

MafI 1 UB07

March 15, 2017

## 11.1 Linearkombinationen und lineare Abhaengigkeit

### 11.1.1

$$\begin{pmatrix} 3 & -3 & 3 & | & 1 \\ 4 & 4 & 4 & | & 1 \\ 5 & 5 & -5 & | & 1 \end{pmatrix} \\
 \begin{pmatrix} 3 & -3 & 3 & | & 1 \\ 4 & 4 & 4 & | & 1 \\ 30 & 0 & 0 & | & 8 \end{pmatrix} \\
 \begin{pmatrix} 3 & -3 & 3 & | & 1 \\ 4 & 4 & 4 & | & 1 \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & -3 & 3 & | & 1 - \frac{12}{15} \\ 0 & 4 & 4 & | & 1 - \frac{16}{15} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & -3 & 3 & | & 1 - \frac{12}{15} \\ 0 & 0 & 24 & | & \frac{9}{15} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & -3 & 3 & | & 1 - \frac{12}{15} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & -3 & 0 & | & 1 - \frac{12}{15} - \frac{3}{40} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & -3 & 0 & | & 1 - \frac{105}{120} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & 1 & 0 & | & -\frac{15}{360} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix} \\
 \begin{pmatrix} 0 & 1 & 0 & | & -\frac{1}{24} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

damit ist  $u = \frac{1}{40} * v_1 - \frac{1}{24} * v_2 + \frac{1}{40} * v_3$

### 11.1.2

$$u - v = -(v - w + w - u)$$

damit dann auch

$$v - w = -(u - v + w - u)$$

$$w - u = -(u - v + v - w)$$

### 11.1.3

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & 2 & 2 & -1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & 1 & -2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 5 & -2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

damit linear unabhaengig.

## 11.2 Untervektorraum und Dimensionssatz

### 11.2.1

$$((0, 0, 0, 0) \in V_1) \Rightarrow V_1 \neq \emptyset$$

$$(x \in \mathbb{R}^4. x_1 = \dots = x_4) \Rightarrow x \in V_1$$

$$(y \in \mathbb{R}^4. y_1 = \dots = y_4) \Rightarrow y \in V_1$$

$$s \in \mathbb{R}$$

$$x + y = (x_1 + y_1, \dots, x_4 + y_4)^t \in V_1$$

$$s * x = (s * x_1, \dots, s * x_4)^t \in V_1$$

seien nun

$$((0, 0, 0, 0) \in V_2) \Rightarrow V_2 \neq \emptyset$$

$$(x \in \mathbb{R}^4. x_1 + \dots + x_4 = 0) \Rightarrow x \in V_2$$

$$(y \in \mathbb{R}^4. y_1 + \dots + y_4 = 0) \Rightarrow y \in V_2$$

$$s \in \mathbb{R}$$

$$x + y = (x_1 + y_1, \dots, x_4 + y_4)^t \in V_2$$

$$s * x = (s * x_1 + \dots + s * x_4)^t \in V_2$$

### 11.2.2

$$B_{V_1} = (1, 1, 1, 1)^t$$

$$\dim V_1 = 1$$

$$B_{V_2} = (1, 0, -1, 0)^t, (0, 1, 0, -1)^t, (1, 0, 0, -1)^t, (0, 1, -1, 0)^t$$

$$\dim V_2 = 4$$

$$B_{V_1 \cap V_2} = (0, 0, 0, 0)^t$$

$$\dim V_1 \cap V_2 = 1$$

$$B_{V_1 + V_2} = (1, 1, 1, 1)^t, (1, 0, -1, 0)^t, (0, 1, 0, -1)^t, (1, 0, 0, -1)^t, (0, 1, -1, 0)^t$$

$$\dim V_1 + V_2 = 5$$

### 11.3 Basis eines EZS

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 4 & 3 & 9 & -2 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 8 & -2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 4 & 3 & 9 & -2 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 0 & 2 & 8 & -2 & 0 \\ 0 & 7 & 11 & 16 & 18 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 17 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 17 & 9 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 0 \\ 0 & 9 & 53 & 0 & 0 \\ 0 & 0 & 17 & 9 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$