

Subject Index

- absolute value, 4
- abundant number, 45
- acute angle, 55
- aggregation bias, *see* Simpson's Paradox
- antiderivative, 209, 231
- apostrophe notation, 92
- argmin, 18
- Aristotle, 38
- arithmetic rules, 2
- asymptote, 180

- basis vectors, 266
- Bayes
 - Reverend Thomas, 315
- Bayes Law, 315
- Bayesianism, 337
- bell shaped
 - stupidity of expression, 381
- Bernoulli PMF, 340
- Bernoulli trials, 339
- Bernoulli, Jacob, 22, 37, 284
- Bernoulli, Johann, 201, 284
- Bhaskara, 200
- Binomial PMF, 340
- Binomial Theorem, 289
- Borel-field, 307
- boundary notation, 17

- Brahmagupta, 200
- breakdown bound, 365

- Cafe Med, 327
- Carl Jacobi, 2
- Cartesian coordinate system, 7, 133
- Cartesian plane, 133
- Cartesian to polar, 53
- Cauchy PDF, 381
- Celsius, Anders, 333
- chain rule, 195, 196
- characteristic equation, 162
- characteristic root, *see* matrix, eigenvalue
- χ^2 PDF, 352
- choose notation, 288, 325
- chord, 244
- Chu Shi-kié, 290
- circular functions, *see* trigonometric functions
- Clement matrix, 173
- collinearity, 135
- column space, 136
- comparison test, 262
- complement, *see* set, complement
- complementary angles, 60
- complex number, 14
- concave, 244
- condition number, 164

- conic section, 62
- constrained optimization, 271
- convex, 244
- convex set, 9
- cosine, 55
- countable, 330
- counting numbers, 14
- counting rules, 285, 402
 - ordered, with replacement, 287
 - ordered, without replacement, 287
 - unordered, with replacement, 289
 - unordered, without replacement, 288, 326
- coupling from the past, 395
- covariance, 367
- Cramer's rule, 177
- craps, 373–378
- cumulative distribution function (CDF), 348, 356
- data
 - categorical, 331
 - continuous, 330
 - definition, 6
 - discrete, 330
 - interval, 332
 - levels of measurement, 330
 - nominal, 331
 - ordinal, 331
 - polychotomous, 331
 - ratio, 333
 - single point: datum, 6
- De Moivre, Abraham, 31, 284
- de Morgan's Laws, 299, 324
- decay rate, 44
- deficient number, 45
- definite integral, 207
- degrees of freedom, 352
- derivative, 185
 - chain rule, 192
 - exponent, 196
 - higher order, 239–241, 280, 282
 - logarithm, 196
 - of a constant, 190
 - partial, 199, 234–239, 279, 280
 - power rule, 190
 - product rule, 191
 - quotient rule, 191
 - second, 239
 - sum, 190
 - trigonometric functions, 225, 234
- derivative notation
 - Leibniz's, 185, 239
 - Newton's, 185, 239
- determinant, *see* matrix, determinant
- differential equations, 217
- differentiation, 185
- Dirichlet, Lejeune, 6, 22
- distribution function, 334
- domain, 23
- double factorial, 232
- eigen-analysis, 160–167
- eigenvalue, *see* matrix, eigenvalue
- eigenvectors, *see* matrix, eigenvectors
- ellipse, 67–69, 282
 - definition, 67
 - foci, 67
 - major axis, 68
 - minor axis, 68
 - standard form, 67
 - vertices, 67
- entropy, 359–361
 - information, 360
 - Shannon, 360
- equation
 - definition, 18
- equitable matrix, 127
- Ergodic Theorem, 415
- Euclid, 46, 290
- Euler's expansion, 37, 46
- Euler's integral, *see* gamma function
- Euler, Leonhard, 20, 22, 37, 185, 222, 284
- even function, 76
- expected value, 370
 - conditional, 372

- finite properties, 372, 373
- exponent
 - properties, 34
- exponential function, 36–40, 71
- exponential PDF, 351
- factor, 32
- factorial function, 29, 285
- Fermat, Pierre, 284
- Fibonacci numbers, 291
- Fibonacci sequence, 42
- field, 83
- Fourier function, 79
- Fourier series, 79
- frequentism, 336
- function
 - asymptotic value, 180
 - continuous, 21
 - definition, 19
 - invertible, 21
 - limit, 179
 - limit examples, 181
 - limit properties, 181
 - linear, 24
 - monotonically decreasing, 183
 - monotonically increasing, 183
 - properties, 21
 - rate of change, 183
 - right-continuous, 348
 - symmetric, 179
 - unimodal, 179
- function root, 247
- Fundamental Theorem of Calculus, 211
- Fundamental Theorem of Counting, 286
- gamma function, 222, 232, 352
- gamma PDF, 351
- Gauss, Carl Friedrich, 284, 354
- Gauss-Jordan elimination, 152–154
- Gaussian PDF, *see* normal PDF
- Glottochronology, 44
- gradient, 270
- Greek symbols, 49–50
- Hamiltonian notation, 266
- Hesse, Ludwig, 270
- Hessian, 270
- Hilbert matrix, 174
- histogram, 364
- Huygens, Christian, 37
- hyperbola, 69–74, 282
 - definition, 70
 - standard form, 70
- idempotent matrix, 118, 126, 130
- identity matrix, 103, 113
- independence, 317
 - conditional, 318
 - pairwise, 317
- independence rule, 341
- indexing, 11
 - computer notation, 13
- inequalities, 378–380
- inequality
 - Berge, 379
 - Cauchy-Schwarz, 96, 126, 151
 - Chebyshev, 378
 - Cramer-Rao, 379
 - Hölder's, 96, 97, 126, 379
 - Jensen's, 378
 - Liapounov's, 379
 - Markov, 378
 - Minkowski, 379
 - Schwarz, 379
 - triangle, 96, 126
- inequality notation, 18
- infinity, 16
- inflection point, 241, 278
- integral
 - definite
 - properties, 212
 - discontinuities, 214
 - indefinite, 217, 232
 - multidimensional, 250–256
 - repeated, 250
 - trigonometric functions, 226
- Integral Test, 260

- integrand, 208
- integration, 205
- integration by parts, 219, 232
- integration symbol, 207
- intersection, *see* set, intersection
- interval colon notation, 10
- interval comma notation, 10
- interval notation, 9
- involutory matrix, 119, 126
- irrational number, 37
- irrational numbers, 14
- iterated integral, 281
- Iterated Integral Theorem, 254

- J matrix, 113

- Kolmogorov probability axioms, *see*
 - probability, axioms
- Kronecker product, *see* matrix, Kronecker
 - product

- Lagrange multiplier, 272, 283
 - example, 273
- Lagrange, Joseph Louis, 284
- Laplace, Pierre-Simon de, 284
- latent characteristics, 332
- law of cosines, 95
- Legendre, Adren-Marie, 284
- Leibniz, Gottfried Wilhelm, 22, 204
- L'Hospital's rule, 201, 223, 231
- L'Hospital, Guillaume, 201
- limit notation, 17
- line
 - parallel, 28
 - perpendicular, 28
 - point-slope form, 42
 - slope-intercept form, 26
- linear algebra
 - definition, 136
- linear combinations, 136
- linear programming, 152
- linear regression, 29, 46, 172
- linear space, 138
- linear subspace, 138
- linear system of equations, 157–160
- linear transformation, 138
 - closed, 138
- linearly dependent columns, 146
- linearly independent, 146
- location-scale distributions, 354
- logarithm, 35–36
 - base, 35
 - change of base, 36, 197
 - natural, 36
 - properties, 36
- logarithmic differentiation, 197, 230
- logical symbols, 15

- M-matrix, 175
- marginal effects, 199
- Markov chain, 393, 394, 409
 - closed, 424
 - absorbing, 409
 - detailed balance equation, 417
 - discrete, 395, 409
 - ergodic, 415
 - hitting times, 409
 - homogeneous, 395
 - j-step transition matrix, 399
 - limiting distribution, 414
 - marginal distribution, 413
 - mixing, 414
 - null recurrent, 410
 - period, 405
 - positive recurrent, 410
 - recurrent, 409–411, 426
 - reducible, 408
 - reversibility, 417
 - stability, 413–415
 - state space, 407, 409
 - absorbing, 409
 - closed, 410
 - irreducible, 407
 - recurrent, 410
 - stationarity, 414
 - stationary distribution, 401, 427
 - transient, 409

- transition kernel, 398, 421
- transition matrix, 398
- Markovian cat, 393
- Markovian property, 394
 - example, 396
- martingale, 394
 - example, 397
- matched pair, 26
- matrix
 - addition, 107
 - block diagonal, 130
 - calculation
 - properties, 108
 - cofactor, 141
 - collinearity, 176
 - decomposition, 104
 - definition, 100
 - determinant, 90, 140, 176
 - graphical interpretation, 145
 - properties, 144
 - diagonal, 102
 - dimensions, 100
 - eigenvalues, 161, 171, 173, 176
 - properties, 163
 - eigenvectors, 161, 173, 175
 - elements, 100
 - entries, 100
 - equality, 101
 - Euclidean norm, 149
 - Frobenius norm, 149
 - full column rank, 146
 - full row rank, 146
 - Hadamard product, 129
 - idempotent
 - rank, 148
 - ill-conditioned, 164
 - indefinite, 168
 - inverse, 173, 176
 - properties, 154
 - Jordan product, 130
 - Kronecker product, 119–124, 131
 - exponents, 144
 - properties, 122
 - trace operation, 144
 - LDU decomposition, 168
 - left eigenvector, 165
 - Lie product, 130
 - lower triangular, 104
 - LU decomposition, 112, 130
 - minor, 141
 - multiplication, 107, 109–115
 - properties, 111
 - negative definite, 168
 - negative semidefinite, 168
 - noninvertible, 152
 - nonnegative definite, 168
 - nonpositive definite, 168
 - norm
 - properties, 150
 - matrix
 - order- k , 101
 - orthogonal, 172
 - p-norm, 126, 149
 - pivots, 143
 - positive definite, 168
 - positive semidefinite, 168
 - post-multiplication, 110, 112, 126
 - pre-multiplication, 110, 112, 126
 - quadratic forms, 167
 - rank, 146, 172
 - properties, 148
 - right eigenvector, 165
 - scalar division, 107
 - short rank, 146
 - singular, 152
 - skew-symmetric, 102, 116
 - square, 101
 - subtraction, 107
 - symmetric, 102, 116
 - trace, 139, 173
 - properties, 140
 - trace norm, 149
 - transposition, 115
 - properties, 116
 - upper triangular, 104
 - vectorization, 119, 131

- matrix inverse, 151
- maximization, 270
- maximum likelihood estimation, 269
- mean, 362
- Mean Value Theorem, 203, 210, 234
- measures of centrality, 361
- measures of dispersion, 365
- median, 362
- median absolute deviation (MAD), 366
- Mercator, Nicolaus, 37
- mode, 362
- modulo function, 32
- moments, 380
 - central, 380
- multinomial choice, 289
- mutually exclusive, *see* set, pairwise disjoint

- Napier, John, 37
- natural exponent, 38
- natural numbers, 14
- nautilus, 38
- nested equation, 20
- Newton's method, 247, 278
 - example, 249
- Newton, Isaac, 204, 247
- Newton-Raphson, *see* Newton's method
- nilpotent matrix, 118, 119
- normal PDF, 354
 - standard, 355
- n th partial sum, 259
- number typology, 14

- odd function, 76
- odds ratio, 321
- order of operations, 3
- ordered pair, 23, 133
- Oughtred, William, 37

- parabola, 64–66
 - definition, 64
 - directrix, 64
 - focus, 64
- Pareto's Law, 352
- partial derivative, *see* derivative, partial

- Pascal's Triangle, 290, 324
- Pascal, Blaise, 284, 290
- perfect number, 45
- permutation matrix, 114
- Plato, 2
- PMF, *see* probability mass function
- point
 - on coordinate system, 7
- Poisson PMF, 344
- Poisson, Siméon Denis, 284
- polar coordinates, 53
- polar to Cartesian, 53
- polynomial function, 33
- power, 35
- principle root, 4
- prisoner's dilemma, 263
- probability, 306–315
 - as degree of belief, 337
 - axioms, 308, 339, 414
 - conditional, 312–315
 - joint, 309, 315, 341
 - multiplication rule, 315
 - objective, 285
 - rules for calculation, 309
 - subjective, 285
 - updating, 312
- probability density function (PDF), 350
- probability function
 - definition, 307
- probability mass function (PMF), 339
- probability measure space, *see* triple
- probability space, *see* triple
- probit, 357
- product operator, 12
- projection, 134
 - orthogonal, 134
- pseudo-random numbers, 407
- Pythagorean Theorem, 133
- Pythagoreans, 38, 45

- quadratic, 33, 46, 63

- radian measurement, 51
- radical index, 4

- radican, 4
- random variable, 338
- randomness, 334, 337
- range, 23
- ratio test, 262
- rational numbers, 14
- real number line, 9
- rectangle rule, 205
- relation, 21, 23
- resistance, 365
- Riemann integral, 231
- Riemann integration, 205
- Riemann sum, 209
- Riemen-Stieltjes integral, 372
- right-hand-rule, 90
- Rolle's Theorem, 203, 234, 246
- root test, 262
- roots of a polynomial, 33
- row space, 136

- Saint-Vincent, Gregory, 37
- sample space, 292
- Schur product, 129
- second derivative, *see* derivative, second
- second derivative test, 245
- series, 256–266
 - alternating, 263
 - convergent, 260, 281
 - divergent, 260, 281
 - finite, 259
 - geometric, 262, 263
 - harmonic, 260, 281
 - infinite, 259
 - Maclaurin, 265, 282
 - Mercator, 282
 - power, 264
 - Taylor, 265
- set
 - cardinality, 294
 - complement, 298, 300
 - containment, *see* subset
 - countable, 293
 - definition, 291
 - difference operator, 298
 - disjoint, 302
 - element, 292
 - empty, 295
 - equality, 296
 - event, 292
 - finite, 293
 - infinite, 293
 - intersection, 297, 300
 - null, 295
 - operations, 295
 - outcome, 292
 - pairwise disjoint, 302
 - partition, 302
 - properties, 300
 - uncountable, 293
 - union, 296, 300
- set notation, 16
- sigma-algebra, 307
- sigma-field, *see* sigma-algebra
- Simpson's Paradox, 316
- sine, 55
- span
 - matrix, 139
 - vectors, 138
- spectral component, 165
- spectral decomposition, 161
- standard deviation, 366
- state space, 393
- stem and leaf plot, 364
- Stirling's Approximation, 31, 46
- stochastic process, 393
- structural equation models, 147
- subjectivism, 337
- submatrix, 141
- subscript notation, 13
- subset, 295
 - proper, 295
- summation operator, 11
- support, *see* domain
- symmetric difference, 298
- tangent function, 55

- tangent line, 185
- Taylor series expansion, 247
- tensor product, *see* matrix, Kronecker product
- theorem, definition, 55
- Total Probability Theorem, 309, 313, 314, 316, 325, 328
- trace, *see* matrix, trace
- transformation, 77
- transition kernel, *see* Markov chain, transition kernel
- transition matrix, *see* Markov chain, transition matrix
- triangular matrix, 104
- trigonometric functions, 55
 - radian measure, 59
 - reciprocal relations, 55
- trigonometry
 - definition, 55
- triple, 308
- uncertainty, 335
- uncountable, 331
- uniform PDF, 358
- uniform PMF, 358, 361
- union, *see* set, union
- unit circle, 62
- unit vector, 96
- unit-radius circle, 53
- utility
 - definition, 5
- Vandermonde matrix, 174
- variable
 - definition, 6
- variance, 366
- vector
 - addition, 84
 - algebra
 - properties, 86
 - basic calculations, 84
 - basis, 138, 139
 - column form, 84
 - conforming, 85
 - cross product, 89
 - properties, 91
 - definition, 83
 - difference norm, 126
 - dot product, 87
 - inner product, 87
 - properties, 88
 - multiplication norm, 126
 - nonconforming, 85
 - norm, 93
 - properties, 94
 - orthogonal, 87
 - outer product, 89
 - p-norm, 96
 - row form, 84
 - scalar division, 85
 - scalar multiplication, 85
 - scalar product, 87
 - subtraction, 85
 - transpose, 92
- vector function, 267
 - differentiation, 268
 - integration, 268
- Venn diagram, 299
- whole numbers, 14
- zero, 3
- zero matrix, 104