a) $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$

$$E(y_t) = E(\alpha + \beta y_{t-1} + \varepsilon_t) = \alpha + \beta E(y_{t-1}) + E(\varepsilon_t)$$

$$\mu = \alpha + \beta\mu + 0$$

$$\mu = \alpha/(1-\beta) \quad (\beta \neq 1 \text{ is required})$$

b) $\alpha = \mu(1-\beta) = \mu - \beta\mu$

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t = \mu - \beta\mu + \beta y_{t-1} + \varepsilon_t$$

$$y_t - \mu = \beta(y_{t-1} - \mu) + \varepsilon_t$$

$$z_t = \beta z_{t-1} + \varepsilon_t$$

$$E(z_t) = E(y_t - \mu) + E(y_t) - \mu = \mu - \mu = 0$$

$$\begin{aligned}
Var(z_t) &= E(z_t - E(z_t))^2 = E(z_t^2) \\
&= E((\beta z_{t-1} + \varepsilon_t)^2) = \beta^2 E(z_{t-1}^2) + E(\varepsilon_t^2) + 2\beta E(z_{t-1}\varepsilon_t) \\
&= \beta^2 var(z_t) + \sigma^2 + 0 \\
&= \sigma^2/(1-\beta^2) \quad (\text{requires } -1 < \beta < 1)
\end{aligned}$$

c)

$$\gamma_1 = E(z_t z_{t-1}) = E(\beta z_{t-1} + \varepsilon_t) z_{t-1}$$
$$= \beta E(z_{t-1}^2) + E(\varepsilon_t z_{t-1})$$
$$= \beta\gamma_0 + 0 = \beta\gamma_0 \quad \text{, hence} \quad \rho_1 = \gamma_1/\gamma_0 = \beta$$

$$\gamma_2 = E(z_t z_{t-2}) = E(\beta z_{t-1} + \varepsilon_t) z_{t-2} = \beta E(z_{t-1} z_{t-2}) + E(\varepsilon_t z_{t-2})$$
$$= \beta\gamma_1 + 0 = \beta(\beta\gamma_0) = \beta^2\gamma_0 \text{. hence} \quad \rho_2 = \gamma_2/\gamma_0 = \beta^2$$

$$\gamma_k = E(z_t z_{t-k}) = E(\beta z_{t-1} + \varepsilon_t) z_{t-k} = \beta\gamma_{k-1} + 0 = \beta\gamma_{k-1}$$
$$= \dots = \beta^k\gamma_0 \text{, so} \quad \rho_k = \beta^k$$

d) correlations are always between -1 and $+1$, so $|\beta| \leq 1$

$$\left.\begin{array}{l}\beta = 1 \text{ is excluded in part (a)} \\ \beta = -1 \text{ is excluded in part (b)}\end{array}\right\} \quad -1 < \beta < 1$$