ENGR 232
Dynamic Engineering Systems

Week 1 Lecture 1

Introduction to Problem Solving
with Differential Equations

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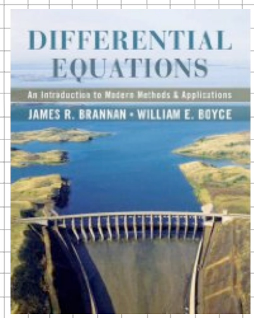
Textbook

- **Text:** James R. Brannan and William E. Boyce, **Differential Equations: An Introduction to Modern Methods & Applications**, John Wiley & Sons, Inc. 2007. ISBN-10 0-471-65141-9



Course Formalities

- Course materials will be on the learning.drexel.edu web site.
- Two in-class midterms, Week 4 and 8
- Two Matlab exams Week 5 and 10
- Seven homework assignments - posted on Tuesday, due next Wednesday by 4 PM at the ECE Lab window on the second floor of Bossone. No HW will be collected at lecture.
- Eight prelabs and labs – second half of recitation.
- You may not change lab/recitation sections.
- No late homework, labs, or exams except with valid prior notification or with documented medical emergency.
- See Syllabus for grading policy and details for lab procedures

How To Succeed

- Read the syllabus and follow instructions.
- Download and print lecture notes prior to class, follow and annotate during lecture.
- Read and study the text, do the homework.
- Read labs prior to recitation/lab session.
- Complete the prelabs and bring hardcopy stapled to verification sheet to lab sessions.
- If you have any questions, see me or any of the instructors or TA's. We are available by e-mail always and by appointment. See syllabus for TA office hours and appointment days.

Other Matters

- Honors credit?
- MATLAB club?
- If you are interested, send an e-mail to tretiak@drexel.edu

Introduction

- **Differential equations** are equations containing derivatives.
- Some examples of physical phenomena involving rates of change:

  Motion of mechanical systems
  Population dynamics
  Electrical circuits

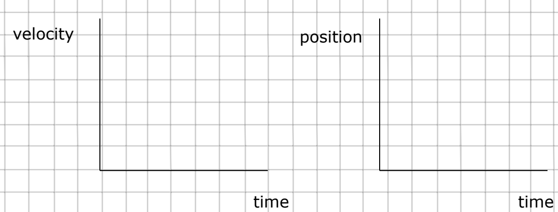
- A differential equation that describes a physical process is often called a **mathematical model**
- **This lecture focuses on Section 1.1 and 1.2 of the text**

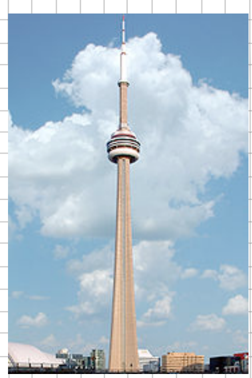
Problem

- We drop a stone from the top of the CN Tower in Toronto (553m)

  How long before it hits the ground? Time impact
  How fast will it go? Terminal speed

- General plot of what will happen





Example 1: Free Fall
Section 1.1

Formulate a differential equation describing motion of an object falling near sea level, *neglect the force of the air*.

- Variables: time t, velocity v, position x
- $v = dx/dt,\ a = dv/dt = d^2x/dt^2$
- Newton's 2nd Law: $F = ma = m(\mathrm{d}v/\mathrm{d}t)$ net force
- Force of gravity: $F = mg$ downward force
- At $t = 0$, $x = 0$, $v = 0$. initial condition

- Plan:
  - Find equation for $v(t)$
  - Find equation for $x(t)$
  - Solve for t_i when $x(t_i) = 0$
  - Compute $v(t_i)$

Example 1: Free Fall Model

Example 2: Increased Model Complexity A Falling Hailstone (1 of 4)

- A hailstone has mass *m=0.025 kg* and drag coefficient *=0.007 kg/s.*
- *Taking g* = 9.8 m/sec^2, the differential equation for the falling hailstone is

$$m\frac{dv}{dt} = mg - \gamma v$$

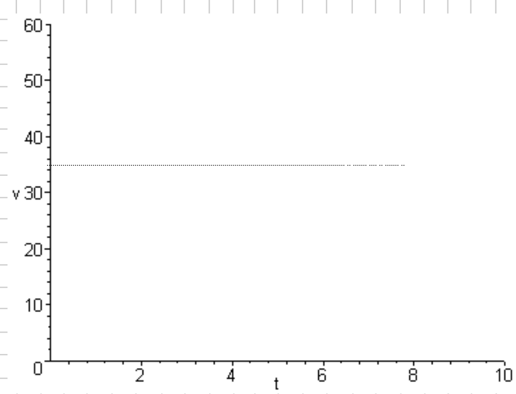
$$\frac{dv}{dt} = 9.8 - 0.28v$$

Example 2: Sketching Direction Field (2 of 4)

- Using differential equation and table, plot slopes (estimates) on axes below. The resulting graph is called a **direction field.** (values of *v′* do not depend on *t*.)

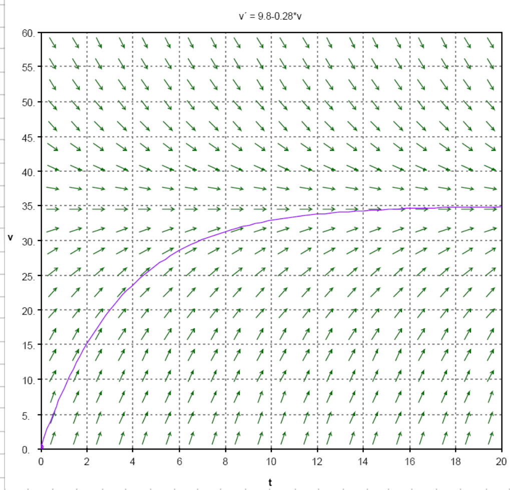
$$v' = 9.8 - 0.28v$$

v	v'
0	9.8
5	8.4
10	7
15	5.6
20	4.2
25	2.8
30	1.4
35	0
40	-1.4
45	-2.8
50	-4.2
55	-5.6
60	-7



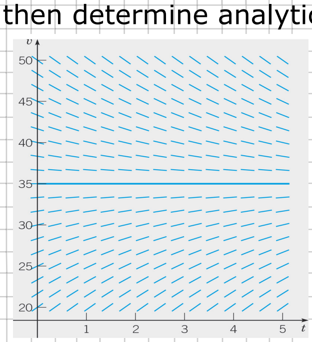
Example 2: Computer Plot of Direction (3 of 4)

http://math.rice.edu/~dfield/dfpp.html



Example 2: Direction Field & Equilibrium Solution (4 of 4)

- Arrows give tangent lines to solution curves, and indicate where soln is increasing & decreasing (and by how much).
- Horizontal solution curves are called **equilibrium solutions**.
- Use the graph below to solve for equilibrium solution, and then determine analytically by setting *v′* = 0.



Set $v' = 0$:

$$v' = 9.8 - 0.28v$$

Mice and Owls – A Model

- Consider a mouse population that reproduces at a rate proportional to the current population (assuming no owls present).
- Let t represent time, $p(t)$ represent the mouse population, and r represent the growth rate (mice/time). Then
- When owls are present, they eat the mice. If the predation rate is a constant, k (mice/time), then

Example 3: Mice and Owls (1 of 2)

$$\frac{dp}{dt} = rp - k$$

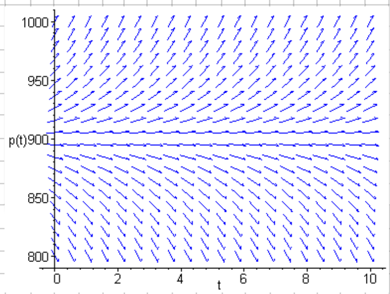
- Consider a mouse population (p(t) is the number of mice at time t) that reproduces at a rate proportional to the current population, with a rate constant equal to 0.5 mice/month (assuming no owls present).
- When owls are present, they eat the mice. Suppose that the owls eat 15 per day (average). Write a differential equation describing mouse population in the presence of owls. (Assume that there are 30 days in a month.)
-

$$\frac{dp}{dt} = 0.5p - 450$$

Example 3: Direction Field (2 of 2)

- Discuss solution curve behavior, and find equilibrium soln.

$$p' = 0.5p - 450$$



Solutions of Some Differential Equations Section 1.2

- Recall the examples from the last section
- Free fall:

$$m\frac{dv}{dt} = mg - \gamma v$$

- Owls and mice:

$$\frac{dp}{dt} = rp - k$$

- These equations have the general form $y' = ay - b$
- We can use methods of calculus to solve differential equations of this form.

Example 1: Mice and Owls (1 of 3)

For the differential equation $p' = 0.5p - 450$

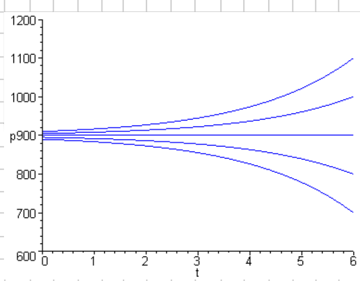
the solution is $p = 900 + ke^{0.5t}$ where k is a constant.

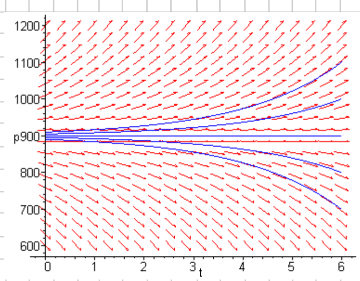
Verify solution is correct:

Example 1: Integral Curves (2 of 3)

- Thus we have infinitely many solutions to our equation,
  $$p' = 0.5p - 450 \quad \Rightarrow \quad p = 900 + ke^{0.5t},$$
  since k is an arbitrary constant.

- Graphs of solutions (**integral curves**) for several values of k, and direction field for differential equation, are given below.
- Choosing k = 0, we obtain the equilibrium solution, while for $k \neq 0$, the solutions diverge from equilibrium solution.

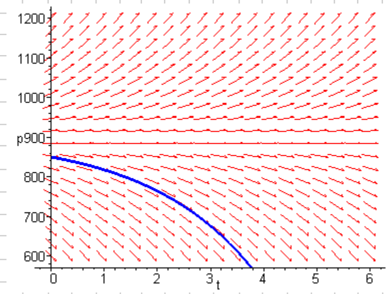




Example 1: Initial Conditions (3 of 3)

- A differential equation usually has infinitely many solutions. If a point on the solution curve is known, *such as an initial condition*, then this determines a unique solution.
- In the mice/owl differential equation, suppose we know that the mice population starts out at 850. Then $p(0) = 850$, and

$$p(t) = 900 + ke^{0.5t}$$



Summary