R&D Lab
Magnetic Inductance

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Through the entire lab, our goal is to develop a close range communication device. In order to design such device, we need to investigate the properties of mutual inductance. Our research will start with the basic properties of RLC circuits, with an emphasis on the internal properties of the measurement devices. We will proceed into coupled RLC circuits, and then we will measure directly the affects of mutual inductance when using the same coils as our product will have. After the research, we will be able to decide the critical behavior of our coil during transmission. The communication will be transmitted by one of the coils and receive by the other coil. Than we will be able to programmatically define the behavior of our device.

Part I
Preparations

As an introduction to the experiment we will perform several measurements in order to determine the internal resistance, capacitance and inductance of our picoscope.

I. MEASURING INTERNAL RESISTANCE

We would like to measure the internal resistance of the picoscope. In order to perform such measurement, we would generate constant voltage by the picoscope, and measure the voltage on a resistor connected to the picoscope as shown in figure ??.

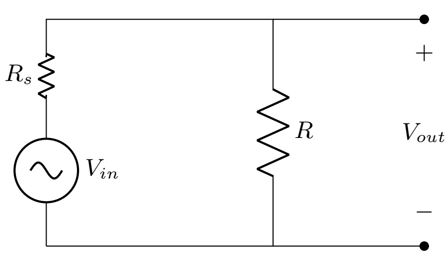


FIG. 1: A circuit with voltage source V_{in}, internal resistance R_s, and a resistor R. Voltmeter measures the voltage V_{out} on the resistor R.

Using Kirchoff's law we get

$$V_{out} = \left(\frac{R}{R+R_s}\right)V_{in} \tag{1}$$

A. Course of The Experiment

In order to get meaningful measurements we would like to measure the voltage of a resistor with resistance R which is in the same order of magnitude of R_s. We tried using resistors with the resistances: $1M\Omega$, $1K\Omega$ and 1Ω. We saw that we get meaningful measurements (V_{out} varies as we change V_{in}) with the $R = 1K\Omega$. Therefore we chose that resistor and proceeded the experiment. We performed a series of measurements for V_{out} as function of V_{in}. We let V_{in} range between 0 to $1.5V$ with steps of $0.1V$. From Equation 1 we would expect to get a linear graph that intersects the origin. The result are shown in figure 2.

B. Results Analysis

We got a fit with $R^2 = 0.9999$ and we got a line equation of the form $y = mx + b$ where

$$m = (617.8 \pm 0.3) \cdot 10^{-3}$$
$$b = 1.7 \pm 0.5\ mV$$

We will extract the internal resistance from the slope using equation 1.

$$m = \tfrac{R}{R+R_s}$$
$$R_s = 618.6 \pm 0.6\Omega$$

We would determine the internal resistance of the PicoScope as

$$\boxed{R_s = 618.6\Omega} \tag{2}$$

C. The Error of The Free Parameter

From the theoretical part, we expected to get a linear graph which crosses the origin. In our measurements we got $b = 0.0017 \pm 0.0005[V]$. This implies that the system has a voltage even when not generating any. In theory, we could subtract it from the graph, but we are only interested in the slope.

II. MEASURING INTERNAL CAPACITANCE AND INDUCTANCE

Now we will measure the internal capacitance and inductance. We will assemble a RLC circuit, with resis-

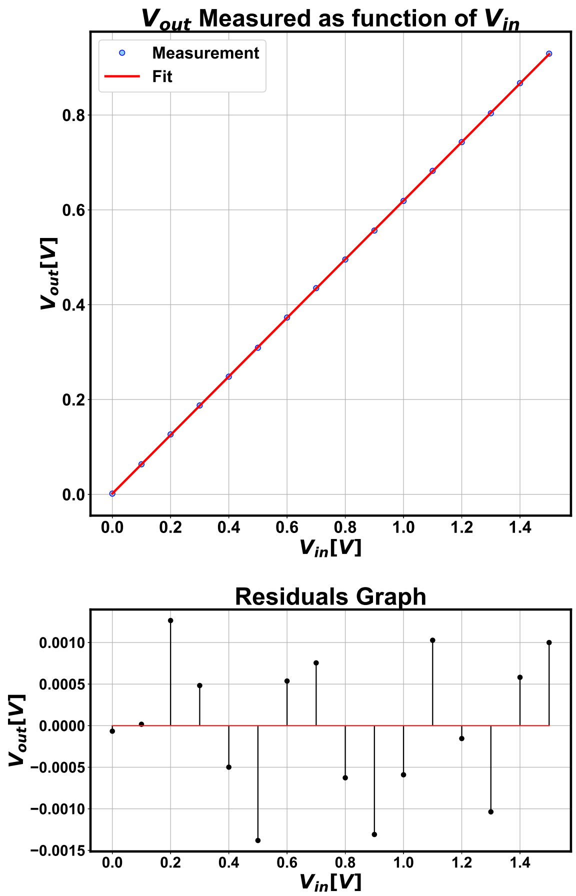


FIG. 2: The results of the internal resistance measurement displayed on a graph of the measured voltage V_{out} as a function of the generated voltage V_{in}. The measurements are displayed in blue, and the linear fit in red.

tances R, R_s, capacitance C_s, and inductance L_s, as seen in figure 3.

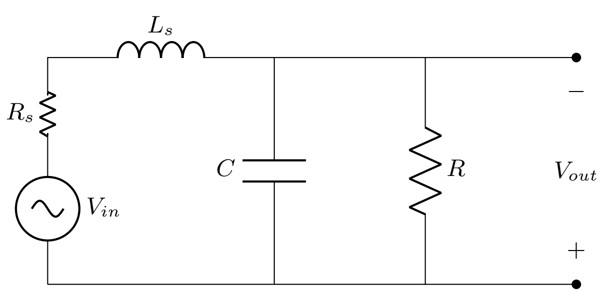


FIG. 3: A circuit with voltage source V_{in}, internal resistance R_s, resistor R, internal capacitance C, internal inductance L_s, when measuring the voltage V_{out} on the resistor R.

From the following kirchoff's equations

$$\begin{cases} R_sI + RI_1 + L_s\dot{I} = V_0 \\ RI_1 = \frac{Q_2}{C_s} \\ I = I_1 + \dot{Q}_2 = \frac{Q_2}{RC} + \dot{Q}_2 \end{cases} \tag{3}$$

We would a get a differential equation for the Charge

$$L_sQ_2 + (R_s + \frac{L_s}{RC})\ddot{Q} + (\frac{R_s}{R} + 1)\frac{\dot{Q_2}}{C} = V_0$$

and the solution $V_{out}(t) = \frac{Q(t)}{C}$ will be

$$V_{out}(t) = Ae^{-\frac{\alpha t}{2}} cos(\frac{1}{2}\cdot\sqrt{4\beta - \alpha^2}t + \phi) + \frac{V_0}{1 + \frac{R_s}{R}} \tag{4}$$

for the parameters

$$\begin{cases} \alpha = \frac{R_s}{L_s} + \frac{1}{RC} \\ \beta = \frac{1}{L_sC}(\frac{R_s}{R} + 1) \end{cases}$$

and the variables A and ϕ are chosen by the boundery values. Now we will be able to measure the voltage and extract the internal capacitance and inductance.

A. Course of The Experiment

We chose a $R = 1K\Omega$ resistor. We generated rectangular pulse oscillating between $2V$ and $-2V$ and measured the voltage on capacitor just after the voltage changed. In other words, we discharged the capacitor under $-2V$ and then charged with $2V$ and measured the voltage on capacitor. The measurements are shown in the figure 4.

B. Results Analysis

We fitted the following function to the measurements

$$f(t) = -\frac{2\cdot c}{\sin(\phi)}e^{-bt}\sin(\omega t + \phi) + c \tag{5}$$

and got the following parameters from the fit

$$\begin{cases} b = (154 \pm 0.8)\cdot 10^5 \; [Hz] \\ \omega = (8 \pm 600)\cdot 10^4 \; [\frac{Rad}{s}] \end{cases}$$

Where b is the decay parameter, i.e. b^{-1} is the lifetime of the oscillation's amplitude. The parameter ω is the angular frequency of the voltage oscillations. One can notice that the relative error of ω is $7.5\cdot 10^3\%$ which means that this result is meaningless. The experiment was designed to measure the capacitance of the PicoScope. Therefore, ω wasn't calculate accurately. Both are determine the oscillations in our circuit (to the extent of boundary values). We would get the following relations between the parameters of the fit to the theoretical expression we showed above 4.

$$\begin{cases} b = \frac{\alpha}{2} \\ \omega = \frac{\sqrt{4\beta - \alpha^2}}{2} \end{cases}$$

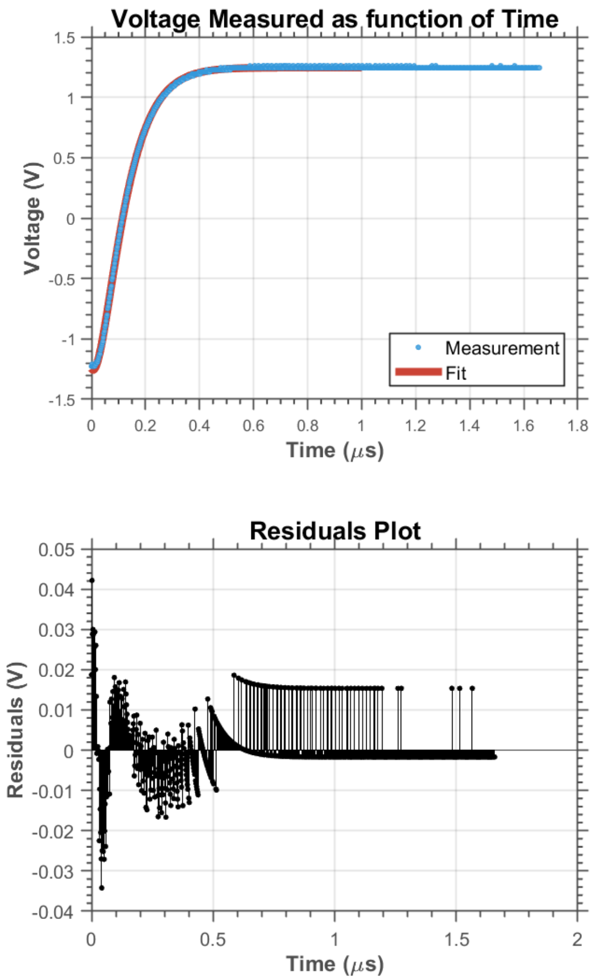


FIG. 4: The results of the internal capacity discharge displayed on a plot of the measured voltage V_{out} as function of time. Blue points represent the measurements and the orange curve represents the fit 5. The bottom graph is the residuals of the fit. One can notice that the residuals graph isn't random as we expected. The reason is that the PicoScope use A2D module that splits the measured voltage to quantization levels. The quantization level is $\Delta v_{quantization} = 0.01\ V$ which is the size of our residuals.

C. Internal Capacitance

In order to extract the internal capacitance from the fitting parameter we will use the equations for L_s, C, β, α when ω, b, R, R_s are known. We get:

$$\frac{\sqrt{\omega^2+b^2}}{\frac{R_s}{R}+1}C^2 - \frac{2b}{R_s}C + \frac{1}{RR_s} = 0 \tag{6}$$

After substitution, we get a quadratic equation and the solution for the capacitance is $C = 32.4 \pm 0.2\ pF$. For the rest of the experiment, we will determine the internal capacitance to be:

$$\boxed{C = 32.4\ pF} \tag{7}$$

D. Internal Inductance

In order to extract L_s we used the relation between L_s and C_s and got $L_s = (45 \pm 7 \cdot 10^{10}) \cdot 10^2\ H$. This is a relative error of $\sim 10^{11}\%$. We conclude that this experiment can't determine the internal inductance and thus another experiment is needed to be done.

Part II
RLC Circuits

III. RLC CIRCUIT

After we measured the internal parameters, we can proceed into building an RLC circuit and measure its transmission function.

A. The Transmission Function

We want to measure the transmission function of the RLC circuit, when measuring the voltage from the resistor R. We will generate voltage from V_{in} and measure the voltage on the resistor R as shown in figure 5

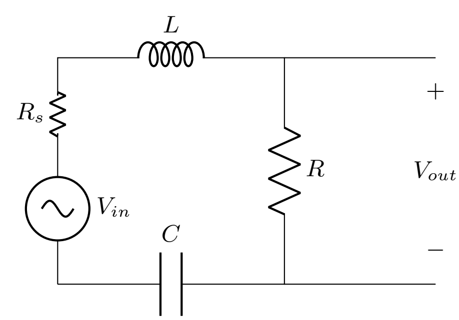


FIG. 5: A RLC circuit with voltage source V_{in}, internal resistance R_s, resistor R, capacitor C and inductor L. We measure the voltage V_{out} on the resistor.

Theoretically we will get

$$H(\omega) = \frac{R}{R+R_s+\frac{1}{i\omega C}+i\omega L} = \frac{1}{(1+\frac{R_s}{R})+i(\frac{L}{R}\omega-\frac{1}{RC\omega})} \tag{8}$$

As we considered R, L, C as the resistance, inductance, and capacitance of the components we connected to the source. R_s is the internal resistance, and the internal inductance and capacitance are redundant.

B. Course of The Experiment

The transmission function defined as follows $V_{out}(\omega) = H(\omega) \cdot V_{in}(\omega)$. Hence, in order to measure $H(\omega)$ we can send different ω to V_{in} and calculate the phase difference between $V_{in}(\omega)$ and $V_{out}(\omega)$, and calculate the ratio between their amplitudes. The phase and amplitude define uniquely the transmission function.

During the experiment we used $R = 1000\ \Omega$, $L = 3\ mH$ and $C = 10^{-10}\ F$. We used a sweep signal in order to measure the amplitude and the phase at various frequencies. The source's voltage amplitude was $V_0 = 2\ V$. We used frequency range of $200\ kHz - 450\ kHz$ in order to capture the theoretical resonance (given by $f_{res} = \frac{1}{2\pi\sqrt{LC}} \approx 300\ kHz$). The chosen frequency rate was $\Delta t = 1\ ms$ in order to let the system stable ($\tau = \frac{1}{b} \approx 6.28 \cdot 10^{-8}\ s \ll 10^{-3}\ s = \Delta t$ is the typical damping period time of mods measured by the voltmeter). We measured the amplitude (A) and the phase (ϕ) of the transmission function as a function of to the power source frequency (f). We expected to get the following mathematical relations:

$$A(f) = |H(2\pi f) \cdot V_{in}(2\pi f)| = \frac{V_0}{\sqrt{(1 + \frac{R_s}{R})^2 + (2\pi f \frac{L}{R} - \frac{1}{2\pi f RC})^2}} \tag{9}$$

$$\phi(f) = \arg\left(H(2\pi f)\right) = -\arctan \frac{2\pi f \frac{L}{R} - \frac{1}{2\pi f RC}}{1 + \frac{R_s}{R}} \tag{10}$$

The voltage we measured on the x is shown in figure 6.

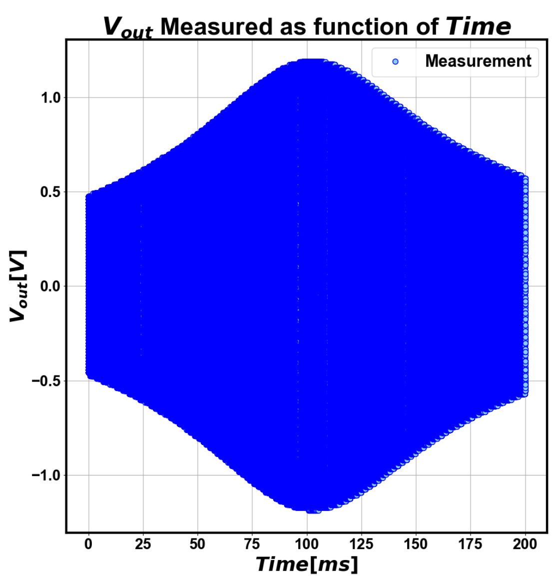


FIG. 6: The results of the RLC resonance measurement displayed on a graph of the measured voltage wave V_{out} as function of time.

C. Results Analysis

While fitting we used the following constants

$$a = 2\pi \frac{L}{R}, \quad b = 2\pi RC, \quad c = 1 + \frac{R_s}{R} \tag{11}$$

and the following functions

$$A(f) = \frac{V_0}{\sqrt{c^2 + (a \cdot f - \frac{1}{b \cdot f})^2}} \tag{12}$$

$$\phi(f) = -\arctan \frac{a \cdot f - \frac{1}{b \cdot f}}{1.618} + constant \tag{13}$$

The constant 1.618 is the value of c extracted from the theory. The graphs are shown in figure 7 and figure 8 and the fitting parameters are shown in I.

Every fitting parameter describes an aspect of the system:

- a - determines the behaviour of the system at large frequencies ($f \gg f_{res}$). The circuit is acting as an LR circuit at this range.
- b - determines the behaviour of the system at small frequencies ($f \ll f_{res}$). The circuit is acting as an RC circuit at this range.

TABLE I: Fitting parameters from the amplitude fitting, phase fitting, and expected theoretical values.

	units	A fitting	ϕ fitting	theory
a	$[s]$	$(168 \pm 0.4) \cdot 10^{-7}$	$(277 \pm 5) \cdot 10^{-7}$	$47 \cdot 10^{-7}$
b	$[s]$	$(557 \pm 1) \cdot 10^{-9}$	$(364 \pm 7) \cdot 10^{-9}$	$157 \cdot 10^{-9}$
c	-	$(169 \pm 0.2) \cdot 10^{-2}$	-	$162 \cdot 10^{-2}$

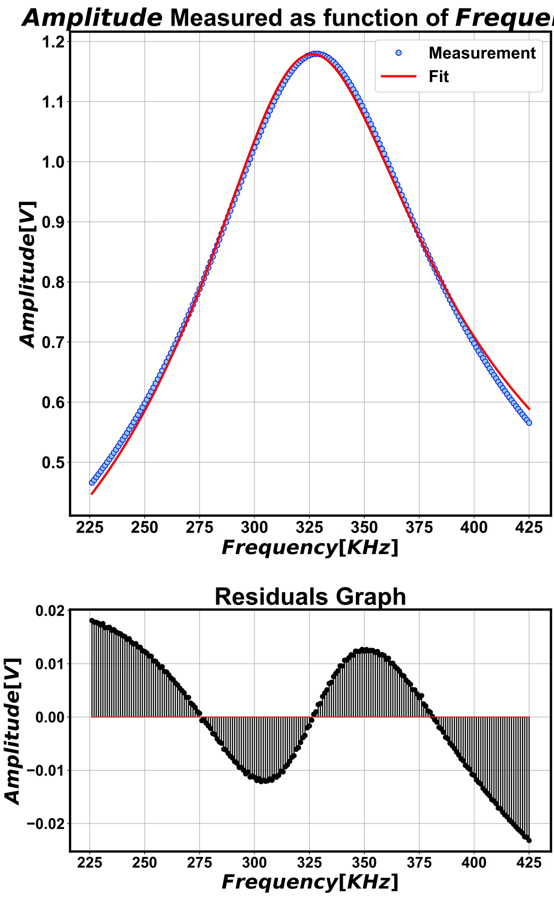


FIG. 7: The results of the matlab script displayed on a graph of the measured voltage $Amplitude$ as function of the generated alternating voltage wave $frequency$. The raw data is displayed in blue, the fit in red. Below, we can see the residuals plot which is not random, as we might expect. We hypothesize that happens because our fit is not optimal.

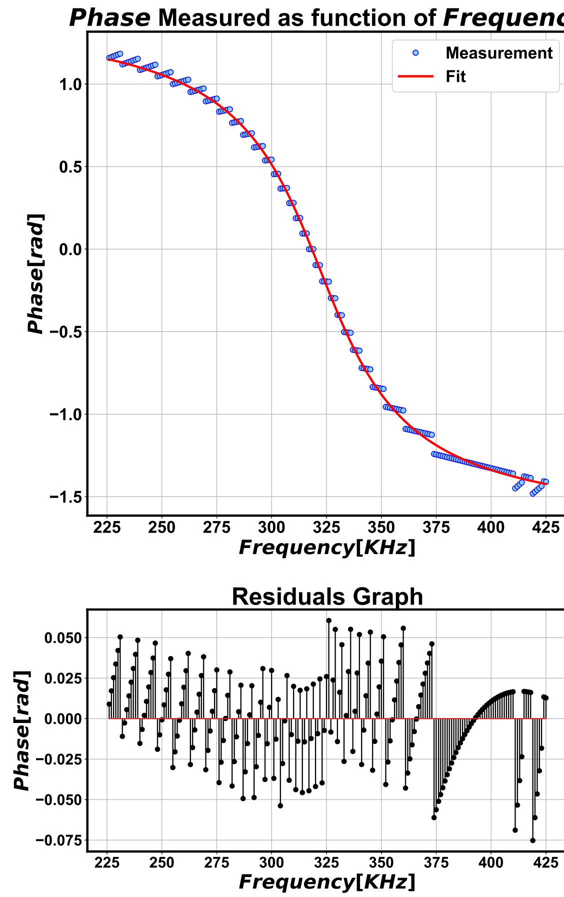


FIG. 8: The results of the Matlab script on a graph of the generated voltage phase on the resistor relative to the excitation voltage's phase $phase$ as a function of the generated alternating voltage wave $frequency$. The measurements are displayed in blue, and the fit is in red. Below, we can see the residuals plot which is not random, as we might expect. We hypothesize that it happens because we calculate the phase using discrete cross-correlation between the samples. Thus the phase is quantized (as you can see the measurements are grouped and not spaced along the y-axis) which makes it hard to fit.

- c - describes the ratio between R and the total resistance in the circuit. This parameter determines the height of the voltage at the resonance frequency.

The results we got are not quite similar to the expected values. We hypothesize that the gap results in the tolerance of the electrical components we used. All of them have a tolerance of 10%. That implies for example that the theoretical resonance frequency is $f_{res} = \frac{1}{2\pi\sqrt{LC}} = 290 \pm 30\ kHz$ while the measured resonance is $f_{res\ measured} = \frac{1}{\sqrt{ab}} = 326 \pm 0.6\ kHz$. In order to check this hypothesis one can conduct an experiment to check each component individually.

IV. COUPLED RLC CIRCUIT

We want to measure the voltage of a resistor in an RLC circuit coupled to another RLC circuit connected to a power supply as shown in Figure 9

We used two identical coupled RLC circuit with $R = 1000\ \Omega,\ L = 1\ mH$ and $C = 10^{-10}\ F$. The sweep signal has the same frequency change rate and the frequency range was $300\ kHz - 800\ kHz$. We measured the amplitude (A) and the phase (ϕ) of the transmission function

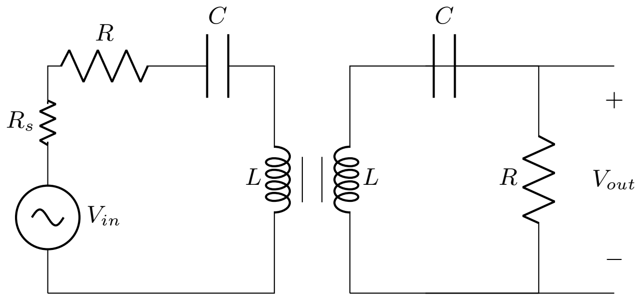


FIG. 9: A RLC circuit with voltage source V_{in}, internal resistance R_s, resistor R, capacitor C and inductor L. Another RLC circuit with a resistor R, an inductor L a capacitor C. Voltmeter V_{out} measures the resistor R. The circuits' inductors are coupled.

as a function of the power source frequency (f). This can be seen in figure 10 and figure 11 accordingly.

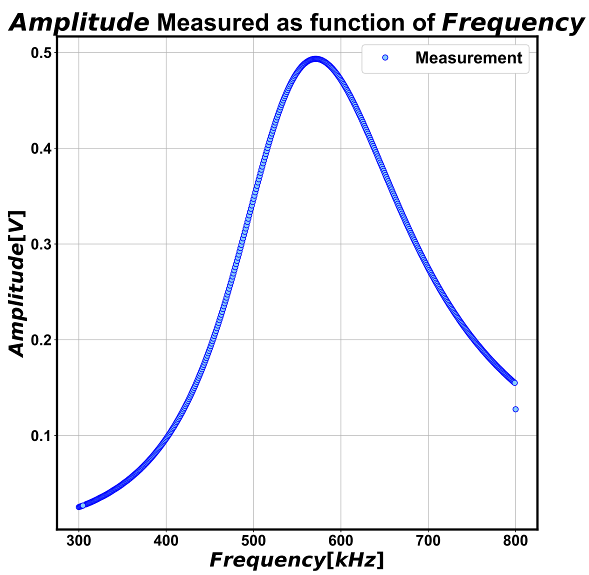


FIG. 10: The results of the Matlab script on a graph of the measured voltage amplitude as a function of the generated alternating voltage wave $frequency$. The measurements are displayed in blue.

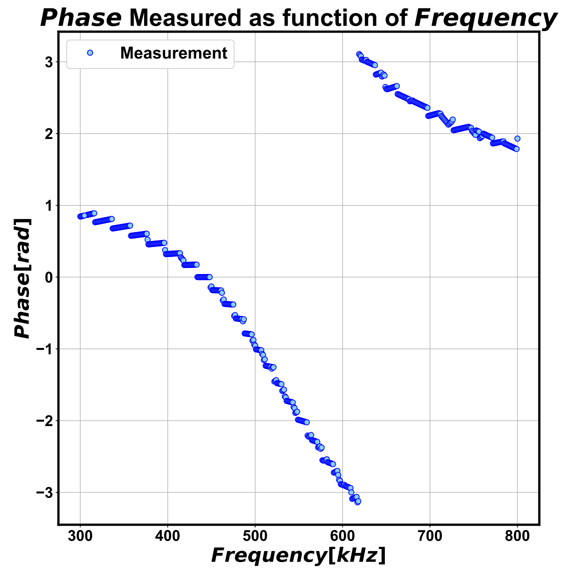


FIG. 11: The results of the Matlab script on a graph of the generated voltage phase on the resistor relative to the excitation voltage's phase $phase$ as a function of the generated alternating voltage wave $frequency$. The measurements are displayed in blue.

Part III
The Coil

V. INVESTIGATING THE COIL PROPERTIES

We are using two different coils with different sizes. The large one has $d = 22$ $[cm]$ and $N \approx 50$. The small one has $d = 30$ $[cm]$ and $N \approx 50$. In order to investigate the coil, we will plug different electrical components in line, generate a voltage in varying frequencies and measure the voltage on the component. We will perform experiments to examine the coil's model. Our broad model 12 has 5 fitting parameters and is chosen to be wide and expressive, but it would be hard to get an accurate fit. We will perform several fits on simplified models, And the model with the best fit will be chosen.

A. Course of The Experiment

We designed two reasonable models of coil impedance. In order to determine which model describes every coil the best we will plug and measure our model on the:

- Resistor connected in series to the coil.
- Capacitor connected in parallel to the coil and measuring on resistor connected in series.

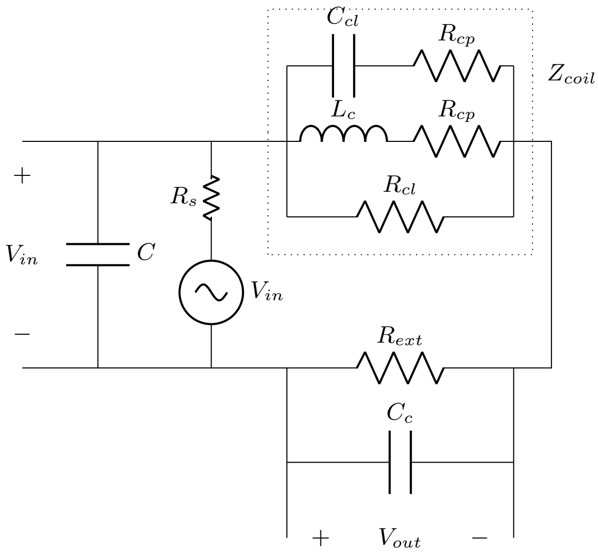


FIG. 12: Broad model, measureing on a resistor

We fit our measures by amplitude and phase according to each of the models and decide which one is more reasonable by the residuals graphs.

We plugged in line a resistor $R = 47$ $[k\Omega]$ and measured the voltage on it. Second measurement we plugged a capacitor $100[pF]$ in parallel and a resistor $R = 47$ $[k\Omega]$ in line and measured the voltage on the resistor.

1. *measuring on a resistor on the large coil (figures 13, 14)*

We will now measure the resistor connected to the large coil as modeled in 13 and 14. The measurements are displayed in 15 and 16.

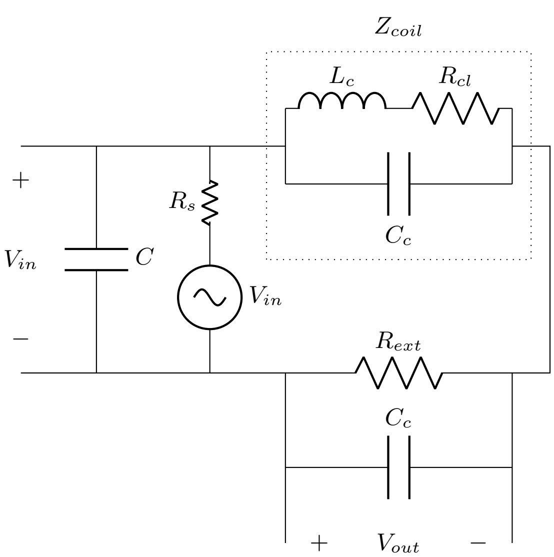


FIG. 13: First model, measuring on a resistor

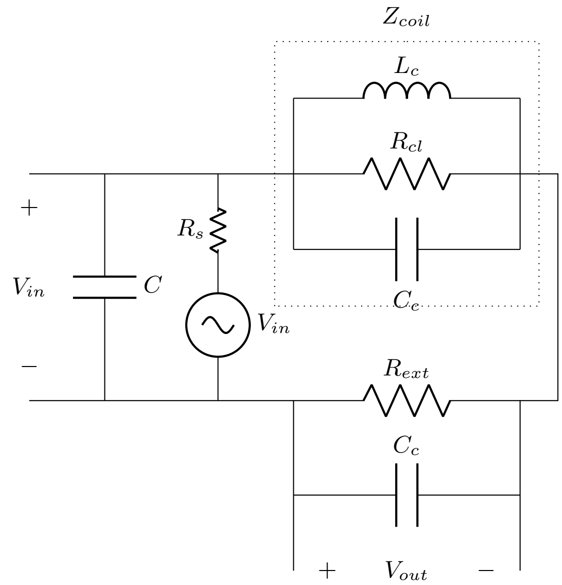


FIG. 14: Second model, measuring on a resistor

2. *measuring on a capacitor on the large coil (figures 17, 18)*

We will now measure on a resistor while connecting a capacitor in parallel into the large coil as modeled in 17 and 18. The measurements are displayed in 19 and 20.

B. Results

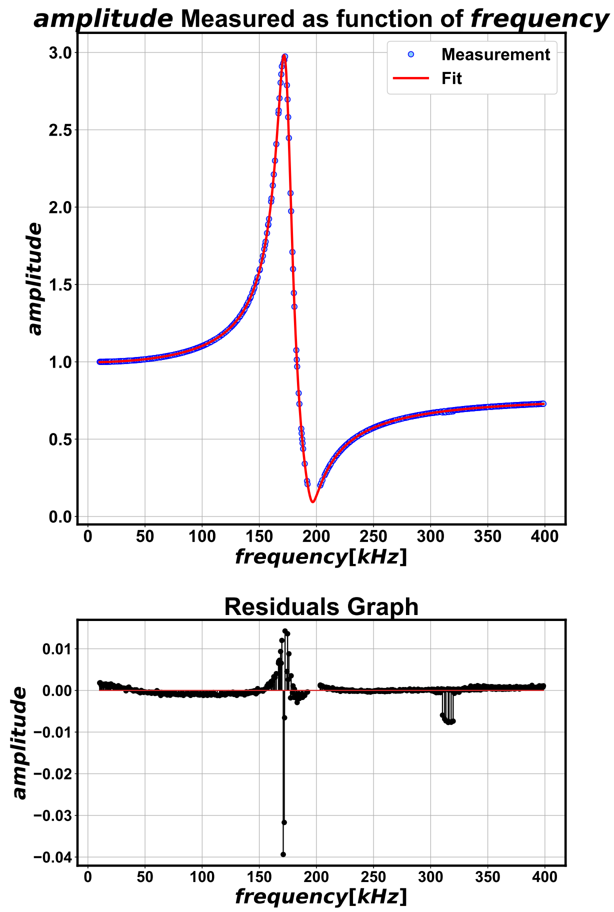
The results of the first model can be seen in the figures 15a, 16a, 19a and 20a and the results of the second model are in the figures 15b, 16b, 19b, 20b.

We will start using the resistor measurements. By comparing the residuals of fig 15a and fig 15b, they are smaller and much more random, we can see that model 1 is more accurate. The residuals of fig 16a and fig 16b are the same. The same is happening by comparing fig 19a and fig 19b and comparing 20a and fig 20b. We fed the fitting constants into tables:

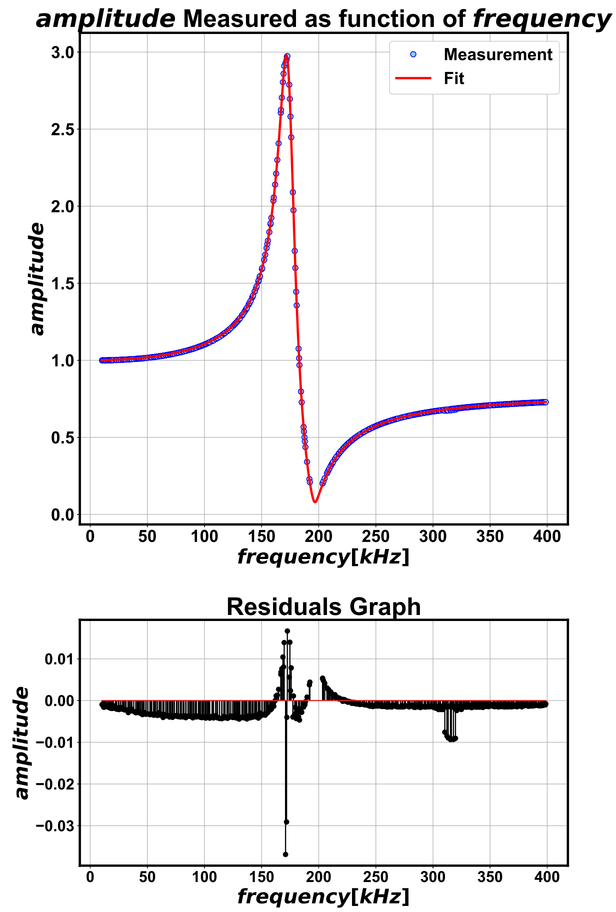
- Table II describes the parameters of the measurements on the resistor, in both models.
- Table III describes the parameters of the measurements on the capacitor, in both models.

This two tables support our selected model. Notice in model 1 in both measurements, on the resistor and the capacitor, we get greater agreement, between the values calculated from the amplitude and the phase, related to model 2. Thus we choose **model 1** to describe the big coil.

In order to verify that model 1 describes the coil good enough, we did the same measurements (connecting a resistor and a capacitor) again on the small coil and tried



(a) The results of the Matlab script on a graph of the generated voltage Amplitude on the external resistor (47 $[k\Omega]$) connected in series to the large coil as a function of the generated alternating voltage wave $frequency$ according to Model 1 (13). The measurements are displayed in blue.



(b) The results of the Matlab script on a graph of the generated voltage Amplitude on the external resistor (47 $[k\Omega]$) connected in series to the large coil as a function of the generated alternating voltage wave $frequency$ according to Model 2 (14). The measurements are displayed in blue.

FIG. 15: Measurements of the amplitude on a large coil with a resistor connected

to fit them only to the first model. We got 22a and 22b. The residuals are not much random but small enough so we deduce that the model fits the small coil as well.

C. Additional Results

While fitting, we used L_c, C_c, R_{cl}, C_s as free parameters. The first three parameters describe the coil and we want to find them (the purpose of this part of the experiment) whereas C_s was measured before. However, we suspect that this is not accurate and when we tried to fix it, we failed to get a reasonable fit. Table IV describes the physical parameters of the small coil.

D. Result Analysis

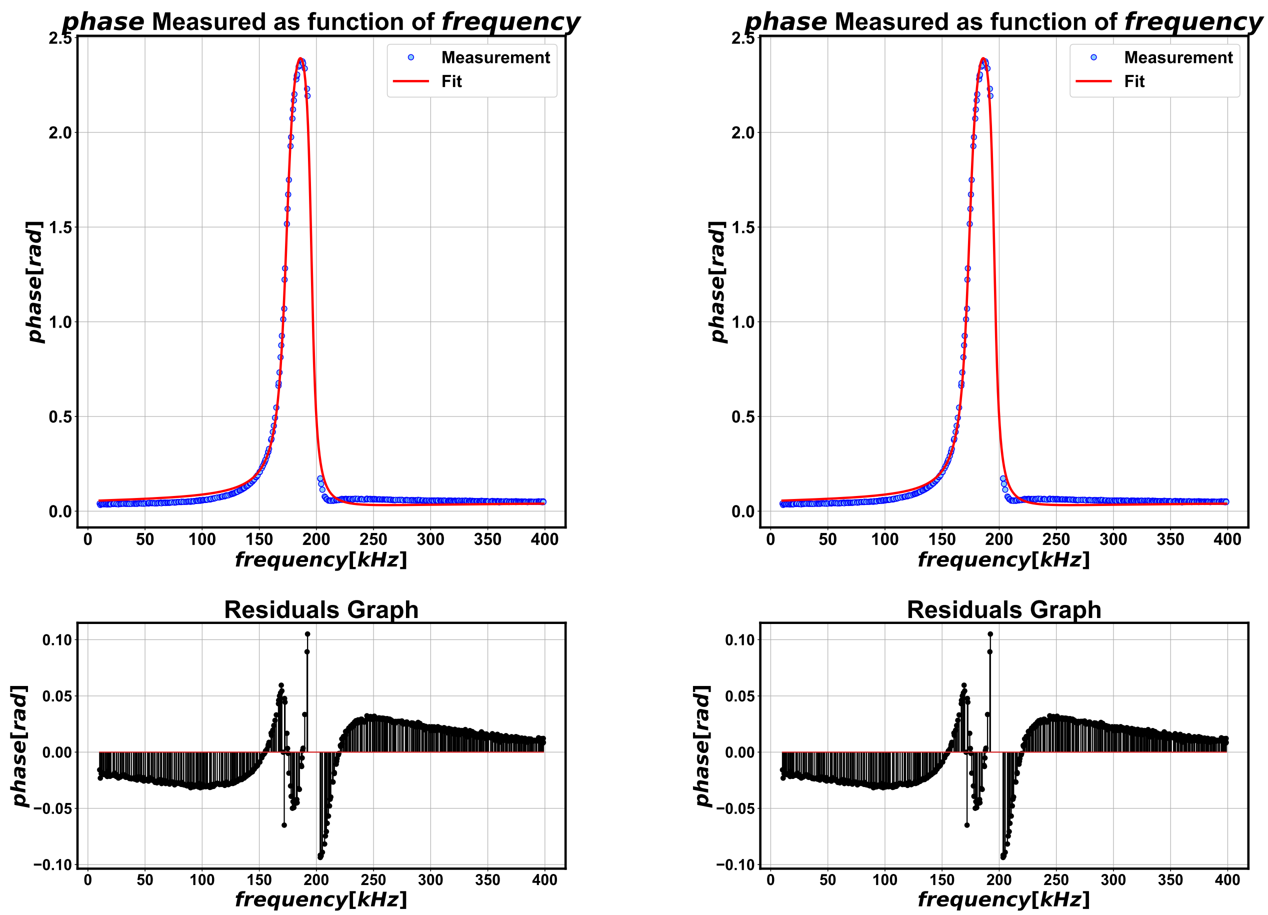
Now we can calculate the physical parameters by taking the average result and the error will be its std.

The large coil's physical parameters are:

$$\boxed{L_{c,l} = 1.6 \pm 0.6 \; [mH]} \tag{14}$$

$$\boxed{R_{cl,l} = 57 \pm 10 \; [\Omega]} \tag{15}$$

$$\boxed{C_{c,l} = 342 \pm 100 \; [pF]} \tag{16}$$



(a) The results of the Matlab script on a graph of the generated voltage phase on the external resistor (47 $[k\Omega]$) connected in series to the large coil relative to the excitation voltage's *phase* as a function of the generated alternating voltage wave *frequency* according to Model 1 (13). The measurements are displayed in blue.

(b) The results of the Matlab script on a graph of the generated voltage phase on the external resistor (47 $[k\Omega]$) connected in series to the large coil relative to the excitation voltage's *phase* as a function of the generated alternating voltage wave *frequency* according to Model 2 (13). The measurements are displayed in blue.

FIG. 16: Measurements of the phase on a large coil with a resistor connected

TABLE II: Fitting parameters from the amplitude and phase fitting from measuring on resistor added to the large coil using different models.

	units	A resistor (model 1)	ϕ resistor (model 1)	A resistor (model 2)	ϕ resistor (model 2)
L_c	mH	1.97 ± 0.03	1.23 ± 0.03	2.43 ± 0.02	0.90 ± 0.07
C_c	pF	333 ± 5	537.2 ± 14.6	269.7 ± 3.2	721 ± 56.9
R_{cl}	Ω	67.3 ± 0.4	52.9 ± 1.1	12780 ± 5811	46.27 ± 3.06
C_s	pF	93.8 ± 1.3	142.7 ± 4.04	75.4 ± 0.9	208.3 ± 16.2

The small coil's physical parameters are:

$$\boxed{L_{c,s} = 0.91 \pm 0.4\ [mH]} \tag{17}$$

$$\boxed{R_{cl,s} = 70 \pm 34\ [\Omega]} \tag{18}$$

$$\boxed{C_{c,s} = 206 \pm 100\ [pF]} \tag{19}$$

Now we can compare our result to the expected values.The theoretical value of the inductance of a coil with thesame radius (r), number of loops (N), and width d ¡¡ r is given by:

TABLE III: Fitting parameters from the amplitude and phase fitting from measuring on capacitor added to the large coil using different models.

	units	A capacitor (model 1)	ϕ capacitor (model 1)	A capacitor (model 2)	ϕ capacitor (model 2)
L_c	mH	2.07 ± 0.03	1.02 ± 0.05	2.33 ± 0.01	8.4 ± 0.4
C_c	pF	315.8 ± 4.6	641.8 ± 37.3	258.9 ± 1.9	$(4.4 \pm 0.05) \times 10^{-15}$
R_{cl}	Ω	65.5 ± 0.6	42.4 ± 2.4	10760 ± 1958	239100 ± 15660
C_s	pF	88.9 ± 1.2	184.9 ± 10.5	78.2 ± 0.4	20.6 ± 1.1

TABLE IV: Fitting parameters from the amplitude and phase fitting from measuring on the small coil using different electrical components.

	units	A resistor	ϕ resistor	A capacitor	ϕ capacitor
L_c	mH	1.37 ± 0.02	0.69 ± 0.05	0.673 ± 0.001	2.7 ± 0.4
C_c	pF	106 ± 2	207 ± 20	306.4 ± 1.1	$4.83 \times 10^{-10} \pm 15.83$
R_{cl}	Ω	82 ± 4	97 ± 7	30.69 ± 0.06	208.6 ± 30.1
C_s	pF	90 ± 2	181 ± 20	184.3 ± 0.5	44.6 ± 7.05

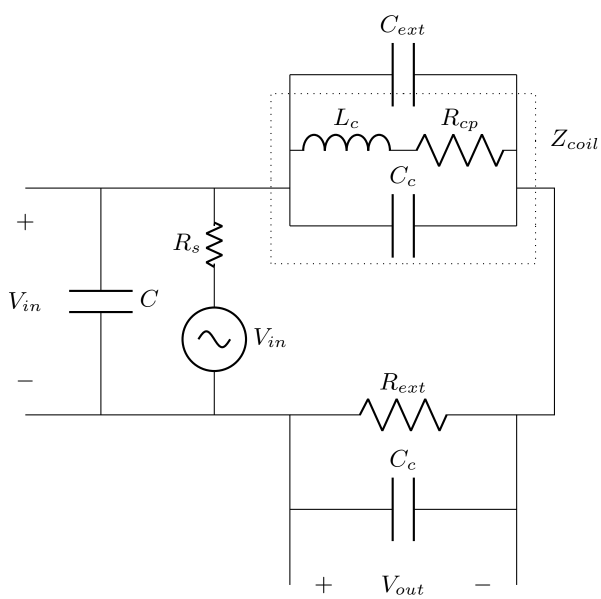


FIG. 17: First model, measuring on a resistor

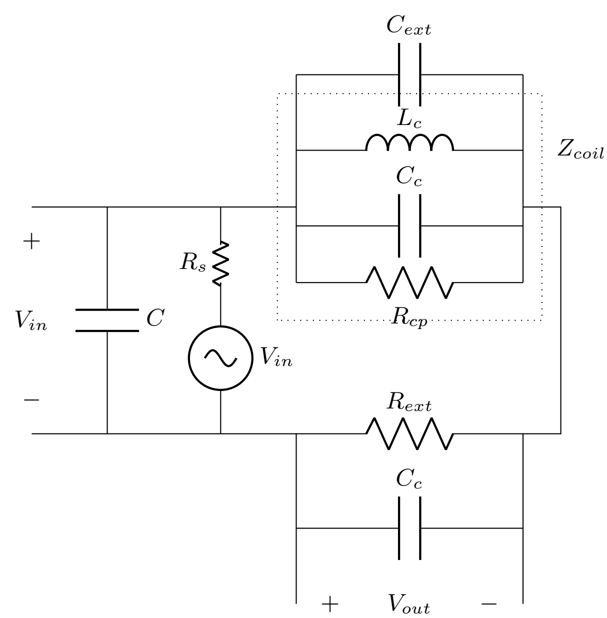


FIG. 18: First model, measuring on a capacitor

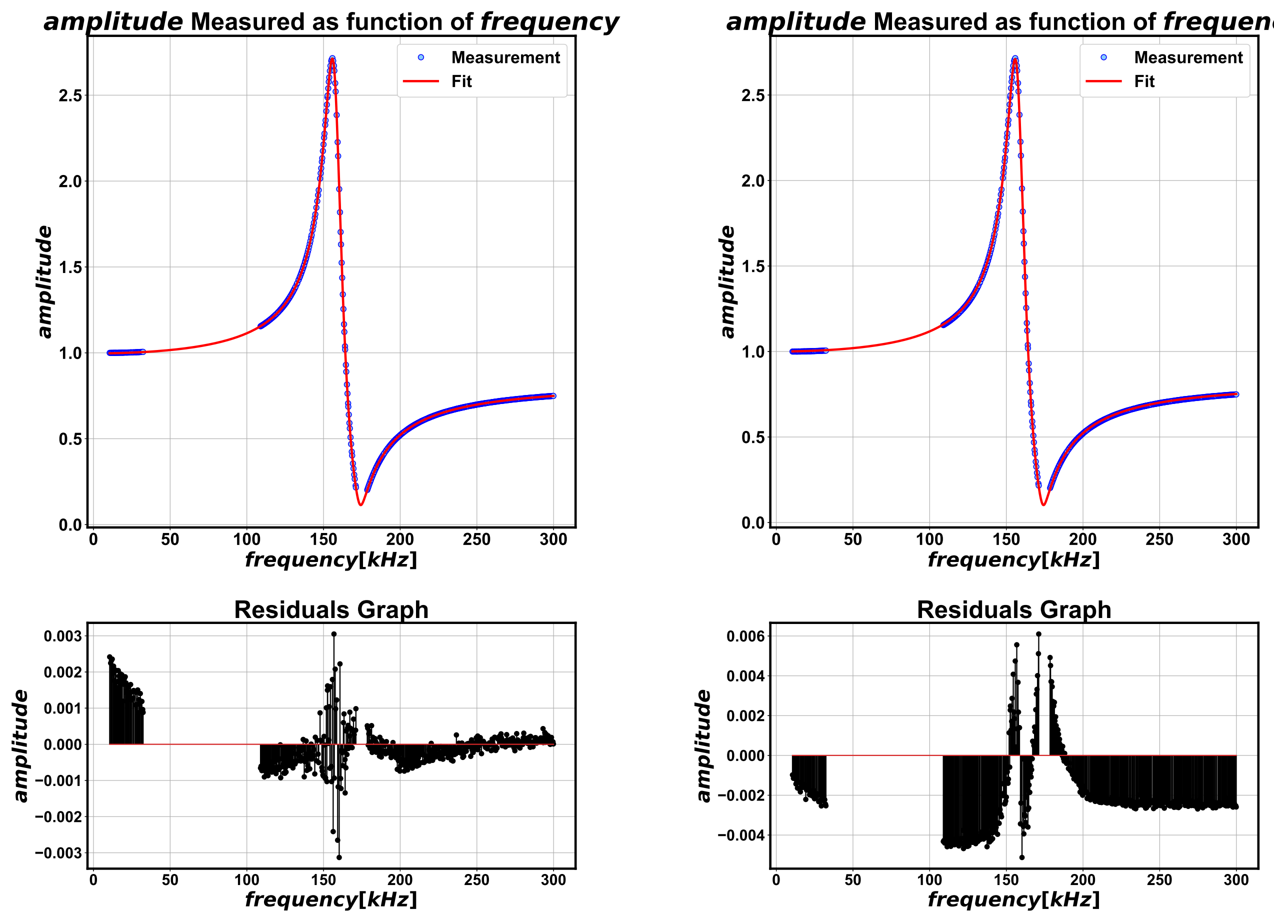
$$L_{coil\ theoretical} = \mu_0 \frac{N^2 A}{2r} = \mu_0 \frac{N^2 \pi r}{2} \tag{20}$$

For the large coil ($R_s \approx 11$ $[cm]$) and the small coil ($R_l \approx 15$ $[cm]$) with $N \approx 50$ and we get:

$$L_{small\ coil\ theoretical} \approx 1.08\ [mH] \tag{21}$$

$$L_{large\ coil\ theoretical} \approx 1.48\ [mH] \tag{22}$$

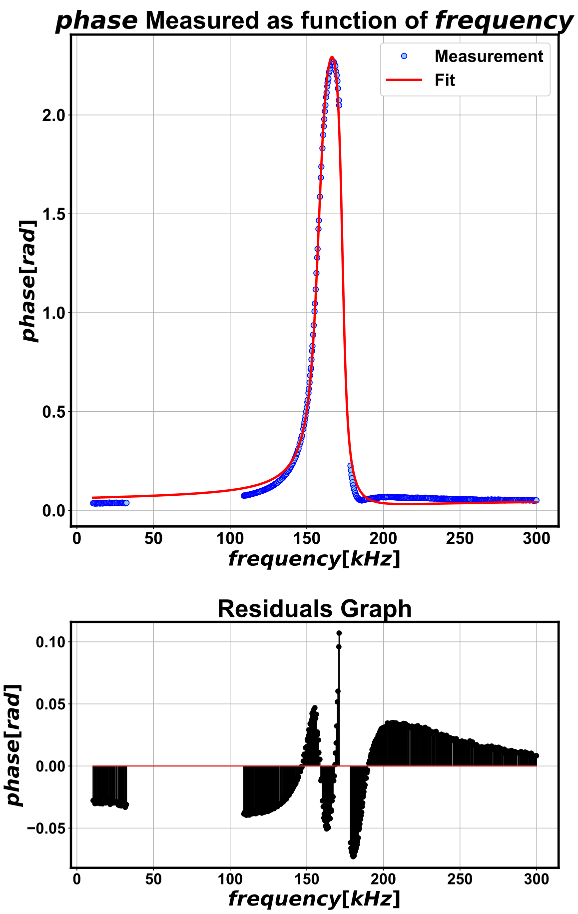
The theoretical values are of the same magnitude of order as our results. More importantly, they are 1 std away from our prediction. This shows we're able to measure it correctly.



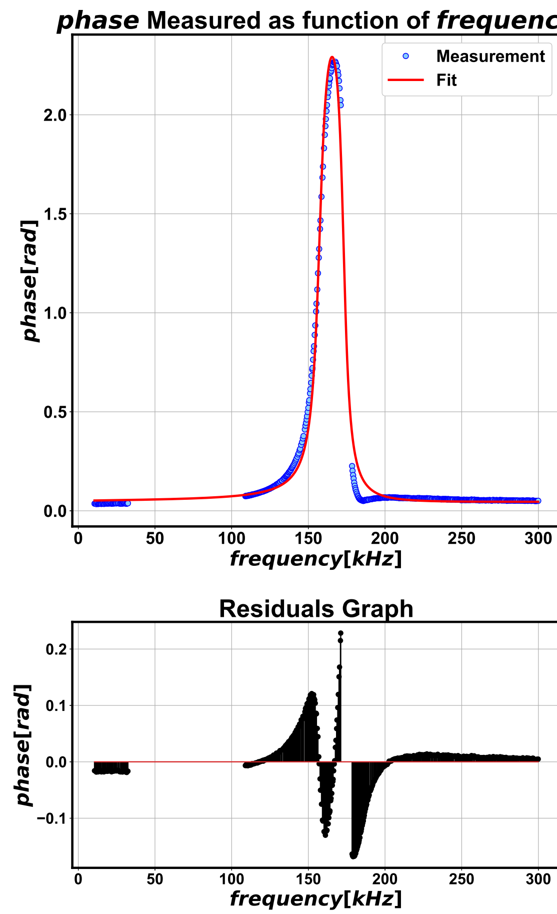
(a) The results of the Matlab script on a graph of the generated voltage Amplitude on the external resistor ($47\ [k\Omega]$) and capacitor ($1 \cdot 10^{-10}\ [F]$) connected in series to the large coil as a function of the generated alternating voltage wave $frequency$ according to Model 1 (17). The measurements are displayed in blue.

(b) The results of the Matlab script on a graph of the generated voltage Amplitude on the external resistor ($47\ [k\Omega]$) and capacitor ($1 \cdot 10^{-10}\ [F]$) connected in series to the large coil as a function of the generated alternating voltage wave $frequency$ according to Model 2 (18). The measurements are displayed in blue.

FIG. 19: Measurements of the amplitude on a large coil with a capacitor connected

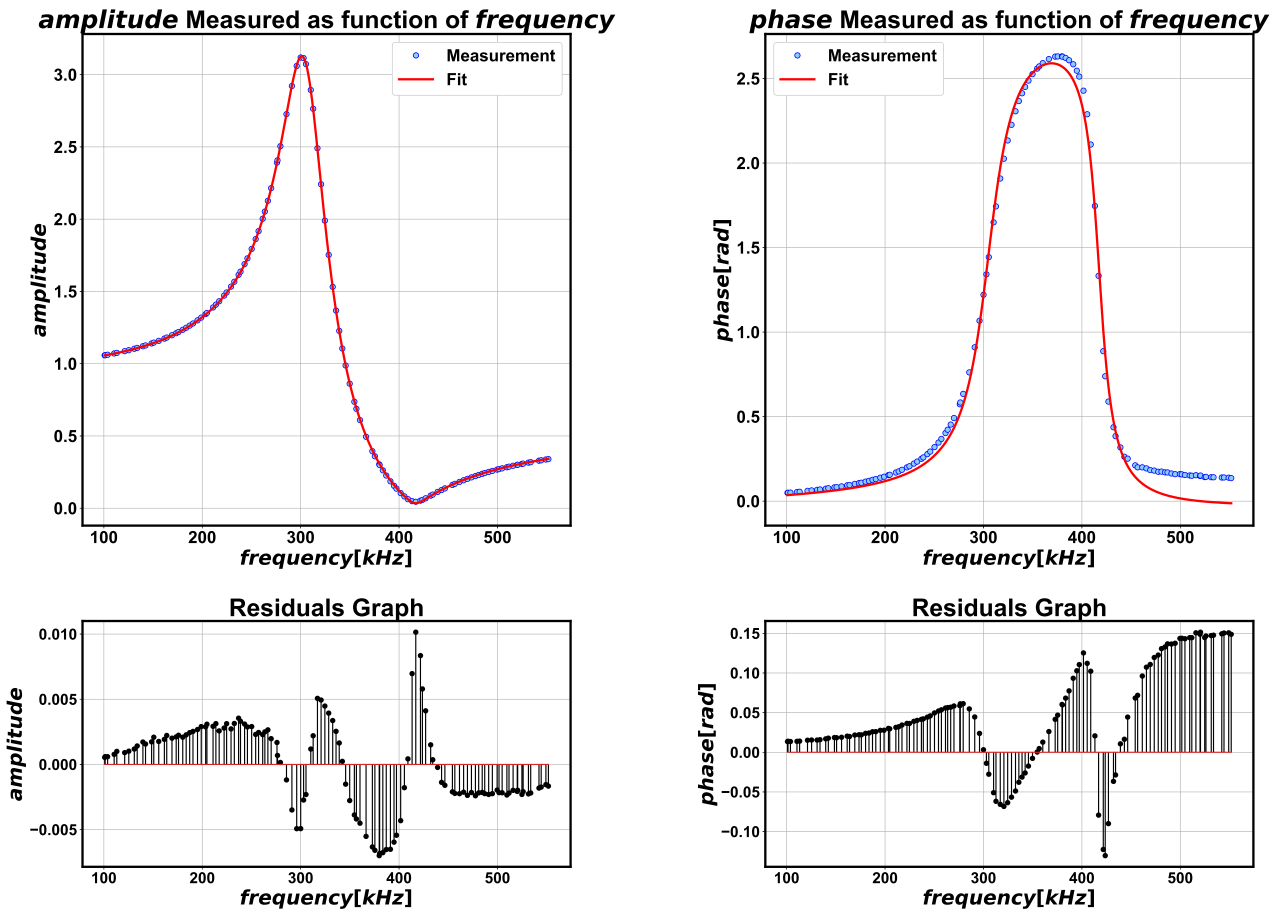


(a) The results of the Matlab script on a graph of the generated voltage phase on the external resistor (47 $[k\Omega]$) and capacitor ($1 \cdot 10^{-10}$ $[F]$) connected in series to the large coil as a function of the generated alternating voltage wave $frequency$ according to Model 1 (17). The measurements are displayed in blue.



(b) The results of the Matlab script on a graph of the generated voltage phase on the external resistor (47 $[k\Omega]$) and capacitor ($1 \cdot 10^{-10}$ $[F]$) connected in series to the large coil as a function of the generated alternating voltage wave $frequency$ according to Model 2 (18). The measurements are displayed in blue.

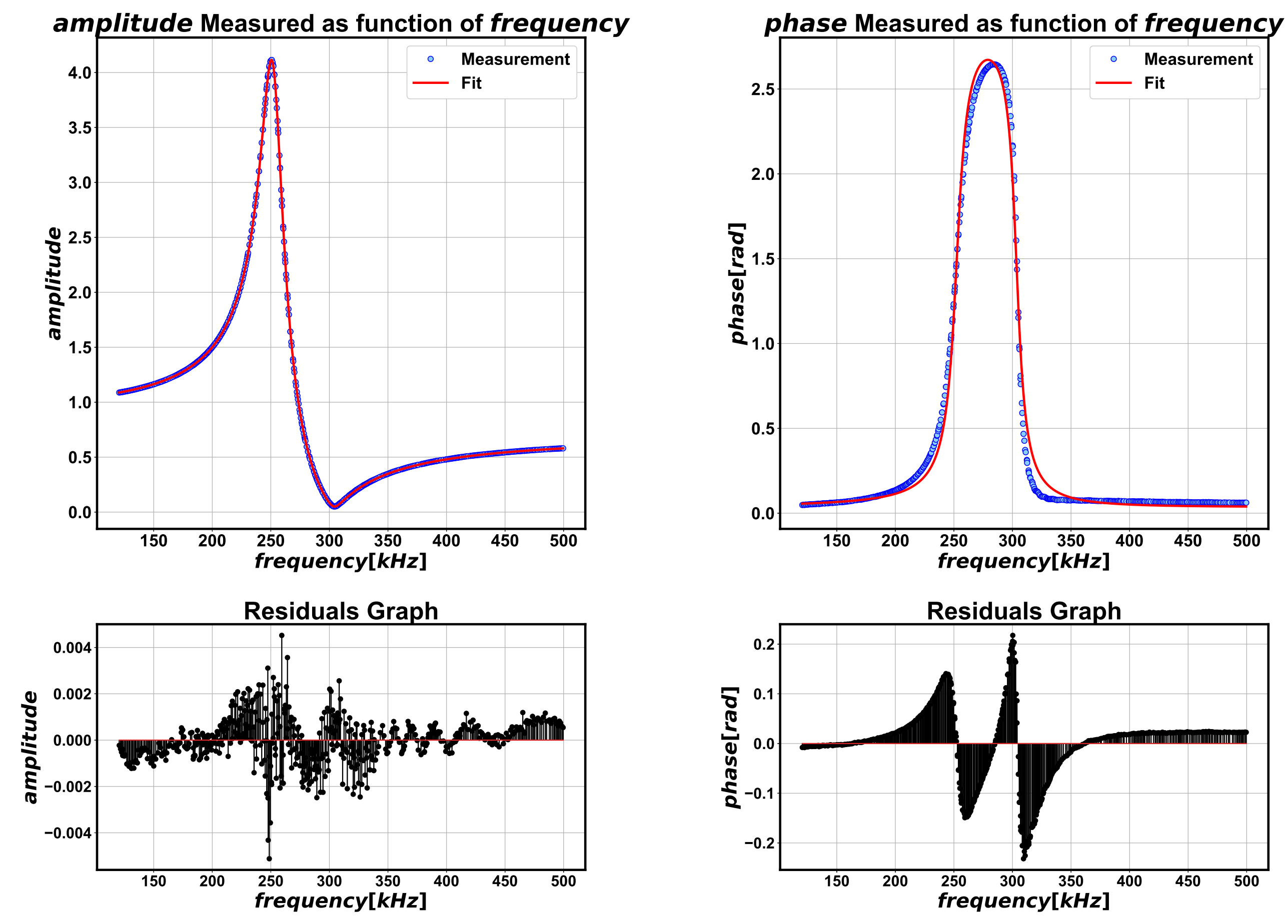
FIG. 20: Measurements of the phase on a large coil with a capacitor connected



(a) The results of the Matlab script on a graph of the generated voltage Amplitude on the external resistor (22 $[k\Omega]$) connected in series to the small coil as a function of the generated alternating voltage wave $frequency$ according to 13. The measurements are displayed in blue.

(b) The results of the Matlab script on a graph of the generated voltage phase on the external resistor (22 $[k\Omega]$) connected in series to the small coil voltage's $phase$ as a function of the generated voltage wave $frequency$ according to Model 1. The measurements are displayed in blue.

FIG. 21: Measurements on the small coil with a resistor connected



(a) The results of the Matlab script on a graph of the generated voltage Amplitude on the external resistor (47 $[k\Omega]$) and capacitor ($1 \cdot 10^{-10}$ $[F]$) connected in series to the small coil as a function of the generated alternating voltage wave $frequency$ according to 17. The measurements are displayed in blue.

(b) The results of the Matlab script on a graph of the generated voltage phase on the external resistor (47 $[k\Omega]$) and capacitor ($1 \cdot 10^{-10}$ $[F]$) connected in series to the small coil as a function of the generated alternating voltage wave $frequency$ according to Model 1. The measurements are displayed in blue.

FIG. 22: Measurements on the small coil with a capacitor connected

Part IV
Coupled Coils

VI. INVESTIGATING THE MUTUAL INDUCTANCE OF THE COILS

After finding the coil's model and physical parameters, we will examine the behavior of two mutually inducted coils as shown in 23.

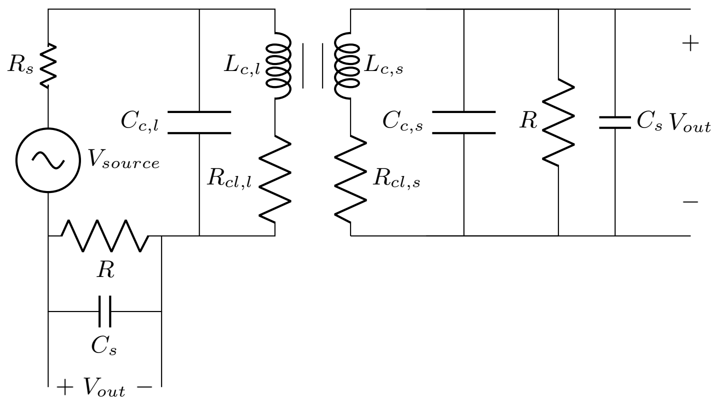


FIG. 23: Coupled coils model

We modeled the mutual inductance as an additional inductor, next to the real ones, with inductance M (by the reciprocity mutual inductance theorem $M_{1,2} = M_{2,1} = M$) and its current is that of the other inductor. The mutual inductance voltage is expressed as follows 23, 24.

$$V_{M,s} = i\omega M I_{L_{c,l}} \tag{23}$$

$$V_{M,l} = i\omega M I_{L_{c,s}} \tag{24}$$

A. Course of The Experiment

In our experiment, we will generate a voltage in the large coil and measure the voltage on the small coil. We put the small coil inside the large coil and align their centers in order to maximize the phenomenon of mutual inductance. The transmission function will be as described in 25.

$$H(\omega) = \frac{V_{out,\ r}}{V_{out,\ t}} = \frac{V_{R,s}}{V_{R,l}} \tag{25}$$

We used Kirchhoff's laws and got 8 linear equations of the currents in the different circuit's components. We created a Mathematica script in order to extract the theoretical relation $H(\omega)$ (it is too long and not informative) and fitted $|H(\omega)|$, $\arg(H(\omega))$ to our results.

$$\begin{cases} \frac{R_{ext}I_1+I_4}{I\omega C_1} = 0 \\ \frac{I_4}{I\omega C_1} = R_1 I_3 + I\omega L_1 I_3 - I\omega M I_5 \\ \frac{R_{ext}I_2+I_6}{I\omega C_2} + R_s(I_2+I_8) = V_0 \\ \frac{I_6}{I\omega C_1} = R_2 I_5 + I\omega L_2 I_5 - I\omega M I_3 \\ I_1 + I_7 = I_3 + I_4 \\ I_2 + I_8 = I_5 + I_6 \\ \frac{I_7}{I\omega C_s} = R_{ext} I_1 \\ \frac{I_8}{I\omega C_s} = R_{ext} I_2 \end{cases} \tag{26}$$

B. Results

The results of the experiment are shown in 24a and 24b.

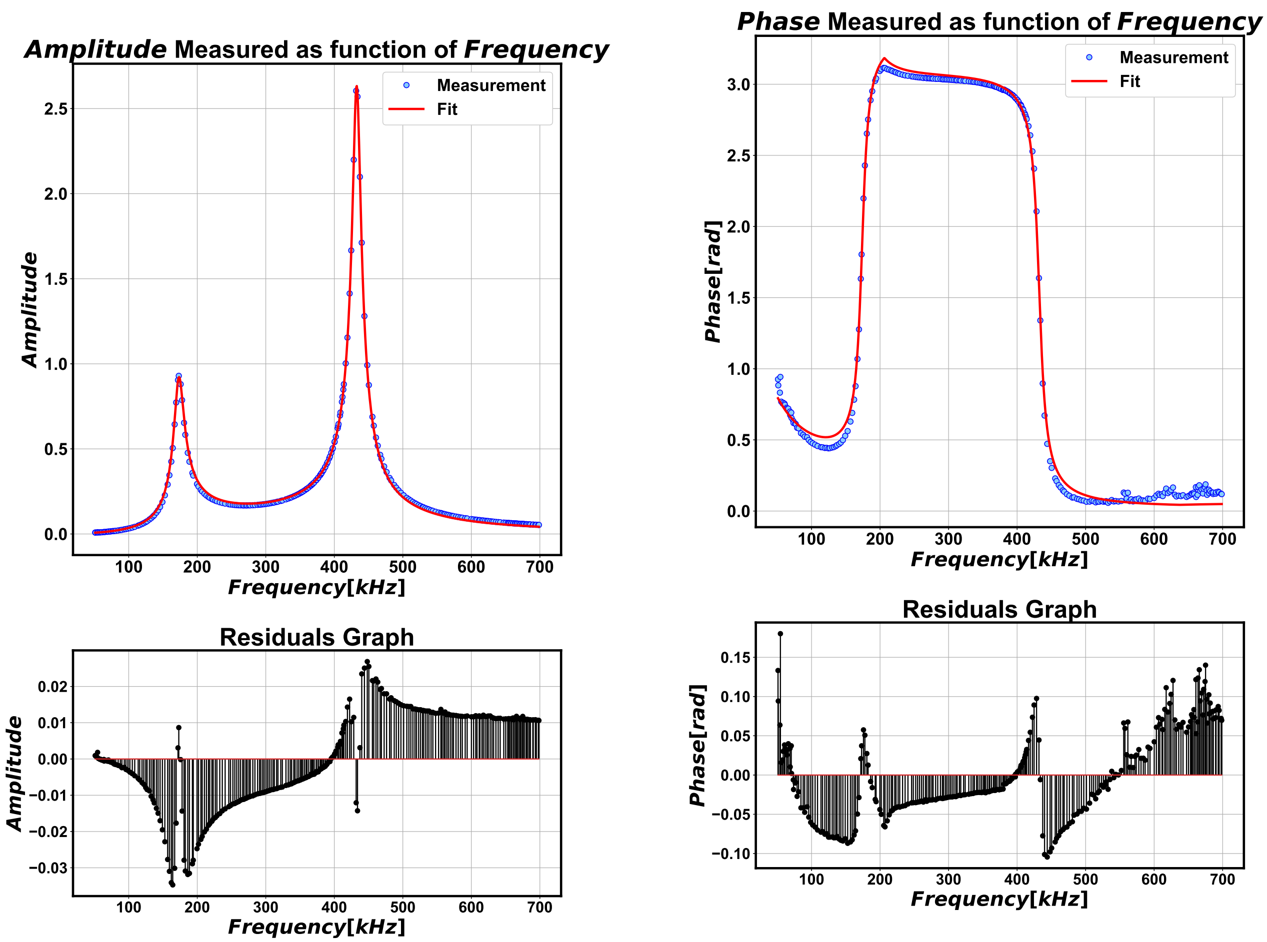
Now we can extract the mutual inductance

$$\boxed{M = 0.50 \pm 0.02\ [mH]} \tag{27}$$

The fitting parameters are described in table V. In order to fit the amplitude graph we free the next variables between and put their std as the bounds: L_s, L_b, C_b, C_s, R_s, R_b. We got different values from what we measured while characterizing the coils (although they are one std away from them we have large errors in those values). While fitting we noticed that the fit values of $L_{s\ coil}$ and $L_{b\ coil}$ are similar for each d. Thus we fixed them on this values. That improved tremendously the accuracy of M. From now on, our new characterization of the coils will be as we achieved in this part.

C. Result Analysis

Now we will compare our result to the theory. Using Biot–Savart law we managed to get an expression for the z component of the magnetic field being produced by the large coil in space. Then, we integrated it over the surface of the small coil in order to get Φ_B. Φ_B is linearly dependent on I. Thus the ratio of the time derivatives is actually the ratio itself. Putting it all together we get equation 28.



(a) The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil and a supply source. The measurements are displayed in blue. The fit is displayed in red.

(b) The results of the Matlab script on a graph of the phase between the voltage measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and the voltage measured on identical resistor connected to the small coil and a supply source. The measurements are displayed in blue. The fit is displayed in red

FIG. 24: Measurements on the coupled coils

$$M_{d,\,theoretical} = \frac{\Phi_B}{I} = \frac{\mu_0 N^2 R_l}{2} \int_0^{\frac{R_s}{R_l}} \int_0^{2\pi} \frac{1 - s\cos t}{(1 + s^2 + \frac{d}{R_l}^2 - 2s\cos t)^{\frac{3}{2}}} dt \cdot s\, ds \tag{28}$$

After simplification, the integral is actually an elliptic integral that can't be solved analytically. So, we substituted N, R_l, R_s and calculated it numerically. For fixing $d = 0$ (the coils are concentric), the radius of the large coil ($R_s \approx 11$ $[cm]$) and the radius of the small coil ($R_l \approx 15$ $[cm]$) with $N \approx 50$ and we got 30.

$$M_{theory} \approx 0.52\ [mH] \tag{29}$$

The theoretical values are of the same magnitude of order as our results and less than one std away from our prediction.

VII. DISTANT COUPLED COILS

A. Course of The Experiment

We extracted the mutual inductance in order to create a communication device. However, if we want the com-

munication device to be useful for communication, the coils shouldn't be that close to each other. If we enlarge the distance between them, the magnetic field they create inside the loops is smaller. Thus the magnetic flux being created by the other coil is also smaller which results in smaller M. In this part, we will verify that hypothesis.
In the previous experiment, we modeled the mutual inductance of two concentric coils. Now, we will elevate the outer coil by 5 $[cm]$, 11 $[cm]$, 14 $[cm]$, 20 $[cm]$, 24 $[cm]$ and perform the same experiment as in section VI.

B. Results

The results of the experiment are shown in 28, 29, 30, 31, 32. The fitting parameters are described in table V.
We did the fit the same as we did the fit for $d = 0$ in the previous section.

C. Results Analysis

The measurements of M are shown in table VI.

TABLE VI: The mutual inductance M of the large and small coil on different d values.

d $[cm]$	M $[mH]$	$M_{theoretical}$ $[mH]$
0 ± 0.1	0.50 ± 0.02	0.52
5 ± 0.1	0.33 ± 0.02	0.36
11 ± 0.1	0.184 ± 0.002	0.18
14 ± 0.1	0.134 ± 0.002	0.13
20 ± 0.1	0.072 ± 0.002	0.073
24 ± 0.1	0.051 ± 0.002	0.051

We did the same as we did in the previous section but fixed d. For substituting the experiment's d values, we got the right column in table VI. The theoretical values are of the same magnitude of order as our results and are less than one std away from our prediction. We can do better and plot our results on top of the theoretical values of M. For $0 \leq d \leq 30$ $[cm]$ we got figure 25.

As we expected, the coupling coefficient decreases as d increases and the measures fit the theory.
While fitting we notice that as M increases the distance between the peaks of the amplitude of the transmission function are more separated. In order to check this observation we created figure 26.

This fact is important for the development of the communication device. We want the data to be received with low SNR. Thus we need to use the frequencies where the transmission function is at maximum. The two peaks are perfect for this purpose if they are separated enough. We can conclude from this section that as the coils are more distant, the mutual inductance is larger and the peaks of the transmission function are less separated which will decrease the SNR of the received data.

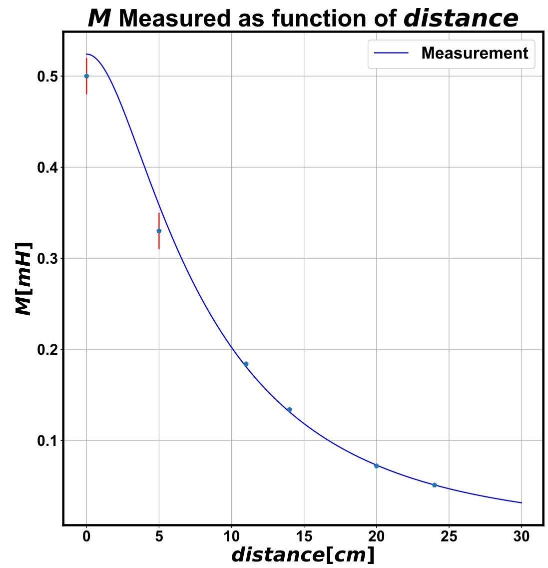


FIG. 25: The results of the numerical integration of the theoretical M. The mutual inductance M as a function of the distance between the coils.

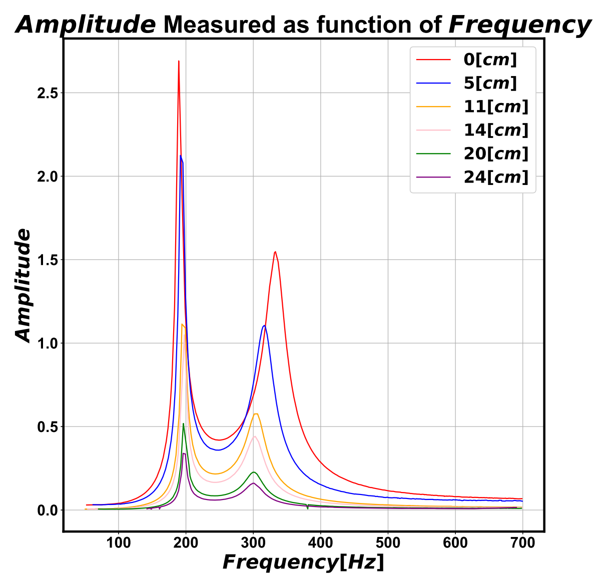
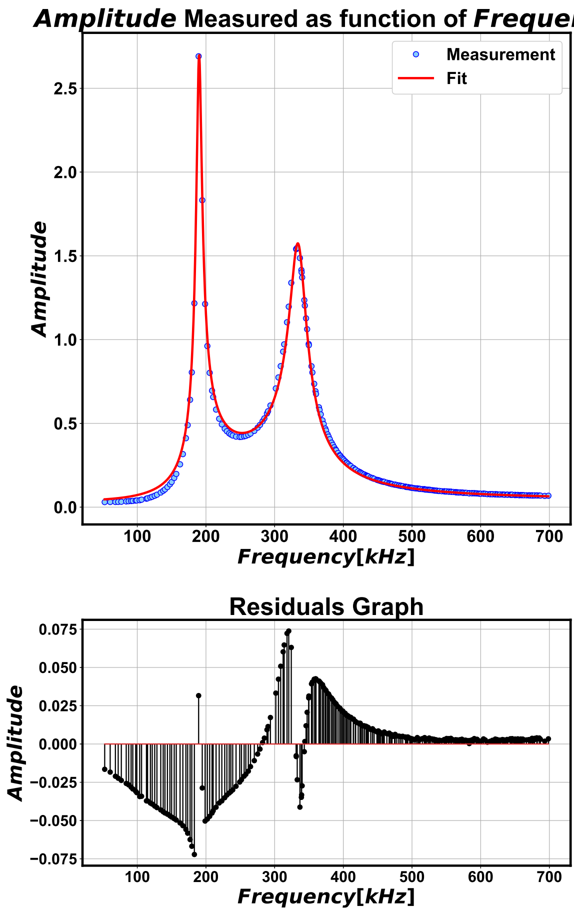
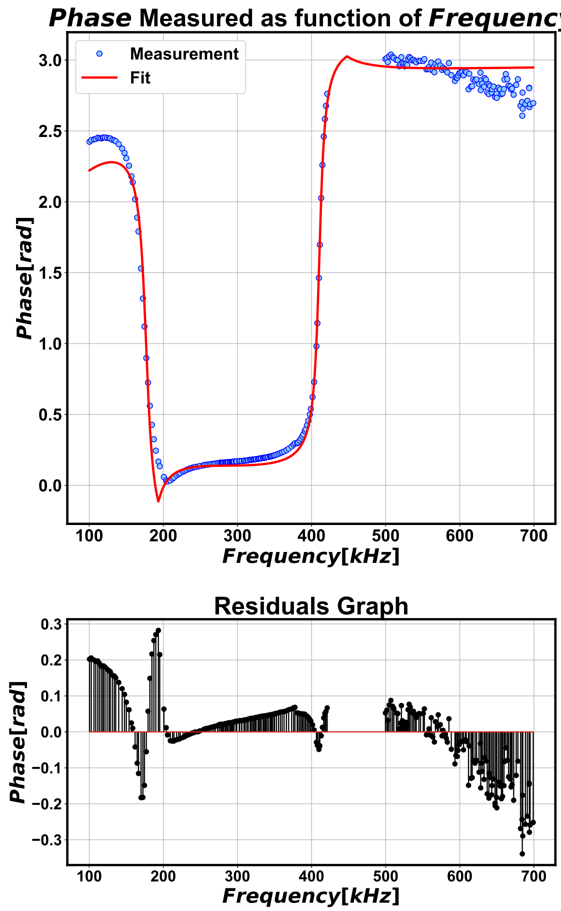


FIG. 26: Comparison of the 6 measurements with different distances: ADD REF. Measurements are performed exactly like previous measures 24 and 27

.



(a) The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with distance of 0 $[cm]$. The measurements are displayed in blue. The fit is displayed in red.



(b) The results of the Matlab script on a graph of the ratio of the voltage phase measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with distance of 0 $[cm]$. The measurements are displayed in blue. The fit is displayed in red.

FIG. 27: Measurements on the coupled-distant coils

TABLE V: The fitting parameters of measuring the mutual inductance M on different values of d as described in figure 23.

d $[cm]$	M $[mH]$	C_s $[pF]$	C_1 $[pF]$	L_1 $[mH]$	L_2 $[mH]$	R_1 $[\Omega]$	R_2 $[\Omega]$
0 ± 0.1	0.50 ± 0.02	105 ± 2	340 ± 1	0.74	1.76	57 ± 2	85 ± 3
5 ± 0.1	0.33 ± 0.02	107 ± 1	350 ± 1	0.65	1.80	61 ± 2	75 ± 2
11 ± 0.1	0.184 ± 0.002	107 ± 1	359 ± 1	0.62	1.79	64 ± 2	77 ± 2
14 ± 0.1	0.134 ± 0.002	95 ± 1	358 ± 1	0.62	1.80	59 ± 2	70 ± 2
20 ± 0.1	0.072 ± 0.002	88 ± 2	361 ± 1	0.62	1.79	59 ± 2	67 ± 3
24 ± 0.1	0.051 ± 0.002	93 ± 2	360 ± 1	0.61	1.80	59 ± 6	78 ± 4

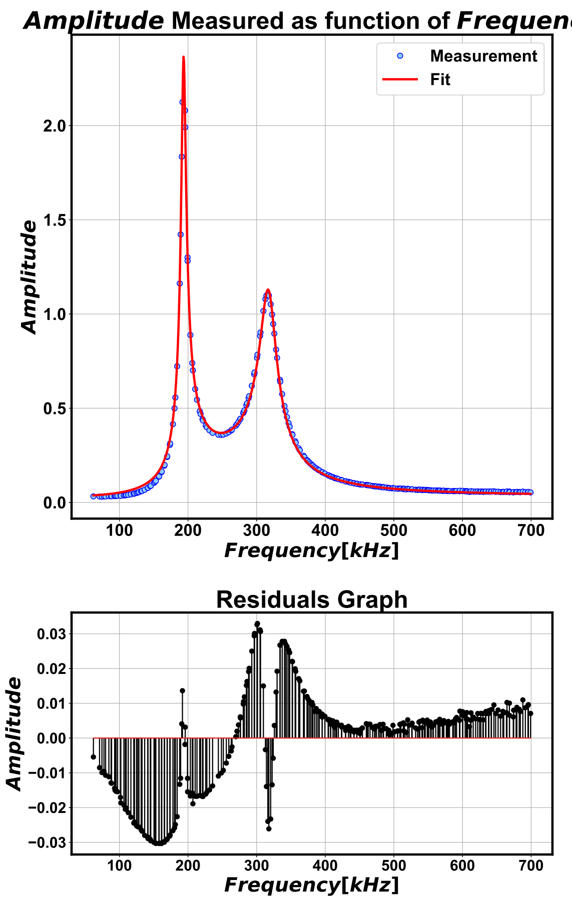


FIG. 28: The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with a distance of $d = 5$ $[cm]$. The measurements are displayed in blue. The fit is displayed in red.

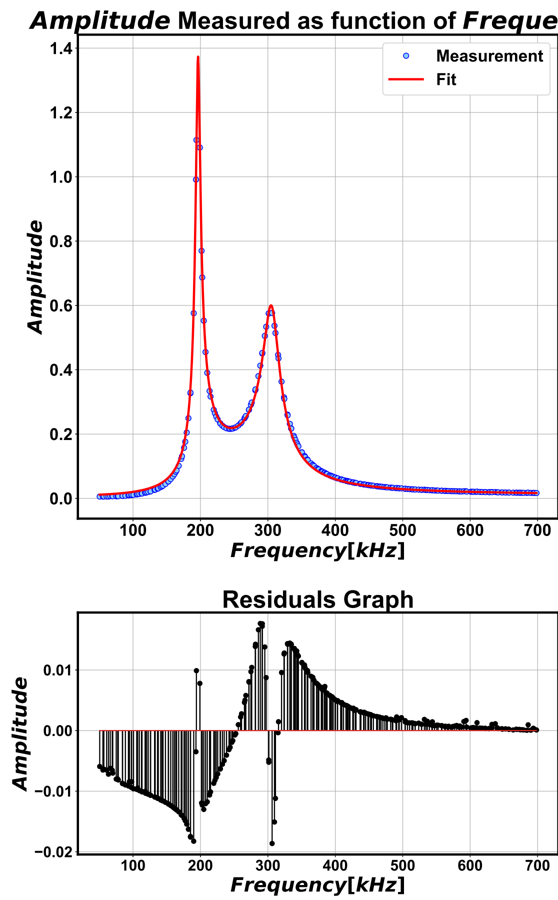


FIG. 29: The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with distance of $d = 11$ $[cm]$. The measurements are displayed in blue. The fit is displayed in red.

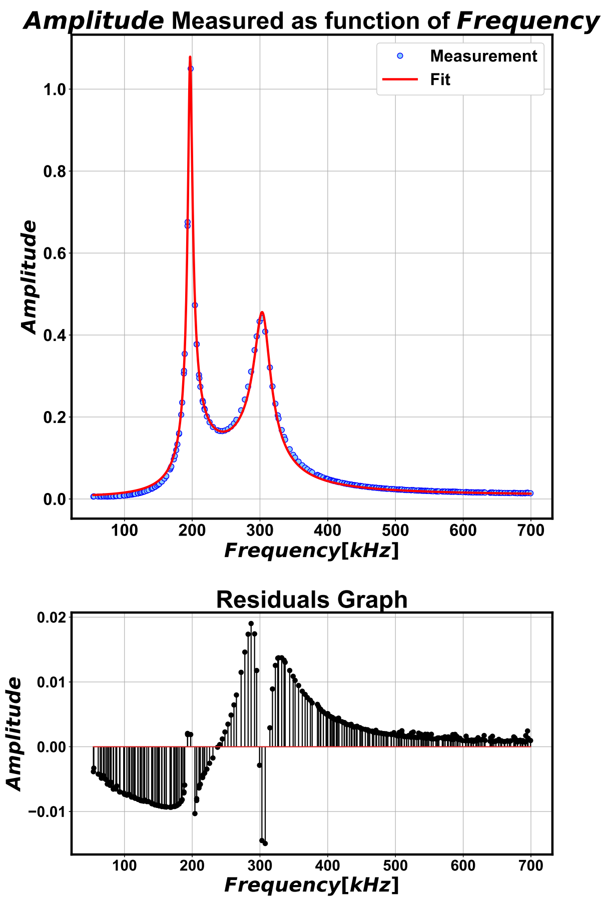


FIG. 30: The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with distance of $d = 14$ $[cm]$. The measurements are displayed in blue. The fit is displayed in red.

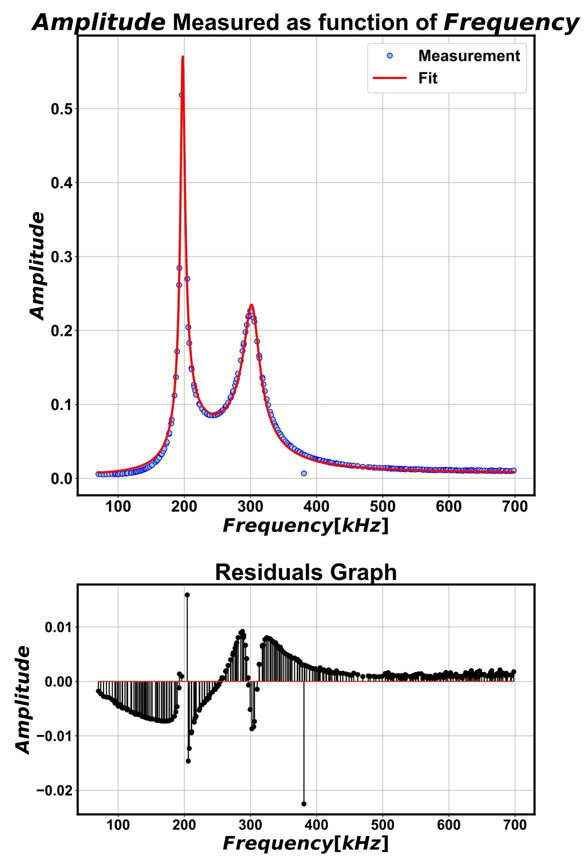


FIG. 31: The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47$ $[k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with distance of $d = 20$ $[cm]$. The measurements are displayed in blue. The fit is displayed in red.

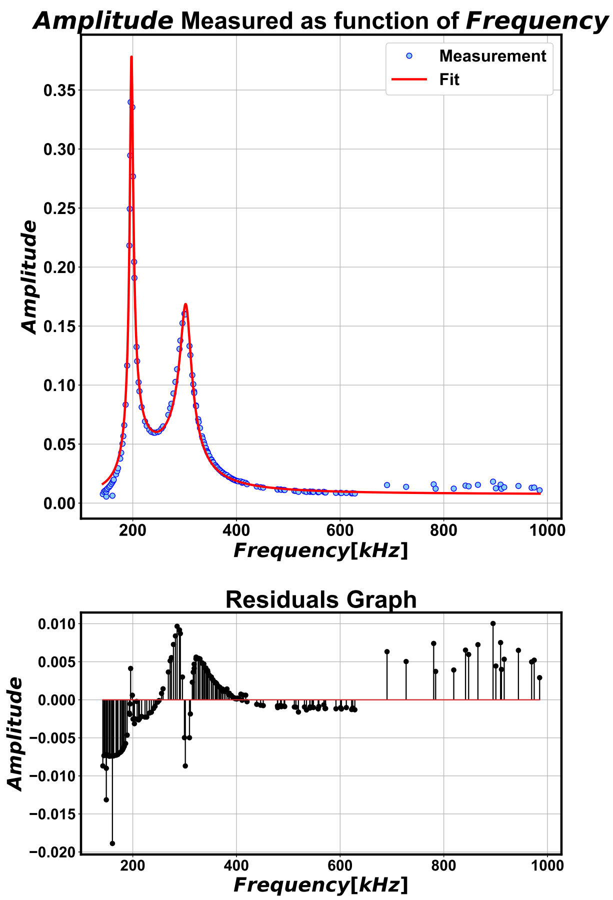


FIG. 32: The results of the Matlab script on a graph of the ratio of the voltage amplitude measured on $R_{ext} = 47\ [k\Omega]$ connected to the large coil and identical resistor connected to the small coil as a supply source with distance of $d = 24\ [cm]$. The measurements are displayed in blue. The fit is displayed in red.

Part V
The Communication Device

In order to develop an End-to-End communication device, we need to design and define several layers in our system:

- A modulation method.
- Modulation parameters - based on the coil's parameters we have been found in previous parts.
- Picoscope interface.
- designing the program that will wrap the physics into a functioning device.

VIII. MODULATION METHOD

We chose to modulate using BASK (Binary Amplitude Shift Keying). We synchronize the received signal by synchronization bit which is transmitted in a slightly bigger amplitude than the amplitude in which the 1's are transmitted. In the following section, we will choose the best T_s and test it on our system.

A. theoretical optimal constellation time

Theoretically, the optimal constellation time depends on the bandwith we transmit our data on. We want to take small bandwidth in order to preserve high SNR (we want to be near the peak), but large bandwidth will allow us to transmit data faster. The bandwidth is a parameter of our system, and for a given bandwidth B the optimal constellation time would be $T_s = \frac{2}{B}$. From the graph of V_R (the voltage on the resistor in the receiver coil) vs f we can estimate $B \approx 30[kHz]$ which results in

$$T_{s,\ theory} \approx 60\ [\mu s] \tag{30}$$

B. Picoscope Interface

We used the original Picotech GitHub library [1] and wrapped the API calls with our own code that enables us send and receive using the picoscoe.

C. implementation

We are modulating the data using BASK, the code is published on github [2]. our program supports sending an array of N bits in a sections of length $T = 0.1[s]$. we compute the constellation time as $T_{const} = \frac{0.1}{N}$ and the frequency of the carrier wave as $f_c = T_{const}^{-1}$. We appended a graph 33 of the bits we send using the pico.

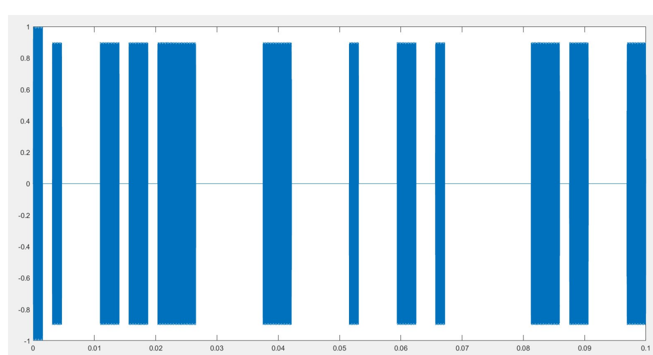


FIG. 33: The modulated signal sent by the pico

D. bit error rate for different T_s

We chose to compare how "good" the system is for different T_s by the BER (bit error rate) rather than the SNR of the received signal.

TABLE VII: The BER for transmitting at time symbol of T_s. f is the data transmitting rate.

T_s [ms]	f_c [bps]	BER [%]
1.56	640	0
0.19	5120	0
0.097	10240	0
0.048	20480	4.34
0.024	40960	Exception

For $T_s = 0.024[ms]$ we couldn't demodulate the received signal because the constellation time was too short and we didn't manage to sync the data using the sync bit.

As shown in the theoretical section, we found the theoretical value for $T_{s,theory} = 60[\mu s]$ and you can see we can see in the table VII that we start to get BER only in the measurements that which has $T_s < T_{s,theory}$. It happens because the sinc's in the waveform domain become too wide.

E. bit error rate for different d

Now we will check how d, the distance between the coils, affects the BER for taking constant time $T_s = 0.097$. The measurements are displayed here VIII.

TABLE VIII: The BER for transmitting at the chosen time symbol $T_s = 0.097$ for distance d between the coils.

d [cm]	BER [%]
2.5	0
5	31.1
7.5	33.3

The BER is prety bad for small distances, we will try with bigger constellation time $T_s = 0.19$. The measurements are displayed here IX.

TABLE IX: The BER for transmitting at the chosen time symbol $T_s = 0.19$ for distance d between the coils.

d [cm]	BER [%]
2.5	0.39
5	6.05
7.5	6.25
10	9.35
12.5	9.37

For $d > 15[cm]$ we didn't manage to measure the sync bit.

We will define the parameters of the system as:

$$\boxed{\begin{cases} N = 512 \\ T_s = 0.19[ms] \\ f_c = 5120[bps] \end{cases}} \tag{31}$$

because this is the fastest method to send which ensures $BER = 0$ and small BER when distancing the coils. We could have chosen a smaller constellation time to achieve faster data transfer speeds but with $BER > 0$.

[1] N. Hamilton, PS 2000 Driver, `https://github.com/picotech/picosdk-ps2000-matlab-instrument-driver` (2013).

[2] E. Nahum, Coils Transciver Pico Wrapper, `https://github.com/ErelNahum/CoilsTransceiver` (2013).