Drexel University, ENGR 232 – Dynamic Engineering Systems
Summer 2010-11
Homework 5 Coversheet

THIS MUST BE STAPLED TO THE FRONT OF YOUR ASSIGNMENT

Due: In Recitation, July 28-29

Name_____ Section_____

The following three problems deal with autonomous differential equations of the form $y' = f(y)$. Draw the function, find the equilibrium points, and indicate which are stable and which are unstable. Draw several solutions in the *t*-*y* plane.

1. $y' = y(y-1)(y-2)$, $-\infty < y_0 < \infty$
2. $y' = \sin \pi y$, $0 \le y_0 \le 2$
3. $y' = y^2(1-y^2)$, $-\infty < y_0 < \infty$.

4. Let $y' = ky^2$, $-\infty < y_0 < \infty$, k is positive.
   (a) Find all critical points.
   (b) Sketch several solutions of this problem. Note that this equation has a semistable critical point.
   (c) Find the general closed-form solution to this problem and compare it with the results from (b).

5. Given the following system of linear equations:

$$\begin{aligned} 3x_1 - 2x_2 &= 0 \\ -6x_1 + 4x_2 &= 0 \end{aligned}.$$

   (a) Find all solutions of this system.
   (b) Sketch the graph of each equations. Are the lines parallel, intersecting, or coincident?

In the following two problems, find the eigenvalues and eigenvectors by hand and verify with MATLAB

6.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$$

7.

$$A = \begin{bmatrix} 4 & -4 \\ 1 & -1 \end{bmatrix}$$

8. Find a formula for the eigenvalues of matrix A and sketch their dependence on α.

$$A = \begin{bmatrix} 1 & \alpha \\ \alpha & 2 \end{bmatrix}$$

In the next three problems state whether or not the system of differential equations is (a) autonomous and (b) homogeneous.

9.

$$\begin{aligned} x' &= x + 2y + \sin t \\ y' &= -x + y - \cos t \end{aligned}$$

10.

$$\begin{aligned} x' &= 2tx + y \\ y' &= -x + y \end{aligned}$$

11.

$$\begin{aligned} x' &= x + 2y + 3 \\ y' &= 2x - y - 5 \end{aligned}$$