Bits, Bytes and Words

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Based on slides by Randal E. Bryant and David R. O'Hallaron

Agenda

Representing information as bits

Bit-level manipulation

Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating

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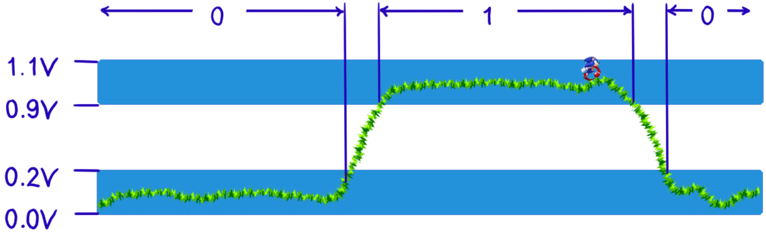
Everything is bits

- Each bit is 0 or 1
- By interpreting sets of bits in various ways...
  - ...computers determine what to do.
  - ...represent and manipulate numbers, sets, strings—*data*.

Why bits? Why not decimals? Could it have been some some other way?

Everything is bits

- **Why bits? Electronic implementation.**
  - Easy to store with bistable elements.
  - Reliably transmitted on noisy and inaccurate wires via *error correction*.



- **... But there exist models that do not use bits.**
  - The Setun computer developed in the Soviet Union used ternary *trits*.
  - Quantum computers use *qubits* that are in a superposition of the two states.
    - ...error correction is the main challenge here.

Everything is bit vectors

A sequence of bits is called a *bit vector*

$$\langle x_{w-1}, \ldots, x_0 \rangle$$

- Number bits from 0 to $w - 1$.
- Bit x_0 typically called *least significant* and x_{w-1} *most significant*.
  - Due to how bit vectors can be interpreted as binary numbers.
- **Bit vectors are not numbers.**
  - Can represent many kinds of objects.
  - ...but we will mostly focus on number representations.

Binary numbers

- **Base 2 numbers.**
  - Represent 15213_{10} as 11101101101101_2
    - $\langle$0011 1011 0110 1101$\rangle$ (with $w = 16$)
  - Represent $\frac{15_{10}}{213_{10}}$ as $\frac{1111_2}{11010101_2}$
    - $\langle$0000 0000 0000 1111 0000 0000 1101 0101$\rangle$ ($w = 32$)
    - 16 bits for each of numerator and denominator.
    - (This is not how we actually represent rational numbers in a computer−we'll see how next week.)
- **Machine numbers are of some finite size.**
  - If we use w bits to represent a number, only 2^w distinct values are possible.
  - How we interpret those bits can vary.
  - **Why do we use finite-sized numbers?**
    - A "w-bit machine" handles numbers of up to w bits "natively" (meaning fast).
    - A bit vector of some natively supported size is called a *word*.

Encoding byte values

Byte = 8 bit word

- (Machine-specific, but is true for all mainstream machines.)
- 256 different values.
- Binary 00000000_2 to 11111111_2.
- Decimal 0_{10} to 255_{10}.
- Hexadecimal 00_{16} to FF_{16}.
  - Base 16 number representation.
  - Uses characters 0—9 and A—F.
  - In C we write $FA1D37B_{16}$ as
    - `0xFA1D37B`
    - `0xfa1d37b` (case does not matter)

Hex	Dec	Bits
0	0	⟨0000⟩
1	1	⟨0001⟩
2	2	⟨0010⟩
3	3	⟨0011⟩
4	4	⟨0100⟩
5	5	⟨0101⟩
6	6	⟨0110⟩
7	7	⟨0111⟩
8	8	⟨1000⟩
9	9	⟨1001⟩
A	10	⟨1010⟩
B	11	⟨1011⟩
C	12	⟨1100⟩
D	13	⟨1101⟩
E	14	⟨1110⟩
F	15	⟨1111⟩

Let’s play a game

http://topps.diku.dk/compsys/integers.html

Example data representations

C data type	Typical 16-bit	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1	1
`short`	1	2	2	2
`int`	2	4	4	4
`long`	4	4	8	8
`int32_t`	4	4	4	4
`int64_t`	8	8	8	8
`float`	4	4	4	4
`double`	8	8	8	8
`pointer`	2	4	8	8

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Boolean algebra

Developed by George Boole in the 19th century

- Algebraic representation of logic (“truth values”).
- Encode *true* as 1 and *false* as 0.

And

$\wedge$	0	1
0	0	0
1	0	1

Or

$\vee$	0	1
0	0	1
1	1	1

Not

$\neg$	
0	1
1	0

Exclusive-or

$\oplus$	0	1
0	0	1
1	1	0

- These operations can be implemented with tiny electronic *gates*.

Boolean algebra on bit vectors

- The truth tables generalise to *bit vectors*, applied elementwise.

$$
\begin{array}{cc} & \langle 01101001 \rangle \\ \wedge & \langle 01010101 \rangle \\ \hline & \langle 01000001 \rangle \end{array}
\quad
\begin{array}{cc} & \langle 01101001 \rangle \\ \vee & \langle 01010101 \rangle \\ \hline & \langle 01111101 \rangle \end{array}
\quad
\begin{array}{cc} & \langle 01101001 \rangle \\ \oplus & \langle 01010101 \rangle \\ \hline & \langle 00111100 \rangle \end{array}
\quad
\begin{array}{cc} \neg & \langle 01101001 \rangle \\ \hline & \langle 10010110 \rangle \end{array}
$$

- This is the form they take when available in programming languages such as C.
- ...although C uses different symbols.

Bit-level operations in C

Operators `&` ($\wedge$), `|` ($\vee$), `^` ($\oplus$), `~` ($\neg$) available in C.

- Apply to any integral type.
  - E.g. `long`, `int`, `short`, `char`...
- Interpret operands as bit vectors.
- Applied element-wise.

Examples

- `~0x41 = 0xBE`
  - $\neg\langle 01000001\rangle = \langle 10111110\rangle$
- `~0x00 = 0xFF`
  - $\langle 00000000\rangle = \langle 11111111\rangle$
- `0x69 & 0x55 = 0x41`
  - $\langle 01101001\rangle \wedge \langle 01010101\rangle = \langle 01000001\rangle$
- `0x69 & 0x55 = 0x7D`
  - $\langle 01101001\rangle \wedge \langle 01010101\rangle = \langle 01111101\rangle$

Contrast: logical operators in C

The logical operators interpret numbers as *single boolean values*, not as bit vectors!

- **`&&`, `||`, `!`**
  - View 0 as false.
  - Anything nonzero as true.
  - Always produce 0 or 1.
  - **Short circuiting:** `1 || (0/0)` is safe.
- **Examples**
  - `!0x41 = 0x00`
  - `!0x00 = 0x01`
  - `!!0x41 = 0x01`
  - `0x69 && 0x55 = 0x01`
  - `0x69 || 0x55 = 0x01`
- **Do not confuse the logical and bitwise operators!**

Shift operations

- **Left shift `x << y`**
  - Shift bit-vector `x` left by `y` positions.
    - Throws away excess bits on the left.
    - Fills with zeroes on right.
- **Right shift `x >> y`**
  - Shift bit-vector `x` right by `y` positions.
    - Throws away excess bits on the left.
  - Logical shift: Fill with 0s on left.
  - Arithmetic shift: Replicate most significant bit on left.
- **Undefined behaviour in C**
  - Shifting a negative amount or by the vector size or more.

x	$\langle 01100010 \rangle$
$x << 3$	$\langle 00010000 \rangle$
$x >> 2$	$\langle 00011000 \rangle$
$x >>^{a} 2$	$\langle 00011000 \rangle$

x	$\langle 10100010 \rangle$
$x << 3$	$\langle 00010000 \rangle$
$x >> 2$	$\langle 00101000 \rangle$
$x >>^{a} 2$	$\langle 11101000 \rangle$

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Encoding integers

Suppose x_i is the ith bit of a w-bit word (with x_0 being the least significant bit).

Unsigned

$$\text{Bits2N}(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's complement (AKA *signed*)

$$\text{TC2Int}(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
int16_t x =  15213;
int16_t y = -15213;
```

	Decimal	Hex	Bits
`x`	15213	`3 B 6 D`	$\langle$0011 1011 0110 1101$\rangle$
`y`	-15213	`C 4 9 3`	$\langle$1100 0100 1001 0011$\rangle$

Sign bit

- For 2's complement, most significant bit (x_{w-1}) indicates sign.
  - 0 for non-negative.
  - 1 for negative.

Two's complement encoding example

```
int16_t x =  15213; // 0011 1011 0110 1101
int16_t y = -15213; // 1100 0100 1001 0011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2047	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric ranges

Unsigned

$$\begin{aligned} \text{UMin} &= & 0 &= 0\ldots0_2 \\ \text{UMax} &= & 2^w - 1 &= 1\ldots1_2 \end{aligned}$$

Two's complement

$$\begin{aligned} \text{SMin} &= & -2^{w-1} &= 10\ldots0_2 \\ \text{SMax} &= & 2^{w-1} - 1 &= 01\ldots1_2 \\ -1 & & &= 1\ldots1_2 \end{aligned}$$

Values for $w = 16$:

	Decimal	Hex	Bits
UMax	65535	F F F F	$\langle$1111 1111 1111 1111$\rangle$
SMax	32767	7 F F F	$\langle$0111 1111 1111 1111$\rangle$
SMin	-32768	8 0 0 0	$\langle$1000 0000 0000 0000$\rangle$
-1	-1	F F F F	$\langle$1111 1111 1111 1111$\rangle$
0	0	0 0 0 0	$\langle$0000 0000 0000 0000$\rangle$

Values for different word sizes

	w			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
SMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
SMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

$$|\mathrm{SMin}| = \mathrm{SMax} + 1$$
$$|\mathrm{UMax}| = 2 \cdot \mathrm{SMax} + 1$$

Note the asymmetric range.

C Programming

- `#include <limits.h>`
- Declares constants, e.g:
  - `ULONG_MAX`
  - `LONG_MAX`
  - `LONG_MIN`
- Values are platform-specific.

Unsigned and signed numeric values (here $w = 4$)

x	$\mathrm{Bits2N}(x)$	$\mathrm{TC2Int}(x)$
$\langle 0000 \rangle$	0	0
$\langle 0001 \rangle$	1	1
$\langle 0010 \rangle$	2	2
$\langle 0011 \rangle$	3	3
$\langle 0100 \rangle$	4	4
$\langle 0101 \rangle$	5	5
$\langle 0110 \rangle$	6	6
$\langle 0111 \rangle$	7	7
$\langle 1000 \rangle$	8	-8
$\langle 1001 \rangle$	9	-7
$\langle 1010 \rangle$	10	-6
$\langle 1011 \rangle$	11	-5
$\langle 1100 \rangle$	12	-4
$\langle 1101 \rangle$	13	-3
$\langle 1110 \rangle$	14	-2
$\langle 1111 \rangle$	15	-1

- **Equivalence**
  - Same encoding for non-negative values.
- **Uniqueness**
  - Every bit vector represents distinct integer value.
  - Each representable integer has unique bit encoding.
  - The representation is **bijective**.
- **Can invert mappings**
  - $\mathrm{N2Bits}(x) = \mathrm{Bits2N}^{-1}(x)$
    - Bit vector for unsigned integer in range.
  - $\mathrm{Int2TC}(x) = \mathrm{TC2Int}^{-1}(x)$
    - Bit vector for Two's Complement integer in range.

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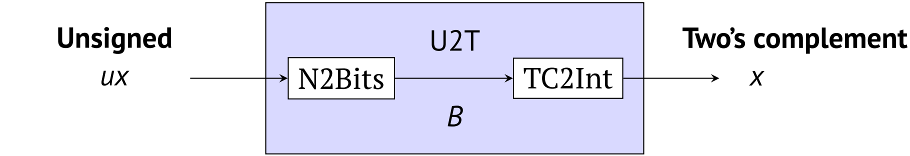
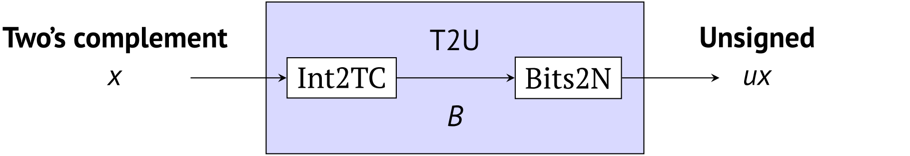
Mapping between signed and unsigned



Maintain same bit pattern



Mapping between unsigned and two's complement numbers:
Keep bit representations and reinterpret.

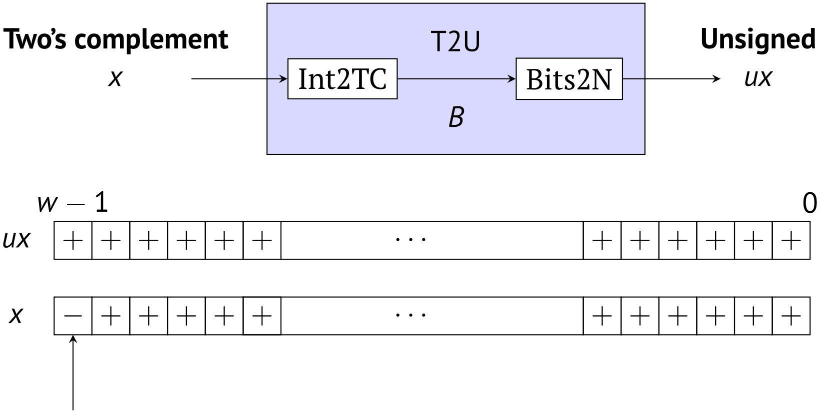
Mapping signed ⇔ unsigned

Bits	Signed	Unsigned
⟨0000⟩	0	0
⟨0001⟩	1	1
⟨0010⟩	2	2
⟨0011⟩	3	3
⟨0100⟩	4	4
⟨0101⟩	5	5
⟨0110⟩	6	6
⟨0111⟩	7	7
⟨1000⟩	-8	8
⟨1001⟩	-7	9
⟨1010⟩	-6	10
⟨1011⟩	-5	11
⟨1100⟩	-4	12
⟨1101⟩	-3	13
⟨1110⟩	-2	14
⟨1111⟩	-1	15

$$\xrightarrow{\text{T2U}}$$

$$\xleftarrow{\text{U2T}}$$

Relation between signed and unsigned

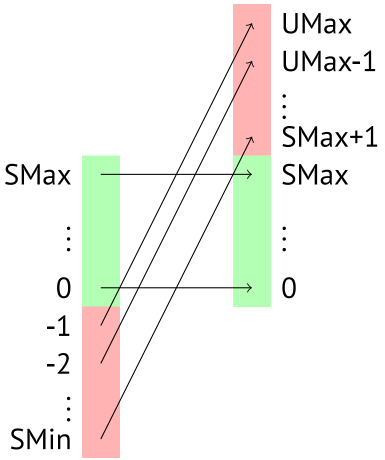


Large negative weight becomes **large positive weight.**

Conversion (that is, *reinterpretation*) visualized

Two's complement to unsigned

- Ordering inversion.
- Negative numbers become large positive numbers.



Signed versus unsigned in C

C makes working with this more error-prone than it should be.

Types
- Signedness part of type: `unsigned int`, `int32_t`, `uint32_t`.

Constants
- By default are considered signed integers.
- Unsigned with `U` suffix: `0U, 4294967259U`

Casting
- Explicit casting between signed and unsigned:

```
int tx, ty;
unsigned int ux, uy;
tx = (int) ux;
uy = (unsigned int) ty;
```

- Implicit casting due to assignments and other expressions:

```
tx = ux;
uy = ty;
```

Casting surprises

Evaluation

- If there is a mix of unsigned and signed in single expression, *signed values implicitly cast to unsigned*.
- Including comparison operations $<$, $>$, $==$, $<=$, $>=$.
- Examples for $w = 32$: $SMin = -2,147,483,648$, $SMax = 2,147,483,647$:

Const LHS	Relation	Const RHS	Evaluation
0	==	0U	

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Const LHS	Relation	Const RHS	Evaluation
0	==	0U	unsigned

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- Examples for $w = 32$: $SMin = -2,147,483,648$, $SMax = 2,147,483,647$:

Const LHS	Relation	Const RHS	Evaluation
0	$==$	0U	unsigned
-1	$<$	0	

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-1	$<$	0	signed

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Const LHS	Relation	Const RHS	Evaluation
0	$==$	0U	unsigned
-1	$<$	0	signed
-1	$>$	0U	

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Const LHS	Relation	Const RHS	Evaluation
0	==	0U	unsigned
-1	<	0	signed
-1	>	0U	unsigned
2147483647	>	-2147483647-1	

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0	$==$	0U	unsigned
-1	$<$	0	signed
-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed

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Const LHS	Relation	Const RHS	Evaluation
0	$==$	0U	unsigned
-1	$<$	0	signed
-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	

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-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
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-1	$<$	0	signed
-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	unsigned
-1	$>$	-2	

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-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	unsigned
-1	$>$	-2	signed
(unsigned int)-1	$>$	-2	

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0	==	0U	unsigned
-1	<	0	signed
-1	>	0U	unsigned
2147483647	>	-2147483647-1	signed
2147483647U	<	-2147483647-1	unsigned
-1	>	-2	signed
(unsigned int)-1	>	-2	unsigned

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Const LHS	**Relation**	**Const RHS**	**Evaluation**
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-1	$<$	0	signed
-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	unsigned
-1	$>$	-2	signed
(unsigned int)-1	$>$	-2	unsigned
2147483647	$<$	2147483648U	

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-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	unsigned
-1	$>$	-2	signed
(unsigned int)-1	$>$	-2	unsigned
2147483647	$<$	2147483648U	unsigned
2147483647	$>$	(int) 2147483648U	

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2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	unsigned
-1	$>$	-2	signed
(unsigned int)-1	$>$	-2	unsigned
2147483647	$<$	2147483648U	unsigned
2147483647	$>$	(int) 2147483648U	signed

Casting surprises

Evaluation

- If there is a mix of unsigned and signed in single expression, *signed values implicitly cast to unsigned*.
- Including comparison operations $<, >, ==, <=, >=$.
- Examples for $w = 32$: $SMin = -2,147,483,648$, $SMax = 2,147,483,647$:

Const LHS	Relation	Const RHS	Evaluation
0	$==$	0U	unsigned
-1	$<$	0	signed
-1	$>$	0U	unsigned
2147483647	$>$	-2147483647-1	signed
2147483647U	$<$	-2147483647-1	unsigned
-1	$>$	-2	signed
(unsigned int)-1	$>$	-2	unsigned
2147483647	$<$	2147483648U	unsigned
2147483647	$>$	(int) 2147483648U	signed

Casting between signed and unsigned: basic rules

- Bit representation is maintained.
- ...but reinterpreted.
- Can have unexpected effects: adding or subtracting 2^w.

- Expression containing signed and unsigned int:
  - `int` is cast to `unsigned int`!
  - **When can this go bad?**

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```

Advice: Avoid arithmetic on unsigned types—only use them for bit operations.

But: Some C operators (`sizeof`) and many functions return unsigned types (e.g. `size_t`). C is always ready to stab you in the back.

Representing information as bits

Bit-level manipulation

Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating

Truncation

Task

- Given $k + w$-bit signed integer x.
- Convert it to w-bit integer x' with same value i possible.

Approach

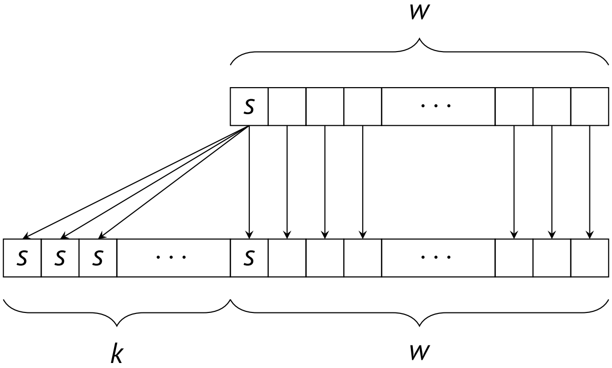
- Remove the k most significant bits.
- Equivalent to computing $x' = x \bmod 2^w$.
- Numerical change if number has no representation in w bits.
- Otherwise safe.

w	x	$TC2Int(x)$
8	$\langle 11111111 \rangle$	-1
4	$\langle 1111 \rangle$	-1
8	$\langle 10000000 \rangle$	-128
4	$\langle 0000 \rangle$	0

Sign extension

Task
- Given w-bit signed integer x.
- Convert it to $w+k$-bit integer x' with same value.

Approach
- Make k copies of sign bit (most significant bit):
- $x' = \langle \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of sign bit.}} x_{w-1} \cdots x_0 \rangle$



Sign extension example

```
short int x = 15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

	Decimal	Hex	Bits
`x`	15213_{10}	`3B 6D`	$\langle$0011 1011 0110 1101$\rangle$
`ix`	15213_{10}	`00 00 3B 6D`	$\langle$0000 0000 0000 0000 0011 1011 0110 1101$\rangle$
`y`	-15213_{10}	`C4 93`	$\langle$1100 0100 1001 0011$\rangle$
`iy`	-15213_{10}	`FF FF C4 93`	$\langle$1111 1111 1111 1111 1100 0100 1001 0011$\rangle$

Summary: basic rules for expanding and truncating

Expanding (e.g. `short` to `int`)

- Unsigned: zeros added.
- Signed: sign extension.
- Both yield expected result.

Truncating (e.g. `unsigned int` to `unsigned short`)

- Bits are truncated.
- Result reinterpreted.
- Unsigned: modulo operation.
- Signed: similar to a modulo operation.
- For small numbers yield expected behaviour.