Erasmus School of Economics

MOOC Econometrics

Training Exercise 5.2

Questions

Suppose that an individual has two choices, denoted by 0 and 1. Let u_i be an unobserved random variable that measures the difference in attractiveness (utility) between the choices 1 and 0. The attractiveness u_i is a linear function of explanatory variables x_{ji} with parameters β_j and an error term, that is,

$$u_i = \beta_1 + \sum_{j=2}^{k} \beta_j x_{ji} + \eta_i,$$

where η_i are independently and identically distributed random terms.

Assume that individual i chooses 1 when $u_i \geq 0$, and chooses 0 when $u_i < 0$. In other words, individual i chooses the most attractive alternative.

(a) Show that the probability that $y_i = 1$ equals $\Pr[-\eta_i \leq \beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}]$.

(b) Assume that the distribution of η_i is symmetric around 0 and hence η_i has the same distribution as $-\eta_i$. Furthermore, assume that η_i has a standard logistic distribution with cumulative distribution function

$$F(\eta_i) = \frac{\exp(\eta_i)}{1 + \exp(\eta_i)}.$$

Show that under this condition the probability that $y_i = 1$ is given by

$$\Pr[y_i = 1] = \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}{1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})}.$$

(c) Consider the logit probability given in (b). Show that

$$\frac{\beta_j}{\beta_l} = \frac{\partial \Pr[y_i = 1]}{\partial x_{ji}} \Big/ \frac{\partial \Pr[y_i = 1]}{\partial x_{li}}.$$

What is the implication of this result?

(d) (Optional) Use the chain rule to show that

$$\frac{\partial \Pr[y_i = 1]}{\partial x_{ji}} = \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})\beta_j}{1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})} - \frac{\exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}) \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji})\beta_j}{(1 + \exp(\beta_1 + \sum_{j=2}^{k} \beta_j x_{ji}))^2}$$

and show that you can rewrite the expression as

$$\frac{\partial \Pr[y_i = 1]}{\partial x_{ji}} = \Pr[y_i = 1] \Pr[y_i = 0] \beta_j$$