Solving Simultaneous Linear Equations

Choose a method wisely: substitute when a variable is "ready eliminate when coefficients line up.

Mini-Review

Substitution: Solve one equation for a variable, plug into the other. **Elimination:** Add/subtract multiples of equations to cancel a variable.
Special cases: After eliminating, a true statement like $0 = 0 \Rightarrow$ infinitely many; a false one like $0 = 5 \Rightarrow$ no solution.

1. Solve the system (fractions):

$$\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 4 \\ \frac{2}{3}x - \frac{1}{2}y = 1 \end{cases}$$

2. Choose a method and solve:

$$\begin{cases} y = 2x + 5 \\ 3x + 2y = 19 \end{cases}$$

3. **Word problem — tickets**. An adult ticket costs \$A and a student ticket costs \$S. On Monday a school bought 8 adult and 12 student tickets for \$164. On Tuesday they bought 5 adult and 15 student tickets for \$150. Find A and S.
中文：成人票价为\$A，学生票价为\$S。周一学校买了8张成人票和12张学生票，共\$164。周二买了5张成人票和15张学生票，共\$150。求A 和S。

4. **Word problem — coins**. You have only quarters (25¢) and dimes (10¢). There are 22 coins total worth \$3.70. How many of each?

Русский: У вас только четвертаки (25¢) и даймы (10¢). Всего 22 монеты на сумму \$3.70. Сколько каких?

5. Solve and identify the case (unique / none / infinitely many):

$$\begin{cases} 2x - 3y = 7 \\ 4x - 6y = 20 \end{cases}$$

6. **Word problem — grocery**. Two apples and three bananas cost \$3.10. Four apples and one banana cost \$3.00. Find the unit prices.
中文：2个苹果和3根香蕉共\$3.10；4个苹果和1根香蕉共\$3.00。求一个苹果和一根香蕉的单价。

7. **Word problem — ages**. The sum of Ali's and Bea's ages is 29. In three years, Ali will be twice Bea's age three years ago. How old are they now?
Русский: Сумма возрастов Али и Беа — 29. Через три года Али будет вдвое старше, чем была Беа три года назад. Сколько им сейчас лет?

8. Solve in terms of parameter k (classify special cases):

$$\begin{cases} x + ky = 5 \\ 2x + 2ky = 10 \end{cases}$$

9. **Word problem — rectangle**. A rectangle's perimeter is 74 cm. The length is 5 cm more than twice the width. Find length and width.
中文：长方形的周长是74厘米，长比宽的两倍多5厘米。求长和宽。

10. **Word problem — mixture**. Mix 8% and 20% saline to get 300 mL of 12%. How many mL of each?
Русский: Смешивают 8% и 20% растворы, чтобы получить 300 мл 12%. Сколько мл каждого?

11. Eliminate strategically:

$$\begin{cases} 4x - 7y = -1 \\ 6x + 7y = 41 \end{cases}$$

12. **Word problem — two numbers**. The difference of two numbers is 9 and their sum is 55. Find the numbers.
中文：两个数的差是9，和是55。求这两个数。

13. **Word problem — motion**. Two cyclists leave together: one east at v mph, one north at $(v + 2)$ mph. After 3 hours they are 39 miles apart (straight-line). Find v.
Русский: Два велосипедиста стартуют одновременно: один на восток со скоростью v, другой на север со скоростью $(v + 2)$. Через 3 часа расстояние между ними 39 миль. Найдите v.

14. Solve (decimals):

$$\begin{cases} 0.6x - 0.4y = 1.8 \\ 1.2x + 0.8y = 4.0 \end{cases}$$

15. **Word problem — bakery**. Bagels \$1.20, donuts \$0.90. On Saturday they sold 140 items for \$150.60. How many of each?

中文：面包店卖百吉饼\$1.20、甜甜圈\$0.90。周六共卖出140件，总额\$150.60。各卖了多少?

16. **Word problem — furniture**. In a classroom there are chairs and tables, each with 4 legs. There are 17 items and 60 legs total. How many chairs and tables?
Русский: В классе есть стулья и столы, у каждого по 4 ножки. Всего 17 предметов и 60 ножек. Сколько стульев и столов?

17. Determine solution type and solve if possible:

$$\begin{cases} 3x - 9y = 12 \\ x - 3y = 4 \end{cases}$$

18. Solve (watch structure):

$$\begin{cases} 2x + y = 5 \\ y = -2x + 5 \end{cases}$$

中文提示：第二个方程已给出y 的表达式。

19. Solve (fractions):

$$\begin{cases} \frac{x}{4} + \frac{y}{3} = 5 \\ \frac{x}{2} - \frac{y}{6} = 1 \end{cases}$$

Русский: решите систему.

20. **Challenge (choose wisely)**:

$$\begin{cases} 5x - 2y = 13 \\ 3(5x - 2y) - 4y = 31 \end{cases}$$

(Hint: one equation already contains the other's left side.)