a) $$\frac{\exp(b_1)}{1+\exp(b_1)} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\exp(b_1) = \frac{1}{n}\sum_{i=1}^{n} y_i + \exp(b_1)\frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\exp(b_1)(1 - \frac{1}{n}\sum_{i=1}^{n} y_i) = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\exp(b_1) = \frac{\sum_{i=1}^{n} y_i}{1-\frac{1}{n}\sum_{i=1}^{n} y_i} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(1-y_i)} \quad (1)$$

$$1 = \frac{1}{n}\sum_{i=1}^{n} 1$$

$$b_1 = \log\left(\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(1-y_i)}\right)$$

b) $$\hat{Pr}[y_i=1] = \frac{\exp(b_1)}{1+\exp(b_1)} = \frac{\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(1-y_i)}}{1+\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(1-y_i)}} \frac{\cdot \sum_{i=1}^{n}(1-y_i)}{\cdot \sum_{i=1}^{n}(1-y_i)}$$

$$\hat{Pr}[y_i=1] = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(1-y_i)+\sum_{i=1}^{n} y_i} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} 1} = \frac{\sum_{i=1}^{n} y_i}{n}$$

c) $$\hat{V} = \left(\sum_{i=1}^{n}\left(\frac{\exp(x_i'b)}{(1+\exp(x_i'b))}\right)\left(\frac{1}{(1+\exp(x_i'b))}\right)x_i x_i'\right)^{-1}$$

$$\hat{V} = \left(\sum_{i=1}^{n} \frac{\sum_{i=1}^{n} y_i}{n}\left(1-\frac{\sum_{i=1}^{n} y_i}{n}\right)11'\right)^{-1}$$

$$= \left(\frac{\sum_{i=1}^{n} y_i}{n}\left(1-\frac{\sum_{i=1}^{n} y_i}{n}\right)\sum_{i=1}^{n} 11'\right)^{-1}$$

$$= \left(\frac{\sum_{i=1}^{n} y_i}{n}\left(1-\frac{\sum_{i=1}^{n} y_i}{n}\right)n\right)^{-1}$$