N_s = N_D \times N_B \times N_z$ .
- 2. Define policy guesses:

$$c_n^{*(0)}(d, b, \mathbf{z}_{z \times 1}) = w\mathbf{z}\bar{l} + r^b b + (1 + r^d)d + \Pi$$

 $<sup>^{20}</sup>$ This matrix will be computed using Rouwenhorst algorithm to discretized a continuous Markov Chain of a AR(1) log-normal process.

$$c_a^{*(0)}(d, b, \mathbf{z}_{z \times 1}) = w\mathbf{z}\bar{l} + (1 + r^b)b + (1 + r^d)d + \Pi$$

- 3. Start iteration with the policy functions  $c_n^{*(0)}(d,b,z)$ ,  $c_a^{*(0)}(d,b,z)$  and  $\psi^{(0)}(d,b,z) = 0$  on a given grid  $(d,b) \in D \times B$ .
- 4. Update  $c_n^*(d_n^*, b, z)$  in  $n^{th}$  iteration implementing EGM. In matrix form, for a given (d', b', z) (on-grid):

$$\frac{\partial u(c_n^{*(n)}(d_n^*(d',b',z),b,z))}{\partial c} = \frac{\beta\left(1+r^d\right)\left(\nu^{*(n-1)}\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_a^{*(n-1)}(d',b',z')\right)}{\partial c} + \left(1-\nu^{*(n-1)}\right)\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_n^{*(n-1)}(d',b',z')\right)}{\partial c}\right) (76)$$

Then, using approximant functions, interpolate  $c_n^{*(n)}\left(d_n^*\left(d',b',z\right),b,\mathbf{z}_{z\times 1}\right)$  at  $c_n^{*(n)}\left(d,b,\mathbf{z}_{z\times 1}\right)$ . Regarding the binding constraints, I identify the points where the policy  $d_n^*<-\underline{d}$  and adjust  $d_n^*=-\underline{d}$  and the consumption policy accordingly.

5. Using EGM evaluated at every  $(b', z) \in \mathcal{H}_b \otimes \mathcal{H}_z$  (on-grid), I find  $\tilde{d}_a^*(b', z)$  (off-grid) that solves the following condition:

$$0 = \beta \nu^{*(n-1)} \left( \left( r^b - r^d \right) \mathbf{P}_{\mathbf{z}} \frac{\partial u \left( c_a^{*(n-1)} \left( \tilde{d}_a^*(b', z), b', \mathbf{z}'_{z \times 1} \right) \right)}{\partial c} \right)$$

$$+ \beta \left( 1 - \nu^{*(n-1)} \right) \left( \left( r^b - r^d \right) \mathbf{P}_{\mathbf{z}} \frac{\partial u \left( c_n^{*(n-1)} \left( \tilde{d}_a^*(b', z), b', \mathbf{z}'_{z \times 1} \right) \right)}{\partial c} \right)$$

$$+ \beta \left( 1 - \nu^{*(n-1)} \right) \mathbf{P}_{\mathbf{z}} \psi^{(n-1)} \left( \tilde{d}_a^*(b', z), b', \mathbf{z}'_{z \times 1} \right)$$

$$(77)$$

If no solution exists, I set  $\tilde{d}_a^*(b',z) = -\underline{d}$ . Also, if solution exists, but if  $\tilde{d}_a^*(b',z) < -\underline{d}$  then I set  $\tilde{d}_a^*(b',z) = -\underline{d}$  and if  $\tilde{d}_a^*(b',z) > d_D$ , then I set  $\tilde{d}_a^*(b',z) = d_D$ .

6. Solve for consumption for a given bond choice b' (on-grid) when there are bond adjustments

 $c_{a}^{*\tilde{(}n)}\left(b^{\prime},z\right)$  using the following condition:

$$\frac{\partial u\left(c_{a}^{*(n)}\left(b',z\right)\right)}{\partial c} = \beta\left(1+r^{d}\right)\left(\nu^{*(n-1)}\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_{a}^{*(n)}\left(\tilde{d}_{a}^{*}\left(b',z\right),b',z'\right)\right)}{\partial c}+\left(1-\nu^{*(n-1)}\right)\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_{n}^{*(n)}\left(\tilde{d}_{a}^{*}\left(b',z\right),b',z'\right)\right)}{\partial c}\right)$$
(78)

Next, find the total expenses  $\mathcal{R}_a(b',z)$  which are consistent to total resources

$$\mathcal{R}_a(b',z) := c_a^{\tilde{*}(n)}(b',z) + d_a^{\tilde{*}(n)}(b',z) + b' - wz\bar{l} - \Pi$$

$$\tilde{\mathcal{R}}_{a}(d(b',z),b(b',z)) = (1+r^{d})d(b',z) + (1+r^{b})b(b',z)$$

Interpolating  $c_a^{\tilde{*}(n)}(b',z')$ ,  $d_a^{\tilde{*}(n)}(b',z')$  at  $\tilde{\mathcal{R}}_a(d,b,z)$  that is consistent with  $\tilde{\mathcal{R}}_a(d(b',z),b(b',z))$ , I get policy functions  $c_a^{*(n)}(d,b,z)$  and  $d_a^{*(n)}(d,b,z)$ .

#### 7. Update $\psi$ such that

$$\psi^{(n)}(d,b,z) = \beta \nu^{*(n)} \left(1 + r^{b}\right) \mathbf{P}_{\mathbf{z}} \frac{\partial u\left(c_{a}^{*(n)}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)\right)}{\partial c}$$

$$+\beta \left(1 - \nu^{*(n)}\right) \left(1 + r^{b}\right) \mathbf{P}_{\mathbf{z}} \frac{\partial u\left(c_{n}^{*(n)}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)\right)}{\partial c}$$

$$+\beta \left(1 - \nu^{*(n)}\right) \mathbf{P}_{\mathbf{z}} \psi^{(n-1)}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)$$

$$(79)$$

8. After convergence of policies, I update the value functions using the following conditions:

$$V_n^{(n)} = u\left(c_n^{*(n)}\right) + \beta \mathbb{E}V\left(d_n^{*(n)}, b^{(n)}, z'\right)$$

$$V_a^{(n)} = u\left(c_a^{*(n)}\right) + \beta \mathbb{E}V\left(d_a^{*(n)}, b^{*(n)}, z'\right)$$

$$\mathbb{E}V = \nu^{*(n-1)}V_a^{(n)} + \left(1 - \nu^{*(n-1)}\right)V_n^{(n)} - \int_0^{V_a^{(n)} - V_n^{(n)}} \chi f(\chi)d\chi$$
(80)

where the expected adjustment cost is defined by  $AC(\nu; \mu_x, \sigma_x) = \int_0^{V_a^{(n)} - V_n^{(n)}} \chi f(\chi) d\chi = \mu_x \nu + \sigma_x (\nu \ln \nu + (1 - \nu) \ln (1 - \nu)) - C.$ 

- 9. Finally, update probabilities of adjustment  $\nu^{*(n)} = F_{\chi} \left[ V_a^{(n)} V_n^{(n)} \right]$  since adjustment costs are distributed logistically.
- 10. Update probabilities of adjustment and iterate until convergence of policies.

### G.2 Firms in the aggregate

1. Start with some guesses for interest rates,  $r^b$  and  $r^d$  and define the law of motion of aggregate stock of capital and firms' networth where  $L_t^B = \int_i b_{it} d\Theta_b$  and  $L_t^D = \int_i d_{it} d\Theta_d$  represents the aggregate supply of bond savings and deposits collected by financial intermediaries in steady state.

$$K_{t+1}^n = N_t^n + L_t^B + L_t^D (81)$$

$$N_{t+1}^{n} = (1 - \tau + \tau \zeta) \left( R_{t}^{*K} K_{t+1}^{n} - R_{t}^{B} \left( r_{t}^{*b} \right) L_{t}^{B} - p_{H} R_{t}^{D} \left( r_{t}^{*d} \right) L_{t}^{D} \right)$$
(82)

Iterate until convergence (i.e.  $|N_{t+1}^n - N_{t+1}^{n-1}| < \epsilon$ ) and obtain stock of capital  $K_{SS}^*$  and networth  $N_{SS}^*$  in steady state. By Corollary 1, to guarantee the convergence of networth and positive returns to households (in equilibrium), the stock of assets must satisfy that  $K_{SS}^* \in (\underline{K}, \overline{K})$ .

2. Define the function of excess of demand of risk-free bonds and risky bank loans and find the interest rates that clears the market in four scenarios: i) both financial constraints are binding, ii) only risk-free bond demand is binding, iii) only risky bank loans is binding and iv) both financial constraint are slack.

$$L^{B}\left(K_{SS}^{*}, r^{b}, r^{d}\right) - B^{*}\left(r^{b}, r^{d}\right) = 0$$
(83)

$$L^{D}\left(K_{SS}^{*}, r^{b}, r^{d}\right) - D^{*}\left(r^{b}, r^{d}\right) = 0$$
(84)

and get the pair  $\{r^b, r^d\}$  that solve the system of equations.

3. Find  $\bar{R}^B$  and  $\bar{R}^D$  and choose the equilibrium using the conditions in 39 and 41.

4. Find wages in equilibrium using 32 and compute the expected labor wage including the probability to have a positive wage:

$$w^{*e} = p_H(1 - \alpha)AK^{*\alpha} \tag{85}$$

5. Update interest rate and wages guesses and iterate households and firms algorithm until convergence.

# **H** Transition Dynamics

### H.1 Summary of equations

1. Income process

$$z_{it} = \frac{\tilde{z}_{it}}{\int \tilde{z}_{it}} \tag{86}$$

$$\tilde{z}_{it} = \exp\left(\rho \log \tilde{z}_{it-1} + \varepsilon_{it}\right), \quad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_t^2\right)$$
 (87)

I implement the discretization of labor productivity  $\tilde{z}_{it}$  using the Rouwenhorst method. Then, using the stationary distribution of income, I calculate the  $ratio_z$  and find  $\sigma_t^2$  that matches the data with the model.

$$ratio_z = \frac{F_z^{-1}(0.9)}{F_z^{-1}(0.1)} \tag{88}$$

2. **Households' policies:** Compute the Euler Equations implementing the backward induction method using the following equations:

$$\beta^{-1}\psi\left(d_{t},b_{t},z_{t}\right) \coloneqq \nu_{t}\mathbb{E}_{t}\left[\frac{\partial u\left(c_{a}^{*}\left(d_{n}^{*},b_{t},z_{t+1}\right)\right)}{\partial c}\left(1+r_{t}^{b}\right)\right] + \left(1-\nu_{t}\right)\mathbb{E}_{t}\left[\frac{\partial u\left(c_{n}^{*}\left(d_{n}^{*},b_{t},z_{t+1}\right)\right)}{\partial c}\left(1+r_{t}^{b}\right)\right] + \left(1-\nu_{t}\right)\mathbb{E}_{t}\left[\psi\left(d_{n}^{*},b_{t},z_{t+1}\right)\right]$$

$$\left(89\right)$$

$$c_{n}^{*} = \frac{\partial u^{-1}}{\partial c} \left( \beta \mathbb{E}_{t} \left( \left( 1 + r_{t}^{d} \right) \left[ \nu_{t} \frac{\partial u \left( c_{a}^{*} \left( d_{n}^{*}, b_{t}, z_{t+1} \right) \right)}{\partial c} + \left( 1 - \nu_{t} \right) \frac{\partial u \left( c_{n}^{*} \left( d_{n}^{*}, b_{t}, z_{t+1} \right) \right)}{\partial c} \right] \right) \right) \tag{90}$$

$$0 = \beta \nu_t \mathbb{E}_t \left( \frac{\partial u \left( c_a^* \left( d_a^*, b_t^*, z_{t+1} \right) \right)}{\partial c} \left( r_t^b - r_t^d \right) \right) + \beta \left( 1 - \nu_t \right) \mathbb{E}_t \left( \frac{\partial u \left( c_n^* \left( d_a^*, b_t^*, z_{t+1} \right) \right)}{\partial c} \left( r_t^b - r_t^d \right) \right) + \beta \left( 1 - \nu_t \right) \mathbb{E}_t \left( \psi \left( d_a^*, b_t^*, z_{t+1} \right) \right)$$

$$(91)$$

$$c_a^* = \frac{\partial u^{-1}}{\partial c} \left( \beta \mathbb{E}_t \left( \left( 1 + r_t^d \right) \left[ \nu_t \frac{\partial u \left( c_a^* \left( d_a^*, b_{t+1}^*, z_{t+1} \right) \right)}{\partial c} + \left( 1 - \nu_t \right) \frac{\partial u \left( c_n^* \left( d_a^*, b_{t+1}^*, z_{t+1} \right) \right)}{\partial c} \right] \right) \right)$$

$$(92)$$

- 3. **Distribution of households:** I assume that prior to t = 0, the economy is in a steady state with returns  $r_t^d$  and  $r_t^b$  and wages  $w_t$ . In this steady state, there is a unique value function, decision rules, and a unique stationary distribution  $\Phi$  of (d, b, z) that solves the general equilibrium. Using Young's (2010) method, I compute the stationary distribution  $\Phi_t$  on each period t.
- 4. Aggregate deposits and bond holdings: Since there is no aggregate uncertainty, conditional on  $\sigma_t$ , the initial distribution of assets should be the same as its final distribution on each t. Let's denote  $\pi_{mnk} \equiv \Phi(d_m, b_n, z_k)$

$$D_t = \sum_{m,n,k} d_{m,t} \pi_{mnk} \tag{93}$$

$$B_t = \sum_{m,n,k} b_{m,t} \pi_{mnk} \tag{94}$$

5. Firms' stock of assets and networth: Assuming that the steady state in t = 0 is in the constrained regime:

$$K_{t+1} = N_t + B_t + D_t (95)$$

$$N_{t+1} = (1 - \tau + \tau \zeta)(R_t^K K_{t+1} - R_t^B B_t - R_t^D D_t)$$
(96)

6. Return of assets, cost of credit and wages:

$$R_t^K = p_H \alpha A_t K_{t+1}^{\alpha - 1} + p_H (1 - \delta) \kappa + (1 - \delta) (1 - \kappa)$$
(97)

and using the optimal borrowing decision of firms explained previously, I find  $R_t^B$  and  $R_t^D$  that clear the market of bonds and bank loans. Then, I update the return of deposits, bonds, and expected wages received by households as follows:

$$r_t^d = p_H R_t^D - \hat{c} - 1 (98)$$

$$r_t^b = R_t^B - 1 (99)$$

$$w_t = p_H A (1 - \alpha) K_{t+1}^{\alpha} \tag{100}$$

# H.2 Algorithm

- 1. Define the transition path where  $t \in \{1, 2, ..., T\}$  and compute the path of labor productivity  $z_{it}$  based on Equation 88.
- 2. Define the sequences of guesses for  $\{r_t^d, r_t^b, w_t\}_{t=1}^{T-1}$ .
- 3. I know the policies in period T (Steady state 2)  $c_i^{*n}(d_T, b_T, z_T)$  and  $c_i^{*a}(d_T, b_T, z_T)$  but I don't know the RHS of Equations 90 92 since they're  $c_i^{*n}(d'_{T-1}, b'_{T-1}, z_T)$  and  $c_i^{*a}(d'_{T-1}, b'_{T-1}, z_T)$ . So, I'll apply EGM-step to recover the policies on each t = 1, 2, 3, ..., T 1.
- 4. Evaluate the RHS of Euler Equations using the policies on T, I'll assume  $\nu_{T-1}$  is  $\nu_T$ .
- 5. Start iteration with the policy functions  $c_{n}^{*(T)}\left(d,b,z\right),c_{a}^{*(T)}\left(d,b,z\right)$  and  $\psi^{(T)}\left(d,b,z\right)$ .
- 6. Update  $c_n^{*T-1}\left(d_n^{T-1},b,z\right)$  implementing EGM. In matrix form, for a given (d',b',z) (on-grid):

$$\frac{\partial u(c_{n}^{*(T-1)}\left(d_{n}^{*}\left(d',b',z\right),b,z\right)\right)}{\partial c} = \beta\left(1 + r_{T-1}^{d}\right)\left(\nu^{*(T)}\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_{a}^{*(T)}\left(d',b',z'\right)\right)}{\partial c} + \left(1 - \nu^{*(T)}\right)\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_{n}^{*(T)}\left(d',b',z'\right)\right)}{\partial c}\right)$$

$$(101)$$

Then, using approximant functions, interpolate  $c_n^{*(T-1)}\left(d_n^*\left(d',b',z\right),b,\mathbf{z}_{z\times 1}\right)$  at  $c_n^{*(T-1)}\left(d,b,\mathbf{z}_{z\times 1}\right)$ . Regarding the binding constraints, I identify the points where the policy  $d_n^*<-\underline{d}$  and adjust  $d_n^*=-\underline{d}$  and the consumption policy accordingly.

7. Using EGM evaluated at every  $(b', z) \in \mathcal{H}_b \otimes \mathcal{H}_z$  (on-grid), I find  $\tilde{d}_a^*(b', z)$  (off-grid) that solves the following condition:

$$0 = \beta \nu^{*(T)} \left( \left( r_{T-1}^b - r_{T-1}^d \right) \mathbf{P}_{\mathbf{z}} \frac{\partial u \left( c_a^{*(T)} \left( \tilde{d}_a^*(b', z), b', \mathbf{z}'_{z \times 1} \right) \right)}{\partial c} \right)$$

$$+ \beta \left( 1 - \nu^{*(T)} \right) \left( \left( r_{T-1}^b - r_{T-1}^d \right) \mathbf{P}_{\mathbf{z}} \frac{\partial u \left( c_n^{*(T)} \left( \tilde{d}_a^*(b', z), b', \mathbf{z}'_{z \times 1} \right) \right)}{\partial c} \right)$$

$$+ \beta \left( 1 - \nu^{*(T)} \right) \mathbf{P}_{\mathbf{z}} \psi^{(T)} \left( \tilde{d}_a^*(b', z), b', \mathbf{z}'_{z \times 1} \right)$$

$$(102)$$

If no solution exists, I set  $\tilde{d}_a^*(b',z) = -\underline{d}$ . Also, if solution exists, but if  $\tilde{d}_a^*(b',z) < -\underline{d}$  then I set  $\tilde{d}_a^*(b',z) = -\underline{d}$  and if  $\tilde{d}_a^*(b',z) > d_D$ , then I set  $\tilde{d}_a^*(b',z) = d_D$ .

8. Solve for consumption for a given bond choice b' (on-grid) when there are bond adjustments  $c_a^{\tilde{r}(n)}(b',z)$  using the following condition:

$$\frac{\partial u\left(c_{a}^{*(\tilde{T}-1)}\left(b',z\right)\right)}{\partial c} = \beta\left(1 + r_{T-1}^{d}\right)\left(\nu^{*(T)}\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_{a}^{*(T)}\left(\tilde{d}_{a}^{*}(b',z),b',z'\right)\right)}{\partial c} + \left(1 - \nu^{*(T)}\right)\mathbf{P}_{\mathbf{z}}\frac{\partial u\left(c_{n}^{*(T)}\left(\tilde{d}_{a}^{*}(b',z),b',z'\right)\right)}{\partial c}\right)$$

$$(103)$$

Next, find the total expenses  $\mathcal{R}_a(b',z)$  which are consistent to total resources

$$\mathcal{R}_a(b',z) := c_a^{*(\tilde{T}-1)}(b',z) + d_a^{*(\tilde{T}-1)}(b',z) + b' - wz\bar{l} - \Pi(z)$$

$$\tilde{\mathcal{R}}_{a}\left(d\left(b',z\right),b\left(b',z\right)\right) = \left(1 + r_{T-1}^{d}\right)d\left(b',z\right) + \left(1 + r_{T-1}^{b}\right)b\left(b',z\right)$$

Interpolating  $c_a^{*(\tilde{T}-1)}\left(b',z'\right)$ ,  $d_a^{*(\tilde{T}-1)}\left(b',z'\right)$  at  $\tilde{\mathcal{R}}_a\left(d,b,z\right)$  that is consistent with  $\tilde{\mathcal{R}}_a\left(d\left(b',z\right),b\left(b',z\right)\right)$ , I get policy functions  $c_a^{*(T-1)}\left(d,b,z\right)$  and  $d_a^{*(T-1)}\left(d,b,z\right)$ .

9. Update  $\psi$  such that

$$\psi^{(T-1)}(d,b,z) = \beta \nu^{*(T)} \left(1 + r_{T-1}^{b}\right) \mathbf{P}_{\mathbf{z}} \frac{\partial u\left(c_{a}^{*(T)}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)\right)}{\partial c}$$

$$+\beta \left(1 - \nu^{*(T)}\right) \left(1 + r_{T-1}^{b}\right) \mathbf{P}_{\mathbf{z}} \frac{\partial u\left(c_{n}^{*(T)}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)\right)}{\partial c}$$

$$+\beta \left(1 - \nu^{*(T)}\right) \mathbf{P}_{\mathbf{z}} \psi^{(T)}\left(d_{n}^{*}\left(d,b,z\right),b,z'\right)$$

$$(104)$$

10. I update the value functions using the following conditions:

$$V_n^{(T-1)} = u\left(c_n^{*(T-1)}\right) + \beta \mathbb{E}V\left(d_n^{*(T-1)}, b^{(T-1)}, z'\right)$$

$$V_a^{(T-1)} = u\left(c_a^{*(T-1)}\right) + \beta \mathbb{E}V\left(d_a^{*(T-1)}, b^{*(T-1)}, z'\right)$$

$$\mathbb{E}V = \nu^{*(T)}V_a^{(T-1)} + \left(1 - \nu^{*(T)}\right)V_n^{(T-1)} - \int_0^{V_a^{(T-1)} - V_n^{(T-1)}} \chi f(\chi)d\chi$$
(105)

where the expected adjustment cost is defined by  $AC(\nu; \mu_x, \sigma_x) = \int_0^{V_a^{(T-1)} - V_n^{(T-1)}} \chi f(\chi) d\chi = \mu_x \nu + \sigma_x \left(\nu \ln \nu + (1-\nu) \ln (1-\nu)\right) - C.$ 

- 11. Finally, update probabilities of adjustment  $\nu^{*(T-1)} = F_{\chi} \left[ V_a^{(T-1)} V_n^{(T-1)} \right]$  since adjustment costs are distributed logistically.
- 12. Update probabilities of adjustment and iterate until convergence of policies.
- 13. Repeat Step 6-12 for each t = 1, 2, 3, ..., T 2.
- 14. Update the distribution of households for each t and compute the aggregate deposits  $D_t$  and bonds  $B_t$ . Then, compute firms' stock of assets  $K_{t+1}$  and networth  $N_t$ .
- 15. Update the return of capital  $R_t^{K,(n)}$ , return of deposits, bonds  $\{r_t^{d,(n)}, r_t^{b,(n)}\}$ , and wages  $w_t^{(n)}$ .
- 16. Stopping rule: Iterate until  $\max\{\frac{||r_t^{d,(n)} r_t^{d,(n-1)}||}{||r_t^{d,(n-1)}||}, \frac{||r_t^{b,(n)} r_t^{b,(n-1)}||}{||r_t^{b,(n-1)}||}, \frac{||w_t^{(n)} w_t^{(n-1)}||}{||w_t^{(n-1)}||}\} < 1e^{-5}$