CS499 Homework 9 (First Draft)

Intersteller

Exercise 9.1

We define $f_1 : N \to N$

$f_1(0) = 0, f_1(1) = 1, \cdots, f_1(n) = n.$

We define $f_2 : N \to N^2$ based on this graph:

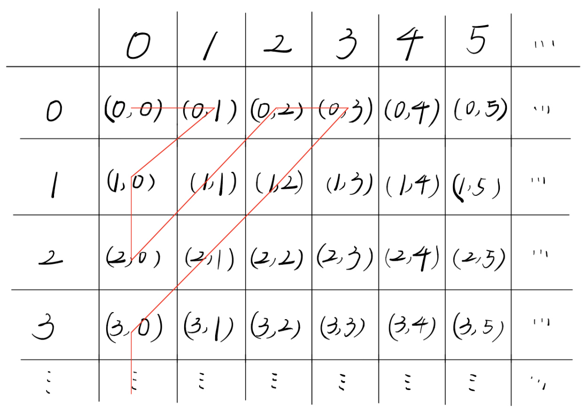

Figure 1:

$f_2(0) = (0,0), f_2(1) = (0,1), f_2(2) = (1,0) \cdots$

We define $f_3 : N \to N^3$ based on this graph:

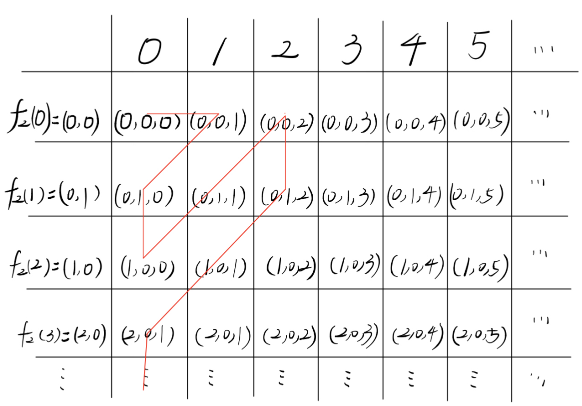

Figure 2:

$f_2(0) = (0,0,0), f_2(1) = (0,0,1), f_2(2) = (0,1,0) \cdots$

And so on, we can define $f_k, k \in N$. Now we can define a bijection $N \to N*$ base on this graph:

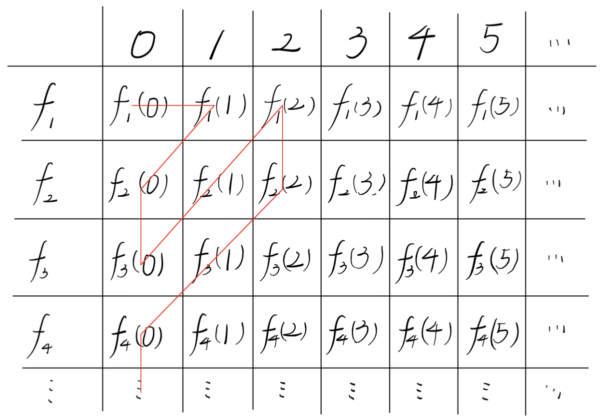

Figure 3:

We have $0 \to f_1(0), 1 \to f_1(1), 2 \to f_2(0) \cdots$. This is a bijection $N \to N*$.

Exercise 9.5

$000\cdots, 100\cdots, 1100\cdots, 11100\cdots$ According to this rule, the first n bits of the n_{th} sequence are 1, and the remaining bits are 0. Obviously, these sequences constitute a countably and infinite chain.

Exercise 9.6

$100\cdots, 0100\cdots, 00100\cdots, 000100\cdots$ According to this rule, the n_{th} bit of the n_{th} sequence is 1, and the remaining bits are 0. Obviously, these sequences constitute a countably and infinite antichain.