

$$a) \quad b_{2sls} = (X' H_z X)^{-1} X' H_z y$$

$$H_z = z(z'z)^{-1}z'$$

$$m = k$$

$$b_{2sls} = \underbrace{(X'z)}_{(k \times k)} \underbrace{(z'z)^{-1}}_{(k \times k)} \underbrace{z'X}_{(k \times k)}^{-1} X'z (z'z)^{-1} z'y$$

→ have an inverse (for n large enough)

$$\boxed{\text{Rule } (ABC)^{-1} = C^{-1} B^{-1} A^{-1}}$$

$$b_{2sls} = (z'X)^{-1} (z'z) (X'z)^{-1} (X'z) (z'z)^{-1} (z'y)$$

$$b_{2sls} = (z'X)^{-1} z'y$$

$$b) \quad m = k$$

$$b_{2SLs} = (z'X)^{-1} z'y$$

$$z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$z'X = \sum z_i x_i \quad \text{and} \quad z'y = \sum z_i y_i$$

$$b_{2SLs} = \frac{\sum z_i y_i}{\sum z_i x_i}$$

$$= \frac{\sum (z_i - \bar{z})(y_i - \bar{y})}{\sum (z_i - \bar{z})(x_i - \bar{x})} = \frac{\text{cov}(y, z)}{\text{cov}(z, x)} \quad \left. \vphantom{\frac{\text{cov}(y, z)}{\text{cov}(z, x)}} \right\} \begin{array}{l} \text{factors } \frac{1}{n-2} \\ \text{cancel against each} \\ \text{other} \end{array}$$

c) $b_{2SLS} = \frac{\text{Cov}(y, z)}{\text{Cov}(z, x)}$

Diagram illustrating the case where both the numerator and denominator are zero:

- An arrow points from $\text{Cov}(y, z)$ to 0.
- An arrow points from $\text{Cov}(z, x)$ to 0.
- A curved arrow points from the 0 in the denominator to the text $2SLS \neq \text{defined}$.

$$b_{2SLS} = \frac{0}{0}$$

practice \rightarrow correlations will never be exactly equal to zero
 consequence! may obtain any number as estimate when
 correlation between z and x is almost zero