a) $e_u = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} y_1 - X_1 b_1 \\ y_2 - X_2 b_2 \end{pmatrix}$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \left(\begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}' \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \right)^{-1} \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}' \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$= \begin{pmatrix} X_1'X_1 & 0 \\ 0 & X_2'X_2 \end{pmatrix}^{-1} \begin{pmatrix} X_1'y_1 \\ X_2'y_2 \end{pmatrix}$$

$$= \begin{pmatrix} (X_1'X_1)^{-1} X_1'y_1 \\ (X_2'X_2)^{-1} X_2'y_2 \end{pmatrix}$$

$e_u = \begin{pmatrix} y_1 - X_1 b_1 \\ y_2 - X_2 b_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$ $e_u'e_u = e_1'e_1 + e_2'e_2 = S_1 + S_2$

b) (1) $y_1 = X_1\beta + \varepsilon_1$

(2) $y_2 = X_2\beta + D\gamma + \varepsilon_2$ $\quad D = I_{n_2}$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1\beta + \varepsilon_1 \\ X_2\beta + D\gamma + \varepsilon_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ X_2 & D \end{pmatrix} \begin{pmatrix} \beta \\ \gamma \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_R \end{pmatrix}$$

$$F_{\{\gamma=0\}} = \frac{(e_R'e_R - e_u'e_u)/n_2}{e_u'e_u/(n_1+n_2-(k+n_2))} = \frac{(e_R'e_R - e_u'e_u)/n_2}{e_u'e_u/(n_1-k)}$$

$e_R'e_R = S_0$ n_1+n_2 model (1)

$$\min_{\{\beta,\gamma\}} \begin{pmatrix} y_1 - X_1\beta \\ y_2 - X_2\beta - D\gamma \end{pmatrix}' \begin{pmatrix} y_1 - X_1\beta \\ y_2 - X_2\beta - D\gamma \end{pmatrix} =$$

$$\min_{\{\beta,\gamma\}} (y_1 - X_1\beta)'(y_1 - X_1\beta) + (y_2 - X_2\beta - D\gamma)'(y_2 - X_2\beta - D\gamma)$$

$\hat{\beta} = (X_1'X_1)^{-1}X_1'y_1$ $\quad \hat{\gamma} = y_2 - X_2\hat{\beta}$

$e_1 = y_1 - X_1\hat{\beta}$ $\quad n_1$ obs

$e_2 = y_2 - X_2\hat{\beta} - D\hat{\gamma} = 0$ $\quad n_2$ obs

$e_u'e_u = e_1'e_1 + e_2'e_2 = e_1'e_1 = S_1$ $\quad n_1$ model (1)

$$F_{\{\gamma=0\}} = \frac{(S_0 - S_1)/n_2}{S_1/(n_1-k)}$$