CS499 Homework 10 (First Draft)

Intersteller

Exercise 10.1

Since
$$\sum_{v\in V} f(s,v) = \sum_{v\in V\setminus S} f(s,v) + \sum_{v\in S} f(s,v)$$
we only need to prove that
$$\sum_{v\in S} f(s,v) = \sum_{u\in S-s,\ v\in V\setminus S} f(u,v)$$
Since
$$\sum_{v\in S} f(s,v) = -\sum_{v\in S-s} f(v,s)$$
we only need to prove that
$$\sum_{u\in S-s,\ v\in s+V\setminus S} f(u,v) = 0$$
It is obvious to see that
$$\sum_{u\in S-s,\ v\in S-s} f(u,v) = 0$$
So, we only need to prove that
$$\sum_{u\in S-s,\ v\in s+V\setminus S} f(u,v) + \sum_{u\in S-s,\ v\in S-s} f(u,v) = \sum_{u\in S-s,\ v\in V} f(u,v) = 0$$
According to the defination,
$$\sum_{u\in S-s,\ v\in V} f(u,v) = 0$$
Done.

Exercise 10.2

Define the minimum cut between i and j as minCut(i,j). According to the Max Flow Min Cut Theorem, minCut(s,r)≥k, minCut(r,t)≥k. Obviously, minCut(s,t)≥min{minCut(s,r),minCut(r,t)}≥k, which means there is a flow from s to r of value k.

Exercise 10.3

Suppose there is a s-t-path in G that has less that k edges. Then, at least one edge in the path moves more than one level forward, which contradicts Definition 10.3. So, $dist(s,t) \geq k$.

Exercise 10.9

proof According to **Exercise 10.8**, a particular layering is no more optimal after at moat m iterations. Since a layering is at least 1-layering and at most n-latering, after at most $m * n$ iterations, there is no optimal layering, which means there is no s-t-path, the algorithm terminates.

Exercise 10.10

proof According to **Exercise 10.9**, the Edmonds-Karp algorithm terminates after nm iterations of the while-loop, which is to say, we can get the max flow f after finite steps by Edmonds-Karp algorithm.

Question