Modeling and Sliding Mode Control of Dc-Dc Buck-Boost Converter

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Abstract**—** In this paper a dc-dc buck-boost converter is modeled and controlled using sliding mode technique. First the buck-boost converter is modeled and dynamic equations describing the converter are derived and sliding mode controller is designed. The robustness of the converter system is tested against step load changes and input voltage variations. Matlab/Simulink is used for the simulations. The simulation results are presented..

Keywords**—**Buck-boost converter, dc-dc converter, sliding mode control

I. INTRODUCTION

Dc-dc converters are widely used in most of the power supply systems such as computers, aircrafts and electronic equipment. They are the most efficient way to implement actuators for electromechanical systems. A dc-dc converter must provide a regulated dc output voltage even subjected to load and input voltage variations.

The control of output voltage of dc-dc converters especially boost and buck-boost type converters is more difficult. The difficulty in the control of this types of converters are due to their non-minimum phase structure that is, the control input appears both in voltage and current equations. Controlling the current can indirectly control the output voltage.

Since the dc-dc converters are nonlinear and time invariant system, the linear conventional control techniques applied for the control of these converters are not suitable. Variation of system parameters and large signal transients produced during start up or against changes in the load cannot be handled with these techniques.

A control technique suitable for dc-dc converters must cope with their intrinsic nonlinearity and wide input voltage and load variations, ensuring stability in any operating condition while providing fast transient response [1]. Therefore, nonlinear and advanced non-conventional robust control structures to improve the performance of the dc-dc converters became the attractive research topic of the researchers.

Sliding Mode Controllers (SMC) are well known for their robustness and stability [2]. It is a nonlinear control approach which complies with the nonlinear characteristic inherent in the dc-dc converters.

Application of sliding mode for dc-dc converters is promising because a switching control strategy is employed in the converters [3].

The sliding mode control of dc-dc converters is simple to implement and it is studied in many papers [4-6]. It is robust against changes in the load and uncertain system parameters, and characterized by a good dynamic response [7].

II. BUCK-BOOST CONVERTER

A buck-boost converter provides an output voltage which can be controlled above and below the input voltage level. The output voltage polarity is opposite to that of the input voltage. The converter topology is shown in Figure 1. The circuit consists of a dc input voltage source (*E*), a controlled switch (*S*), a diode (*D*), a filter inductor (*L*), a filter capacitor (*C*) and a load resistor (*R*).

When the switch (*S*) is on for a time *DT*, *D* is the duty ratio and *T* is the period, the switch conducts the inductor current and the diode becomes reverse biased. A voltage $V_L=E$ occurs across the inductor. This voltage causes an increase in the inductor current I_L. When the switch is turned off, the inductor current I_L continues to flow because of the stored energy in the inductor. This current flows through the diode, and the voltage across the inductor becomes $V_L=-V_o$ for the time *(1-D)T* until the switch is turned on again. The voltage and current wave shapes are shown in Figure x for times *DT* and *(1-D)T*.

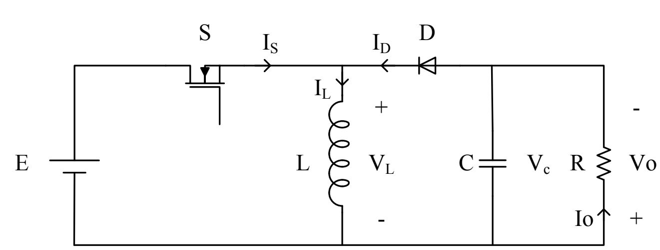

Figure 1: Buck-boost converter circuit

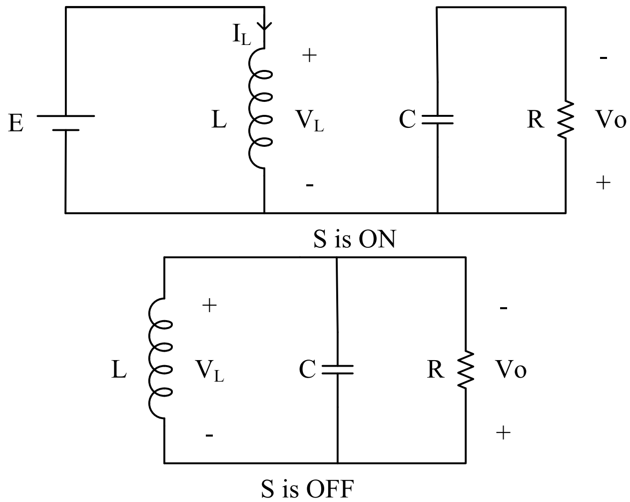

Figure 2: The converter circuit when the switch is on and off

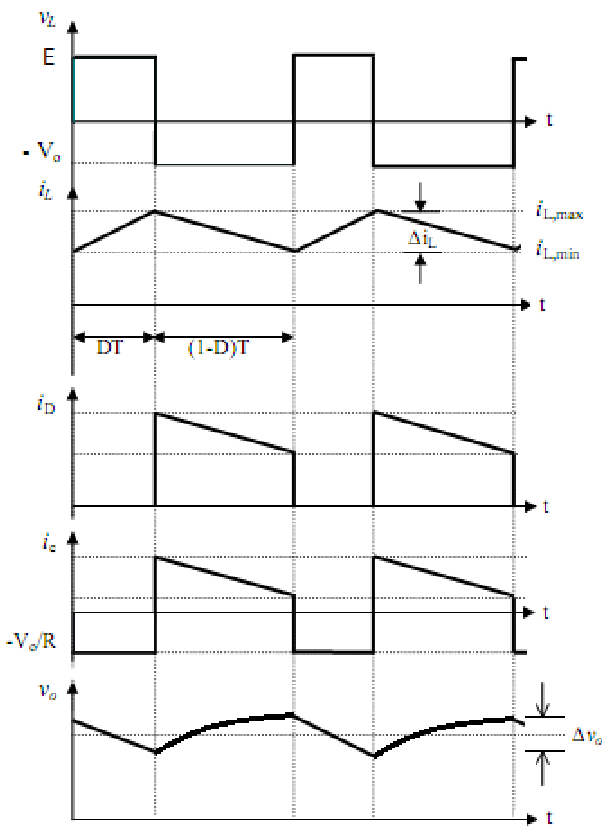

Figure 3: The voltages and currents waveforms in a buck-boost converter

The mathematical model of the buck-boost type dc-dc converter in state space form is obtained by application of basic laws governing the operation of the system. The circuit is redrawn in Figure x for obtaining the state equations with the control input (u) of the converter. The control input u, representing the switch position function, is a discrete-valued signal taking values in the set {0;1}. The dynamics of this converter is obtained by assuming the converter is operating in the continuous conduction mode.

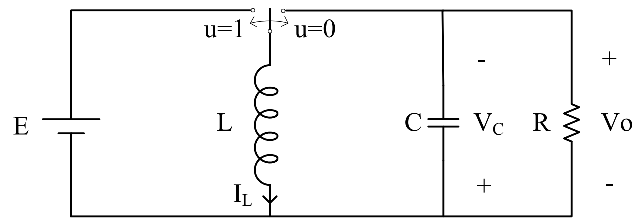

Figure 4: Buck-boost converter circuit is redrawn with control input u

When the switch is on ($u=1$) the system is linear and the state equations can be written as

$$\frac{di_L}{dt} = \frac{E}{L} \tag{1}$$

$$\frac{dv_c}{dt} = -\frac{V_o}{RC} \tag{2}$$

When the switch is off ($u=0$), the system is also linear and the state equations are

$$\frac{di_L}{dt} = \frac{v_c}{L} \tag{3}$$

$$\frac{dv_c}{dt} = -\frac{i_L}{C} - \frac{v_c}{RC} \tag{4}$$

Combining these two set of equations with the control input u, gives the following state equations of the buck-boost converter.

$$\frac{di_L}{dt} = (1-u)\frac{v_c}{L} + \frac{E}{L}u \tag{5}$$

$$\frac{dv_c}{dt} = -(1-u)\frac{i_L}{C} - \frac{v_c}{RC} \tag{6}$$

Figure 5 shows the block diagram representing the solution of the state equations given by equation (5-6).

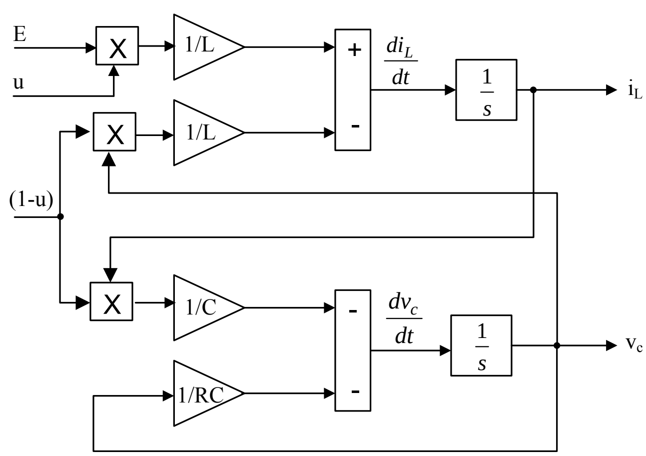

Figure 5: Block diagram of the buck-boost converter

III. SLIDING MODE DESIGN METHODOLOGY

The sliding mode theory provides a method to design a controller for a system so that the controlled system is to be insensitive to parameter variations and external load disturbances [5, 6]. The details of the Sliding Mode control method, equivalent control and application to converters can be found in references [8-12]. The approach is realized by the use of a high speed switching control law which forces the trajectory of the system to move to a predetermined path in the state variable space called Sliding Surface and to stay in that surface thereafter. Before the system reaches the switching surface, there is a control directed towards the switching surface which is called reaching mode. The regime of a control system in the sliding surface is called Sliding Mode. In sliding mode a system's response remains insensitive to certain parameters variations and unknown disturbances.

One of the main features of this method is that one only needs to drive the error to a switching surface, after which the system is in sliding mode and robust against modeling uncertainties and disturbances [13, 14]. A Sliding Mode Controller is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that map plant state to a control surface, and the switching among different functions is determined by plant state that is represented by a switching function.

If the following linear time invariant system state equation is given

$$\dot{x}(t) = Ax(t) + B(x)u(t) \tag{7}$$

which can be rewritten as

$$\dot{x}(t) = f(x,t,u) \tag{8}$$

where x is the state vector of the system, u is the control input and f is a function vector. If the function vector f is discontinuous on a surface *S(x)=0* called sliding surface in the sliding mode theory then

$$f(x,t,u) = \begin{cases} f^{+}(x,t,u^{+}) & if \quad S > 0 \\ f^{-}(x,t,u^{-}) & if \quad S < 0 \end{cases} \tag{9}$$

The system is in sliding mode if its representative point moves on the sliding surface *S(x)=0*. The sliding surface is also called as switching function because the control action switches depending on its sign on the two sides of the sliding surface. The sliding mode exists on the manifold *S(x)=0*; if for the motion in subspace S with

$$\dot{S} = CAx + CBu \tag{10}$$

the origin is asymptotically stable with finite time of convergence [13].

The equivalent control method was developed to drive the sliding mode equations in the manifold *S(x)=0*. The solution $\dot{S} = 0$ is called equivalent control (u_{eq}) and takes the following form.

$$u_{eq} = -(CB)^{-1}(CAx) \tag{11}$$

In sliding mode theory, the control problem is to find a control input u such that the state vector x tracks a desired trajectory x^* in the presence of model uncertainties and external disturbance. The sliding surface may then be set to be of the form

$$S(x) = x - x^{*} \tag{12}$$

If the initial condition *S(0)=0* is not satisfied then the tracking can only be achieved after a transient phase called *reaching mode*.

Since the aim is to force the system states to the sliding surface, the adopted control strategy must guarantee the system trajectory move toward and stay on the sliding surface from any initial condition if the following condition meets [13],

$$S\dot{S} \le -\eta|S| \tag{13}$$

where η is a positive constant that guarantees the system trajectories hit the sliding surface in finite time [14]. The required sliding mode controller achieving finite time convergence to the sliding surface is given by

$$u = \begin{cases} 1 & for \quad S > 0 \\ 0 & for \quad S < 0 \end{cases} \tag{14}$$

IV. SLIDING MODE CONTROLLER DESIGN

The control problem is to provide the following condition:

$$\lim_{t\to\infty} v_c(t) = V^*$$

where V* is the reference input voltage. The controlled transient performance should be insensitive to parameter variation of the buck-boost converter and external disturbances.

The state equations of the buck-boost converter given by equations (5) and (6) can be expressed in the following matrix form as

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where

$$A = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}; \qquad B = \begin{bmatrix} \frac{E - x_2}{L} \\ \frac{x_1}{C} \end{bmatrix};$$

$$x = [x_1 \quad x_2]^T = [i_L \quad v_c]^T \tag{15}$$

A sliding surface with the desired dynamics of the corresponding sliding motion need to be determined for the sliding mode control. For this purpose the following sliding surface is chosen.

$$S(x) = k_1(x_1 - I^*) + k_2(x_2 - V^*) \tag{16}$$

Where k_1 and k_2 are the sliding coefficients i.e. the design parameters and I^* and V^* are the desired output current and voltage respectively.

When the sliding mode exists then

$$S(x) = \dot{S}(x) = 0 \tag{17}$$

The general control structure u consists of two components. A nonlinear component u_n and an equivalent control component u_{eq}.

$$u = u_n + u_{eq} \tag{18}$$

The equivalent control is obtained using the equations (7), (16) and (17) as follows.

$$\begin{aligned} u_{eq} &= -\left[\frac{\partial S}{\partial x} B(x)\right]^{-1} \left(\frac{\partial S}{\partial x} Ax\right) \\ &= \frac{-k_1 RCx_2 + k_2 L(Rx_1 + x_2)}{k_1 RC(E - x_2) + k_2 LRx_1} \end{aligned} \tag{19}$$

where

$$\frac{\partial S}{\partial x} B(x) = k_1\left(\frac{E - x_2}{L}\right) + k_2 \frac{x_1}{C} \neq 0 \tag{20}$$

The inductor current x_1 is always positive and the output voltage x_2 is negative. Therefore the condition in equation (20) is satisfied when the design parameters k_1 and k_2 are chosen to be positive.

The objective of sliding mode control is to force the system states to the sliding surface from any initial condition and remain on it. This is satisfied by the Lyapunov function of the form [13]

$$V = \frac{1}{2} S(x)^2 \tag{21}$$

The control input u have to be chosen so that the time derivative of V must be negative definite i.e. $\dot{V} < 0 \;\; for \;\; S(x) \neq 0$ to ensure the stability of the system and to make the surface S attractive.

$$\dot{V} = S\,\dot{S} < 0 = S\left[\frac{\partial S}{\partial x} Ax + \frac{\partial S}{\partial x} B(x)u\right] \tag{22}$$

To satisfy this condition the nonlinear control component can be defined as

$$u_n = k_3 \operatorname{sgn}(S) \tag{23}$$

Where k_3 is chosen to be negative [15], and sgn(.) is a sign function defined as

$$\operatorname{sgn}(S(x)) = \begin{cases} +1 & if \quad S(x) > 0 \\ -1 & if \quad S(x) < 0 \end{cases} \tag{24}$$

Thus

$$\dot{V} = k_3 S \operatorname{sgn}(S) = k_3 |S| < 0 \tag{25}$$

implies that the equilibrium S=0 is globally asymptotically stable [13]. The following control law can be obtained combining the equations (18), (19) and (23).

$$u = \frac{1}{k_1 RC(E - x_2) + k_2 LRx_1} \left[-k_1 RCx_2 + k_2 L(Rx_1 + x_2) + RLCk_3 \operatorname{sgn}(S(x))\right] \tag{26}$$

If the sliding mode is enforced on the sliding line $S(x) = (x_2 - V^*) = 0$, then in the sliding mode $v_c = V^* > 0$ now the current equation is [15]

$$\frac{di_L}{dt} = \frac{E}{L} - \frac{V^*(E + V^*)}{LRi_L} \tag{27}$$

which has an equilibrium point

$$I_L = \frac{V^*}{R}\left(\frac{V^*}{E} - 1\right) \tag{28}$$

Similarly for the switching line $S(x) = (x_1 - I^*) = 0$ then $x_1 = I^* > 0$ then the voltage equation is

$$\frac{dv_c}{dt} = -\frac{v_c}{RC} + \frac{EI^*}{(E + v_c)C} \tag{29}$$

and the equilibrium point in sliding mode is [16]

$$Vc = \frac{-E + \sqrt{E^2 + 4ERI^*}}{2} \tag{30}$$

V. SIMULATIONS

The block diagram of the sliding mode controlled buck-boost converter used in simulations is shown in Figure 3.

Figure 6 shows output voltage and current transient response during a change in the reference voltage from 10V to 20V at time t=2.5ms.

Figure 7 shows the performance of the sliding mode based control scheme, when the input voltage E is increased 50% from 12V to 18V at the time t=2.5 ms with a desired steady state output voltage of 20 V.

It is known that the sliding mode control is regarded as a robust feedback control technique with respect to matched unmodeled external perturbation signals and plant parameter variations. The robustness of the sliding mode control scheme is tested against the load resistor R. The load R has been let to change 100% of its nominal value of 5 Ω to 10Ω . This variation took place, at time, t = 2.5 ms, while the system was already stabilized to the desired voltage value of 20 V.

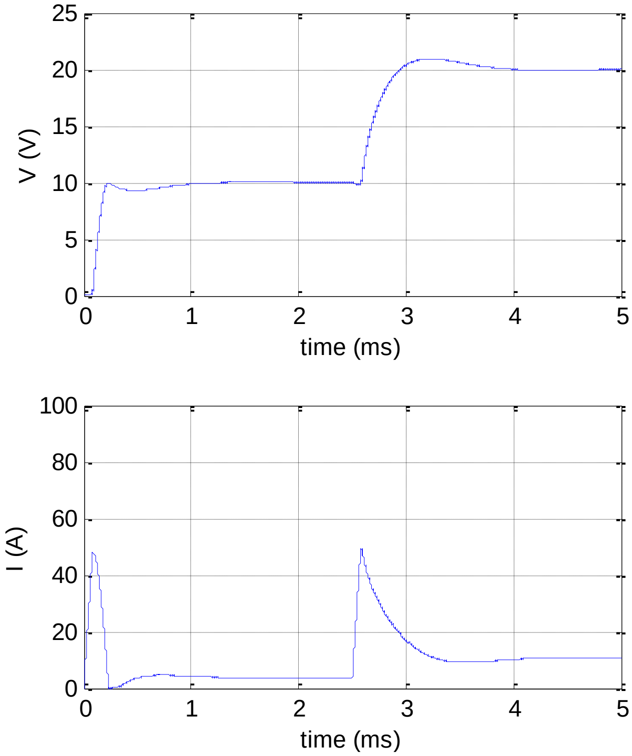

Figure 6: Desired voltage changed from 10 to 20 at t=2.5 ms

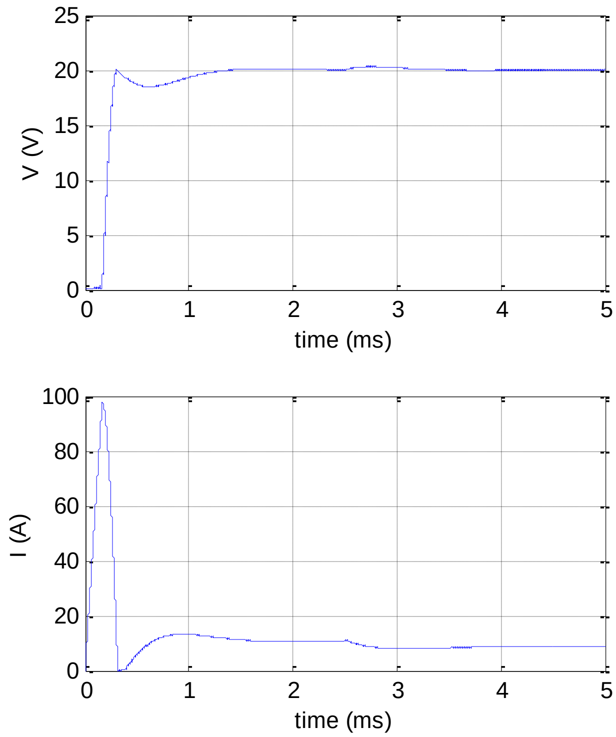

Figure 7: Input voltage of the converter is increased 50% at t=2.5 ms from 12 to 18 V

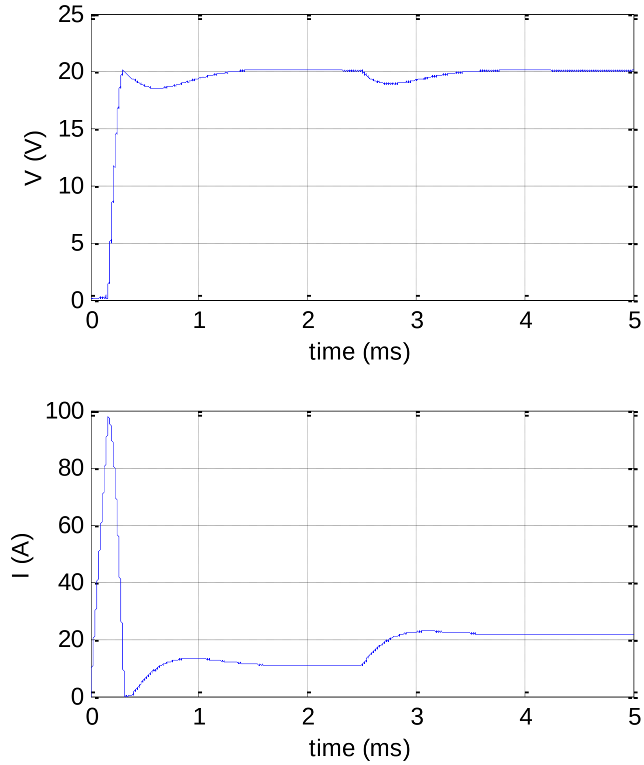

Figure 8: Load R is decreased from 5ohm to 2.5 ohm at t=2.5 ms

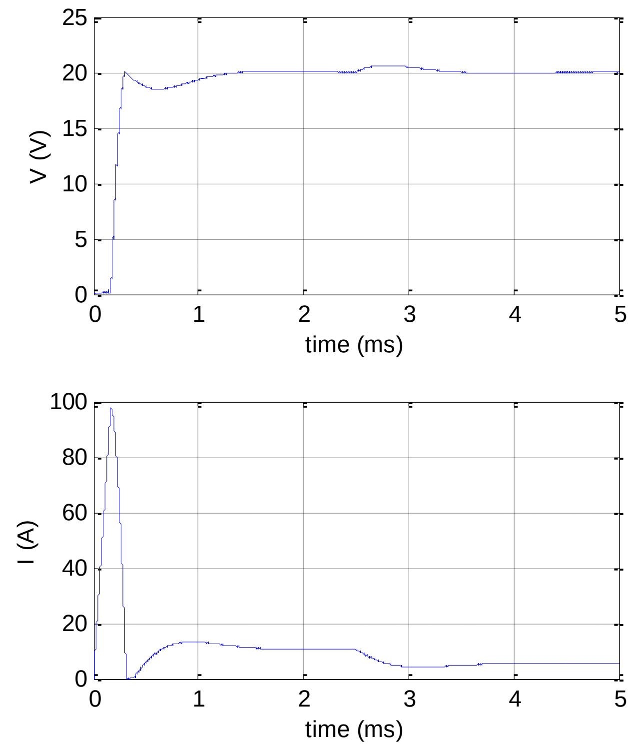

Figure 9: Load R is increased from 5 ohm to 10 ohm at t=2.5 ms

The Figure 8 and 9 show the recovering features of the proposed controller to the imposed load variation. As expected, the output voltage is robust when the load resistance was subject to a sudden unmodelled variation from R = 5 Ω to R= 2.5 Ω at time t = 2.5 ms in Figure 9.

Figures 6-9 prove the robustness of the sliding mode control against changes in the load and variations in the input voltage.

VI. CONCLUSIONS

The modeling and simulation of a sliding mode controlled dc-dc buck-boost converter is presented. The robustness with respect to load and supply parameter step variations of the control system is shown with the simulations. The stability, robustness and good dynamic behavior is obtained even for large supply and load variations.

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