A Discrete Sliding Mode Control of a Buck-Boost Inverter

Jose Matas, Luis Garcia de Vicuña, Josep M. Guerrero*, Jaume Miret, Oscar López

Departament d'Enginyeria Electrónica. Universitat Politècnica de Catalunya
Av. Víctor Balaguer S/N. 08800 – Vilanova i la Geltrú, Spain. matas@eel.upc.es

(*) Departament d'Enginyeria de Sistemes, Automática i Informàtica Industrial.
Escola d'Enginyeria Técnica Industrial de Barcelona. Universitat Politècnica de Catalunya
C/Comte d'Urgell 187. 08036 – Barcelona, Spain.

Abstract – **This paper proposes the design of a discrete sliding control of a buck-boost inverter. The sliding control surfaces are designed by imposing a desired dynamic behavior on the system, which allows us to determine the main parameters of the sliding mode controller, and is specially interesting in tracking problems. This procedure leads to discrete-time switching surfaces, which provide robustness with regard to external disturbances, and a good dynamic response of the output voltage.**

I. INTRODUCTION

Power Electronics systems can be classified as variable structure systems for the fact that the structure of the converter changes with every control action. In variable structure systems, the sliding mode theory was initially applied to system stabilization or regulation type problems. Nevertheless, this technique has already been extended to other issues like tracking problems [1].

The use of the sliding mode control is well known in many applications of power electronics, and most of the research work is based on continuous-time sliding mode control. The drawbacks in this kind of control are due to the hysteresis type control and the consequent variable frequency operation of the converter. However, the discrete time implementation has attractive features as fixed frequency operation and versatility in control.

The motivation of this work is the study of a discrete-time sliding control of a MIMO system as the buck-boost dc-ac converter, which is based on two buck-boost dc-dc converters with a load differentially connected between them [2-5].

The systematic general method to design the sliding mode control is presented in section II. In section III this method is applied to a dc-ac buck-boost converter. Finally, the simulation results for output voltage tracking are presented in section IV.

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II. SLIDING MODE CONTROL

Consider the following MIMO system:

$$\begin{aligned} x(k+1) &= f(x(k),u(k)) \\ y_i(k) &= h_i(x(k)) \qquad \text{for } i=1,2,\dots,m \end{aligned} \tag{1}$$

where $x(k)$ is an analytic n-dimensional manifold, $u(k)$ is a m-dimensional control vector $[u_1(k),\dots,u_m(k)]^T$, $y(k)=[y_1(k),\cdots,y_m(k)]^T$ is the output vector, $f: \mathrm{M} \times \mathrm{R}^m \to \mathrm{M}$, and h_i are analytic functions with respect to their arguments. The multivariable discrete-time system (1) has a vector relative degree $r=(r_1,r_2\dots,r_m)$ at the equilibrium pair (x_o, u_o)=(0, 0) if:

$$\frac{\partial}{\partial u_j} h_i \circ f^k(x,u) = 0 \tag{2}$$

for all $1 \le j \le m$, for all $k < r_i - 1$, for all $1 \le i \le m$, and for all x in a neighborhood of x_o, and if the matrix

$$\begin{pmatrix} \frac{\partial}{\partial u_1} h_1 \circ f^{r_1-1} & \cdots & \frac{\partial}{\partial u_m} h_1 \circ f^{r_1-1} \\ \frac{\partial}{\partial u_1} h_2 \circ f^{r_2-1} & \cdots & \frac{\partial}{\partial u_m} h_2 \circ f^{r_2-1} \\ \vdots & \vdots & \vdots \\ \frac{\partial}{\partial u_1} h_m \circ f^{r_m-1} & \cdots & \frac{\partial}{\partial u_m} h_m \circ f^{r_m-1} \end{pmatrix} (x_o, u_o) \tag{3}$$

is nonsingular [4]. From (2) and (3), we can derive that the relative degree r_i is the number of times one has to delay the i-th output, y_i(k), in order to have al least one component of the input vector $u(k)$ explicitly appearing.

If the nonlinear system (1) has a vector relative degree $r=(r_1,r_2,\ldots,r_m)$ such that $r_1+r_2+\ldots+r_m=n$, where n is the dimension of the state space, it can be changed into a lineal system by the following local coordinates transformation [6]:

$$\begin{aligned}\phi_1^i(x)&=h_i(x)\\ \phi_2^i(x)&=h_i\circ f(x)\\ &\vdots\\ \phi_{r_i}^i(x)&=h_i\circ f^{r_i-1}(x)\end{aligned}\qquad \text{for}\quad 1\le i\le m \tag{4}$$

Setting

$$\xi^i(k)=col\left(\xi_1^i(k),\xi_2^i(k),\cdots,\xi_{r_i}^i(k)\right)==col\left(\phi_1^i(x),\phi_2^i(x),\cdots,\phi_{r_i}^i(x)\right)$$

for all $1\le i\le m$, the system dynamics can be transformed into a normal form:

$$\begin{aligned}&\xi_1^i(k+1)=\xi_2^i(k)\\ &\quad\ldots\\ &\xi_{r_i-1}^i(k+1)=\xi_{r_i}^i(k)\\ &\xi_{r_i}^i(k+1)=\psi_i(\xi^i(k),u(k))\\ &y_i(k)=\xi_1^i(k)\end{aligned}\qquad \text{for } i=1,\cdots,m \tag{5}$$

The relationships between the future outputs of the system (1) and the states of (5) may be derived as:

$$\begin{aligned}&y_i(k)=\phi_1^i(x)=h_i(x(k))\\ &y_i(k+1)=\phi_2^i(x)=h_i\circ f(x(k))\\ &\vdots\\ &y_i(k+r_i-1)=\phi_{r_i-1}^i(x)=h_i\circ f^{r_i-1}(x(k))\end{aligned}\qquad \text{for } i=1,\ldots,m \tag{6}$$

In a similar manner like it is described in [7-9], a discrete sliding mode control can be designed to make the system outputs y_i to satisfy the following r_i-order dynamic equation:

$$y_i(k+r_i)+\sum_{j=0}^{r_i-1}\lambda_{i,j}\cdot y_i(k+j)-y_{i_{ref}}(k)=0\qquad \text{for}\quad i=1,\ldots,m \tag{7}$$

where $y_{i_{ref}}(k)$ are the desired references. Taking into account the future outputs of $y_{i_{ref}}(k)$ the outputs tracking error equations can be derived as:

$$e_i(k+r_i)+\lambda_{i,r_i-1}\cdot e_i(k+r_i-1)+\cdots+\lambda_{i,0}\cdot e_i(k)=0\qquad \text{for}\quad i=1,\ldots,m \tag{8}$$

where $e_i(k)=y_i(k)-y_{i_{ref}}(k)$ are the output tracking errors. If the coefficients $\lambda_{i,0},\cdots,\lambda_{i,r_i-1}$ are chosen such that the z-polynomials

$$z^{r_i}+\lambda_{i,r_i-1}\cdot z^{r_i-1}+\cdots+\lambda_{i,0}=0\qquad \text{for}\quad i=1,\ldots,m \tag{9}$$

have all their zeros inside the unit circle in the z-plane, then all the outputs $y_i(k)$ of the system will track asymptotically the desired reference outputs $y_{i_{ref}}(k)$:

$$\lim_{k\to\infty}\left(y_i(k)-y_{i_{ref}}(k)\right)=0 \tag{10}$$

The relation (7) must be satisfied when the discrete sliding regime is achieved. This regime can be identified with the invariance conditions [10]:

$$s_i(k+1)-s_i(k)=0\qquad \text{for}\quad i=1,\ldots,m \tag{11}$$

The sliding surfaces, $s_i(k)$, can be deduced by identifying (11) with (7):

$$s_i(k+1)-s_i(k)=y_i(k+r_i)+\sum_{j=0}^{r_i-1}\lambda_{i,j}\cdot y_i(k+j)-y_{i_{ref}}(k)\qquad \text{for}\quad i=1,\ldots,m \tag{12}$$

III. BUCK-BOOST INVERTER

The buck-boost inverter [2] is composed by two bi-directional buck-boost dc-dc converters, with a load, R, differentially connected across the converters, see Fig. 1. This circuit generates an AC output voltage by modulating in each boost dc-dc converter a sinusoidal output voltage, with a 180° out of phase with respect the other and with the same dc-bias. A DC bias appears at each end of the load with respect to ground, so that differential DC voltage across the load is zero.

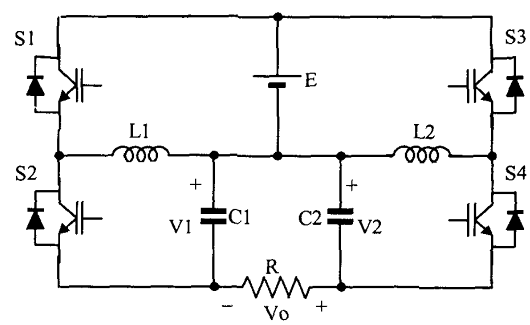

Fig. 1: DC-AC Buck-boost converter.

Assuming that the two buck-boost dc-dc converters are 180° out of phase, their respective output voltages *V1(t)* and *V2(t)* are defined as:

$$V_1(t) = V_{DC} + \frac{A}{2}\sin(\omega \cdot t) \qquad (13)$$

$$V_2(t) = V_{DC} - \frac{A}{2}\sin(\omega \cdot t) \qquad (14)$$

Thus, the differential voltage across the load is defined as:

$$Vo(t) = V_1(t) - V_2(t) = A \cdot \sin(\omega \cdot t) \qquad (15)$$

The buck-boost dc-ac converter, Fig. 1, is a four-order MIMO system with two control inputs, $u_1(t)$ and $u_2(t)$, and two outputs, $V_1(t)$ and $V_2(t)$, whose dynamic behavior can be described by the following bilinear state-space equation:

$$\dot{x}(t) = f(x) + g(x) \cdot u \qquad (16)$$

Applying the Forward difference method, this bilinear system can be discretized into:

$$x[(k+1)T] = f(x(kT)) + g(x(kT)) \cdot u(kT) \qquad (17)$$

where $x(kT) = [i_1(kT), V_1(kT), i_2(kT), V_2(kT)]^T$ is the state vector, T is the sampling period, $f(x(kT))$ and $g(x(kT))$ are:

$$f(x(kT)) = \begin{pmatrix} i_1(kT) - \frac{T}{L_1}V_1(kT) \\ \frac{T}{C_1}i_1(kT) + V_1(kT)\left(1 + \frac{T}{RC_1}\right) - \frac{T}{RC_1}V_2(kT) \\ i_2(kT) - \frac{T}{L_2}V_2(kT) \\ \frac{T}{C_2}i_2(kT) + V_2(kT)\left(1 + \frac{T}{RC_2}\right) - \frac{T}{RC_2}V_1(kT) \end{pmatrix}$$

$$g(x(kT)) = T \cdot \begin{pmatrix} \frac{E + V_1(kT)}{L_1} & 0 \\ -\frac{i_1(kT)}{C_1} & 0 \\ 0 & \frac{E + V_2(kT)}{L_2} \\ 0 & -\frac{i_2(kT)}{C_2} \end{pmatrix}$$

and $u(kT) = |u_1(kT) \quad u_2(kT)|$ is the input vector where the control actions $u_1(kT)$ and $u_2(kT)$ can take the following values:

$$u_1(kT) = \begin{cases} 0 & \text{when S1 is OFF and S2 is ON} \\ 1 & \text{when S1 is ON and S2 is OFF} \end{cases}$$

$$u_2(kT) = \begin{cases} 0 & \text{when S3 is OFF and S4 is ON} \\ 1 & \text{when S3 is ON and S4 is OFF} \end{cases}$$

For this converter, by taking the outputs functions as:

$$\phi_1^1(kT) = y_1(kT) = V_1(kT)$$
$$\phi_1^2(kT) = y_2(kT) = V_2(kT)$$

the obtained vector relative degree is $k = (1, 1)$ and then the dynamic imposition method can not be applied to this system. In order to avoid this obstacle, the vector relative degree can be extended to $r = (2, 2)$, if we take the following functions as an output function for each converter:

$$\phi_1^1(kT) = y_1(kT) = \frac{C_1}{2}V_1(kT)(2E + V_1(kT)) + \frac{L_1}{2}(i_1(kT))^2 \qquad (18)$$

$$\phi_1^2(kT) = y_2(kT) = \frac{C_2}{2}V_2(kT)(2E + V_2(kT)) + \frac{L_2}{2}(i_2(kT))^2 \qquad (19)$$

From these relations can be easily deduced:

$$\phi_2^1(kT) = y_1(kT+T) = V_1(kT)(E + V_1(kT))C_1\left(1 - \frac{T}{RC_1}\right) + \qquad (20)$$
$$+ i_1(kT)^2 L_1 + i_1(kT)ET - V_1(kT)V_2(kT)\frac{T}{R} - \frac{ET}{R}V_2(kT)$$

$$\phi_2^2(kT) = y_2(kT+T) = V_2(kT)(E + V_2(kT))C_2\left(1 - \frac{T}{RC_2}\right) + \qquad (21)$$
$$+ i_2(kT)^2 L_2 + i_2(kT)ET - V_2(kT)V_1(kT)\frac{T}{R} - \frac{ET}{R}V_1(kT)$$

Then, the vector relative has been extended to degree r = (2, 2), since relations (20) and (21) do not have any component of the input vector *u(k)*. According to (7), the system can satisfy the following second order dynamic difference equations:

$$e_1(k+2T) + \lambda_{1,1} \cdot e_1(k+T) + \lambda_{1,0} \cdot e_1(kT) = 0 \qquad (22)$$

$$e_2(k+2T) + \lambda_{2,1} \cdot e_2(k+T) + \lambda_{2,0} \cdot e_2(kT) = 0 \qquad (23)$$

where $e_1(kT)$ and $e_2(kT)$ are the system tracking errors defined as:

$$e_1(kT) = y_1(kT) - g_1(kT) \qquad (24)$$

$$e_2(kT) = y_2(kT) - g_2(kT) \qquad (25)$$

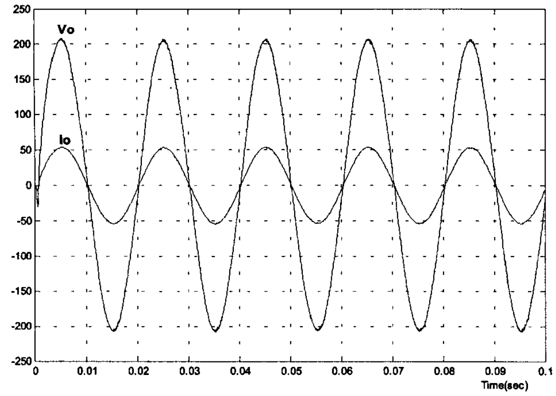

Fig. 2: Output voltage Vo and output current Io transient responses. (Vo 50 V/div, Io 2A/div).

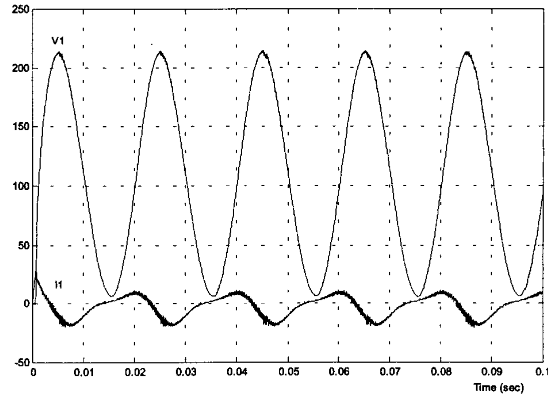

Fig. 3: Voltage across the capacitor C_1 and inductor current I_1 transient responses. (V_1 50 V/div, I_1 1 A/div).

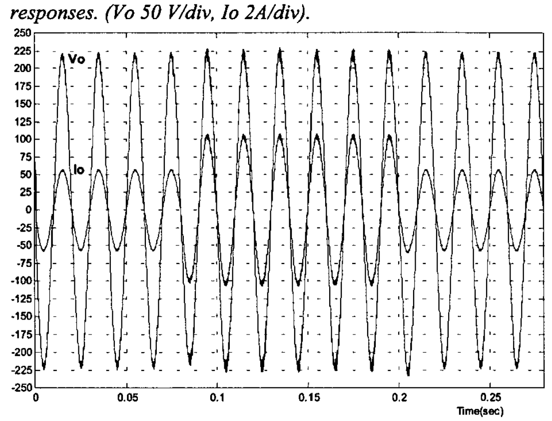

Fig. 4. Output voltage V_o and output current I_o transient responses for a sudden change in the load from 96Ω to 43Ω and then back to 96Ω. (V_o 25 V/div, I_o 1A/div).

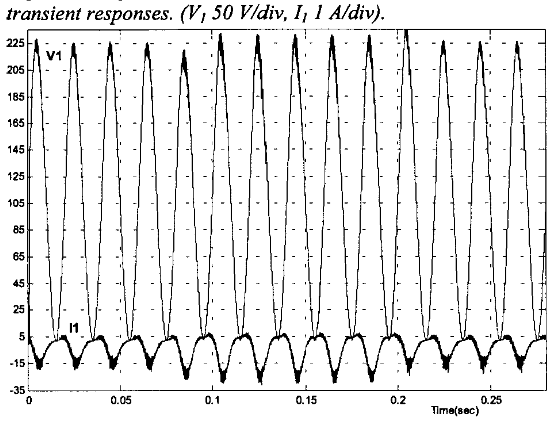

Fig. 5. Voltage V_1 and inductor current I_1 transient responses for a sudden change in the load from 96Ω to 43Ω and then back to 96Ω.

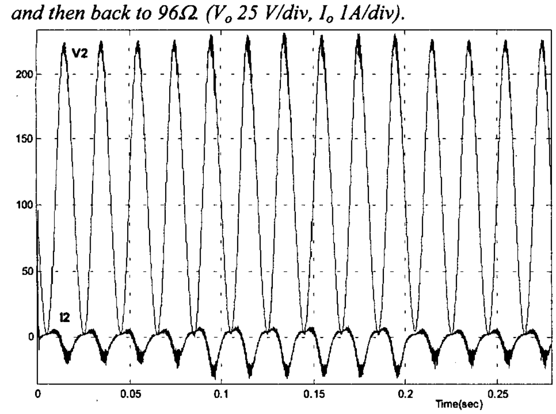

Fig. 6. Voltage V_2 and inductor current I_2 transient responses for a sudden change in the load from 96Ω to 43Ω and then back to 96Ω.

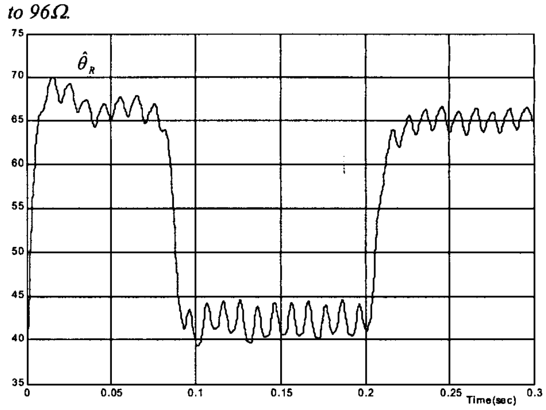

Fig. 7. Estimated $\hat{\theta}_R$ transient response for a sudden change in the load from 96Ω to 43Ω and then back to 96Ω.

and $g_1(kT)$ and $g_2(kT)$ are the reference output functions:

$$g_1(kT) = \frac{C_1}{2} V_{1ref}(kT)^2 + \frac{L_1}{2} i_{1ref}(kT)^2 \tag{26}$$

$$g_2(kT) = \frac{C_2}{2} V_{2ref}(kT)^2 + \frac{L_2}{2} i_{2ref}(kT)^2 \tag{27}$$

where the inductor current references of (26) and (27) can be approximated by its value when the system (17) is in steady state, and $V_1(kT) \cong V_{1ref}(kT)$ and $V_2(kT) \cong V_{2ref}(kT)$:

$$g_1(kT) = \frac{C_1}{2} V_{1ref}(kT)^2 + \frac{L_1}{2} \left(\frac{V_{2ref} - V_{1ref}}{E \cdot R} \right)^2 \left(E + V_{1ref} \right)^2 \tag{28}$$

$$g_1(kT)=\frac{C_1}{2}V_{1ref}(kT)^2+\frac{L_1}{2}\left(\frac{V_{1ref}-V_{2ref}}{E\cdot R}\right)^2\left(E+V_{2ref}\right)^2 \quad (29)$$

Finally, the sliding surfaces can be obtained by identifying the invariance conditions $s_1(kT+T)-s_1(kT)=0$ and $s_2(kT+T)-s_2(kT)=0$ with (22) and (23), respectively:

$$\begin{aligned} &s_1(kT+T)= \\ &=s_1(kT)+e_1(kT+2T)+\lambda_{1,1}\cdot e_1(kT+T)+\lambda_{1,0}\cdot e_1(kT) \end{aligned} \quad (30)$$

$$\begin{aligned} &s_2(kT+T)= \\ &=s_2(kT)+e_2(kT+2T)+\lambda_{2,1}\cdot e_2(kT+T)+\lambda_{2,0}\cdot e_2(kT) \end{aligned} \quad (31)$$

IV. SIMULATION RESULTS

The discrete sliding surfaces (30) and (31) have been simulated for the buck-boost dc-ac converter with the following parameters:

V_o=160Vrms, E=50V, f_o=50Hz, L_1=L_2=1mH, C_1=C_2=47μF, and R=96Ω.

The output voltages V1(t) and V2(t) must follow the reference signals:

$$V_{1ref}(kT)=110+110\sin(2\pi f_o\cdot kT)$$
$$V_{2ref}(kT)=110-110\sin(2\pi f_o\cdot kT)$$

The switching frequency has been set to 25 kHz, and the parameters of the second order difference equations (22) and (23) have been selected with one pole at the origin $z_1=1$, and the second pole at $z_2=0.78$, for both equations. Fig. 2 shows the transient responses of the output voltage Vo, and of the output current Io with a resistive load R=30Ω. Fig. 3 shows the transient responses of the voltage V1 and the inductor current I1 with a resistive load R=30Ω.

In order to achieve load regulation, the value of R in (28) and (29) can be estimated from the following adaptive law:

$$\begin{aligned} &\hat{\theta}_R(kT+T)= \\ &=\hat{\theta}_R(kT)-\gamma\cdot T\cdot\left(Vo(kT)-Voref(kT)\right)\cdot \operatorname{sgn}\left(\frac{i_1(kT)}{E+V_1(KT)}\right) \end{aligned} \quad (32)$$

where γ is a designer chosen constant that fixes the adaptation speed, which is set to $\gamma=650$. The estimated value of R is affected by noise, due to the switching frequency and to the output ac voltage, and has been filtered with a discrete first order IIR low-pass filter with a 50Hz cutoff frequency. The expressions (28) and (29), by denoting $\hat{\theta}_R(kT)$ as the -filtered estimate of the load, are:

$$g_1(kT)=\frac{C_1}{2}V_{1ref}(kT)^2+\frac{L_1}{2}\left(\frac{V_{2ref}-V_{1ref}}{E\cdot\hat{\theta}_R(kT)}\right)^2\left(E+V_{1ref}\right)^2 \quad (33)$$

$$g_1(kT)=\frac{C_1}{2}V_{1ref}(kT)^2+\frac{L_1}{2}\left(\frac{V_{1ref}-V_{2ref}}{E\cdot\hat{\theta}_R(kT)}\right)^2\left(E+V_{2ref}\right)^2$$

(34)

This system has been simulated for a sudden change in the load from 96Ω to 43Ω and then back to 96Ω. The following figures depict the transient responses of the output voltage Vo, the output current I_o, the voltages V_1 and V_2, the inductor currents I_1 and I_2, and the filtered estimate $\hat{\theta}_R(kT)$.

V. CONCLUSIONS

In this paper the design of a discrete sliding control of a dc-ac buck-boost converter for the output tracking of two sinusoidal references has been proposed. The design of the discrete sliding surfaces is based on the method of dynamic imposition. This method is applied to a dc-ac buck-boost converter, which provides the system with the following characteristics: fast transient response, absence of steady-state errors in the tracking of the sinusoidal references, and robust performance in relation to input voltage and load disturbances.

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