a) True relation : $y_i = \alpha + \beta x_i^* + u_i$

Test score $x_i = x_i^* + w_i$ (1)

OLS : $y_i = \alpha + \beta x_i + \varepsilon_i$

$$\alpha + \beta x_i^* + u_i = \alpha + \beta x_i + \varepsilon_i$$

$$\varepsilon_i = \beta(x_i^* - x_i) + u_i = -\beta w_i + u_i \quad (2)$$

b) Using (1) and (2) we can show that $cov(x_i, \varepsilon_i)$:

$$cov(x_i, \varepsilon_i) = cov(x_i^* + w_i, -\beta w_i + u_i)$$
$$= -\beta \underbrace{cov(x_i^*, w_i)}_{0} + \underbrace{cov(x_i^*, u_i)}_{0} - \beta\, cov(w_i, w_i) + \underbrace{cov(w_i, u_i)}_{0}$$

$$= -\beta\sigma_w^2$$

c) x_i is endogenous if $cov(x_i, \varepsilon_i) \neq 0$

Here: $\sigma_w^2 \neq 0$ and $\beta \neq 0$