67355 Introduction to Speech Processing

Exercise 1

The following exercise is structured from two parts: Theoretical and Practical.
The exercise should be done **in pairs** and is to be submitted via moodle by the deadline appearing under the subission box.
See submission guidelines for further instructions

1 Theoretical part (55 Points)

1.1 10 points

Prove: Multiplication over the time domain is equivalent to applying convolution over the frequency domain, i.e.

$$F[x_1(t) \cdot x_2(t)] = \frac{1}{2\pi} \left(X_1^F(\omega) * X_2^F(\omega) \right).$$

1.2 Proving Nyquist sampling theorem - 35 points

Recall:

1. Impulse sampling with rate $\frac{1}{T}$ is in practice a discrete sampling $x_d(t)$ of a continuous signal $x(t)$

$$x_d(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT).$$

2. Impulse train $s_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_n \delta(t - nT)$; $\delta(x) = \{1 \text{ if } x = 0 \text{ else } 0\}$.
3. $X^F(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$.
4. Convolution: $\{x * t\}(t) = \int_{-\infty}^{\infty} x(\tilde{t})y(t - \tilde{t})d\tilde{t}$.

In the next sections assume that the Fourier transform of an impulse train is $S_T^F(\omega) = \frac{2\pi}{T} \sum_n \delta\left(\omega - \frac{2\pi n}{T}\right)$,

1.2.1 10 points

Prove:

$$X_d^F(\omega) = \frac{1}{2\pi} \left(X^F(\omega) * S_T^F(\omega) \right).$$

1.2.2 5 points

Prove:

$$\sum_n \int_{-\infty}^{\infty} X^F(\tilde{\omega})\delta\left(\tilde{\omega} - \left(\omega - \frac{2\pi n}{T}\right)\right) d\tilde{\omega} = \sum_n X^F\left(\omega - \frac{2\pi n}{T}\right).$$

1.2.3 10 points

Prove that $X_d^F(\omega) = \frac{1}{T}\sum_n X^F\left(\omega - \frac{2\pi n}{T}\right)$.

1.2.4 10 points

Definition: A signal is called "band limited" if $\exists\omega_{max} \geq 0$ s.t. $\forall\omega, |\omega| > \omega_{max} : X^F(\omega) = 0$
Show that every band limited signal $x(t)$ bounded by ω_{max}, $x(t)$ could be reconstructed with sampling frequency

$$\frac{1}{T} = f_s > 2f_{max} \ ; \ f_{max} = \frac{\omega_{max}}{2\pi},$$

i.e. show that for the above, the following holds:

$$\forall|\omega| \leq \omega_{max} : X_d^F(\omega) = \frac{1}{T}X^F(\omega),$$

To simplify things, assume that $\forall w \in [-\omega_{max}, \omega_{max}] : X^F(\omega) \neq 0$.

Hint: prove by negation.

1.3 10 points

Let $x(t) = sin(2\pi \cdot 1000 \cdot t) + sin(2\pi \cdot 5000 \cdot t)$. We sample $x(t)$ with sampling rate of $8[KHz]$.
What frequencies would appear in the Fourier transform of the discrete measured signal? Assume $\forall|\omega| > 2\pi \cdot 5000$: $X^F(\omega) = 0$.

Hints: (i) use what you proved in section 1.2.3; (ii) think what frequencies would have $X^F(\cdot) \neq 0$. Meaning consider the values where ω and n equals the signal frequencies; (iii) recall that Fourier transform is symmetric around zero (we also have negative frequencies).

2 Practical part (45 Points)

In Ex1.zip you will find two .ipynb notebooks with coding questions.
Please place your answers in the designated locations, and submit your filled notebooks as well as your generated files.
The Points for these notebooks will be divided as follows:

1. Basic audio handling - 25 Points
2. Naive audio manipulation - 20 Points

3 Submission Guidelines

- All theoretical questions are to be typed (i.e. not handwritten) and submitted as a single pdf.
- The header of your submitted solution pdf should include your IDs.
- Please include graphs and final results for code-related questions in your pdf file.
- All used code pieces should be submitted, alonside a README.txt file with a single line containing your IDs separated by commas.
- Please submit a single zip/tar file containing all relevant files.