※ Kinematic Analysis ※ 3 D.O.F ☐1
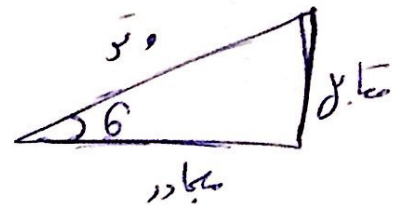
$Sin \theta = \dfrac{\text{قابل}}{\text{وتر}}$

$Cos \theta = \dfrac{\text{مجاور}}{\text{وتر}}$

$tan \theta = \dfrac{\text{قابل}}{\text{وتر}}$

وتر

قابل

θ

مجاور

$a^2 + b^2 = h^2$ ← نظرية فيثاغورس ؟

$(a+b)^2 = a^2 + b^2 + 2ab$ قانون فك الأقواس الكامل :-

$$Sin^2 \theta + Cos^2 \theta = 1$$

∴ منطقات جمع زوايا المثلث و ترابها :-

$$\begin{cases} Sin(A+B) = Sin A \, Cos B + Cos A \, sin B \\ Cos(A+B) = Cos A \, Cos B - sin A \, sin B \end{cases}$$

※ Forward kinematic ※

المدخلات

θ_1
θ_2
θ_3
L_1
L_2
L_3

$(X_3, Y_3) = ?$

المخرجات

X_3
Y_3
$\emptyset$

$$\emptyset = \theta_1 + \theta_2 + \theta_3$$

※ 1

$$\cos \theta = \frac{\text{المجاور}}{\text{الوتر}}$$

$$\left[X_3 = L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3) \right]$$

※ 2

$$\sin \theta = \frac{\text{المقابل}}{\text{الوتر}}$$
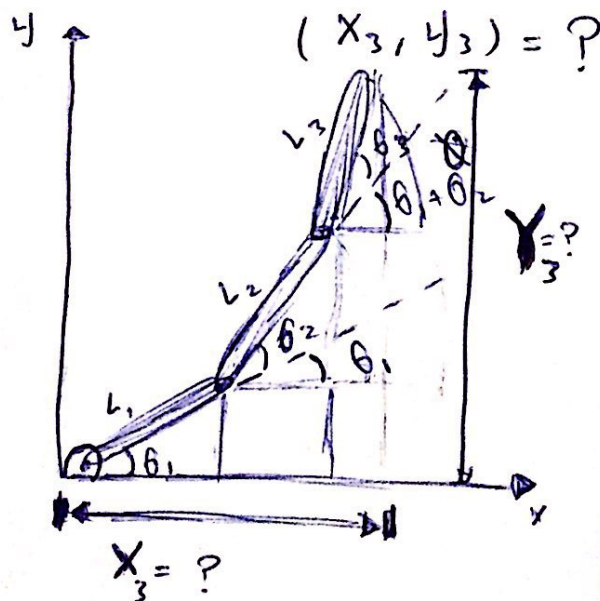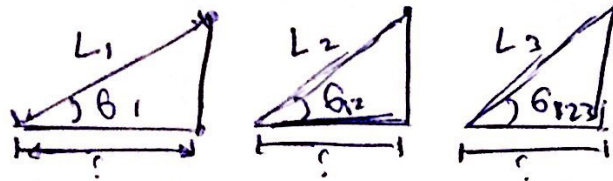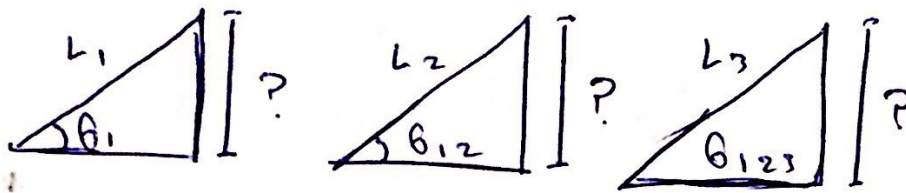
$$\left[Y_3 = L_1 \sin \theta_1 + L_1 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3) \right]$$

(X_3, Y_3)

$Y_3 = \checkmark$

$X_3 = \checkmark$

$$\therefore \boxed{\emptyset = \theta_1 + \theta_2 + \theta_3}$$

※ Kinematic Analysis ※   3 DOF   [3]

※ Inverse Kinematic ※

المخرجات
X_3
Y_3
θ
L_1
L_2
L_3

$(X_3, Y_3) \Rightarrow$ given

$\theta = \checkmark$

$Y_3 = \checkmark$

$X_3 = \checkmark$

المدخلات
θ_1
θ_2
θ_3

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[4]

$Y_3 = \checkmark$

$X_3 = \checkmark$

$$\boxed{y_2 = Y_3 - L_3 \sin(\theta_1 + \theta_2 + \theta_3)}$$

$$\boxed{X_2 = X_3 - L_3 \cos(\theta_1 + \theta_2 + \theta_3)}$$

∴ بعد أن عرفنا قيمة $[y_2, X_2]$ سوف نتخلص من $[L_3]$ ونوجد $[\theta_2]$
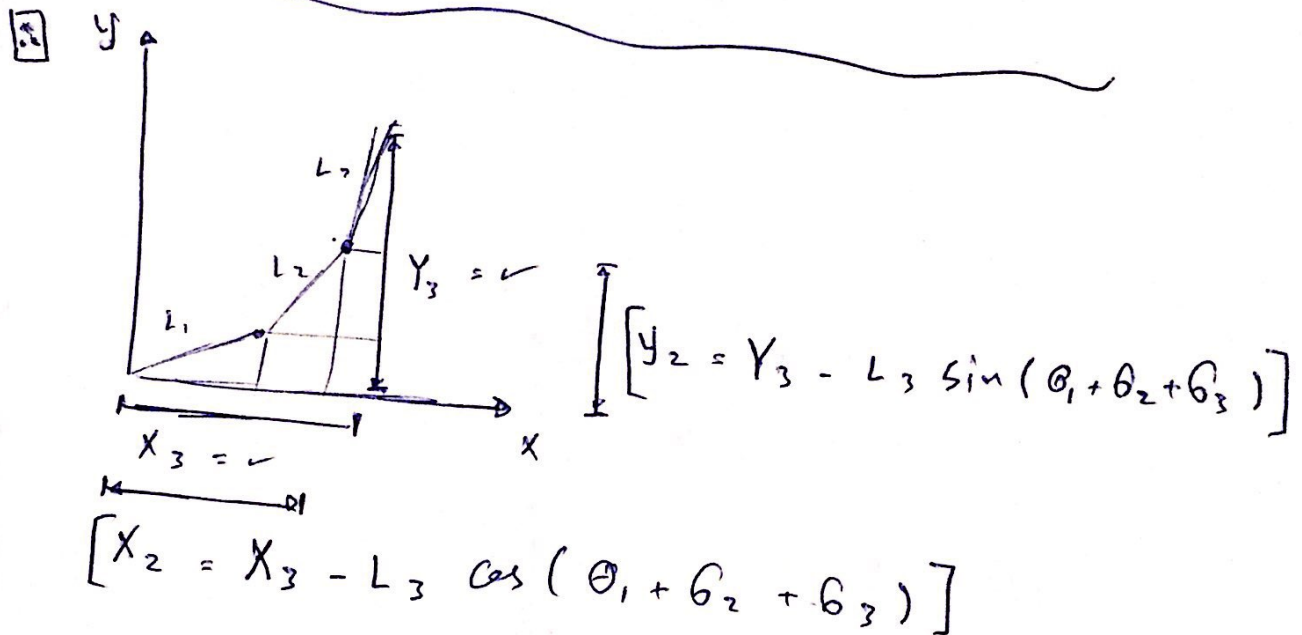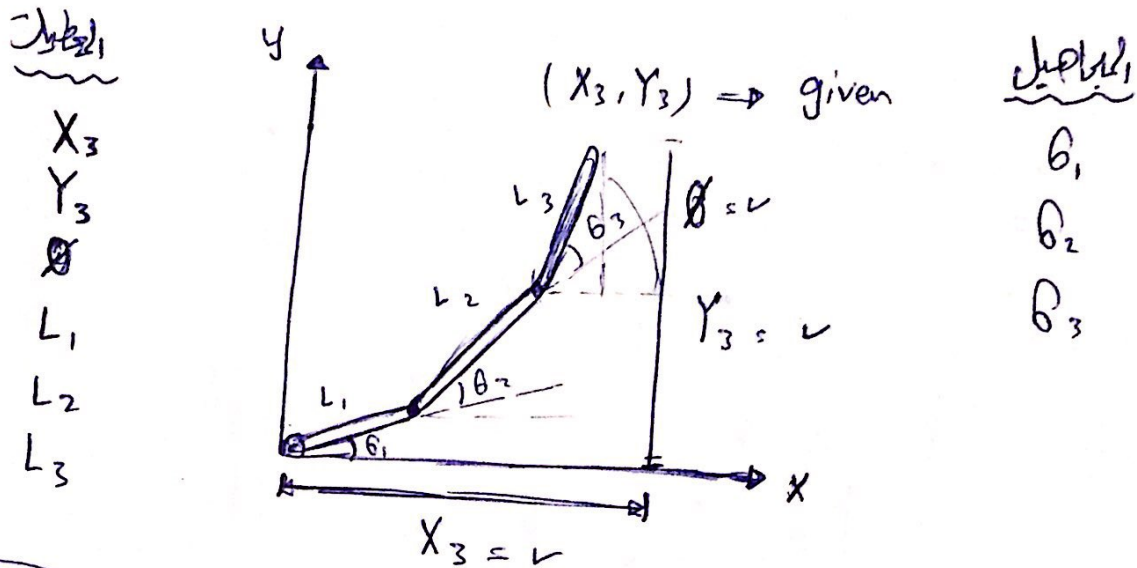
شغل المسألة في 2DOF -

∴ After we know the value of $[X_2, Y_2]$ we will neglect $[L_3]$ and Find $[\theta_2]$ like the previous in 2 DOF.

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✳ Kinematic Analysis ✳   3 DOF   4

$$Cos\, \theta_2 = \frac{X_2^2 + Y_2^2 - L_1^2 - L_2^2}{2\, L_1 L_2} \quad , \quad sin\, \theta_2 = \pm \sqrt{1 - cos^2 \theta_2}$$

$$\theta_2 = Cos^{-1}\left(\frac{X_2^2 + Y_2^2 - L_1^2 - L_2^2}{2\, L_1 L_2} \right) \qquad \begin{bmatrix} two \\ solutions \end{bmatrix}$$

$$\theta_2 = sin^{-1}(6) = \checkmark$$

To Find $\theta_1 = ?$

$$X_2 = L_1\, Cos\, \theta_1 + L_2\, Cos(\theta_1 + \theta_2)$$

$$Y_2 = L_1\, sin\, \theta_1 + L_2\, sin(\theta_1 + \theta_2)$$

نستعين بالمتطابقات المثلثية لفك التعويض:-

$$X_2 = L_1\, Cos\, \theta_1 + L_2\,(Cos\, \theta_1\, Cos\, \theta_2 - sin\, \theta_1\, sin\, \theta_2)$$

$$Y_2 = L_1\, sin\, \theta_1 + L_2\,(sin\, \theta_1\, cos\, \theta_2 + Cos\, \theta_1\, sin\, \theta_2)$$

بحل عامل مشترك:-

$$\begin{cases} X_2 = Cos\, \theta_1\,(L_1 + L_2\, Cos\, \theta_2) - (L_2\, sin\, \theta_2)\, sin\, \theta_1 \\ Y_2 = sin\, \theta_1\,(L_1 + L_2\, Cos\, \theta_2) + (L_2\, sin\, \theta_2)\, cos\, \theta_1 \end{cases}$$

نعامل المعادلتين بنظام معادلتين

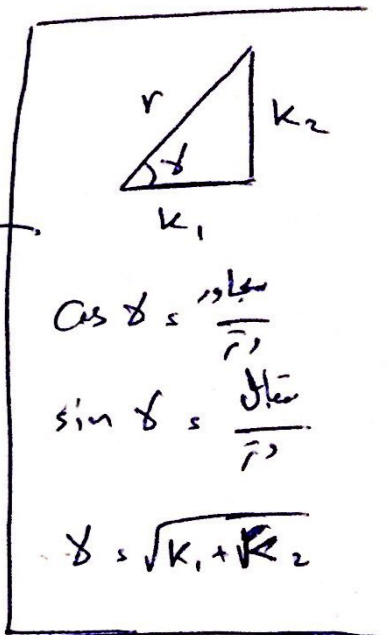※ kinematic Analysis ※ 3 DOF ⑤

<u>Let say</u>:- $k_1 = L_1 + L_2 \cos \theta_2$ ---- ①

$k_2 = L_2 \sin \theta_2$ ---- ②

$x_2 = \cos \theta_1 (k_1) - (k_2) \sin \theta_1$

$y_2 = \sin (k_1) + (k_2) \cos \theta_1$

بالتعويض

$\dfrac{x}{r} = \dfrac{r}{r} \cos \gamma \cos \theta_1 - \dfrac{r}{r} \sin \gamma \sin \theta_1$

$\dfrac{y}{r} = \dfrac{r}{r} \cos \gamma \sin \theta_1 + \dfrac{r}{r} \sin \gamma \cos \theta_1$

$\cos \gamma = \dfrac{\text{مجاور}}{r}$

$\sin \gamma = \dfrac{\text{مقابل}}{r}$

$\gamma = \sqrt{k_1 + k_2}$

بالقسمة على r

$\dfrac{x}{r} = \underline{\cos \gamma} \ \underline{\cos \theta_1} - \underline{\sin \gamma} \ \underline{\sin \theta_1}$

$\dfrac{y}{r} = \underline{\cos \gamma} \ \underline{\sin \theta_1} + \underline{\sin \gamma} \ \underline{\cos \theta_1}$

بالاستعلام عن المتطابقات المثلثية للزاوية المجموع وأوجد:-

$\dfrac{x}{r} = \cos (\gamma + \theta_1)$

$\dfrac{y}{r} = \sin (\gamma + \theta_1)$

$\left[\tan = \dfrac{\sin \theta}{\cos \theta} \right]$

$\tan (\gamma + \theta_1) = \dfrac{\dfrac{y}{r}}{\dfrac{x}{r}}$

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then :-

$$\forall + \theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \forall$$

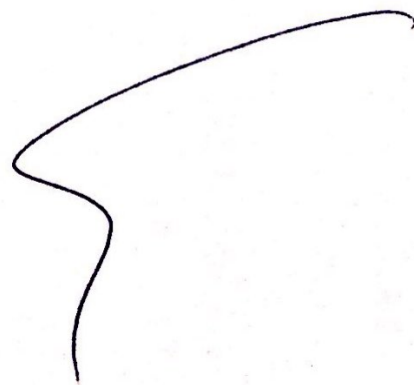$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{k_2}{k_1}\right) = \checkmark$$

$$\theta_1 = \checkmark \qquad \theta_2 = \checkmark$$

$$\forall = \theta_1 + \theta_2 + \theta_3$$

$$\theta_3 = \forall - \theta_1 + \theta_2 \qquad = \checkmark$$

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هناك حليل لـ θ_2 تحته لاشكل اخرى قيمه $\pm \sin\theta_2$

</div>

$$\theta_2 \begin{cases} + \sin\theta_2 \longrightarrow k_2 \ (+ve) \longrightarrow \forall' \longrightarrow \theta_1' \\ - \sin\theta_2 \longrightarrow k_2 \ (-ve) \longrightarrow \forall'' \longrightarrow \theta_1'' \end{cases}$$

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مام عبد الكريم

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