a) let $\hat{y}_i = x_i' b$ → expl. part of log-wage, from the model with c, Female, Age, Educ, parttime

$e_i = y_i - \hat{y}_i$, where $y_i = \log(\text{wage}_i)$

$e_i > 0$: actual wage > predicted
$e_i < 0$: actual wage < predicted

For educ level = 1 , $DE_2 = DE_3 = DE_4 = 0$, so

$e_i = 0.03 + res_i$

$\overline{res} = 0$
$\bar{e} = 0.03$
} meaning: actual wage is about 3% higher than predicted by model

For educlevel = 2 , $DE_2 = 1$ and $DE_3 = DE_4 = 0$, so

$e_i = 0.03 - 0.06 + res_i = -0.03 + res_i$

$\overline{res} = 0$
$\bar{e} = -0.03$
} meaning: actual wage is about 3% lower than predicted by model

For educ level = 3 : actual wage is about 6% lower than predicted

For educlevel = 4 : " " " " 9% higher " "

b) If $\beta_2 = \beta_3 = \beta_4 = 0$, the model becomes

$e_i = \beta_1 + res_i$

$e = X\beta_1 + res$, where $X = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$ $(n \times 1)$

OLS with X and with $y = e$ gives

$$\hat{\beta}_1 = (X'X)^{-1} X'y = \left((1 \dots 1) \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \right)^{-1} (1 \dots 1) \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix}$$

$$= \frac{1}{n} \sum_{i=1}^{n} e_i = \bar{e}$$

$res_i = e_i - \hat{\beta}_1 = e_i - \bar{e}$

$$\left.\begin{array}{l} SSR = \sum (e_i - \bar{e})^2 \\ SST = \sum (e_i - \bar{e})^2 \end{array}\right\} SSR = SST$$

$$R_0^2 = 1 - \frac{SSR}{SST} = 1 - 1 = 0$$

F-test with $R_1^2 = 0.04$, $R_0^2 = 0$, $g = 3$, $n = 500$, $k = 4$

$$F = \frac{(R_1^2 - R_0^2)/g}{(1-R_1^2)/(n-k)} = \frac{R_1^2/3}{(1-R_1^2)/496} = \frac{0.04/3}{0.96/496} = 6.89$$

As $F = 6.89 > 2.6$, we reject H_0

c) The model with fixed educ. level effects gives systematically biased wage forecasts per educ. level

- $y_i > \hat{y}_i$ (level 1 & 4)
- $y_i < \hat{y}_i$ (level 2 & 3)

Fixed educ. level effect : 26%

per educ. level:

1 → 2 : 19% (< 26%)

2 → 3 : 23% (< 26%)

3 → 4 : 47% (> 26%)

d) $X'e = 0$

$e = res$

$$X = \begin{array}{cl} \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 1 \end{array} \right) & \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{educ level} = 1 \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \quad \text{"} \qquad = 2 \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \quad \text{"} \qquad = 3 \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \quad \text{"} \qquad = 4 \end{array} \end{array}$$

Let $r_i = res_i$ $\quad$ r $(n \times 1)$ vector

$$X'r = \begin{pmatrix} \Sigma r_i \\ \Sigma_2 r_i \\ \Sigma_3 r_i \\ \Sigma_4 r_i \end{pmatrix} = 0$$

$$\Sigma r_i = \Sigma_1 r_i + \Sigma_2 r_i + \Sigma_3 r_i + \Sigma_4 r_4$$

$$0 = \underbrace{\Sigma_1 r_i}_{0} + 0 + 0 + 0$$