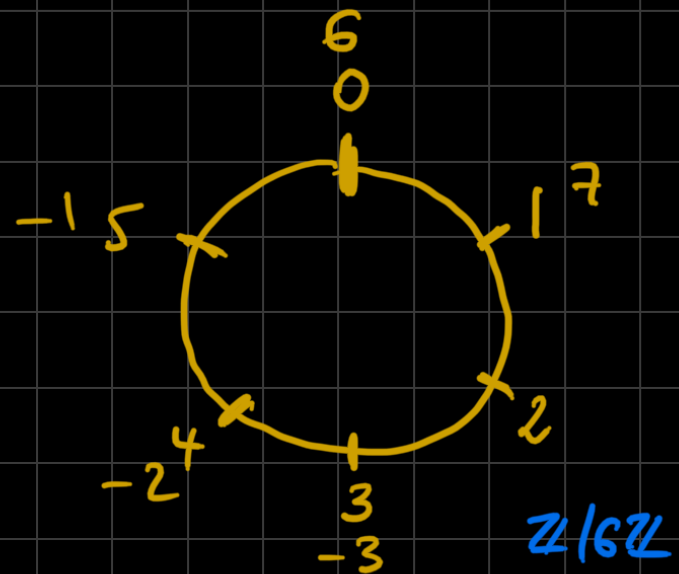


arithmétique vs arithmétique modulaire



critères de divisibilité

$$(a_p \dots a_0)_{10} \equiv 0[2] \Leftrightarrow (a_p \dots a_1)_{10} \times 10 + a_0 \equiv 0[2]$$

$\equiv 0[2]$

$$\Leftrightarrow a_0 \equiv 0[2]$$

$$(15554)_{10} \equiv 0[2] \Leftrightarrow (1555)_{10} \times 10 + 4 \equiv 0[2]$$

$$(a_p \dots a_0)_{10} \equiv 0[4] \Leftrightarrow (a_p \dots a_1)_{10} \times 100 + (a_1 a_0)_{10} \equiv 0[4]$$

$\equiv 0[4]$

$$\Leftrightarrow (a_1 a_0)_{10} \equiv 0[4]$$

$$(15554)_{10} \equiv 0[2] \Leftrightarrow (155)_{10} \times 100 + (54)_{10} \equiv 0[4]$$

$$\Leftrightarrow (54)_{10} \equiv 0[4]$$

donc $15554 \not\equiv 0[4]$.

$$(a_p \dots a_0)_{10} \equiv 0 [3] \Leftrightarrow \sum_{i=0}^p a_i \underbrace{10^i}_{\equiv 1 [3]} \equiv 0 [3]$$

$$\Leftrightarrow \sum_{i=0}^p a_i \equiv 0 [3]$$

$$(a_p \dots a_0)_{10} \equiv 0 [7] \Leftrightarrow \underbrace{5}_{2^*} \times (a_p \dots a_1)_{10} \times \underbrace{10}_{\equiv 1 [7]} + \underbrace{5}_2 a_0 \equiv 0 [7]$$

$$\Leftrightarrow (a_p \dots a_1)_{10} + 5 a_0 \equiv 0 [7]$$

$$\Leftrightarrow (a_p \dots a_1)_{10} - 2 a_0 \equiv 0 [7]$$

$$\begin{array}{r|l} 1 & 664 \\ \times 5 & \\ \hline 5 & 3320 \end{array} \begin{array}{l} \equiv 0 [7] \\ \equiv 0 [7] \\ \equiv 0 [7] \end{array}$$

$$\begin{array}{r|l} 1 & 664 \\ \times 2 & \\ \hline 2 & 1328 \end{array} \begin{array}{l} \equiv 0 [7] \\ \equiv 0 [7] \\ \equiv 0 [7] \end{array}$$

$$(1664)_{10} \not\equiv 0 [7]$$

Calcul sur un petit Reste :

$$b^k \equiv 0[d] : (a_p \dots a_0)_b \equiv 0[d] \Leftrightarrow (a_{k-1} \dots a_0)_b \equiv 0[d]$$

il suffit de regarder les k derniers chiffres.

Calcul sur toute la Longueur :

$$b \equiv \pm 1[d] : (a_p \dots a_0)_b \equiv 0[d] \Leftrightarrow (\pm 1)^p a_p + \dots \pm a_1 + a_0 \equiv 0[d]$$

il suffit de faire une somme (alternée) sur les chiffres.

Calcul Mixte :

$$m b^k \equiv \pm 1[d] : (a_p \dots a_0)_b \equiv 0[d] \Leftrightarrow (a_p \dots a_k)_b \pm m (a_{k-1} \dots a_0)_b \equiv 0[d]$$

Calcul basé sur la Conjonction de R, L et/ou M.

Méthode par recomposition / méthode de Horner

$$(0,4312)_5 = (0,9312)_{10}$$

$$0, \frac{1}{5} \times \left(4 + \frac{1}{5} \times \left(3 + \frac{1}{5} \times \left(1 + \frac{1}{5} \times (2) \right) \right) \right)$$

Diagram illustrating the step-by-step calculation of the decimal equivalent of the base-5 number $0,4312_5$ using Horner's method. The expression is written in green, and the intermediate results are written in yellow below each term, connected by vertical arrows.

- $0,4312_5$ is converted to $0,9312_{10}$.
- The expression is expanded as $0, \frac{1}{5} \times \left(4 + \frac{1}{5} \times \left(3 + \frac{1}{5} \times \left(1 + \frac{1}{5} \times (2) \right) \right) \right)$.
- Intermediate results (in yellow) are calculated from the innermost parentheses outwards:
 - $2 \times \frac{1}{5} = 0,4$
 - $1 + 0,4 = 1,4$
 - $1,4 \times \frac{1}{5} = 0,28$
 - $3 + 0,28 = 3,28$
 - $3,28 \times \frac{1}{5} = 0,656$
 - $4 + 0,656 = 4,656$
 - $4,656 \times \frac{1}{5} = 0,9312$

Depliage / repliage.

$$(0, 32133)_4 = (0, 7174)_8$$

dep ↘

$$(0, \underbrace{11}_{\text{green}} \underbrace{10}_{\text{green}} \underbrace{01}_{\text{green}} \underbrace{11}_{\text{green}} \underbrace{11}_{\text{green}} 00)_2$$

↗ rep

attention : en vérifiant avec bc, on peut être trompé.

Convertir en utilisant la périodicité :

$$\text{ex1)} \quad (1, 12\overline{31})_4 = (112)_4 / (16)_{10} + (0, 00\overline{31})_4$$

$$\text{Convertissons } \alpha = (0, 00\overline{31})_4 = (\quad ? \quad)_{10}$$

$$\alpha + (0, 31)_4 = (16)_{10} \alpha = (0, \overline{31})_4$$

$$\alpha = \frac{(0, 31)_4}{15}$$

$\overline{2}$ terminer...

(ex2)

$$\beta = (\overline{1}, 2)_3 = (?)_{10}$$

$$3\beta = (\overline{1}, 2)_3$$

$$\overset{\times 2}{\curvearrowright} 3\beta - 1 = (\overline{1})_3 \overset{\times 3}{\curvearrowright}$$

$$6\beta - 2 = (\overline{2})_3 = -1$$

$$6\beta - 2 = -1$$

$$3(3\beta - 1) = (\overline{1}, 0)_3$$

$$3(3\beta - 1) + 2 = 3\beta$$

$$\boxed{\beta = 1/6} = (0, \overline{1\overline{6}})_{10}$$

(ex3)

On peut montrer $(\overline{w})_b + (-\overline{w})_b = 0$.