Name (please print): Solution January 31, 2008

ENGR 232: Dynamic Engineering Systems I: Exam 1

Section No.:

Instructions:

1. Please write your name and section number in the space provided.
2. This exam is closed book and notes.
3. Calculators may be used.
4. Points will be deducted if the work is unclear and the answers are not justified.

Evaluate the following integrals

1. (4 points) $\int xe^{3x}dx$

$\Rightarrow u = x \quad dv = e^{3x}$

$\Rightarrow du = dx \quad v = \int e^{3x}dx = \frac{e^{3x}}{3}$

$\int u\,dv = uv - \int v\,du$

$= x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}dx$

$= \boxed{\frac{xe^{3x}}{3} - \frac{1}{9}e^{3x} + C}$

For version (B), Answer is $\frac{xe^{4x}}{4} - \frac{1}{16}e^{4x} + C$

2. (4 points) $\int 3x\sqrt{(1+x^2)}dx$

let $\sqrt{1+x^2} = t = (1+x^2)^{1/2}$

$\Rightarrow 1+x^2 = t^2$

$\Rightarrow 2x\,dx = 2t\,dt$

$\Rightarrow x\,dx = t\,dt$

$\Rightarrow I = 3\int t \cdot t\,dt$

$= 3\int t^2 dt$

$= \cancel{3}\frac{t^3}{\cancel{3}} + C$

$\Rightarrow \boxed{I = (1+x^2)^{3/2} + C}$

For version (B), Answer is $\frac{4}{3}(1+x^2)^{3/2} + C$

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3. (5 points) The number of bacteria in a culture increases at a rate proportional to the number of bacteria present at any time. Initially the number of bacteria is 200. After 2 hours it is observed that there are 400 bacteria present. What is the number of bacteria after 5 hours ?

Let x = no. of bacteria $\quad x(0) = 200 \quad | \; x(2) = 400 \; | \; x(5) = ??$

$$\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$$

$$\Rightarrow \int \frac{dx}{x} = k \int dt$$

$$\Rightarrow \ln|x| = kt + c$$

$$\Rightarrow x = e^{kt+c} = e^{kt} \cdot e^{c} = c \cdot e^{kt}$$

$$\Rightarrow x(t) = ce^{kt} \quad @\, t=0 \Rightarrow 200 = c$$

$$\Rightarrow \boxed{x(t) = 200e^{kt}} \quad @\, t=2 \Rightarrow 400 = 200e^{k(2)}$$

$$\Rightarrow \boxed{k = \frac{\ln|2|}{2}}$$

$$\Rightarrow x(t) = 200e^{\frac{\ln|2|}{2}t}$$

$$\Rightarrow x(5) = 200e^{\frac{\ln|2|}{2} \cdot 5}$$

$$= 200e^{\frac{\ln 2}{2} \cdot 5}$$

$$= \boxed{1131.37}$$

ANSWER

4. (7 points) The differential equation for the volume of a spherical drop whose evaporation rate is proportional to the area is

$$\frac{dV}{dt} = -kV^{2/3}$$

(a) The volume of a sphere is $\frac{4\pi r^3}{3}$, where r is the radius of the sphere. Find the differential equation for the radius of the evaporating drop.

(b) Find the formula for $r(t)$, the radius of the drop as a function of time.

$$V = \frac{4\pi r^3}{3} \Rightarrow \frac{dv}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \text{——(i)}$$

Also given: $\frac{dv}{dt} = -kv^{2/3} = -k\left(\frac{4\pi}{3}\right)^{2/3}(r^3)^{2/3}$

$$= -k\left(\frac{4\pi}{3}\right)^{2/3} \cdot r^2 \quad \text{——(ii)}$$

From (i) & (ii),

$$4\pi \cancel{r^2}\frac{dr}{dt} = -k\left(\frac{4\pi}{3}\right)^{2/3}\cancel{r^2}$$

$$\Rightarrow \boxed{\frac{dr}{dt} = -k\left(\frac{4\pi}{3}\right)^{2/3} \cdot \frac{1}{4\pi} = -kC}$$

↑ Answer (a)

where $C = \left(\frac{4\pi}{3}\right)^{2/3} \cdot \frac{1}{4\pi}$ = constant

$$\Rightarrow \int dr = -kC\int dt$$

$$\Rightarrow \boxed{r(t) = -kCt + c} \leftarrow \text{Answer (b)}$$