Quick Reference (What students saw)

Domain: all allowed inputs. **Range:** all outputs. **x-ints:** $f(x)=0$. **y-int:** $f(0)$.
Inc/Dec: compare $f(x_2)$ vs $f(x_1)$ for $x_2>x_1$. **AROC:** $\dfrac{f(b)-f(a)}{b-a}$.

1. (8 points) Domain & Range; intercepts.

Solution: Points: $\{(-2,1),(0,3),(1,3),(3,-1),(4,0)\}$.

Domain $=\{-2,0,1,3,4\}$, Range $=\{1,3,-1,0\}=\{-1,0,1,3\}$.

x-intercepts: where $y=0 \Rightarrow (4,0)$ only. y-intercept: where $x=0 \Rightarrow (0,3)$.

2. (8 points) Table $\to$ inc/dec/const.

Solution: Table

x	-3	-1	0	2	4
$f(x)$	5	4	4	3	3

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From $x=-3$ to -1: $5\to4$ (decreasing). From -1 to 0: $4\to4$ (constant). From 0 to 2: $4\to3$ (decreasing). From 2 to 4: $3\to3$ (constant).

So f is **decreasing** on $[-3,-1]$ and $[0,2]$; **constant** on $[-1,0]$ and $[2,4]$; not increasing anywhere.

Justification: for each step with $x_2>x_1$, we compared $f(x_2)$ to $f(x_1)$.

3. (10 points) $f(x)=4-2x$.

Solution: x-int: set $f(x)=0 \Rightarrow 4-2x=0 \Rightarrow x=2 \Rightarrow (2,0)$.

y-int: $f(0)=4 \Rightarrow (0,4)$.

AROC on $[1,4]$: $\dfrac{f(4)-f(1)}{4-1}=\dfrac{(-4)-2}{3}=\dfrac{-6}{3}=-2$ (units: output per input; here "y per x").

4. (12 points) $p(x)=\dfrac{x^2-9}{x-3}$.

Solution: Factor: $x^2-9=(x-3)(x+3)$, so for $x\neq3$, $p(x)=x+3$.

Domain: all real x *except* $x=3$ (denominator cannot be 0).

y-int: $p(0)=\dfrac{-9}{-3}=3 \Rightarrow (0,3)$.

x-ints: solve $p(x) = 0 \Rightarrow \dfrac{x^2-9}{x-3} = 0 \Rightarrow x^2 - 9 = 0 \Rightarrow x = \pm 3$. But $x = 3$ is not allowed, so the only x-int is $x = -3 \Rightarrow (-3, 0)$.

Is $y = 6$ in the range? Since $p(x) = x + 3$ for $x \neq 3$, getting $y = 6$ would require $x = 3$, which is excluded. **So $y = 6$ is not in the range.**

5. (10 points) $q(x) = \sqrt{7 - x}$.

Solution: Inside the root must be ≥ 0: $7 - x \geq 0 \Rightarrow x \leq 7$. Domain $= (-\infty, 7]$.

Outputs are ≥ 0: Range $= [0, \infty)$.

x-int: set $\sqrt{7-x} = 0 \Rightarrow 7 - x = 0 \Rightarrow x = 7 \Rightarrow (7, 0)$.

y-int: $q(0) = \sqrt{7} \Rightarrow (0, \sqrt{7})$.

6. (8 points) AROC from a table.

Solution: $[0, 2]$: $\dfrac{10-4}{2-0} = \dfrac{6}{2} = 3$ (units: output per input).

$[2, 5]$: $\dfrac{25-10}{5-2} = \dfrac{15}{3} = 5$.

$[5, 6]$: $\dfrac{28-25}{6-5} = \dfrac{3}{1} = 3$.

7. (10 points) Scoring pace.

Solution: $[0, 16]$: $\dfrac{23-0}{16-0} = \dfrac{23}{16} \approx 1.44$ pts/min.

$[16, 32]$: $\dfrac{50-23}{32-16} = \dfrac{27}{16} = 1.6875$ pts/min.

Higher pace in the second half (larger average rate of change).

8. (8 points) Free throws $F(n) = 0.75n$.

Solution: Domain: nonnegative integers $\{0, 1, 2, \dots\}$ (attempts).
Range: $\{0.75n : n \in \mathbb{Z}_{\geq 0}\}$ (made shots).

AROC $[20, 28]$: $\dfrac{0.75 \cdot 28 - 0.75 \cdot 20}{8} = \dfrac{0.75 \cdot 8}{8} = 0.75$ makes/attempt. Interpretation: about 0.75 made per attempt (i.e., 75%).

9. (8 points) $s(x) = -3x + 1$ increasing or decreasing?

Solution: For $x_2 > x_1$,

$$s(x_2) - s(x_1) = -3x_2 + 1 - (-3x_1 + 1) = -3(x_2 - x_1) < 0.$$

So $s(x_2) < s(x_1)$ when $x_2 > x_1$: the function is **decreasing** on all real numbers.

10. (8 points) Intercepts from a table.

Solution: x-ints where $y = 0$: $x = -2$ and $x = 2 \Rightarrow (-2, 0), (2, 0)$.
y-int where $x = 0$: $(0, 3)$.

Increasing from $x = -2$ to $x = 0$ (values $0 \to 2 \to 3$). Decreasing from $x = 0$ to $x = 2$ ($3 \to 2 \to 0$).

11. (8 points) General AROC for $r(x) = ax + b$.

Solution:

$$\frac{r(x_2) - r(x_1)}{x_2 - x_1} = \frac{(ax_2 + b) - (ax_1 + b)}{x_2 - x_1} = \frac{a(x_2 - x_1)}{x_2 - x_1} = a.$$

So the average rate of change of a line on any interval is its slope a (constant).

12. (10 points) Score difference $S(t) = 12 - 0.5t$.

Solution: y-int: $S(0) = 12 \Rightarrow (0, 12)$ meaning the home team leads by 12 at tip-off (or baseline time).

Tie: set $S(t) = 0 \Rightarrow 12 - 0.5t = 0 \Rightarrow t = 24$ minutes.

S is **decreasing** (slope -0.5), meaning the visitors are catching up by 0.5 point per minute.