$$\frac{d}{dx}\cos(x) = -\frac{x}{1!} + \frac{x^3}{3!} - \frac{x^5}{5!} + \dots$$

we see that $\frac{x}{1!} = \frac{x}{1} = x$ and

$$= -\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)$$

which is

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

□