a) $b_{2SLS} = (X' H_z X)^{-1} X' H_z y$

$H_z = Z(Z'Z)^{-1} Z'$

$m = k$

$b_{2SLS} = (\underbrace{X'Z}_{(k \times k)} (\underbrace{Z'Z}_{(k \times k)})^{-1} \underbrace{Z'X}_{(k \times k)})^{-1} X'Z(Z'Z)^{-1} Z'y$ $\longrightarrow$ have an inverse (for n large enough)

$\boxed{\text{Rule} \quad (ABC)^{-1} = C^{-1} B^{-1} A^{-1}}$

$b_{2SLS} = (Z'X)^{-1} (Z'Z)(X'Z)^{-1} (X'Z)(Z'Z)^{-1} (Z'y)$

$b_{2SLS} = (Z'X)^{-1} Z'y$

b) $m = k$

$$b_{2SLS} = (Z'X)^{-1} Z'y$$

$$Z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$Z'X = \sum z_i x_i \quad \text{and} \quad Z'y = \sum z_i y_i$$

$$b_{2SLS} = \frac{\sum z_i y_i}{\sum z_i x_i}$$

$$= \frac{\sum (z_i - \bar{z})(y_i - \bar{y})}{\sum (z_i - \bar{z})(x_i - \bar{x})} = \frac{\operatorname{cov}(y, z)}{\operatorname{cov}(z, x)}$$

$\Big\}$ factors $\frac{1}{n-2}$ cancel against each other

c) $$b_{2SLS} = \frac{Cov(y, z) \longrightarrow 0}{Cov(z, x) \longrightarrow 0} \rightarrow \text{2SLS} \neq \text{defined}$$

$$b_{2SLS} = \frac{0}{0}$$

practice → correlations will never be exactly equal to zero

consequence! may obtain any number as estimate when correlation between z and x is almost zero