a) F-test i.t.o $e'e$

we need F-test i.t.o R^2

$$R^2 = 1 - \frac{e'e}{SST}$$

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$e'e = SST(1 - R^2)$$

$e_0'e_0 = SST(1 - R_0^2)$ → restricted

$e_1'e_1 = SST(1 - R_1^2)$ → unrestricted

$$F = \frac{(e_0'e_0 - e_1'e_1)/g}{e_1'e_1/(n-k)} = \frac{(SST(1-R_0^2) - SST(1-R_1^2))/g}{SST(1-R_1^2)/(n-k)}$$

$$= \frac{(R_1^2 - R_0^2)/g}{(1-R_1^2)/(n-k)}$$

b) $t = \frac{b_j}{s\sqrt{a_{jj}}}$ a_{jj} is the (j,j)-th element of $(X'X)^{-1}$

to test $H_0: R\beta = r$:

$F = \frac{1}{s^2}(Rb-r)' V^{-1} (Rb-r)/g$, with $V = R(X'X)^{-1}R'$

to test $H_0: \beta_j = 0$, so with $g=1$ restriction, $R = (0 \ldots 0 1 0 \ldots 0)$, $r = 0$

$$V = R(X'X)^{-1}R' = (0 \ldots 0 1 0 \ldots 0)(X'X)^{-1}\begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = a_{jj}$$

$$F = \frac{1}{s^2}(b_j - 0)' \frac{1}{a_{jj}} (b_j - 0) = \frac{b_j^2}{s^2 a_{jj}} = t^2$$