Introduction to Speech processing 67355

Exercise 1

Alon Ziv (), Amit Roth (213212798)

1.1 the convolution theorem

$$\mathcal{F}[x_1(t)\cdot x_2(t)] = \int_{-\infty}^{\infty} x_1(t)\cdot x_2(t)\cdot e^{-i\omega t}dt$$

We will re-write $x_1(t)$ using the inverse FT:

$$x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(k)\cdot e^{ikt}\cdot dk$$

So we will get

$$= \int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty}\frac{1}{2\pi}\cdot X_1(k)\cdot e^{ikt}\cdot dk\Big]\cdot x_2(t)\cdot e^{-i\omega t}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(k)\cdot dk\cdot\int_{-\infty}^{\infty} x_2(t)\cdot e^{-it(\omega-k)}\cdot dt =$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(k)\cdot X_2(\omega-k)\cdot dk = \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$$

1.2 Proving Nyquist sampling theorem

1.2.1

Given $x_d(t) = \sum_{n=-\infty}^{\infty} x(t)\cdot\delta(t-nT) = x(t)\cdot comb(\frac{t}{T})$

In order to calculate the Fourier transform of $x_d(t)$ we just need to use the convolution theorem and to use the known former transform of the $comb$ function which is $FT[comb(\frac{t}{T})] = \frac{2\pi}{T}comb(\frac{\omega T}{2\pi})$.

We will get

$$FT[x_d(t)] = \frac{1}{2\pi}X(\omega) * \frac{2\pi}{T}comb(\frac{T\omega}{2\pi})$$

1.2.2

The $delta$ function defined as follows:

$$\delta(t) = \begin{cases}\infty & t=0\\ 0 & else\end{cases}, \qquad \int_{-\infty}^{\infty}\delta(t)dt = 1$$

Which means that taking an integral of a function times delta, is equal to take the value of the function at the same point.

Using the convolution theorem we can also change the integral with a convolution and get the an integral with a delta is time shifting the function.

$$\sum_n\int_{-\infty}^{\infty} X(\bar{\omega})\cdot\delta(\bar{\omega}-(\omega-\frac{2\pi n}{T})d\bar{\omega} = \sum_n X(\bar{\omega}) * \delta(\bar{\omega}-(\omega-\frac{2\pi n}{T}) = \sum_n X(\omega-\frac{2\pi n}{T}) =$$

1.2.3

As a direct continuation from previous sections

$$FT[x_d(t)] = \frac{1}{T}X(\omega) * comb(\frac{T\omega}{2\pi})$$

As stated before, convolution with a delta is exactly like time shifting, we can take out the convolution with the comb and to write it as a sum of dilated functions.

$$FT[x_d(t)] = \frac{1}{T}X(\omega) * \sum_n \delta(\omega - \frac{2\pi}{T}n) = \frac{1}{T}\sum_n X(\omega - \frac{2\pi}{T}n)$$

1.2.4

Let $x(t)$ be a a limited signal bounded by ω_{max} and lets define $f_{max} = \frac{\omega_{max}}{2\pi}$.

We will sample at $\frac{1}{T} = f_s > 2f_{max}$.

We showed that $X_d(\omega) = \frac{1}{T}\sum_n X(\omega - \frac{2\pi}{T}n)$, and we know that $X_d(\omega)$ is counted by ω_{max}, which means that the all of the shifted $X(\omega - \frac{2\pi}{T}n)$ won't intersect each other because $\frac{2\pi}{T} > 4\pi f_{max} > 2\omega_{max}$.

Which means that we will get identical functions at different places in the waveform domain, with spaces of $4\pi f_s$ and width of $2\omega_{max}$. After applying Anti aliasing filter we will get only the first delta (which is around 0), so we will get the following equivalence:

$$X_d(\omega) = \frac{1}{T}X(\omega)$$

1.3 Fourier

Given $x(t) = sin(2\pi \cdot 1000 \cdot t) + sin(2\pi \cdot 5000 \cdot t)$ and $f_s = 8[Khz]$

The discrete measured signal would be $x_d(t) = x(t) \cdot comb(tf_s)$.

In order to get the frequencies we would apply FT and use the convolution theorem.

$$X_d(\omega) = X(\omega) * comb(\frac{\omega}{2\pi f_s})$$

$X(\omega)$ is a simple FT that contains 4 delta functions ($X(\omega)$ is complex because sin has a phase of $\frac{\pi}{2}$ but we will deal only with the absolute value $|X(\omega)|$):

$$\delta(\omega - 2\pi \cdot 1000) + \delta(\omega - 2\pi \cdot 5000) + \delta(\omega + 2\pi \cdot 1000) + \delta(\omega + 2\pi \cdot 5000)$$

The comb function duplicates these 4 deltas with spaces of $2\pi \cdot 8{,}000$. Because that our 4 deltas has a range of $2\pi \cdot 10{,}000$ we would get an aliasing effect.

It is very clear since we sample at f_s lower then the Nyquist frequency.

After applying anti aliasing filter (at $\omega = 2\pi \cdot 4000$) we would get only the $2\pi \cdot 1000$ frequencies because the $2\pi \cdot 5000$ frequencies will merge with them. Note that if the frequency was different then $2\pi \cdot 5000$ we were also could here different frequencies, but in our cases the were "folded" into the same frequency.