CS499 Homework 7 (First Draft)

Intersteller

Exercise 7.1

(1) Since e is in the minimum spanning tree, we split the minimum spanning tree into two components by deleting e. Let the vertices in the two components consist S and $V\backslash S$ respectively. Since there is no circle in a tree, obviously e is the only edge which is good and cross this cut, which means no edge from X crosses this cut.

(2) Suppose e is not the minimum weight edge crossing this cut, assume there is an edge e' which has less weight and crosses this cut. e' can replace e and consists a spanning tree with less weight. This means e is not in the minimum spanning tree, which means e is not good, which contradicts the condition.

Exercies 7.4

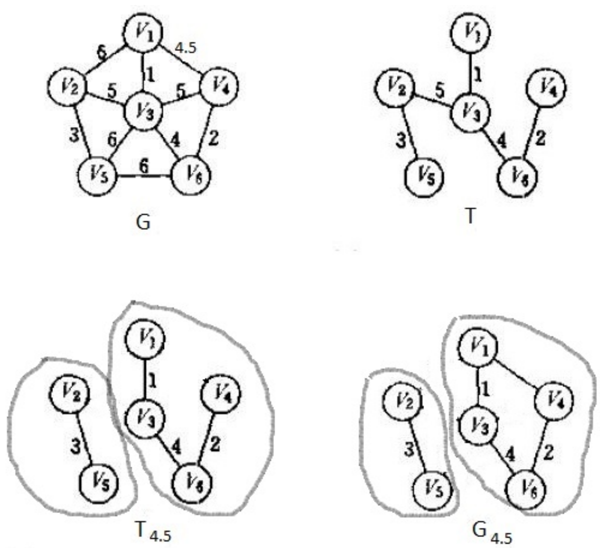

Figure 1:

Exercies 7.5 Obviously, if two vertices are connected in T_c, they are connected in G_c, since T_c is in G_c.
Suppose u,v are connected in G_c, but not connected in T_c. Let two connected components in T_c contain u and v respectively be A and B. Let e be an edge in G_c that connect A and B. Using defination, $w(e) \leq c$. Since A and B are not connected in T_c, there must be an edge e' in T that connects A and B, and $w(e') > c$. So, e'¿e. Obviously T which contains e' is not the minimum spanning tree, since e' can be replaced by e with less weight. This contradicts the condition. So, if two vertices are connected in G_c, they are connected in T_c.

Question:

1. In Exercise 7.11 & 7.12, how can we compute the number of functions with a core of size k? $(1 \leq k \leq n)$