Geometry in Context — Word Problems

Show clear steps, label units, and box final answers. Unless told otherwise, use π exactly.

Formulas & Concepts (for reference)

Perimeter: rectangle $P = 2(\ell + w)$; square $P = 4s$; triangle $P = a + b + c$; circle $C = 2\pi r$ (semicircle perimeter $= \pi r + 2r$).

Area: rectangle $A = \ell w$; square $A = s^2$; triangle $A = \frac{1}{2}bh$; circle $A = \pi r^2$; sector $A = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2}r^2\theta$ (with θ in radians).

Surface Area: cube $S = 6s^2$; right cylinder $S = 2\pi r^2 + 2\pi rh$; sphere $S = 4\pi r^2$; cone $S = \pi r^2 + \pi r\ell$ (lateral + base).

Volume: rectangular prism $V = \ell wh$; cube $V = s^3$; right cylinder $V = \pi r^2 h$; sphere $V = \frac{4}{3}\pi r^3$; cone $V = \frac{1}{3}\pi r^2 h$.

Scaling idea: If all lengths scale by k, then perimeter/lengths scale by k, areas by k^2, volumes by k^3.

Units: area in unit^2, volume in unit^3; surface area in unit^2.

Testing Mindset

When a problem looks "plug-and-play," pause: draw a quick sketch, mark what is known/unknown, and check whether you need the whole figure or only part of it (e.g., a semicircle edge includes the diameter).

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1. **Fence and arc.** A garden is shaped like a rectangle of length ℓ and width w topped with a semicircle of radius $\frac{w}{2}$ (the diameter sits on the rectangle's top). Only the outer boundary is fenced. Write the total fence length P in terms of ℓ and w. Then evaluate P for $\ell = 10$ m and $w = 6$ m.

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2. **Same area, different shapes.** A square and a circle have the same area. If the square has side s, find the circle's radius r in terms of s, then compute the numerical value when $s = 8$ cm (exact form with π).

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3. **You changed the wrong number.** A student doubles the radius of a cylinder instead of doubling the height. By what factor does the volume change? By what factor does the surface area change? Briefly justify using scaling ideas.

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4. **Paint budget.** You can paint at most $600\,\text{ft}^2$ of surface. A cylindrical water tank has radius 5 ft and height h ft. You must paint the entire outside (top + bottom + curved side). What is the largest whole number h you can fully paint? (Use $\pi = 3.14$)

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5. **Two-box shipping.** A company ships using either a cube box of side s or a rectangular prism of dimensions $2s \times s \times s$.

   (a) Compare their *volumes*. Which holds more (or are they equal)? Show work.

   (b) Compare their *surface areas*. Which uses more cardboard?

   Georgian (transliteration) for Problem 5: kartuli targmani (Problem 5): kompania agzavnis an kubisebur quts gverdit s, an martkutkha parallelepiped zomebit $2s \times s \times s$.

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6. **Trim the slice.** A circular pizza of radius 10 in has a sector slice with central angle 72°. What fraction of the pizza is this? What is the area and arc length of the slice

(leave π)?

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7. **Semicircle or three sides?** A park border is composed of a straight segment of length 12 m, a straight segment of 8 m, and a semicircular arc whose diameter is 8 m. What is the total border length? (Explain why it is not simply a perimeter of a standard shape.)

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8. **Hidden height.** A right circular cone has the same volume as a cylinder with the same radius r and height 10 cm. What must the cone's height be? (Use exact relationship, no decimals.)

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9. **Sphere surprise.** If a sphere has radius r, how many times larger is its surface area than the area of a circle with the same radius? How many times larger is its volume than the volume of a hemisphere with radius r? Explain briefly.

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10. **Lawn path.** A rectangular lawn is 20 m by 12 m. A path of uniform width x m runs along the outside edge around all four sides (a larger rectangle). The path area is $136\,\mathrm{m}^2$.

Find x.

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11. **Only the curved part.** A silo is a cylinder of radius r and height h with a hemispherical cap (radius r) on top. You only need to paint the curved surfaces (no flat bases). Write the total painted area in terms of r, h. Then evaluate for $r = 4$ m, $h = 9$ m.

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12. **Arched doorway.** An arched doorway is a rectangle 3 m wide by 2.2 m tall topped by a semicircle whose diameter equals the width. Find the total area of the opening. Then, if trim goes only around the outer edge, how long is the trim?

    中文翻译（题目12）： 一个拱形门洞由一个宽3 米、高2.2 米的长方形和上方一个直径为3 米的半圆组成。求门洞的总面积。若装饰条只沿外边缘一圈，求装饰条的总长度。

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13. **Same volume, different shapes.** A cube and a sphere have the same volume. Express the sphere's radius r in terms of the cube's side s. Then determine which has the smaller

surface area for $s = 6$ cm (compute both, exact in π).

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14. **Sector design.** A circular garden of radius 9 m has a pie-slice flower bed that covers 35% of the circle. Find the central angle in degrees and radians, and the area of the bed (leave π).

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15. **Not all triangles are alike.** Two triangles share the same base b, but Triangle A has height h and Triangle B has height $1.5h$. Compare their areas. Then, if $A_\mathrm{B} - A_\mathrm{A} = 18 \,\mathrm{cm}^2$ and $h = 6$ cm, find b.

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16. **Packaging tradeoff.** You must design a right cylinder of fixed volume $V = 1000\pi \,\mathrm{cm}^3$.

    (a) Express the height h in terms of the radius r.

    (b) Write the total surface area $S(r)$ and identify (without calculus) whether smaller r or larger r tends to reduce S when V is fixed; justify with a brief comparison idea.

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17. **Arc-only fencing.** A dog run is made from two straight segments of 7 ft and 5 ft that meet at a point, plus a circular arc connecting their free ends centered at the meeting point (i.e., a sector boundary). If the angle between the straight segments is 60°, how much fencing is used in total?

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18. **A cone inside a sphere.** A right circular cone is inscribed in a sphere of radius R so that the cone's base lies on the sphere's "equator" and its tip is at the "north pole." Express the cone's height and base radius in terms of R, then its volume in terms of R.

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Sanity check: Units tell a story. Make sure yours match the story.