CS499 Homework 7 (First Draft)

Intersteller

Exercise 7.1

(1) Since e is in the minimum spanning tree, we split the minimum spanning tree into two components by deleting e. Let the vertices in the two components consist S and $V \backslash S$ respectively. Since there is no circle in a tree, obviously e is the only edge which is good and cross this cut, which means no edge from X crosses this cut.

(2) Suppose e is not the minimum weight edge crossing this cut, assume there is an edge e’ which has less weight and crosses this cut. e’ can replace e and consists a spanning tree with less weight. This means e is not in the minimum spanning tree, which means e is not good, which contradicts the condition.

Exercises 7.4

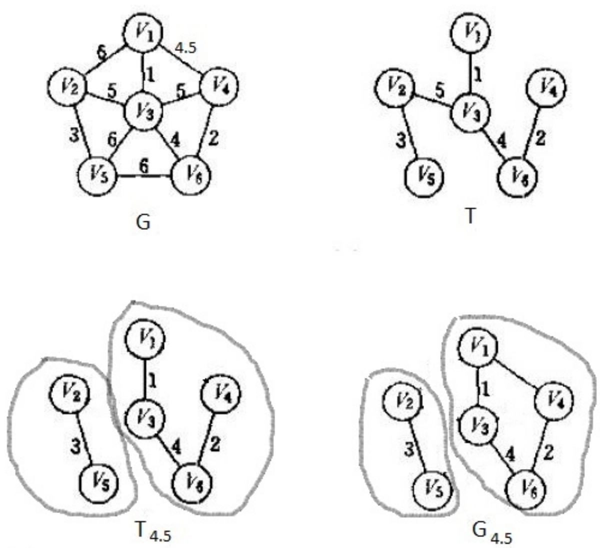

Figure 1:

Exercises 7.5 Obviously, if two vertices are connected in T_c, they are connected in G_c, since T_c is in G_c.
Suppose u,v are connected in G_c, but not connected in T_c. Let two connected components in T_c contain u and v respectively be A and B. Let e be an edge in G_c that connect A and B. Using defination, $w(e) \leq c$. Since A and B are not connected in T_c, there must be an edge e’ in T that connects A and B, and $w(e') > c$. So, e’¿e. Obviously T which contains e’ is not the minimum spanning tree, since e’ can be replaced by e with less weight. This contradicts the condition. So, if two vertices are connected in G_c, they are connected in T_c.

Exercise 7.8

As the picture shows , for $\forall c, m_c(T) = m_c(T\prime)$.

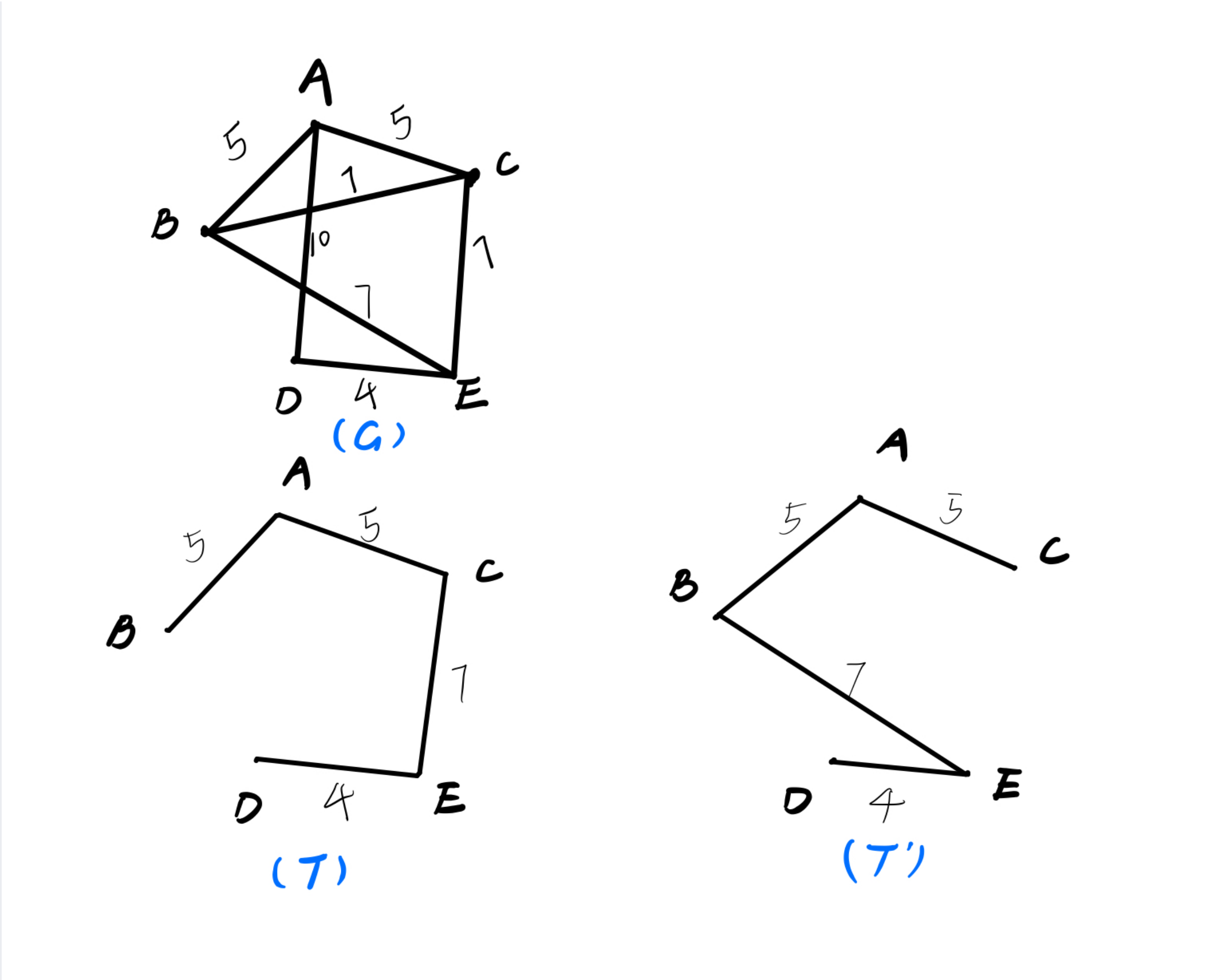

Figure 2:

Exercise 7.9

Sort by the weight of $T's$ edges and $T\prime s$ edges , we have $(a1, a2, a3, \cdots, a_{n-1})$, $(b1, b2, b3, \cdots, b_{n-1})$. Suppose $a_i \neq b_i$, $\forall k < i, a_k = b_k$ and $w(a_i) \geq w(b_i)$, there are two situations:

(1) edge b_i exists in the T , then we can find $j(j > i)$ and $a_j = b_i$. Because $w(b_i) = w(a_j) \geq w(a_i) \geq w(b_i), w(a_i) = w(b_i) = w(a_j)$.So we can exchange a_i and a_j and new sequence is still ordered . $T's$ and $T\prime s$ i position is the same edge.

(2) edge b_i doesn't exist in the T , then we add b_i to T to form a cycle . Because T is a minimum spanning tree , w(edge in the cycle)$\leq w(b_i)$. And we can find $a_j(j > i$ and a_j doesn't exist in the $T\prime$ and a_j in the cycle) . Because $w(b_i) \geq w(a_j) \geq w(a_i) \geq w(b_i), w(b_i) = w(a_i) = w(a_j)$. So we can change a_j with b_i . Turn to the situation (1).

So we know the ordered edge weight list of any two minimum spanning trees is the same.

Obviously, $m_c(T) = m_c(T\prime)$.

Exercise 7.10

Suppose there are two minimum spanning tree , sort by the weight of $T's$ edges and $T\prime's$ edges , we have$(a1, a2, a3, \cdots, a_{n-1})$, $(b1, b2, b3, \cdots, b_{n-1})$.$\exists i, a_i \neq b_i$,based on the 7.9,the ordered edge weight list of any two minimum spanning trees is the same,so $w(a_i) = w(b_i)$.But no two edges of G have the same weight , so there is contradiction . So G has exactly one minimum spanning tree!

Question:

1. In Exercise 7.11 & 7.12, how can we compute the number of functions with a core of size k? $(1 \leq k \leq n)$