MafI 1 UB09

March 22, 2017

9.1 Ringe, Ringhomorphismen

9.1.1

$U = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} | a, b, c \in \mathbb{R} \right\}$

z.z.:
$< U, +, * >$ ist Unterring von $< R^{2\times 2}, +, * >$

Abgschlossenheit:
$" + "$:

$u_1, u_2 \in U$
$u_1 + u_2 = \begin{pmatrix} a + a' & b + b' \\ 0 & c + c' \end{pmatrix}$
$(i) \forall x, x' \in \mathbb{R}. x + x' \in \mathbb{R} \Rightarrow (u_1 + u_2) \in U$

$" * "$:

$u_1 * u_2 = \begin{pmatrix} a * a' & (a * b' + b * c') \\ 0 & c * b' \end{pmatrix}$
$(ii) \forall x, x' \in \mathbb{R}. x * x' \in \mathbb{R} \wedge (i) \Rightarrow (u_1 * u_2) \in U$

neutrl. Element bzgl. "+":
$e = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$u \in U$
$e + u = \begin{pmatrix} a + 0 & b + 0 \\ 0 + 0 & c + 0 \end{pmatrix}$
$= u$

inv. Element v bzgl. "+":
$u - v = e$
$\Rightarrow v = e - u$
$\Rightarrow v = -u$

neutrl. Element bzgl. "*":

$$E_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$u * E_2 = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

9.1.2

bzgl. "+":

$$h(u_1 + u_2) = h(\begin{pmatrix} a + a' & b + b' \\ 0 & c + c' \end{pmatrix})$$
$$= a + a'$$
$$= h(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) + h(\begin{pmatrix} a' & b' \\ 0 & c' \end{pmatrix})$$
$$= h(u_1) + h(u_2)$$

bzgl. "*":

$$h(u_1 * u_2) = h(\begin{pmatrix} a * a' & (a * b' + b * c') \\ 0 & c * c' \end{pmatrix})$$
$$= a * a'$$
$$= h(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}) * h(\begin{pmatrix} a' & b' \\ 0 & c' \end{pmatrix})$$
$$= h(u_1) * h(u_2)$$

9.2 Ideale

9.2.1

9.2.2

$_n\mathbb{Z} =^! I$

$_n\mathbb{Z} \subseteq I:$
$n * x \in \_n\mathbb{Z}$
$n \in I \Rightarrow n * x \in I$

$I \subseteq \_n\mathbb{Z}:$
$z \in I \Rightarrow \exists q, r.z = q * n + r$

$LemmavonBezout:$
$(0 \leq r < n) \Rightarrow r = z - q * n$

$z, q * n \in I \Rightarrow r \in I$

$n = min(\mathbb{N}) \Rightarrow r = 0 \wedge z \in \mathbb{Z}$