Use these power definitions

$$\exp(x) := \sum_{n=0}^{\infty} \frac{x^n}{n!} ,$$

$$\sin(x) := \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} ,$$

$$\cos(x) := \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} .$$

to prove that

$$\frac{d}{dx} \exp(x) = \exp(x) ,$$

$$\frac{d}{dx} \sin(x) = \cos(x) ,$$

$$\frac{d}{dx} \cos(x) = -\sin(x) .$$

First we recognize exp(x), sin(x), cos(x) as the (1), (2) and (3).

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad (1)$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (2)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (3)$$

Starting of with the exponential equation we get

$$(1.1) \quad \frac{d}{dx} \exp(x) = \frac{d}{dx} 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

for the derivative of an added constant is zero. For the next element one takes x out and derives:

$$(1.2) \quad \frac{d}{dx} \frac{x}{1!} = \frac{d}{dx} \frac{1}{1!} \cdot x = \frac{d}{dx} 1 \cdot x = 1$$

For the third element:

$$(1.3) \quad \frac{d}{dx} \frac{x^2}{2!} = \frac{d}{dx} \frac{1}{2!} \cdot x^2 = \frac{1}{2!} 2x = \frac{2x}{2!} = \frac{x}{1!}$$

the fourth element:

$$(1.4) \quad \frac{d}{dx} \frac{x^3}{3!} = \frac{d}{dx} \frac{1}{3!} x^3 = \frac{1}{3!} 3x^2 = \frac{3x^2}{3 \cdot 2 \cdot 1} = \frac{x^2}{2!}$$

replacing (1.2), (1.3) and (1.4) into their appropriate spaces in (1.1) we get

$$\frac{d}{dx} \exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$