CS499 Homework 9 (First Draft)

Intersteller

Exercise 10.5

Since dist$(s,t)=k$, we have a path $s \to u_1 \to u_2 \cdots u_{k-1} \to t$.For convenience, we call $s : u_0, t : u_k$. We construct $V_0, V_1 \cdots V_K$ as follows:

Step1:$u_0 \in V_0, u_1 \in V_1, u_2 \in V_2 \cdots u_{k-1} \in V_{K-1}, u_k \in V_K$.

Step2:$\forall v \in V \backslash V_k$, if there is an edge $(v,t), v \in V_{k-1}$. Obviously, $\forall n, 0 \le n \le k-2, u_n$ can't link to t. Otherwise, we can skip u_{k-1} and get a shorter path from s to t, which contradicts $dist(s,t)=k$.

Step3:$forallv \in V \backslash (V_k \cup V_{k-1})$, if $\exists w \in V_{k-1}, \exists edge(v,w), v \in V_{k-2}$. Similarly, $\forall n, 0 \le n \le k-3$, u_n can't link to any vertex in V_{k-1}.

$\vdots$

Step$k$:$\forall v \in V \backslash \bigcup_{i=2}^{k} V_i$, if $\exists w \in V_2, \exists edge(v,w), v \in V_1$. Similarly, u_0 can't link to any vertex in V_2.

Step$k+1$:$\forall v \in V$ and $v \notin V_i, i \ne 1,2,3 \cdots k, v \in V_0$.

Therefore, if dist$(s,t)=k$, (G,s,t,c) has a k-layering.

Exercise 10.7

Because there is a path from s to t, we suppose $dist_G(s,t)=k$. Based on the construction method in 10.5, we can find a k-layering and it is an optimal layering. Therefore, every network(G,s,t,c) has an optimal layering, provided there is a path from s to t.

Exercise 9.9

$\forall S \in \{0,1\}^{\mathbb{N}}$, define $f(S)=T$ as follows.

$$t_n = \sum_{i=1}^{n} (s_i+1)3^{i-1}$$

. Obviously, f is a bijection. Let $X = \{f(S) | \forall S \in \{0,1\}^{\mathbb{N}}\}$. $X \cong \{0,1\}^{\mathbb{N}}$ is uncountable. And $f(S)$ is infinite. Whenever distinct $x, y \in X (x = \{x_1, x_2, ...\})$, suppose $m = f^{-1}(x), n = f^{-1}(y)$ and assume the first different digit between m and n is the k_{th} digit. Then $x \cap y = \{x_1, x_2, x_3, \cdots, x_{k-1}\}$, which is finite.

Question

How to prove that $\mathbb{R}$ is smaller than $2^{\mathbb{R}}$.