Logik TUT 10

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10.1 Normalformen

$\psi = \forall x' \exists y \exists y' \forall z' ((P(x', g(y)) \vee P(z, y') \wedge P(x, z')))$
$\equiv \forall x' \exists y \exists y' \forall z' ((P(x', g(y)) \vee P(k, y') \wedge P(c, z')))$ x, z frei
$\equiv \forall x' \forall z' ((P(x', g(f(x'))) \vee P(k, h(x')) \wedge P(c, z')))$ $f(y) = h(y') = x'$
$= \psi'$

als Matrixklauselform:

$\{$
$\quad \{P(x', g(f(x')), P(k, h(x'))\},$
$\quad \{P(c, z')\}$
$\}$

10.2 Grundresolution

als Matrixklauselform:

$$
\begin{array}{l}
\{ \\
\quad \{Q(y,x)\}, \\
\quad \{\neg Q(x,y), \neg Q(x, f(a)), P(g(b), f(a))\} \\
\quad \{R(y)\}, \\
\quad \{\neg P(x,y), \neg R(x), \neg R(g(f(b)))\} \\
\}
\end{array}
$$

Herbrand Universum:
$H(\varphi) = \{f(a), g(b), f(f(b)), ...\}$

$$
\begin{array}{ll}
\{ & \\
\quad \{Q(b,a)\}, & y \mapsto b, x \mapsto a \\
\quad \{Q(b, f(a))\}, & y \mapsto b, x \mapsto f(a) \\
\quad \{\neg Q(b,a), \neg Q(b, f(a)), P(g(b), f(a))\}, & x \mapsto b, y \mapsto a \\
\quad \{R(g(b))\} & y \mapsto g(b) \\
\quad \{R(g(f(b)))\}, & y \mapsto g(f(b)) \\
\quad \{\neg P(g(b), f(a)), \neg R(g(b)), \neg R(g(f(b)))\}, & x \mapsto g(b), y \mapsto f(a) \\
\quad ... & \\
\} &
\end{array}
$$

$$
\begin{array}{lll}
\{\neg P(g(b), f(a)), \neg R(g(f(b)))\} & \text{aus } \{\neg P(g(b), f(a)), \neg R(g(b)), \neg R(g(f(b)))\}, & \{R(g(b))\} \\
\{\neg P(g(b), f(a))\} & \text{aus } \{\neg P(g(b), f(a)), \neg R(g(f(b)))\}, & \{R(g(f(b)))\} \\
\{\neg Q(b, f(a)), P(g(b), f(a))\} & \text{aus } \{\neg Q(b,a), \neg Q(b, f(a)), P(g(b), f(a))\}, & \{Q(b,a)\} \\
\{P(g(b), f(a))\} & \text{aus } \{\neg Q(b, f(a)), P(g(b), f(a))\}, & \{Q(b, f(a))\} \\
\Box & \text{aus } \{P(g(b), f(a))\}, & \neg\{P(g(b), f(a))\}
\end{array}
$$

10.3 Grundresolution 2

10.3.1

Praenexform:
$\varphi = \exists y[P(y) \rightarrow \forall z \neg Q(z, f(y))] \wedge \forall z[\exists y Q(y, f(z)) \wedge P(z)]$
$\equiv \exists y[\neg P(y) \vee \forall z \neg Q(z, f(y))] \wedge \forall z[\exists y Q(y, f(z)) \wedge P(z)] = \varphi'$

SkolemForm:
$\varphi' = \exists y[\neg P(y) \vee \forall z \neg Q(z, f(y))] \wedge \forall z[\exists y Q(y, f(z)) \wedge P(z)]$
$\equiv [\neg P(c) \vee \forall z \neg Q(z, f(c))] \wedge \forall z[Q(g(z), f(z)) \wedge P(z)]$
$\equiv \forall z[(\neg P(c) \vee \neg Q(z, f(c))) \wedge Q(g(z), f(z)) \wedge P(z)] = \varphi''$

Matrixklauselform:

$$
\begin{array}{l}
\{ \\
\quad (\neg P(c) \vee \neg Q(z, f(c)), \\
\quad Q(g(z), f(z)), \\
\quad P(z) \\
\}
\end{array}
$$

10.3.2

Herbrand Universum:
$H(\varphi'') = \{c, f(c), g(c), ...\}$

$$
\begin{array}{lll}
\{ & & \\
\quad & (\neg P(c) \vee \neg Q(g(c), f(c))), & z \mapsto g(c) \\
\quad & Q(g(c), f(c)), & z \mapsto c \\
\quad & P(c) & z \mapsto c \\
\} & &
\end{array}
$$

$$
\begin{array}{lll}
\{\neg Q(g(c), f(c))\} & \text{aus } \{\neg P(c) \vee \neg Q(g(c), f(c)\}, & \{P(c)\} \\
\Box & \text{aus } \{\neg Q(g(c), f(c))\}, & \{Q(g(c), f(c))\}
\end{array}
$$