A Survey on Vehicle Routing Problem with Loading Constraints

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Abstract

In the classical Vehicle Routing Problem (VRP), a fleet of vehicles is available to serve a set of customers with known demand. Each customer is visited by exactly one vehicle as well as the objective is to minimize the total distance or the total charge incurred. Recent years have seen increased attention on VRP integrated with additional loading constraints, known as 2L-CVRP or 3L-CVRP. In this paper we present a survey of the state-of-the-art on 2L-CVRP/3L-CVRP.

1. Introduction

In recent years, new research studies have appeared that combine the capacitated vehicle routing problem (CVRP) and a class of additional loading constraints which are frequently encountered in freight transportation. The problem is known as two-dimensional or three-dimensional loading capacitated vehicle routing problem (2L-CVRP or 3L-CVRP).

In the field of combinatorial optimization, vehicle routing and loading problems have been investigated widely but independently. However in recent years some attention has been devoted to the optimization of their integration. 3L-CVRP (2L-CVRP) calls for a determination of a set of optimal routes traveled by a fleet of identical and capacitated vehicles for delivering items to customers with known demand, while minimizing the total distance or the total cost incurred. Items consist of rectangular boxes of given shape and weight, and must be feasibly loaded into vehicles with three (two) dimensional loading spaces (surfaces) before they are shipped.

Many real-world transportation applications indicate the significance in practice of this problem, especially when the shippers should deal with many large items and the loading aspect is not trivial. Examples include the distributions of furniture, household appliances, mechanical components and others goods. The theoretical significance lies in its generalization of two most widely studied problems in combinatorial optimization: the CVRP and the Bin Packing Problem (BPP).

In the well-known CVRP, a set of identical vehicles, based at a central depot, is to be optimally routed to supply customers with known demand subject to vehicle capacity constraints. Among the various surveys on the CVRP are the book by Toth and Vigo [2] and the more recent update by Cordeau *et al.* [3]. For recent overview on heuristic and meta-heuristic approaches, we mention Cordeau *et al.* [5] and Cordeau and Laporte [4].

The problem of loading items to vehicles is closely related to various packing problems, especially to BPP or Container Loading Problem (CLP). The BPP calls for packing a given set of rectangular items into the minimum number of identical rectangular boxes (bins). Exact approaches for Two-dimensional BPP (2BPP) are able to solve instances with up to 100 items but fail in many cases with 20 items. Exact algorithms, lower bounds and heuristic solutions have been proposed by Martello and Vigo [6], Fekete and Schepers [7, 8], Pisinger and Sigurd [9], Fekete, Schepers, Van der Veen [10], Hadjiconstantinoua, Iori [13], Hayeka, Moukrima, Negre [14], L. Wei, D. Zhang, Q. Chen [18]. Meanwhile, several instances with less than 50 items cannot be solved to optimality for Three-dimensional Bin Packing Problem (3BPP). We refer readers to Martello *et al.* [11, 12], Faroe *et al.* [15] and Crainic *et al.* [16, 17] for recent exact and heuristic solutions for 3BPP. When derived with some additional side constraints, BPP are generally included in the family of the container loading problem.

The 3L-CVRP was first proposed in 2006 by Gendreau *et al.* [1] which was motivated by a distribution problem in furniture distribution. Other combinations of routing and three-dimensional loading problems have been studied very recently. S. Chen [19] and Fuellerer [20] addressed exactly the same problem as [1] and they both considered the same loading constraints. The work in Doerner *et al.* [21] arose from a real-world transportation problem occurring to a wood-products retailer, which delivers its timber

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chipboards to customers. They disregarded the constraint on the weight capacity of the vehicle and grouped their products into four main types, which had to be placed on three different piles contained in particular vehicles. Their problem was also called the Multi-pile Vehicle Routing Problem (MP-VRP). The loading/unloading operations should be on only one side of each vehicle. Moura *et al.* [23, 24] presented VRTWLP, which denotes vehicle routing with time windows and loading problem. They both disregarded the capacity constraint. Moura [24] addressed the VRTWLP as a multi-objective optimization problem.

If the items couldn't be stacked on top of each other (such as refrigerators or some other big and heavy boxes), we can then derive 2L-CVRP from 3L-CVRP, which means all the items should only be placed directly on the surface of vehicles but not on other ones. 2L-CVRP was first introduced by Iori's [22] in 2004 and solved with a exact method by the same author [25, 26]. Then this problem was addressed by Gendreau *et al.* [27], Zachariadis *et al.* [28] and Fuellerer *et al.* [29] based on different combinations of loading constraints.

It is clear that the 2L-CVRP or 3L-CVRP is NP-hard in the strong sense because it generalizes the CVRP which is already NP-hard. Moreover, it is extremely hard to solve in practice when attached with loading problem.

The rest of our paper is organized as follows. In section 2 we elaborate the loading constraints concerned with 2L-CVRP/3L-CVRP. In Section 3 we will provide a formal description of the problem. Exact and metaheuristic approaches are described in Section 4. Finally, we conclude our survey and indicate future research directions in Section 5.

2. Loading constraints

In practice, we face a couple of additional loading constraints to apply CVRP to real-world transportation problems. Particularly, in the CVRP, customer demands are expressed by an integer value representing the total weight of the items to be delivered, while in real-world instances, demands consist of items characterized by both a weight and a shape. Moreover, constraints on the supporting surface or on the fragility of the items are frequently considered. 2L-CVRP and 3L-CVRP take into account these facts.

Loading items into a two-dimensional or three-dimensional container is more complex than standard BPP or CLP. We specify the constraints on loading aspects considered in the literature as follows:

a) Classical BPP Constraints

Items cannot overlap and should be completely contained by the bins/vehicles;

b) Capacity Constraint

The total sum of weights in a vehicle should not exceed its weight capacity;

c) Orthogonality Constraint

Every item should be loaded with its edges parallel to the edges of the vehicles;

d) Item Orientation Constraint

Items have a fixed orientation with respect to the height (i.e., as usual in transportation they have a fixed top), and can or can't be rotated by 90° on the width-length plan;

e) Item Fragility Constraint

Items can be divided into two groups: fragile and non-fragile. Non-fragile items can't be stacked on top of fragile ones, while fragile items can be placed on top of each other. Also non-fragile items can be placed on top of each other;

f) Item Stability Constraint

When an item is placed on top of other ones, its base must be supported by a minimum supporting area;

g) LIFO Policy

This constraint is also called Sequential Loading Constraint or Rear Loading Constraint. When unloading the items of a customer, no item belonging to customers served later along the route may either be moved or block the removal of the items of current customer. In other words, no lateral movement of the items is allowed inside the vehicle when serving a customer.

In the practical viewpoint, not all the constraints above should be fulfilled simultaneously to any problem. We can obtain different vehicle routing and loading problems by removing one or more constraints at a time. All these problems are of interest, since they all represent different practical distribution problems.

Table 1 Variants of routing and loading problem that have been considered in the literature.

Constraints / References	a	b	c	d	e	f	g
[1]	√	√	√	√	√	√	√
[19]	√	√	√	√	√	√	√
[20]	√	√	√	√	√	√	√
[21]	√	×	√	×	×	×	√
[23]	√	×	√	√	×	√	√
[24]	√	×	√	√	×	√	√
[22][25][26]	√	√	√	√	−	−	√
[27]	√	√	√	√	−	−	(√)
[28]	√	√	√	√	−	−	(√)
[29]	√	√	√	(√)	−	−	(√)

Several works have dealt with various problems when considering different loading constraints configurations. We summarize them in Table 1(A Tick indicates the reference considered the corresponding constraint and a Cross means it didn't; A Hyphen indicates the problem in the reference fulfilled the constraint naturally ; A Tick between the parentheses indicates the reference addressed both of the situations where the constraint existed or not).

3. Problem Description

In this section, we describe the routing and loading problem and take 3L-CVRP as an example. The notation is based on Gendreau *et al.* [1].

3L-CVRP can be defined over a graph $G = (V, E)$ with vertex set $V = \{0,1,...,n\}$, where 0 denotes the central depot and the other vertices represent the n customers to be served. E is the edge set connecting all the vertex pairs. The cost or length $c_{ij}(i, j = 0,...,n)$ of each edge (i, j) in E is set to be non-negative.

Each customer i requires a set of m_i three-dimensional items $I_{ik}(i = 1,...,n, k = 1,...,m_i)$ with width w_{ik}, height h_{ik} and length l_{ik}. The total weight of customer i is denoted by d_i. Let $s_i = \sum_{k=1}^{m_i} w_{ik}h_{ik}l_{ik}$ be the total amount of space needed by customer i.

The problem is given a fleet of v identical vehicles, each of which has a weight capacity D and a three-dimensional shape loading space of width W, height H and length L. There is only one opening in the rear of each vehicle and it is assumed to be as large as $W \times H$.

The 3L-CVRP calls for a determination of a set of no more than v routes traveled by a fleet of vehicles, such that each route starts and terminates at the central depot, each customer is visited exactly once, a three-dimensional feasible loading solution exists and the total cost is minimized.

Other than the classical CVRP, which only requests the total weight of a vehicle not exceed its weight capacity, a solution to the 3L-CVRP is feasible if there exists an arrangement of the items into the vehicle that satisfies a series of loading constraints according to the specific transportation instance besides the classical BPP constraints.

Items may have fixed orientation and can be rotated on their basis, although in some rare cases this can't happen. An oriented loading means items can't be rotated while non-oriented loading allows them to be rotated by 90° on $w-l$ plane. In addition, the fragility of items may differ. We assign to each item I_{ik} a fragility flag $f_{ik}(i = 1,...,n, k = 1,...,m_i)$, equal to 1 if fragile and to 0 otherwise. No non-fragile item can be placed over a fragile one, although fragile items can be stacked. Some instances may evaluate the supporting area when an item I_{ik} is placed over another one. Let $\overline{h}$ be the height at which the bottom of item I_{ik} is placed and $\overline{A}$ (supporting area) the total area of the bottom of I_{ik} that lies on items whose top is at height $\overline{h}$. A loading is feasible only if the supporting area is no less than a given parameter $\alpha(0 < \alpha < 1)$ of the base of the item, that is, if $\overline{A} \geq \alpha w_{ik}l_{ik}$ (note that the constraint is always satisfied when $\overline{h}$=0). Finally, almost all the papers on routing and loading problem took into account LIFO Policy which is a common request in distribution industry.

4. Solution Approaches

As mentioned above, vehicle routing and loading problem is NP-hard and extremely hard to solve. Therefore, most of the researchers seek for meta-heuristic approaches instead of exact methods. The only exact approach was proposed by Iori *et al.* [26] for 2L-CVRP.

To the best of our knowledge, all the solutions took a two-stage way where the routing problem acts as the main problem and iteratively calls for specific procedures to deal with the loading subproblem. The approaches for the main routing problem were generally the basis and extensions of methods for the CVRP and various means, including exact or heuristic, were proposed to check the feasibility of the loading subproblem. We summarize the procedure in Figure 1.

Iori *et al.* [22, 25, 26] developed an exact approach which uses a branch-and-cut algorithm embedded in the framework provided by ILOG CPLEX 9.0 to deal with the routing characteristics of the problem. Their method is based on the classical two-index vehicle flow formulation, with an additional separation procedure for checking eventual loading infeasibility.

The loading check is done by means of lower bounds, a simple bottom-left heuristic and a branch-and-bound algorithm, in which the enumeration tree follows the principles proposed by Martello *et al* .[6, 11]. Their exact solution methodology could solve to optimality all 2L-CVRP instances with up to 25 customers and 91 items in one day of CPU time per instance.

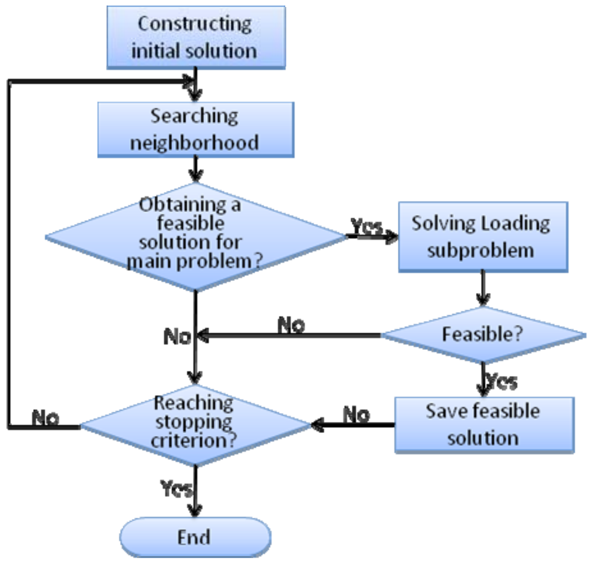

Figure 1 Two-stage procedure for solving the vehicle routing problem and loading constraints

To deal with large scale data in practice, Gendreau *et al.*[27] proposed a tabu search for the main routing problem and considered both conditions where sequential loading constraint existed or not. In their algorithm infeasible solutions, both in terms of weight and volume excess, are accepted but are assigned a penalty term proportional to the level of violation in the objective. Customers are relocated by means of GENI heuristic (Gendreau *et al.* [30]). To escape from a poor or already extensively searched area, an intensification phase is often performed to accentuate the search in the neighborhood. The loading subproblem is taken as the two-dimensional strip packing problem. Their algorithm was tested on instances with up to 255 customers and 786 items.

Zachariadis *et al.* [28] present another metaheuristic methodology for exactly the same problem as [27]. Their algorithm was called Guided Tabu Search (GTS). Regarding the routing aspects, it explored the solution space by employing a search strategy based on tabu search, guided by an objective function alteration mechanism. While Fuellerer *et al.* [29] made use of an Ant Colony Optimization (ACO) for the overall optimization. To ensure the feasible loading of the items into the vehicles, they both employed a bundle of packing heuristics, like Bottom-Let Fill, Max Touching Peri-meter Heuristic etc. Their solution approaches were evaluated on the data sets introduced by Iori *et al.* [22, 25, 26] and Gendreau *et al.* [27]. Fuellerer *et al.* [29] compared the cost of four different loading configurations which are rear oriented loading(|RO|L), unrestricted oriented loading(|UO|L), rear non-oriented loading(|RN|L) and unrestricted non-oriented loading (|UN|L).

The algorithm in [27] was generalized by Gendreau *et al* .[1] so as to address the three-dimensional loading case. They proposed a tabu search algorithm that iteratively invokes an inner tabu search and local search procedure for the solution of the loading subproblem. Fuellerer [20](under review) presented the same problem as [1]. They solved the problem by means of an ant colony optimization algorithm, which made use of fast packing heuristics derived from [1] for the loading. S. Chen [19] also addressed the same problem as [1, 20]. The only difference from [1] is [19] only employed loading heuristic in [1] for local search in the solution approach and it turned out to outperform the solution quality from the algorithm of [1]. The authors in [1, 19, 20] all experimentally evaluated their algorithms on the same data sets introduced by Gendreau *et al.* [1] and compared the solution values of five situations with different constraints configurations. The results indicated the solution approach in [19] outperformed that in [1], while the method in [1] is superior to that in [20]. Doerner *et al.* [21] solved the MP-VRP by means of two different metaheuristics: tabu search and ACO. The tabu search accepted infeasible solutions, while ACO remains inside the space of feasible solutions. Both algorithms use the same loading heuristics and they claimed ACO outperformed the tabu search both in terms of solution accuracy and running time.

Moura *et al.*[23, 24] addressed a particular com-bination of VRP with time window and loading pro-blem. They employed two classes of heuristics based on hierarchical and sequential methods respectively.

5. Conclusions

In this survey, we overviewed the vehicle routing problem with loading constraints, known as 2L-CVRP and 3L-CVRP. First, we classified the two main variants and specified the loading constraints considered in the literature. We then described the problem definition and illustrated both the exact and metaheuristic approaches for solving the problem and compared their performances in brief.

In terms of future research area, researchers may consider even more loading constraints which derive from real world. Another possible direction is new algorithms combining exact and metaheuristic methods. Last but not the least it might be more effective if the loading subproblem could feedback to the main routing problem once it was called.

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