Methods 2 – Portfolio Assignment 1

- *Type:* Group assignment
- *Due:* 13 March 2022, 23:59

Please submit your answers on GitHub Classroom. The best way to give your answers is in a LaTeX or R Markdown file, but you can also create a PDF in some other way.

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1) Given the vectors $\vec{u} = (1, 1, 1)$, $\vec{v} = (2, 3, 1)$, and $\vec{w} = (-1, -1, 2)$, compute the following products:
   a) $\vec{u} \cdot \vec{v}$
   b) $\vec{u} \cdot \vec{w}$
   c) $\vec{v} \cdot \vec{w}$
   d) $\vec{u} \times \vec{v}$
   e) $\vec{u} \times \vec{w}$
   f) $\vec{v} \times \vec{w}$

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2) Vectors are *orthogonal* (or *perpendicular*) when their dot product is zero: $\vec{x} \perp \vec{y} \iff \vec{x} \cdot \vec{y} = 0$. A *unit vector* is a vector with norm 1: $\|\vec{x}\| = 1$.
   a) Find a unit vector that is perpendicular to both $\vec{u} = (1, 0, 1)$ and $\vec{v} = (1, 2, 0)$.
   b) Find a vector that is orthogonal both to $\vec{u}_1 = (1, 0, 1)$ and $\vec{u}_2 = (1, 3, 0)$, and whose dot product with the vector $\vec{v} = (1, 1, 0)$ is equal to 8.

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3) Prove the geometric formula for the dot product $\vec{x} \cdot \vec{y} = \|\vec{x}\|\|\vec{y}\| \cos \varphi$, where φ is the angle between the vectors $\vec{x}$ and $\vec{y}$.

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4) For the matrix

$$X = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

calculate X^n for $n = 2, 3, 4, 5$. Write a rule for calculating higher values of n.