Introduction to Speech processing 67355

Exercise 1

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Theoretical Solution

1.1 the convolution theorem

$$\mathcal{F}[x_1(t) \cdot x_2(t)] = \int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) \cdot e^{-i\omega t} dt$$

We will re-write $x_1(t)$ using the inverse FT:

$$x_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(k) \cdot e^{ikt} \cdot dk$$

So we will get

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \frac{1}{2\pi} \cdot X_1(k) \cdot e^{ikt} \cdot dk \right] \cdot x_2(t) \cdot e^{-i\omega t} dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(k) \cdot dk \cdot \int_{-\infty}^{\infty} x_2(t) \cdot e^{-it(\omega - k)} \cdot dt =$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(k) \cdot X_2(\omega - k) \cdot dk = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$

1.2 Proving Nyquist sampling theorem

1.2.1

Given $x_d(t) = \sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT) = x(t) \cdot comb(\frac{t}{T})$

In order to calculate the Fourier transform of $x_d(t)$ we just need to use the convolution theorem and to use the known former transform of the $comb$ function which is $FT[comb(\frac{t}{T})] = \frac{2\pi}{T} comb(\frac{\omega T}{2\pi})$.

We will get

$$FT[x_d(t)] = \frac{1}{2\pi} X(\omega) * \frac{2\pi}{T} comb(\frac{T\omega}{2\pi})$$

1.2.2

The $delta$ function defined as follows:

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & else \end{cases}, \qquad \int_{-\infty}^{\infty} \delta(t) dt = 1$$

Which means that taking an integral of a function times delta, is equal to take the value of the function at the same point.

Using the convolution theorem we can also change the integral with a convolution and get the an integral with a delta is time shifting the function.

$$\sum_{n} \int_{-\infty}^{\infty} X(\bar{\omega}) \cdot \delta(\bar{\omega} - (\omega - \frac{2\pi n}{T})) d\bar{\omega} = \sum_{n} X(\bar{\omega}) * \delta(\bar{\omega} - (\omega - \frac{2\pi n}{T})) = \sum_{n} X(\omega - \frac{2\pi n}{T})$$

1.2.3

As a direct continuation from previous sections

$$FT[x_d(t)] = \frac{1}{T}X(\omega) * comb(\frac{T\omega}{2\pi})$$

As stated before, convolution with a delta is exactly like time shifting, we can take out the convolution with the comb and to write it as a sum of dilated functions.

$$FT[x_d(t)] = \frac{1}{T}X(\omega) * \sum_n \delta(\omega - \frac{2\pi}{T}n) = \frac{1}{T}\sum_n X(\omega - \frac{2\pi}{T}n)$$

1.2.4

Let $x(t)$ be a a limited signal bounded by ω_{max} and lets define $f_{max} = \frac{\omega_{max}}{2\pi}$.

We will sample at $\frac{1}{T} = f_s > 2f_{max}$.

We showed that $X_d(\omega) = \frac{1}{T}\sum_n X(\omega - \frac{2\pi}{T}n)$, and we know that $X_d(\omega)$ is counted by ω_{max}, which

means that the all of the shifted $X(\omega - \frac{2\pi}{T}n)$ won't intersect each other because $\frac{2\pi}{T} > 4\pi f_{max} > 2\omega_{max}$.

Which means that we will get identical functions at different places in the waveform domain, with spaces of $4\pi f_s$ and width of $2\omega_{max}$. After applying Anti aliasing filter we will get only the first delta (which is around 0), so we will get the following equivalence:

$$X_d(\omega) = \frac{1}{T}X(\omega)$$

1.3 Sampling below Nyquist's frequency

Given $x(t) = sin(2\pi \cdot 1000 \cdot t) + sin(2\pi \cdot 5000 \cdot t)$ and $f_s = 8[Khz]$

The discrete measured signal would be $x_d(t) = x(t) \cdot comb(tf_s)$.

In order to get the frequencies we would apply FT and use the convolution theorem.

$$X_d(\omega) = X(\omega) * comb(\frac{\omega}{2\pi f_s})$$

$X(\omega)$ is a simple FT that contains 4 delta functions ($X(\omega)$ is complex because sin has a phase of $\frac{\pi}{2}$ but we will deal only with the absolute value $|X(\omega)|$):

$$\delta(\omega - 2\pi \cdot 1000) + \delta(\omega - 2\pi \cdot 5000) + \delta(\omega + 2\pi \cdot 1000) + \delta(\omega + 2\pi \cdot 5000)$$

The comb function duplicates these 4 deltas with spaces of $2\pi \cdot 8,000$. Because that our 4 deltas has a range of $2\pi \cdot 10,000$ we would get an aliasing effect.

It is very clear since we sample at f_s lower then the Nyquist frequency.

After applying anti aliasing filter (at $\omega = 2\pi \cdot 4000$) we would get 4 deltas:
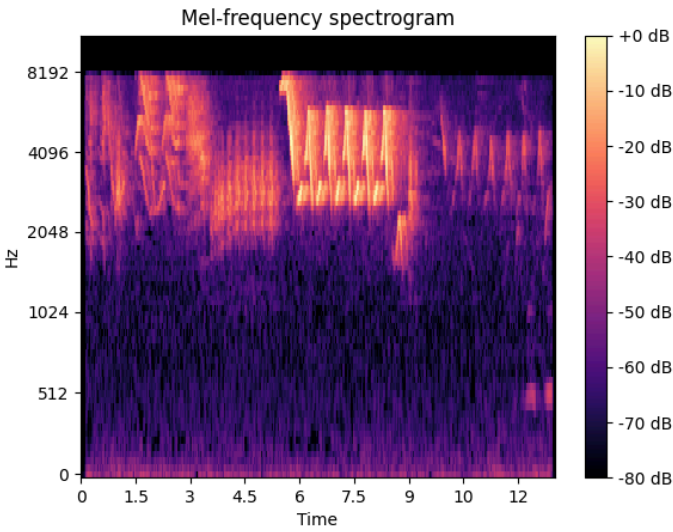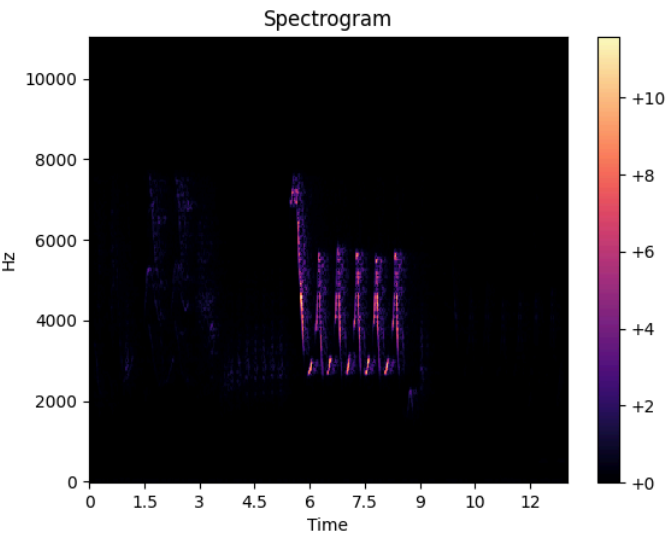
$$\delta(\omega - 2\pi \cdot 1000) + \delta(\omega - 2\pi \cdot 3000) + \delta(\omega + 2\pi \cdot 1000) + \delta(\omega + 2\pi \cdot 3000)$$

The $3000[Khz]$ is an aliased frequency that appeared due to the low sampling rate.
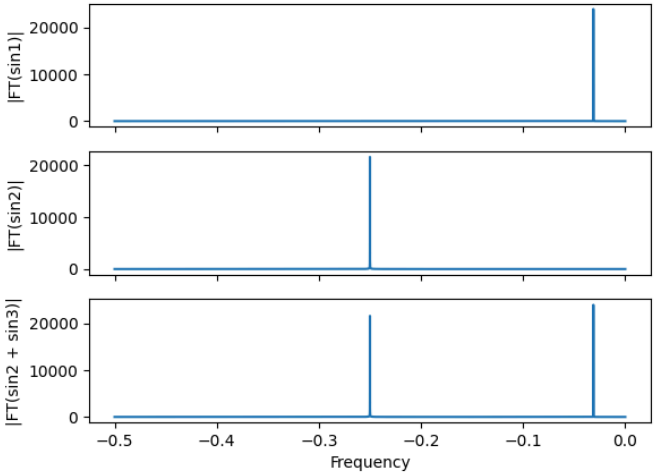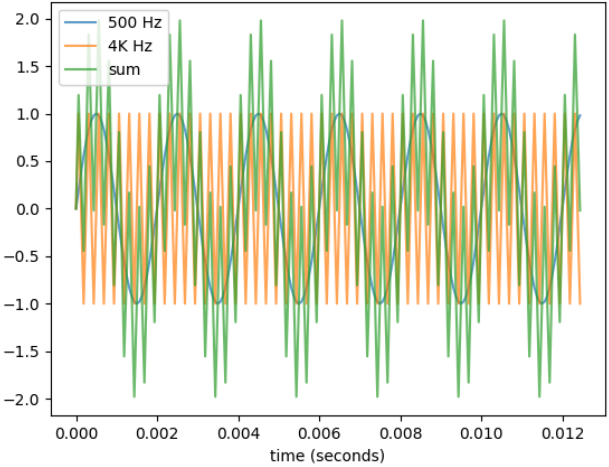
Practical Solution Graphs

1. Basic Audio Handling

• Part A

• Part B

- Part C

Fourier of up sampled waveforms