CS499 Homework 5 (First Draft)

Intersteller

Exercise 5.1

(1) The degree of a vertex is defined as the number of edges linked to this vertex. And the score of a graph is a sequence ranking degree of all vertices from small to big.

(2) Graph score theorem states that, if we can find a graph for graph score $(d_1, \cdots, d_{n-1}, d_n)$, then we can find a graph for graph score $(d_1, \cdots, d_{n-d_n-1}, d_{n-d_n} - 1, \cdots d_{n-1} - 1)$, and vice versa. If we finally get graph score (ϕ), the graph exists.

(3) Graph score algorithm:
First, we get a graph score $(d_1, \cdots, d_{n-1}, d_n)$.
If $d_N > n - 1$, we cannot find a graph. Otherwise, we add an edge from d_n to $d_{n-d_n}, \cdots d_{n-1}$, and check the graph score $(d_1, \cdots, d_{n-d_n-1}, d_{n-d_n} - 1, \cdots d_{n-1} - 1)$ after it is sorted.
We repeat the previous step. If the graph score finally comes to (ϕ), the graph exists.

(4) The most difficult part is to prove if we can find a graph for graph score $(d_1, \cdots, d_{n-1}, d_n)$, then we can find a graph for graph score $(d_1, \cdots, d_{n-d_n-1}, d_{n-d_n} - 1, \cdots d_{n-1} - 1)$. We can suppose there is a solution without edge between n and k $(n - d_n \le k \le n - 1)$, so n must have another link with j $(j \le n - d_{n-1} < k)$. As $j < k$, we know $d_j \le d_k$, so k must have edge with some point l and $l \neq k$. We change the edges (n, j) (k, l) to (n, k) (j, l), and we add an edge between n and k without changing the score. In this way, we can transform the answer to make sure there is an edge from d_n to $d_{n-d_n}, \cdots, d_{n-1}$. Then we delete these edges, we get a graph for score $(d_1, \cdots, d_{n-d_n-1}, d_{n-d_n} - 1, \cdots d_{n-1} - 1)$.

Exercise 5.11

(For convenience, we ignore the 0 (a dot) in the following)

ID:517030910250

(1)(2) It is neither a graph score nor a multigtaph score because the sum of the ID is an odd number.
(3) It is a weighted graph score, as is shown in the following figure.

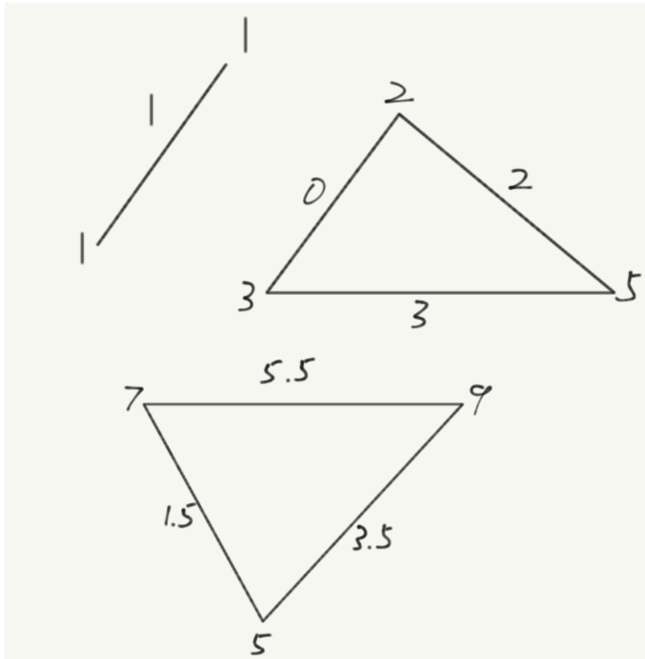

Figure 1:

4Since it is a weighted graph score, it is the score of a graph with real edge weights.

ID:517030910258

(1)(2) It is neither a graph score nor a multigtaph score because the sum of the ID is an odd number.
(3) It is a weighted graph score, as is shown in the following figure.

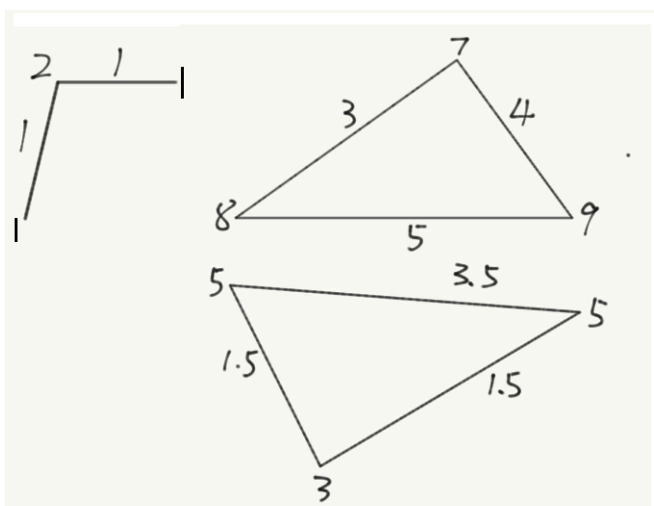

Figure 2:

4Since it is a weighted graph score, it is the score of a graph with real edge weights.

ID:517030910029

(1)(2) It is neither a graph score nor a multigtaph score because the sum of the ID is an odd number.
(3) It is a weighted graph score, as is shown in the following figure.

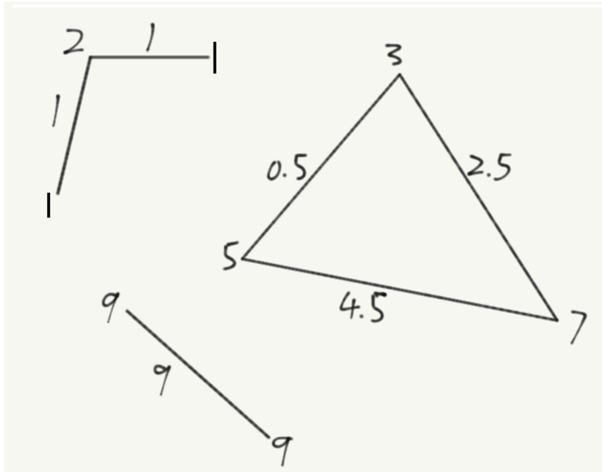

Figure 3:

4Since it is a weighted graph score, it is the score of a graph with real edge weights.

ID:517030910227

(1)(2) It is neither a graph score nor a multigtaph score because the sum of the ID is an odd number.
(3) It is a weighted graph score, as is shown in the following figure.

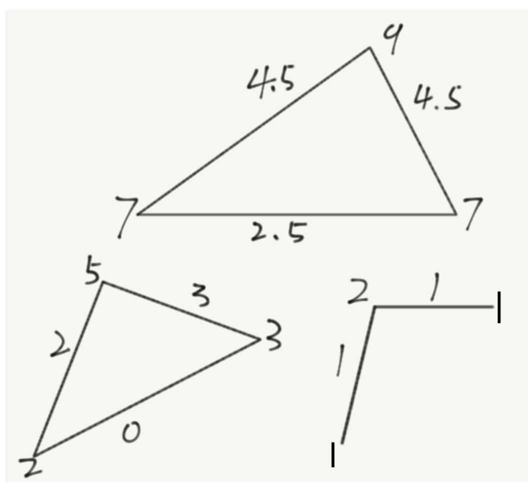

Figure 4:

4Since it is a weighted graph score, it is the score of a graph with real edge weights.

ID:517030910263

(1)(2) It is neither a graph score nor a multigtaph score because the sum of the ID is an odd number.
(3) It is a weighted graph score, as is shown in the following figure.

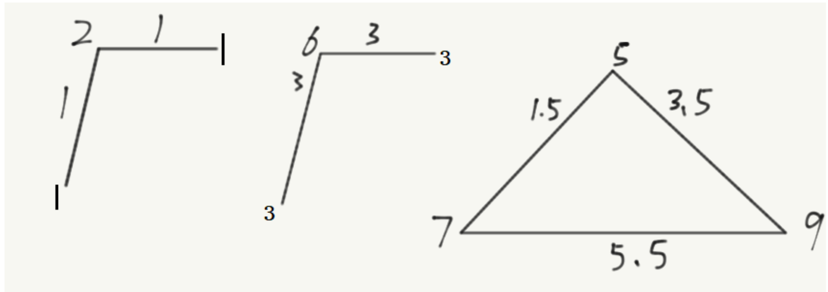

Figure 5:

4Since it is a weighted graph score, it is the score of a graph with real edge weights.

Questions