Kelvin Rosario

Project 1 - part 2

1a) $15 \cdot 29 \bmod 13 = (15 \bmod 13)(29 \bmod 13) \bmod 13$

$= (2)(3) \bmod 13$

$= 6.$

1b) $2 \cdot 29 \bmod 13 = (2)(3) \bmod 13$

$= 6.$

1c) $2 \cdot (-3) \bmod 13 = -6 \bmod 13$

$\equiv 7$, move within the same equivalence class by adding 13 to (-6).

1d) $(-11) \cdot 3 \bmod 13 = (-11 \bmod 13)(3 \bmod 13) \bmod 13$

$= (2)(3) \bmod 13$

$= 6.$

We computed with the same technique from 1c)

$-11 \bmod 13 \equiv 2$, by adding 13

2a) $\frac{1}{5} \bmod 13 = x$

$= 5x$

Can we find an x such that $1 \bmod 13 \equiv 5x$?

From just thinking about the multiples of 5, we realize that <u>40</u> would do the trick. So we let $x = 8$

So, $\frac{1}{5} \bmod 13 = 8.$

2b) $\frac{1}{5} \bmod 7 = x$

$= 5x$

Using the same thinking as above, we find that <u>15</u> would fit the bill, so we let $x = 3$

So, $\frac{1}{5} \bmod 7 = 3.$

2c) $3 \cdot \frac{2}{5} \bmod 7 = \overset{(3)}{\cancel{(3 \bmod 7)}} \overset{(2)}{\cancel{(\frac{1}{5} \bmod 7)}} (\frac{1}{5} \bmod 7) \bmod 7$

$= (3)\,(\overset{2}{\cancel{3}})\,(3) \bmod 7$

$= 18 \bmod 7$

$= 4.$

3a) $5x = 4 \mod 3$

We can shift into

$5x = 1 \mod 3$

and here, we can see that 10 is a multiple of 5 which would give a remainder of 1 when divided by 3. So we solve with $\boxed{x = 2}$

3b) $7x = 6 \mod 5$

again, moving within the equivalance class:

$7x = 1 \mod 5$

We note that 21 is a multiple of 7 that gives a remainder of 1 when divided by 5, so $\boxed{x = 3}$

3c) $9x = 8 \mod 7$

using the same approach:

$9x = 1 \mod 7$

36 is a multiple of 9 that gives a remainder of 1 when divided by 7, so $\boxed{x = 4}$

4a) $3^2 \bmod 13 = (3 \bmod 13)(3 \bmod 13) \bmod 13$

$= (3)(3) \bmod 13$

$= 9 \bmod 13$

$\boxed{= 9}$

4b) $7^2 \bmod 13 = (7 \bmod 13)(7 \bmod 13) \bmod 13$

$= 49 \bmod 13$

$\boxed{= 10}$

4c) Let's compute a few exponents first:

$3^2 \bmod 13 = 9$ (from 4a)

$3^4 \bmod 13 = (3^2 \bmod 13)(3^2 \bmod 13) \bmod 13$

$= (9)(9) \bmod 13$

$= 81 \bmod 13$

$= 3$

$3^8 \bmod 13 = (3^4 \bmod 13)(3^4 \bmod 13) \bmod 13$

$= (3)(3) \bmod 13$

$= 9$

So, $3^{10} \bmod 13 = (3^2 \bmod 13)(3^8 \bmod 13) \bmod 13$

$= (9)(9) \bmod 13$

$\boxed{= 3}$

4d) We compute some exponents, like before:

$$7^2 \bmod 13 = (7 \bmod 13)(7 \bmod 13) \bmod 13$$
$$= 49 \bmod 13$$
$$= 10$$

$$7^4 \bmod 13 = (7^2 \bmod 13)(7^2 \bmod 13) \bmod 13$$
$$= (10)(10) \bmod 13$$
$$= 9$$

$$7^8 \bmod 13 = (7^4 \bmod 13)(7^4 \bmod 13) \bmod 13$$
$$= (9)(9) \bmod 13$$
$$= 3$$

$$7^{16} \bmod 13 = (7^8 \bmod 13)(7^8 \bmod 13) \bmod 13$$
$$= (3)(3) \bmod 13$$
$$= 9$$

$$7^{32} \bmod 13 = (7^{16} \bmod 13)(7^{16} \bmod 13) \bmod 13$$
$$= (9)(9) \bmod 13$$
$$= 3$$

$$7^{64} \bmod 13 = (7^{32} \bmod 13)(7^{32} \bmod 13) \bmod 13$$
$$= (3)(3) \bmod 13$$
$$= 9$$

$$7^{100} \bmod 13 = (7^4 \bmod 13)(7^{32} \bmod 13)(7^{64} \bmod 13) \bmod 13$$
$$= (9)(3)(9) \bmod 13$$
$$\boxed{= 9}$$

4e) In the prior problem, I noticed even exponents of $7^{\text{mod } 13}$ were alternating between 3 and 9, so I figured an odd exponent would be the key to finding x.

x = 1 and x = 3 were not it:

$$7^1 \bmod 13 = 7$$

$$7^3 \bmod 13 = 5$$

$$7^5 \bmod 13 = (7^4 \bmod 13)(7 \bmod 13) \bmod 13$$
$$= (9)(7) \bmod 13$$
$$= 63 \bmod 13$$
$$= 11$$

So, $\boxed{x = 5}$ works out.