MafI 1 UB07

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11.1 Linearkombinationen und lineare Abhaengigkeit

11.1.1

$$\begin{pmatrix} 3 & -3 & 3 & | & 1 \\ 4 & 4 & 4 & | & 1 \\ 5 & 5 & -5 & | & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & 3 & | & 1 \\ 4 & 4 & 4 & | & 1 \\ 30 & 0 & 0 & | & 8 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & 3 & | & 1 \\ 4 & 4 & 4 & | & 1 \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & 3 & | & 1-\frac{12}{15} \\ 0 & 4 & 4 & | & 1-\frac{16}{15} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & 3 & | & 1-\frac{12}{15} \\ 0 & 0 & 24 & | & \frac{9}{15} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & 3 & | & 1-\frac{12}{15} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & 0 & | & 1-\frac{12}{15}-\frac{3}{40} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & -3 & 0 & | & 1-\frac{105}{120} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & | & -\frac{15}{360} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & | & -\frac{1}{24} \\ 0 & 0 & 1 & | & \frac{1}{40} \\ 1 & 0 & 0 & | & \frac{4}{15} \end{pmatrix}$$

damit ist $u = \frac{1}{40} * v_1 - \frac{1}{24} * v_2 + \frac{1}{40} * v_3$

11.1.2

$u - v = -(v - w + w - u)$
damit dann auch
$v - w = -(u - v + w - u)$
$w - u = -(u - v + v - w)$

11.1.3

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & 2 & 2 & -1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & 1 & -2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 5 & -2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

damit linear unabhaengig.

11.2 Untervektorraum und Dimensionssatz

11.2.1

$((0,0,0,0) \in V_1) \Rightarrow V_1 \neq \emptyset$
$(x \in \mathbb{R}^4 . x_1 = ... = x_4) \Rightarrow x \in V_1$
$(y \in \mathbb{R}^4 . y_1 = ... = y_4) \Rightarrow y \in V_1$
$s \in \mathbb{R}$

$x + y = (x_1 + y_1, ..., x_4 + y_4)^t \in V_1$
$s * x = (s * x_1, ..., s * x_4)^t \in V_1$

seien nun
$((0,0,0,0) \in V_2) \Rightarrow V_2 \neq \emptyset$
$(x \in \mathbb{R}^4 . x_1 + ... + x_4) \Rightarrow x \in V_2$
$(y \in \mathbb{R}^4 . y_1 + ... + y_4) \Rightarrow y \in V_2$
$s \in \mathbb{R}$

$x + y = (x_1 + y_1, ..., x_4 + y_4) \in V_2$

$s * x = (s * x_1 + ... + s * x_4) \in V_2$

11.2.2

$B_{V_1} = (1,1,1,1)^t$
$dim V_1 = 1$

$B_{V_2} = (1,0,-1,0)^t, (0,1,0,-1)^t, (1,0,0,-1)^t, (0,1,-1,0)^t$
$dim V_2 = 4$

$B_{V_1 \cap V_2} = (0,0,0,0)^t$
$dim V_1 \cap V_2 = 1$

$B_{V_1 + V_2} = (1,1,1,1)^t, (1,0,-1,0)^t, (0,1,0,-1)^t, (1,0,0,-1)^t, (0,1,-1,0)^t$
$dim V_1 + V_2 = 5$

11.3 Basis eines EZS

$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 4 & 3 & 9 & -2 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 8 & -2 & 4 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 4 & 3 & 9 & -2 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 0 & 2 & 8 & -2 & 0 \\ 0 & 7 & 11 & 16 & 18 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 1 & 1 & 0 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 17 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 17 & 9 & 0 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 1 & 1 & 0 & 0 \\ 0 & 9 & 53 & 0 & 0 \\ 0 & 0 & 17 & 9 & 0 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$