Vehicle routing problem and its solution methodologies: a survey

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Abstract: This paper presents survey of some of the recently employed solution methodologies in the field of vehicle routing problem (VRP) and its variants. VRP and its variants are first briefly discussed and then some of the recently most cited papers have been discussed and critically analysed. We conclude the paper by presenting some future possibilities in VRP.

Keywords: vehicle routing problem; VRP; exact methods; meta-heuristics.

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1 Introduction

Vehicle routing problem (VRP) is standard name that refers to combinatorial optimisation problems where a set of customers are to be serviced by available vehicles. The term 'vehicle routing' was coined by Golden et al. (1975). In 1970s, other versions of VRP such as, 'dial-a-bus system' (Wilson and Sussman, 1971), and 'distribution management' (Ellon et al., 1971; Newton and Thomas, 1974) were also proposed from the application point of view. Dantzig and Ramser (1959) formulated mathematical model of VRP in 1959. 'Truck dispatching problem' was modelled as a generalisation of travelling salesman problem (TSP) presented by Flood (1956). Clarke and Wright (1964) proposed a seminal heuristic method for solving linear optimisation problem of serving a set of geographically scattered customers from a central depot with a limited fleet of vehicles having varying capacities.

Beyond the classical VRP, some other variants have also been studied. Capacited VRP (CVRP), VRP with time windows (VRPTW), VRP with split delivery (VRPSD), VRP with back-haul (VRPB), multi-depot VRP (MDVRP) are some of these. Real life variants such as open VRP, stochastic VRP and rich VRP etc. also have got attention of researchers from realistic situations.

This survey classifies the VRP from the perspective of solution methodologies as well as objective to be optimised. After a brief description of VRP and its variants, we present a comprehensive survey of state of art strategies currently being used for their solution. The main objective of this work is to provide the reader the recent trends and solution methodologies in the field of VRP and some of its well known variants. The survey is expected to help future researchers in identifying the problem domain and the challenges for further research.

1.1 Motivation

VRP is one among the most widely studied topics in the field of operations research. With the advancement and availability of personal computers since late '90s a large amount of work has been done in the field of VRP. Most of this work on VRP is available in the form of journal articles, conference proceedings, thesis, technical reports/books etc. Inspite of such a large amount of literature, not much effort seems to have been made on reviewing VRP. Gendreau and Potvin (1998) discussed stochastic VRP and solution methodologies. Laporte and Osman (1995) is the most cited contribution in the field of review of VRP. They have provided a bibliography of more than 500 studies done till that time.

The taxonomic review of VRP provided by Eksioglu et al. (2009) revealed more than a thousand articles published on VRP till that date. According to Eksioglu et al. (2009), VRP is an evolving field of operations research and every year there is approximately 6% of growth in related literature. Due to its practicality and popularity, it is important to keep track of modelling of VRP and emerging solution techniques. More recently, Baldacci et al. (2012) presented survey of various exact methods used for the solution of VRPTW and other variants. In addition to survey, they compared their developed algorithms with other existing exact algorithms. Lin et al. (2014) presented a survey on green VRP by classifying the problem into three sub-classes namely: green-VRP, pollution routing VRP and reverse logistics. They also provide possibilities for future research in this field. Lahyani et al. (2015) present a detailed review on rich VRP. In their work they developed the taxonomy for rich VRP and then validated the taxonomy against a number of selected papers from the recent literature. They classified the overall literature into three clusters using K-means clustering and finally implied cluster analysis for reviewing the findings of research. Karakatič and Podgorelec (2015) presented a review of genetic algorithms used for solving VRP with multiple depots. They also analysed the effects of various mutation operators on the quality of solution obtained. Finally they compare various GA's with other heuristic and exact algorithms by applying them on a number of test instances.

However, most of the recent surveys/reviews of the available literature mainly focus on the specific variants of VRP and/or specific solution techniques. Pillac et al. (2013) and Laporte (2007) presented surveys on CVRP. Vidal et al. (2013) focus on VRPTW and so on. The purpose of present survey is to enlist different well known variants of VRP and their solution techniques to solve them.

1.2 Survey methodology

Keeping in view the exhaustive review of Eksioglu et al. (2009), here, we have tried to review the work available on VRP and its variants from 2009 onwards. References prior to 2009 have only been given where felt absolutely necessary for clarity purpose. This paper presents a review of relevant work on VRP and the solution basic meta-heuristics procedures. An exhaustive search has been conducted using the goggle search. All of the results of query 'VRP' have been analysed. More than 300 articles were revealed by this search for the survey. Articles from the fields of operations research, transportation sciences, computational mathematics etc. are selected. We primarily considered papers that appeared in journals having SCI index and Thomas Reuter's impact factor greater than six. The other relevant literature from other journals has also been selected provided the citations were greater than 20. The initially collected data was firstly filtered out on the basis of relevancy and significance of contribution to the problem. For non-exhaustiveness only the selected publications of leading researchers/groups have been considered in details.

Rest of the paper is organised as follows. Section 2 presents introduction to VRP and some common variants. Section 3 deals with the most commonly used solution methodologies. Critical analysis of the available literature and the scope of possible future work in this field have been briefly carried out in Section 4.

2 VRP and related work

VRP can be defined as the problem of planning a set of routes with minimum cost from base station to a group of geographically dispersed locations, subject to additional constraint (Laporte and Osman, 1995). A large range of VRPs as well as a huge literature on VRP and related problems (see, for example, work of Raff, 1983; Laporte and Nobert, 1987; Laporte and Martello, 1990), as well as the other classifications presented by Desrochers et al. (1990).

VRP can be modelled as a directed weighted graph $G(V, E)$ where $V = \{v_0, v_1, \ldots, v_n\}$ be the set of nodes i.e. customers to be visited from the central depot v_0. Also $E = [\{v_i, v_j\}, (i, j) = 0, 1, 2, \ldots, n, i \neq j)]$ is the set of arcs interlinking two locations i, j.

Furthermore, a set of vehicle having homogeneous capacity is available to serve all the customers.

Mathematically, VRP can be represented as:

$$\text{Minimise } F = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{K=1}^{V} c_{ij} x_{ij}^{v} \tag{1}$$

Subject to:

$$\sum_{v=1}^{V} \sum_{j=1}^{N} x_{ij}^{v} \le V \text{ for } i = 0 \tag{2}$$

$$\sum_{v=1}^{V} x_{ij}^{v} = \sum_{j=1}^{N} x_{ji}^{v} \le 1 \quad \text{for } i = 0 \text{ and } v \in \{1, \ldots, V\} \tag{3}$$

$$\sum_{v=1}^{V} \sum_{j=0}^{N} x_{ij}^{v} = 1 \text{ for } i \in \{1, \ldots, N\} \tag{4}$$

$$\sum_{v=1}^{V} \sum_{i=0}^{N} x_{ij}^{v} = 1 \text{ for } j \in \{1, \ldots, N\} \tag{5}$$

$$\sum_{i=0}^{N} c_i \sum_{j=0}^{N} x_{ij}^{v} \le q_v \text{ for } v \in \{1, \ldots, V\} \tag{6}$$

$x_{ij} = 1$ if customer j is served after serving customer i and 0 otherwise ($i \neq j$; $i, j = 0, 1, \ldots, N$).

Here:

V is total fleet size

N number of locations/customers to be visited

c_i customer i ($i = 1, 2, \ldots, N$)

c_0 central depot

d_{ij} travelling distance between customer i and customer j

q_i total servings for customer i

q_v upper limit for capacity of vehicle.

Here the objective function given by (1) is to be optimised satisfying constraints (2) to (6).

Objective function (1) corresponds to minimisation of total travelled distance. The first constraint (2) ensures that all of the tour must be completed with at most V vehicles. Beginning and completion of tour at central depot is ensured by (3). Constraints (4) and (5) restrict the partial servings i.e. every location must be visited by exactly one vehicle. Constraint (6) ensures that the net demand on every route must be within vehicle's capacity.

In practice, the basic VRP can be associated with constraints, for instance, maximum allowed capacity of the vehicle, length of route, arrival/departure time at each location and service time, collection or delivery of goods. The extended classes of VRP are VRP having constraints of delivery within specified time windows (VRPTW), VRP with backhauls (VRPB), and simultaneous pickup and delivery VRP (VRPSPD). In general, the primary objective of VRP including different variants problems is to minimise total transportation cost in terms of travelled distance and/or total fleet size. Some of commonly studied variants of VRP are as follows:

2.1 VRP with time windows

VRPTW is the extension of CVRP with specified time interval called time windows are imparted on each delivery location. In case of hard time windows the customer is bounded to be serviced only within its time window. However in VRP with soft time windows, there is a penalty associated with the violation of time windows. These constraints restrict the times at which a location is available to receive a delivery. The existence of additional constraints in terms of time window compels the overall routing having some precedence on visits. As a result problem becomes asymmetric, even if the distance and time matrices were symmetric formally. Pureza et al. (2012) presented a new variant of VRPTW with multiple delivery men. The proposed model fitted to the situations where huge amount goods are to be delivered and delivery requires less time in order to reduce the overall tour time. Tabu search and ant colony-based heuristics were developed to test the proposed model.

2.2 VRP with pick-up and delivery

The pickup and delivery problem (VRPDP) deals with delivery as well as collection of items from the customers, aiming to minimise the total travelled distance. Each location is associated with the items either to be recollected or delivered or both. There is also a precedence associated with each of the location to be visited. Moreover, the pairing constraints bound the set of routes so that one vehicle has to do both the pickup and the delivery of the load of one transportation request. More work on strategies applied for the solution of VRPPD is presented in Parragh et al. (2008).

2.3 Time dependent VRP

More realistic extension of the VRPTW in congestion situations is the time dependent VRPTW. In such scenarios the route costs on the paths depends on time during which that path is being used for routing. This problem is somewhat general in urban areas since the time taken to travel along a link/path is directly dependent on the traffic load, which further depends on the time of the day. Mousavipour and Hojjati (2014) proposed FIFO-based travel time function for TDVRP. They proposed an efficient PSO heuristic to deal with large sized problem instances. A more comprehensive review of all the major work done in field of TDVRP can be found in Gendreau et al. (2015).

2.4 VRP with backhauls

In this customers are divided into two subsets: one set requires delivery while other has to return/supply items to central warehouse. Normally the routes are planned in such a way that pick-ups should be made only after all deliveries on each route are expected to be completed. The total supply and the total pick-ups for each route must separately be in the range of storing space of serving vehicle. Some articles dealing with this variant are Brandao (2006), Ropke and Pisinger (2006) and Salhi et al. (2013) presented mixed fleet size VRPB having heterogeneous vehicles. An integer linear programming (ILP) based on three frameworks was proposed and tested against other existing approaches.

2.5 Dynamic VRP

Another extension of VRP from real world perspective is dynamic VRP. All of the services requests are not completely known at the route planning time but, dynamic requests come while serving the customers. Since new requests arrive dynamically, re-planning of routes is to be done while on the way. The articles dealing with the DVRP include those of Gendreau and Potvin (1998) and Montemanni et al. (2005). Pillac et al. (2013) thoroughly discussed the applications of DVRP and analysed the various solution strategies for the same. Interested readers can refer to Psaraftis et al. (2016) for the comprehensive review for the advancements done in field of DVRP in last three decades.

2.6 Stochastic VRP

In this version no exact information about customer's actual demands is available before starting the tour. Moreover, the travels times are also stochastic due to varying traffic, accidents etc. This is in a way stochastic VRP where routes need to be planned based on the assumed probability distribution of the demand of the customer. In such problems, a strategy needs to be specified as to what would happen when a vehicle fails to follow its committed route due to shortage of goods to be delivered. Should it return back simply or convey this message to nearby vehicles? Some articles dealing with this variant are Marinakis et al. (2013) and Juan et al. (2013).

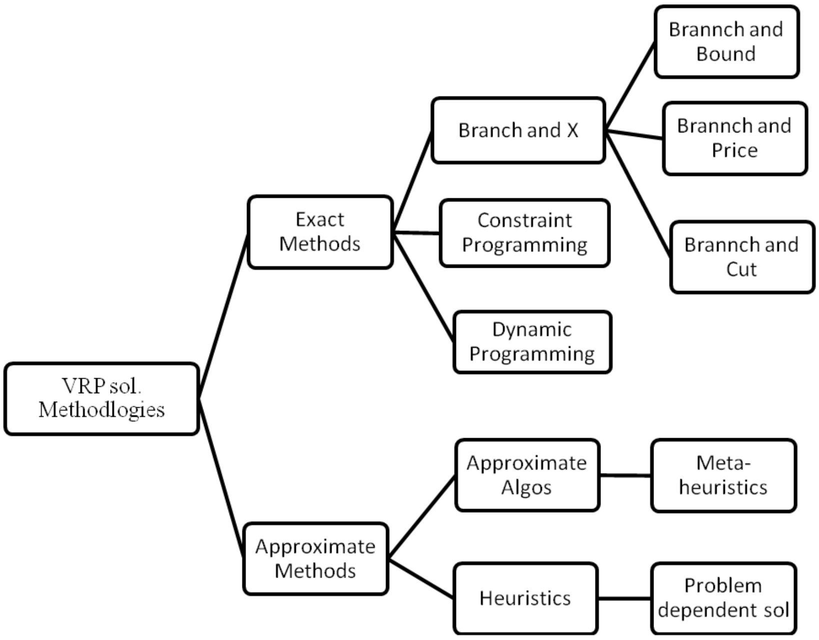
2.7 Other variants

Many other variants of the basic CVRP have been also studied. These include combination of some of the above discussed variants. These include multi-depot VRP, open VRP where a vehicle is not bounded to return back at depot, periodic VRP, split deliver VRP.

3 Solution methodologies

A large variety of VRP solution strategies are available in literature. These range from exact methods to the approximate solution methods. Whereas exact methods provide the optimal i.e. the best solutions, approximate methods often called meta-heuristics generally yield near optimal solutions. Generally exact methods are suitable for only small sized problems (up-to 50–70 customers). However there are no bounds on the problem size when solving with a meta-heuristic. Meta-heuristics-based methods can also handle a large number of constraints efficiently. Figure 1 represents the various approaches to solve VRP.

Figure 1 VRP variants



3.1 Exact methods

"Exact methods obtain optimal solutions and guarantee their optimality" (Jourdan et al., 2009). Exact methods include a variety of approaches mainly Branch and X (X: cut, bound, price etc.), dynamic programming, column generation methods. Whereas Branch and X treat VRP as integer linear programming (ILP) or mixed ILP (MILP), dynamic programming breaks the complex problem into a number of simpler sub-problems. Elhallaoui et al. (2005) suggested a column generation method consisting of dividing the given problem into two problems: the master problem and the sub-problem.

Constraint programming (CP) is a model to interrelate different variables using constraints (Van Hentenryck, 1989). Here search space is reduced (by reducing the problem domain) to make problem simpler. The relatively simpler problem is then solved by other search algorithms. Main emphasis in CP is on variable and relationship among variables rather than to specify a sequence of steps to be executed to solve the given problem. CP-based hybridised method was presented by Guimarans et al. (2011) in 2011. Each route was constructed by applying Lagrange relaxation and finally CP model was applied to check feasibility of obtained routes.

Set partitioning for CVRP was first proposed by Agarwal et al. (1989) each route is formulated as a binary variable. In this technique several constraints are considered simultaneously. Interested reader may refer to Baldacci et al. (2010, 2012) for more details on exact methods for VRP and other variants. However due to high time complexity the exact methods are not now considered very suited for NP-hard problems.

3.2 Heuristics

Some of the approximate methods called heuristics are designed to solve a specific problem. A heuristics focuses to systematically find an acceptable solution within a limited number of iterations. However, optimal solutions are not guaranteed by them. For instance, savings method of Clarke and Wright (1964) is one of the earliest example of heuristic method used to solve the CVRP. In the starting solution each of the routes contains only single customer. The heuristics searches for merging two routes for maximising the distance savings while preserving the feasibility of the merged routes. ‘Route-first cluster-second’ approach (Newton and Thomas, 1974; Bodin and Berman, 1983) and ‘cluster first route second’ (Fisher and Jaikumar, 1981) are the other heuristics for solving CVRP and variants. Similarly the ‘sweep’ by Gillett and Miller (1974) is to simply insert customers into a route on the basis of their polar angle around the depot. A heuristics yield solutions comparatively faster than exact methods. However, optimality is not guaranteed.

3.3 Meta heuristics

Glover (1986) coined the term ‘meta heuristics’ as a class of heuristics which search beyond the local optima (if encountered). In general, meta-heuristics are referred to as ‘heuristics guiding other heuristics’. A meta-heuristic may be referred to as “an intelligent strategy combining the subordinate heuristics for exploration and exploitation”. A meta-heuristic manipulates the solution partially or completely. During exploitation (intensification), promising regions are adopted more while during exploration (diversification) unexplored areas must be visited to ensure that all regions of the search space are uniformly investigated and that the search is not restrained to some specific regions (Caceres-Cruz et al., 2015). Genetic algorithms (GA), simulated annealing (SA), tabu search (TS), ant colony system (ACS), particle swarm optimisation (PSO) are some of the popular meta-heuristics which have been used for solving VRP and other combinatorial optimisation problems. The strength of meta-heuristics lies in their ability to easily handle NP-hard problems. Whereas, GA, PSO, ACS are some of the nature inspired meta-heuristics, TS, greedy randomised adaptive search (GRASP) are examples of memory-based meta-heuristics.

Szeto et al. (2011) developed an enhanced bee colony-based named ABC algorithm to solve CVRP. The authors suggested that the solution may be swapped even with worst solutions if there is no solution improvement in last k iterations. It explores the solution space. The results obtained are not equivalent to the best known results. However these are better than results obtained by other available approximate methods. A parallel version of ACO was proposed by Yu et al. (2011). The parallel ACO is implemented in distributive manner using coarse-grain strategy. The information exchange among different colonies is done after certain number of epochs. The performance is tested on classically available datasets of MDVRP (Yu and Yang, 2011). An improved version of ACO was presented to solve periodic VRPTW. Multi-dimension pheromone information and two improved crossover operations are used to solve PVRPTW. The results show an average deviation of –1.67% from the best known solutions. Balseiro et al. (2011) presents an algorithm where insertion heuristics is merged with multiple ant colony system (called MACS-IH). The advantage is that insertion heuristics explores the number of feasible solutions at the final stages of proposed method. The method resulted in new improved results for TDVRPTW.

Xiao et al.'s (2012) FCVRP model – a fuel consumption-oriented model of a real situation as a CVRP has been presented. A hybrid exchange rule is associated with string-model-based simulated annealing algorithm (SMSAH) for solving the FCVRP model. Efficiency and effectiveness of developed model is tested on 27 well-known CVRP benchmark datasets.

Gomez et al. (2013) proposed a modified bee algorithm for solving CVRP. Sweep process along with six different operators was employed and the results obtained are compared with best known solutions. Baños et al. (2013) proposed a multi-start multi-objective evolutionary algorithm with simulated annealing 'MMOEASA' algorithm to solve multi objective VRPTW. They accounted not only for minimising the total travelled distance, but also considered the load imbalance in terms of distance travelled as well as load on every individual vehicle separately. Zhou et al. (2013) proposed a GA-based approach to solve the bi-objective VRP having time windows where they simultaneously consider the total distance and distance balance of active vehicle fleet. Tournament selection, one-point crossover, and migrating mutation operators have been used to solve.

In Archetti et al. (2014), incomplete and split delivery VRP with objective of maximising profit is presented. Branch and Price method as well as the effect of TS and VNS are separately studied. They concluded the paper with the observation that if the customer's requirements limits within specific ranges there is substantial improvement in profit unlike negligible improvement as in randomly generated scenario. Ganesh et al. (2014) modelled the blood collection and delivery problem as VRPCD. A hybrid of GA and SA namely GASA technique was proposed based on route first and cluster second strategy. Due to absence of standard benchmarks for the said problem new test instances were generated using combination of VRP and VRPB problems.

Tang et al. (2014) developed beam search technique combined with max-min ant system to solve CVRP. The objective of this paper was to minimise the total travelled distance while reducing the carbon emission. The carbon emission of cargo trucks is directly influenced by the loading constraints. The second objective was to balance the load amongst the trucks. From experimentation it was found that the proposed method produces high quality results for medium instances, while MMAS alone provides comparable results within less time as compared to BEAM-MMAS. A solution strategy based on GA was adopted to solve real life VRPTW by Ghannadpour et al. (2014). The customer's requests are non-deterministic hence, travelling times are also fuzzy. Whenever new orders are received, the algorithm tries to find a feasible spot to add the additional customer without rearranging the customers who are already in the planned route. It may degrade the quality of solutions. The dispatcher will immediately be able either to admit or refuse new customers calling in for service.

Belhaiza et al. (2014) hybridised VNS with TS to solve VRP with multiple time windows. The arrival and departure times are adjusted backward. The proposed method is claimed to outperform than ant colony method proposed by Favaretto et al. (2007). Furthermore it is claimed this method is also suitable for multiple time windows. However procedure proposed by Tricoire et al. (2010) is suitable only for two time windows per customer.

Teoh et al. (2015) embedded local search procedures in the original differential evolution (DE) procedure to enhance the exploitation capacity for solving CVRP.

Mutation and crossover operators have been used for the evolution process. Solutions obtained by DE are further improved by applying local search operators. Finally they concluded that efficiency of DELS is limited for small size problems and results may degrade for larger instances. Katiyar (2015) tested relative performances of various meta heuristics such as ACS, SA, GA etc. for the solution of VRPTW. Solution quality and convergence rate were taken as the comparison parameters. Feng et al. (2015) presented a study on the newly developed evolutionary paradigm with learning capabilities. Different knowledge memes of related problem domains are evolved to enhance the search space. Two NP-hard combinatorial optimisation problems, namely CVRP and CARP are represented as common problems and their useful knowledge meme representation was identified. It was also suggested as to how to identify and then transmit the knowledge meme between CVRP and CARP evolutionary search.

Marinaki and Marinakis (2016) hybridised GSO with VNS and path relinking (PR) for solving VRP with stochastic demands. A number of tests were performed with different parameters for finding the best set of parameters. The stability of the algorithm is tested with best set of parameters on a number of benchmark problems.

Lai et al. (2016) formulated time constrained heterogeneous VRP as a mixed-integer linear programming model. A TS-based heuristic is proposed as main solution approach. FSAS a polynomial time heuristic procedure for arc selection is developed to search the neighbourhood efficiently. The results suggest that transportation costs can be considerably reduced by adopting the substitute route structure, especially if the goods are to be delivered in a controlled time limit.

Some of the important references of specific types of studies are listed in appendix A for ready references.

Lalla-Ruis et al. (2016) proposed a new mathematical model for multi-depot open VRP MDOVRP. They proposed the new constraints in terms of sub-tour elimination and showed that developed method outperforms then the existing mixed integer programming method.

4 Concluding observations and comments

On account of practical importance of VRP in real life, the problem has attracted the attention of a number of researchers in the recent past. Most of the work has been devoted to classical objectives e.g. minimisation of total travelled distance, fleet size or time. Some work also has been done considering multiple objectives such as load balancing, distance/time balancing etc. The papers dealing with profit maximisation, penalty consideration, customer's satisfaction etc. usually tend to optimise multiple objectives simultaneously. It is also interesting to observe that the exact methods (Branch & Bound, sweep, mathematical programming) are employed only in 18% of reviewed study. Moreover these methods are employed for single objective and small sized problems. Other classical heuristics such as λ-interchange, local search are employed in 20% of surveyed papers. Most of recent work (62%) is devoted to use of meta-heuristics. Amongst these GA, ACO, SA and TS are the commonly used approaches. Some of the papers have also analysed the effect of relatively new approaches such as GSO, Artificial bee colony, particle swarm optimisation PSO etc. on real life VRPs. Based on the analysis, the selected articles that considers VRP and related variants as well as the solution methodologies to handle them are presented in Table 1 in Appendix. The second column enlists the author/s and third column represents the year of publishing the work. The problem constraints and primary objective/s to be optimised are given in columns 3 and 4 respectively. Second last column presents the main approach adopted for the problem solution.

This paper surveys the recent work on VRP and its most popular variants focusing primarily on the solution strategies used for solving them. A lot of research is still being done in this field. However to the best of our knowledge no efforts have been done to systematically review this work after 2009. We have tried to bridge the gap by listing some of the significant contributions in this field. Based on the detailed analysis of articles being studied some of the new possibilities for the further work in this field are as follows:

In view of the limitations of the exact methods, a greater stress needs to be given of problems on developing meta-heuristics-based new approaches for solving larger instances within short time. For this one can also think of the hybridisation of exact methods with meta-heuristics as well as adaptation of relatively new meta-heuristics such as harmony search, DNA optimisation etc. to this field.

Secondly, it has been observed that most of the recent literature focuses on real VRPTW having uncertain and/or fuzzy constraints rather than classical VRP. As yet no appropriate model seems to have been developed to deal with these fuzzy or uncertain variants of the problem.

Moreover it is also been observed that there are no benchmarks available for more realistic versions of VRP. As a result there is still a scope for further work in the field. So the researchers are motivated to develop publically available datasets and effective and efficient methods to deal with VRPs.

Finally, primarily the main focus is on static routing plan, however due to technology advancements such as GPS, mobile communication there may be a need of dynamic routing algorithms which enable changes in the routing decisions while drivers are on move.

The gaps in the available literature mentioned above may motivate further work in these directions some of the researchers in this field.

References

Adelzadeh, M., Asl, V.M. and Koosha, M., (2014) 'A mathematical model and a solving procedure for multi-depot vehicle routing problem with fuzzy time window and heterogeneous vehicle', *The International Journal of Advanced Manufacturing Technology*, Vol. 75, Nos. 5–8, pp.793–802.

Agarwal, Y., Mathur, K. and Salkin, H.M. (1989) 'A set-partitioning based exact algorithm for the vehicle routing problem', *Networks*, Vol. 19, No. 7, pp.731–749.

Almoustafa, S., Hanafi, S. and Mladenović, N. (2013) 'New exact method for large asymmetric distance-constrained vehicle routing problem', *European Journal of Operational Research,* Vol. 226, No. 3, pp.386–394.

Archetti, C., Bianchessi, N., Speranza, M.G. and Hertz, A. (2014) 'Incomplete service and split deliveries in a routing problem with profits', *Networks*, Vol. 63, No. 2, pp.135–145.

Baldacci, R., Mingozzi, A. and Roberti, R. (2012) 'Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints', *European Journal of Operational Research*, Vol. 218, No. 1, pp.1–6.

Baldacci, R., Toth, P. and Vigo, D. (2010) 'Exact algorithms for routing problems under vehicle capacity constraints', *Annals of Operations Research*, Vol. 175, No. 1, pp.213–245.

Balseiro, S.R., Loiseau, I. and Ramonet, J. (2011) 'An ant colony algorithm hybridised with insertion heuristics for the time dependent vehicle routing problem with time windows', *Computers & Operations Research*, Vol. 38, No. 6, pp.954–966.

Baños, R., Ortega, J., Gil, C., Márquez, A.L. and De Toro, F. (2013) 'A hybrid meta-heuristic for multi-objective vehicle routing problems with time windows', *Computers & Industrial Engineering*, Vol. 65, No. 2, pp.286–296.

Beheshti, A.K., Hejazi, S.R. and Alinaghian, M. (2015) 'The vehicle routing problem with multiple prioritized time windows: a case study', *Computers & Industrial Engineering*, Vol. 90, No. 12, pp.402–413.

Belfiore, P. and Yoshizaki, H.T. (2013) 'Heuristic methods for the fleet size and mix vehicle routing problem with time windows and split deliveries', *Computers & Industrial Engineering*, Vol. 64, No. 2, pp.589–601.

Belhaiza, S., Hansen, P. and Laporte, G. (2014) 'A hybrid variable neighborhood tabu search heuristic for the vehicle routing problem with multiple time windows', *Computers & Operations Research*, Vol. 52, pp.269–281.

Bodin, L.D. and Berman, L. (1979) 'Routing and scheduling of school buses by computer', *Transportation Science*, Vol. 13, No. 2, pp.113–129.

Brandao, J. (2006) 'A new tabu search algorithm for the vehicle routing problem with backhauls', *European Journal of Operational Research*, Vol. 173, No. 2, pp.540–555.

Caceres-Cruz, J., Arias, P., Guimarans, D., Riera, D. and Juan, A.A (2015) 'Rich vehicle routing problem: survey', *ACM Computing Surveys*, Vol. No.CSUR, 47, No. 2, p.32.

Clarke, G.U. and Wright, J.W. (1964) 'Scheduling of vehicles from a central depot to a number of delivery points', *Operations Research*, Vol. 12, No. 4, pp.568–581.

Cordeau, J.F. and Maischberger, M. (2012) 'A parallel iterated tabu search heuristic for vehicle routing problems', *Computers & Operations Research*, Vol. 39, No. 9, pp.2033–2050.

Dantzig, G.B. and Ramser, J.H. (1959) 'The truck dispatching problem', *Management Science*, Vol. 6, No. 1, pp.80–91.

Desrochers, M., Lenstra, J.K. and Savelsbergh, M.W. (1990) 'A classification scheme for vehicle routing and scheduling problems', *European Journal of Operational Research*, Vol. 46, No. 3, pp.322–332.

Eksioglu, B., Vural, A.V. and Reisman, A. (2009) 'The vehicle routing problem: a taxonomic review', *Computers & Industrial Engineering*, Vol. 57, No. 4, pp.1472–1483.

Elhallaoui, I., Villeneuve, D., Soumis, F. and Desaulniers, G. (2005) 'Dynamic aggregation of set-partitioning constraints in column generation', *Operations Research*, Vol. 53, No. 4, pp.632–645.

Ellon, S., Watson-Gandy, C.D.T. and Christofides, N. (1971) *Distribution Management*, Griffin, London.

Favaretto, D., Moretti, E. and Pellegrini, P. (2007) 'Ant colony system for a VRP with multiple time windows and multiple visits', *Journal of Interdisciplinary Mathematics*, Vol. 10, No. 2, pp.263–284.

Feng, L., Ong, Y.S., Lim, M.H. and Tsang, I.W. (2015) 'Memetic search with interdomain learning: a realisation between CVRP and CARP', *IEEE Transactions on Evolutionary Computation*, Vol. 19, No. 5, pp.644–658.

Fisher, M.L. and Jaikumar, R. (1981) 'A generalised assignment heuristic for vehicle routing', *Networks*, Vol. 11, No. 2, pp.109–124.

Flood, M.M. (1956) 'The traveling-salesman problem', *Operations Research*, Vol. 4, No. 1, pp.61–75.

Ganesh, K., Narendran, T.T. and Anbuudayasankar, S.P. (2014) 'Evolving cost-effective routing of vehicles for blood bank logistics', *International Journal of Logistics Systems and Management*, Vol. 17, No. 4, pp.381–415.

Gendreau, M. and Potvin, J.Y. (1998) 'Dynamic vehicle routing and dispatching', *Fleet Management and Logistics*, pp.115–126, Springer, USA.

Gendreau, M., Ghiani, G. and Guerriero, E. (2015) 'Time-dependent routing problems: a review', *Computers & Operations Research*, Vol. 64, No. 1, pp.189–197.

Ghannadpour, S.F., Noori, S., Tavakkoli-Moghaddam, R. and Ghoseiri, K. (2014) 'A multi-objective dynamic vehicle routing problem with fuzzy time windows: model, solution and application', *Applied Soft Computing*, Vol. 14, pp.504–527.

Ghoseiri, K. and Ghannadpour, S.F. (2010) 'Multi-objective vehicle routing problem with time windows using goal programming and genetic algorithm', *Applied Soft Computing*, Vol. 10, No. 4, pp.1096–1107.

Gillett, B.E. and Miller, L.R. (1974) 'A heuristic algorithm for the vehicle-dispatch problem', *Operations Research*, Vol. 22, No. 2, pp.340–349.

Glover, F. (1986) 'Future paths for integer programming and links to artificial intelligence', *Computers & Operations Research*, Vol. 13, No. 5, pp.533–549.

Golden, B.L., Magnanti, T.L. and Nguyen, H.Q. (1975) *Implementing Vehicle Routing Algorithms*, Massachusetts Inst of Tech Cambridge Operations Research Center, No. TR-115.

Gomez, A., Imran, A. and Salhi, S. (2013) 'Solution of classical transport problems with bee algorithms', *International Journal of Logistics Systems and Management*, Vol. 15, Nos. 2–3, pp.160–170.

Guimarans, D., Herrero, R., Riera, D., Juan, A.A. and Ramos, J.J. (2011) 'Combining probabilistic algorithms, constraint programming and Lagrangian relaxation to solve the vehicle routing problem', *Annals of Mathematics and Artificial Intelligence*, Vol. 62, Nos. 3–4, pp.299–315.

Hashimoto, H., Yagiura, M., Imahori, S. and Ibaraki, T. (2013) 'Recent progress of local search in handling the time window constraints of the vehicle routing problem', *Annals of Operations Research*, Vol. 204, No. 1, pp.171–187.

Hernandez, F., Feillet, D., Giroudeau, R. and Naud, O. (2014) 'A new exact algorithm to solve the multi-trip vehicle routing problem with time windows and limited duration', *4OR*, Vol. 12, No. 3, pp.235–259.

Jourdan, L., Basseur, M. and Talbi, E.G. (2009) 'Hybridising exact methods and metaheuristics: a taxonomy', *European Journal of Operational Research*, Vol. 199, No. 3, pp.620–629.

Juan, A.A., Faulin, J., Jorba, J., Caceres, J. and Marquès, J.M. (2013) 'Using parallel & distributed computing for real-time solving of vehicle routing problems with stochastic demands', *Annals of Operations Research*, Vol. 207, No. 1, pp.43–65.

Karakatič, S. and Podgorelec, V. (2015) 'A survey of genetic algorithms for solving multi depot vehicle routing problem', *Applied Soft Computing*, February, Vol. 27, pp.519–532.

Katiyar, V. (2015) 'Relative performance of certain meta heuristics on vehicle routing problem with time windows', *International Journal of Information Technology and Computer Science*, Vol. 7, No. 12, pp.40–49.

Lahyani, R., Khemakhem, M. and Semet, F. (2015) 'Rich vehicle routing problems: from a taxonomy to a definition', *European Journal of Operational Research*, Vol. 241, No. 1, pp.1–14.

Lai, D.S., Demirag, O.C. and Leung, J.M. (2016) 'A tabu search heuristic for the heterogeneous vehicle routing problem on a multigraph', *Transportation Research Part E: Logistics and Transportation Review*, February, Vol. 86, pp.32–52.

Lalla-Ruiz, E., Expósito-Izquierdo, C., Taheripour, S. and Voß, S. (2016) 'An improved formulation for the multi-depot open vehicle routing problem', *OR Spectrum*, Vol. 38, No. 1, pp.175–187.

Laporte, G. (2007) 'What you should know about the vehicle routing problem', *Naval Research Logistics*, Vol. 54, No. 8, pp.811–819.

Laporte, G. and Martello, S. (1990) 'The selective travelling salesman problem', *Discrete Applied Mathematics*, Vol. 26, Nos. 2–3, pp.193–207.

Laporte, G. and Nobert, Y. (1987) 'Exact algorithms for the vehicle routing problem', *North-Holland Mathematics Studies*, December, Vol. 132, pp.147–184.

Laporte, G. and Osman, I.H. (1995) 'Routing problems: a bibliography', *Annals of Operations Research*, Vol. 61, No. 1, pp.227–262.

Lin, C., Choy, K.L., Ho, G.T., Chung, S.H. and Lam, H.Y. (2014) 'Survey of green vehicle routing problem: past and future trends', *Expert Systems with Applications*, Vol. 41, No. 4, pp.1118–1138.

Liu, Y., Wang, S., Dong, F. and Ren, D. (2013) 'A two stage method for VRP based on the improved ant colony algorithm', *International Journal of Modelling, Identification and Control*, Vol. 18, No. 2, pp.174–181.

Malandraki, C. and Daskin, M.S. (1992) 'Time dependent vehicle routing problems: formulations, properties and heuristic algorithms', *Transportation Science*, Vol. 26, No. 3, pp.185–200.

Marinaki, M. and Marinakis, Y. (2016) 'A glowworm swarm optimisation algorithm for the vehicle routing problem with stochastic demands', *Expert Systems with Applications*, March, Vol. 46, pp.145–163.

Marinakis, Y., Iordanidou, G.R. and Marinaki, M. (2013) 'Particle swarm optimisation for the vehicle routing problem with stochastic demands', *Applied Soft Computing*, Vol. 13, No. 4, pp.1693–1704.

Montemanni, R., Gambardella, L.M., Riszoli, A.E. and Donati, A.V. (2005) 'Ant colony system for a dynamic vehicle routing problem', *Journal of Combinatorial Optimisation*, Vol. 10, No. 4, pp.327–343.

Mousavipour, S. and Hojjati, S.M.H. (2014) 'A particle swarm optimisation for time-dependent vehicle routing problem with an efficient travel time function', *International Journal of Operational Research*, Vol. 20, No. 1, pp.109–120.

Nazif, H. and Lee, L.S. (2012) 'Optimised crossover genetic algorithm for capacitated vehicle routing problem', *Applied Mathematical Modelling*, Vol. 36, No. 5, pp.2110–2117.

Newton, R.M. and Thomas, W.H. (1974) 'Bus routing in a multi-school system', *Computers & Operations Research*, Vol. 1, No. 2, pp.213–222.

Parragh, S.N., Doerner, K.F. and Hartl, R.F. (2008) 'A survey on pickup and delivery problems', *Journal für Betriebswirtschaft*, Vol. 58, No. 1, pp.21–51.

Pillac, V., Gendreau, M., Guéret, C. and Medaglia, A.L. (2013) 'A review of dynamic vehicle routing problems', *European Journal of Operational Research*, Vol. 225, No. 1, pp.1–11.

Psaraftis, H.N., Wen, M. and Kontovas, C.A. (2016) 'Dynamic vehicle routing problems: three decades and counting', *Networks*, Vol. 67, No. 1, pp.3–31.

Pureza, V., Morabito, R. and Reimann, M. (2012) 'Vehicle routing with multiple deliverymen: modeling and heuristic approaches for the VRPTW', *European Journal of Operational Research*, Vol. 218, No. 3, pp.636–647.

Raff, S. (1983) 'Routing and scheduling of vehicles and crews: the state of the art', *Computers & Operations Research*, Vol. 10, No. 2, pp.63–211.

Ropke, S. and Pisinger, D. (2006) 'A unified heuristic for a large class of vehicle routing problems with backhauls', *European Journal of Operational Research*, Vol. 171, No. 3, pp.750–775.

Salhi, S., Wassan, N. and Hajarat, M. (2013) 'The fleet size and mix vehicle routing problem with backhauls: formulation and set partitioning-based heuristics', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 56, No. 1, pp.22–35.

Subramanian, A., Penna, P.H.V., Uchoa, E. and Ochi, L.S. (2012) 'A hybrid algorithm for the heterogeneous fleet vehicle routing problem', *European Journal of Operational Research*, Vol. 221, No. 2, pp.285–295.

Szeto, W.Y., Wu, Y. and Ho, S.C. (2011) 'An artificial bee colony algorithm for the capacitated vehicle routing problem', *European Journal of Operational Research*, Vol. 215, No. 1, pp.126–135.

Tang, J., Guan, J., Yu, Y. and Chen, J. (2014) 'Beam search combined with MAX-MIN ant systems and benchmarking data tests for weighted vehicle routing problem', *IEEE Transactions on Automation Science and Engineering*, Vol. 11, No. 4, pp.1097–1109.

Tasan, A.S. and Gen, M. (2012) 'A genetic algorithm based approach to vehicle routing problem with simultaneous pick-up and deliveries', *Computers & Industrial Engineering*, Vol. 62, No. 3, pp.755–761.

Teoh, B.E., Ponnambalam, S.G. and Kanagaraj, G. (2015) 'Differential evolution algorithm with local search for capacitated vehicle routing problem', *International Journal of Bio-Inspired Computation*, Vol. 7, No. 5, pp.321–342.

Tricoire, F., Romauch, M., Doerner, K.F. and Hartl, R.F. (2010) 'Heuristics for the multi-period orienteering problem with multiple time windows', *Computers & Operations Research*, Vol. 37, No. 2, pp.351–367.

Van Hentenryck, P. (1989) *Constraint Satisfaction in Logic Programming*, Vol. 5, MIT Press, Cambridge.

Vidal, T., Crainic, T.G., Gendreau, M. and Prins, C. (2013) 'Heuristics for multi-attribute vehicle routing problems: a survey and synthesis', *European Journal of Operational Research*, Vol. 231, No. 1, pp.1–21.

Wang, J., Zhou, Y., Wang, Y., Zhang, J., Chen, C.P. and Zheng, Z. (2016) 'Multiobjective vehicle routing problems with simultaneous delivery and pickup and time windows: formulation, instances, and algorithms', *IEEE Transactions on Cybernetics*, Vol. 46, No. 3, pp.582–594.

Wilson, N. and Sussman, J. (1971) 'Implementation of computer algorithms for the dial-a-bus system', *39th National Meeting of ORSA/TIMS*.

Xiao, Y., Zhao, Q., Kaku, I. and Xu, Y. (2012) 'Development of a fuel consumption optimisation model for the capacitated vehicle routing problem', *Computers & Operations Research*, Vol. 39, No. 7, pp.1419–1431.

Yu, B. and Yang, Z.Z. (2011) 'An ant colony optimisation model: The period vehicle routing problem with time windows', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 47, No. 2, pp.166–181.

Yu, B., Yang, Z.Z. and Xie, J.X. (2011) 'A parallel improved ant colony optimisation for multi-depot vehicle routing problem', *Journal of the Operational Research Society*, Vol. 62, No. 1, pp.183–188.

Zhou, W., Song, T., He, F. and Liu, X. (2013) 'Multiobjective vehicle routing problem with route balance based on genetic algorithm', *Discrete Dynamics in Nature and Society*, No. 1, pp.1–9.

Zhou, Y. and Wang, J. (2015) 'A local search-based multiobjective optimization algorithm for multiobjective vehicle routing problem with time windows', *IEEE Systems Journal*, Vol. 9, No. 3, pp.1100–1113.

Appendix

Table 1 VRP solution approaches

S. no.	*Author/s*	*Year*	*Constraint*	*Objective*	*Approach*	*Journal*
1	Lai et al.	2016	Time-constrained heterogeneous VRP	TD	Tabu search for fixed sequence arc selection problem (FSASP)	*Transportation Research Part E*
2	Marinaki and Marinakis	2016	VRPSD	TD	Glowworm swarm optimisation (GSO) hybridised with VNS	*Expert Systems with Applications*
3	Beheshti et al.	2015	Multiple prioritised time windows	M.O. (TD and customers satisfaction)	Cooperative co-evolutionary multi-objective quantum-genetic algorithm (CCMQGA)	*Computers & Industrial Engineering*
4	Teoh et al.	2015	CVRP	TD	Differential evolution algorithm with local search	*Int. J. Bio-Inspired Computation*
5	Lalla-Ruiz et al.	2016	MDOVRP	TD	Mixed integer programming	*OR Spectrum*
6	Feng et al.	2015	CVRP and CARP	M.O. (TV, TD, TT, WT)	Evolutionary memetic approach	*IEEE Transactions on Evolutionary Computation*
7	Wang et al.	2016	MO-VRPSDPTW	M.O. (TV, TD, TT, WT)	Memetic algorithm with local search	*IEEE Transactions on Cybernetics*
8	Zhou and Wang	2015	MO-VRPTW	M.O. (TV, TD, TT, WT)	Parallel local search	*IEEE Systems Journal*
9	Archetti et al.	2014	CVRP-split delivery	Profit	Exact methods and tabu search with VNS	*Networks*
10	Tang et al.	2014	Weighted VRP	M.O. (TD, load balance)	Beam search with MMACS	*IEEE Transactions on Automation Science and Engineering*
11	Ganesh et al.	2014	VRP with pick-up and delivery	TD	SA hybridised with GA	*International Journal of Logistics Systems and Management*
12	Adelzadeh et al.	2014	Fuzzy heterogeneous VRPTW	M.O. (TD, service level)	Three-level solution approach based on SA	*Int. J. Adv. Manuf. Technology*
13	Ghannadpour et al.	2014	Dynamic VRP having fuzzy TW	M.O. (TD, TV, TT, and customers satisfaction)	GA with search space	*Applied Soft Computing*
14	Gomez et al.	2013	CVRP	TD	Artificial bee colony	*International Journal of Logistics Systems and Management*
15	Hernandez et al.	2013	Multi trip VRPTW	TD	Two-phase exact method based on Branch and Price	*4OR-Q J Opr. Res*
16	Zhou et al.	2013	VRPTW	M.O. (TD, route length balance)	GA	*Discrete Dynamics in Nature and Society*

Notes: M.O.: multiple objective; TV: total number of vehicles used; TD: total travelled distance; TT: total travel time; WT: total waiting time

Table 1 VRP solution approaches (continued)

S. no.	*Author/s*	*Year*	*Constraint*	*Objective*	*Approach*	*Journal*
17	Hashimoto et al.	2013	VRPTW	M.O. (TD, TV)	Local search	*Annals of Operations Research*
18	Liu et al.	2013	VRPD	TD	Two-stage ACO	*Int. J. Modeling, Identification and Control*
19	Belfiore and Yoshizaki	2013	Fleet size and mixed VRPTW with split delivery	TD	Scatter search	*Computers & Industrial Engineering*
20	Tasan and Gen	2012	VRP-SPD	TD	GA	*Computers & Industrial Engineering*
21	Nazif and Lee	2012	CVRP	TD	Optimised crossover genetic algorithm	*Applied Mathematical Modeling*
22	Cordeau and Maischberger	2012	CVRP, MDVRP, site dependent VRPTW	M.O. (TD, TV)	Parallel tabu search	*Computers & Operations Research*
23	Almoustafa et al.	2013	Distance constrained VRP	TD	Branch & Bound with randomness	*European Journal of Operational Research*
24	Xiao et al.	2012	Fuel consumption CVRP	M.O. (TD, TV, fuel consumption)	Simulated annealing with hybrid exchange rules	*Computers &Operations Research*
25	Yu et al.	2011	MDVRP	TD	Parallel ACO hybridised with improved strategies like coarse-grain parallel strategy, ant weight strategies strategy and the local search strategy and the local search strategy	*Journal of the Operational Research Society*
26	Yu and Yang	2011	Periodic VRPTW	TD	ACO with multi-dimension pheromone information and two-crossover operator	*Transportation Research Part E*
27	Balseiro et al.	2011	Time dependent VRPTW	M.O. (TV, TT)	ACO with aggressive insertion heuristics	*Computers &Operations Research*
28	Szeto et al.	2011	CVRP	TD	Enhanced artificial bee colony accepting worse solutions probabilistically	*European Journal of Operational Research*
29	Ghoseiri and Ghannadpour	2010	MO-VRPTW	M.O. (TD, TV)	GA	*Applied Soft Computing*

Notes: M.O.: multiple objective; TV: total number of vehicles used; TD: total travelled distance; TT: total travel time; WT: total waiting time