ENGR 232 Dynamic Engineering Systems:Mock Final

1. Solve the following first-order differential equations:
   (a) $(1-x^3)\frac{dy}{dx} = 3x^2y$
   (b) $t^3y' + 4t^2y = e^{-t}$, $y(-1) = 0$
   (c) $\frac{dy}{dx} = \frac{xy^2-x-y^2+1}{xy-3y+2x-6}$
   (d) $(x^3 - y^3)dx - 2x^2ydy = 0$
   (e) $(1+e^x)\frac{dy}{dx} + e^xy = 0$
   (f) $cosx\frac{dy}{dx} + ysinx = 1$

2. For the following system of two first-order differential equations, determine the
   - Eigen Values
   - Corresponding eigen vectors
   - General Solutions
   - Constants subject to the initial conditions

   (a) $\frac{dx}{dt} = \begin{pmatrix} -1 & 1 \\ 0 & -4 \end{pmatrix}\mathbf{x}$, $\mathbf{x(0)} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$
   (b) $\frac{dx}{dt} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\mathbf{x}$, $\mathbf{x(0)} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
   (c) $\frac{dx}{dt} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}\mathbf{x}$

3. Transform the following second order differential equations to a system of first order differential equations. Find the solution by determining the Eigen values and corresponding eigen vectors, the general solution and the value of the constants based on the initial conditions
   (a) $y'' - 2y' - 3y = 0$, $y(0) = 1$, $y'(0) = 0$
   (b) $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = -2$

4. A large tank is filled with 500 gallons of pure water. Brine containing 2 lb of salt per gallon is pumped into the tank at a rate of 5 gallons per minute. The well-mixed solution is pumped out at the same rate.
   (a) Formulate the differential equation that describes the rate of change of the amount of salt in the tank.
   (b) Find the number of pounds of salt $A(t)$ in the tank any time.
   (c) What will be the amount of salt in the tank after 5 mins?

5. A thermometer is removed from a room where the air temperature is 70°F to the outside where the temperature is 10°F. After $\frac{1}{2}$ minute the thermometer reads 50°F.
   (a) Formulate the differential equation that describes the rate of change of the temperature measured by the thermometer.
   (b) Solve the differential equation to find out the temperature at any given time.
   (c) What is the reading at $t = 1$ minute?
   (d) How long will it take for the thermometer to reach 15°F?

6. A ball with mass 0.25 kg is thrown upward with initial velocity 20 m/sec from the roof of a building 30m high. Neglect air resistance.
   (a) Formulate the differential equation that describes the rate of change of the velocity of the ball.
   (b) Solve the differential equation to determine the velocity of the ball at any time.
   (c) Find the maximum height above the ground that the ball reaches.

7. A weight of 4 lb stretches a spring by 2 ft. (Use this information to determine the spring constant). The previous weight is removed from the spring and a body with weight 64 lbs is attached to the spring. The body is also attached to a viscous damper with a damping coefficient of $4\frac{lb-sec}{ft}$. The body is released from the equilibrium position with an initial velocity of 0.25 ft/sec. Formulate the differential equation that governs the motion of the body. $(g = 32ft/sec^2)$

8. A mass of 1/2 slug is suspended on a spring whose constant is 6 lb/ft. The system is set in motion in a medium offering a damping force numerically equal to twice the instantaneous velocity. The body is released from the equilibrium position with an initial velocity of 0.5 ft/sec. Formulate the differential equation that governs the motion of this mass. $(g = 32ft/sec^2)$.

9. A 30-year old woman has $\$150,000$ of savings in an account that pays 4% annual interest, compounded continually. She hopes to have \$3 million in this account when she is 70, at which time she will retire. Suppose she deposits 30% of her earnings every year into the saving account.

(a) How much money does she need to earn in order to reach the goal?

(b) How much money would she be able to withdraw every year from her account after age 70, given that she wants the account to last until she is 100?