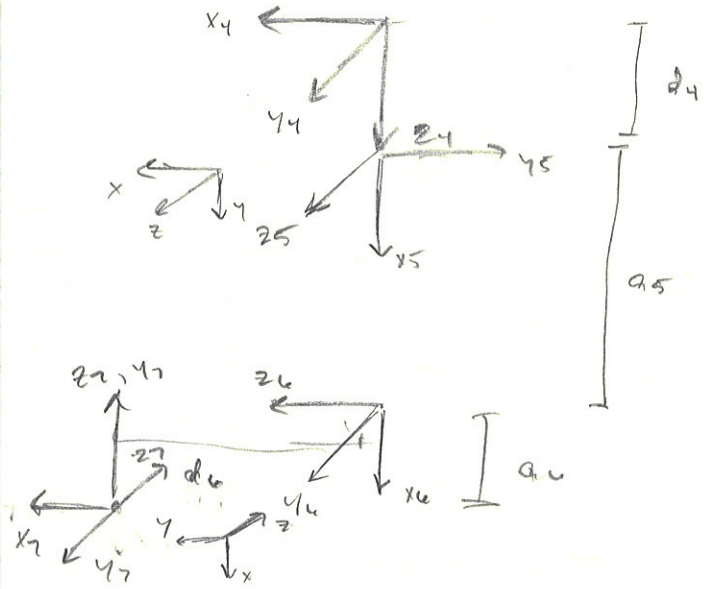
FORWARD KINEMATICS – FINAL DESIGN 3/18/13



	d	θ	a	α
4-5	d_4	θ_4	–	$+90^\circ$
	–	$+90^\circ$	–	–
5-6	–	θ_5	a_5	$+90^\circ$
6-7	d_6	θ_6	a_6	$+90^\circ$
		$+90^\circ$		-90°

D-H

	d	θ	a	α
4-5	d_4	θ_4	–	$+90^\circ$
5-6	–	θ_5+90°	a_5	$+90^\circ$
6-7	d_6	θ_6+90°	a_6	$+90^\circ$
7-8	–	–	–	-90°

$d_4 = 46.2612$ mm

$a_5 = 127.8881$ mm

$a_6 = 42.5911$ mm

$d_6 = 48.7004$ mm

$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_5 & 0 & S_5 & a_5 C_5 \\ S_5 & 0 & -C_5 & a_5 S_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{45} = \begin{bmatrix} C_4C_5 & S_4 & C_4S_5 & a_5C_4C_5 \\ S_4C_5 & -C_4 & S_4S_5 & a_5S_4C_5 \\ S_5 & 0 & -C_5 & a_5S_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & a_6C_6 \\ S_6 & C_6 & 0 & a_6S_6 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{45_6} = \begin{bmatrix} c_4c_5c_6 + s_4s_6 & -c_4c_5s_6 + s_4c_6 & c_4s_5 & a_6c_4c_5c_6 + a_6s_4s_6 + d_6c_4s_5 + a_5c_4c_5 \\ s_4c_5c_6 - c_4s_6 & -s_4c_5s_6 - c_4c_6 & s_4s_5 & a_6s_4c_5c_6 - a_6c_4s_6 + d_6s_4s_5 + a_5s_4c_5 \\ s_5c_6 & -s_5s_6 & -c_5 & a_6s_5c_6 - d_6c_5 + a_5s_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

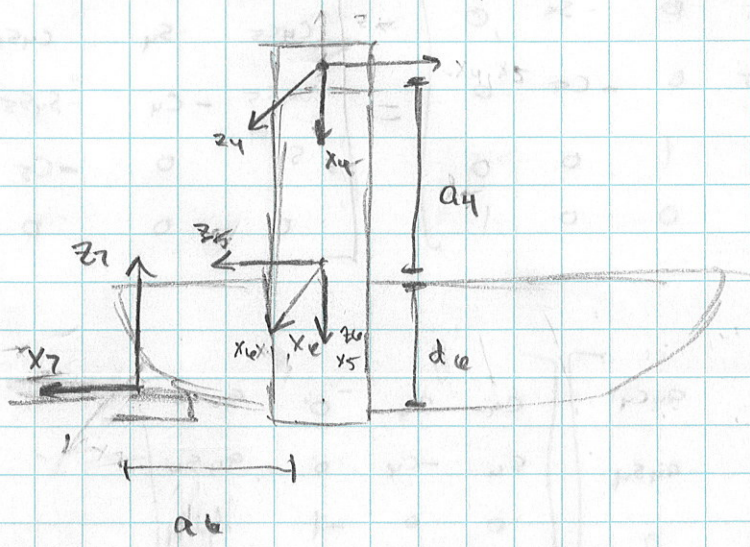
$$A_{4new} = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -c_4 & s_4 & 0 \\ 0 & -s_4 & -c_4 & 0 \\ 1 & 0 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{6new} = \begin{bmatrix} c_6 & -s_6 & 0 & a_6c_6 \\ s_6 & c_6 & 0 & a_6s_6 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_6 & 0 & -c_6 & a_6c_6 \\ c_6 & 0 & -s_6 & a_6s_6 \\ 0 & -1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{45} = \begin{bmatrix} -c_4s_5 & s_4 & c_4c_5 & -a_5c_4s_5 \\ -s_4s_5 & -c_4 & s_4c_5 & -a_5s_4s_5 \\ c_5 & 0 & s_5 & a_5c_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{45_6} = \begin{bmatrix} c_4s_5s_6 + s_4c_6 & -c_4c_5 & c_4s_5c_6 - s_4s_6 & -a_6c_4s_5c_6 + a_6s_4s_6 + d_6c_4c_5 - a_5c_4s_5 \\ s_4s_5s_6 - c_4c_6 & -s_4c_5 & s_4s_5c_6 + c_4s_6 & -a_6s_4s_5c_6 - a_6c_4s_6 + d_6s_4c_5 - a_5s_4s_5 \\ -c_5s_6 & -s_5 & -c_5c_6 & a_6c_5c_6 + d_6s_5 + a_5c_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics #3 — Ball/Socket Design



D-H

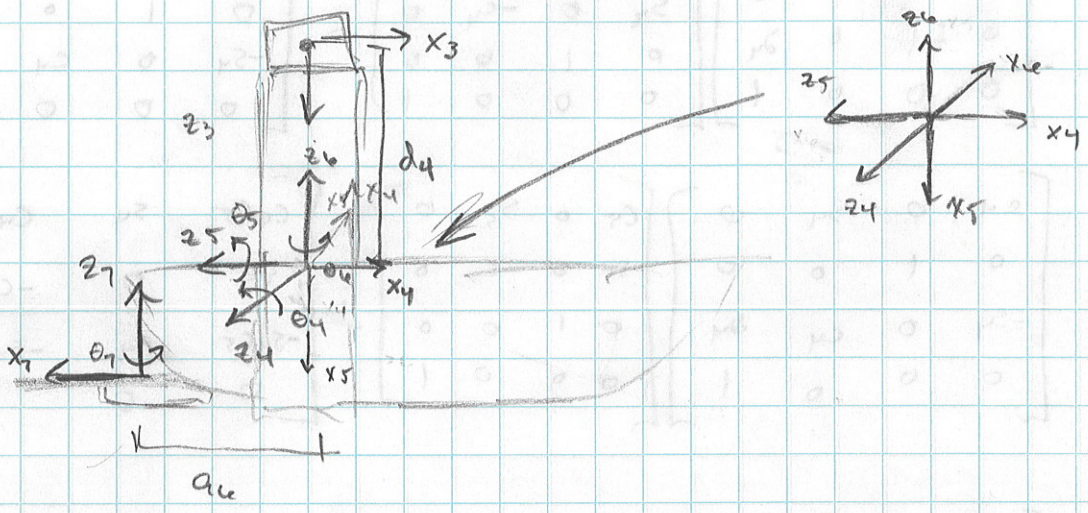
	d	θ	a	α
4-5	–	θ_4	a_4	$+90^\circ$
5-6	–	$\theta_5 + 90^\circ$	–	$+90^\circ$
6-7	d_6	$\theta_6 + 90^\circ$	a_6	$+180^\circ$

$$A_4 = \begin{bmatrix} c_4 & 0 & s_4 & a_4 c_4 \\ s_4 & 0 & -c_4 & a_4 s_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & s_6 & 0 & a_6 c_6 \\ s_6 & -c_6 & 0 & a_6 s_6 \\ 0 & 0 & -1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics #2 — Yoke Design



D-H

	d	θ	a	α
3-4	d_4	—	—	-90°
4-5	—	$\theta_4 - 90^\circ$	—	$+90^\circ$
5-6	—	$\theta_5 - 90^\circ$	—	$+90^\circ$
6-7	—	$\theta_6 + 90^\circ$	a_6	—

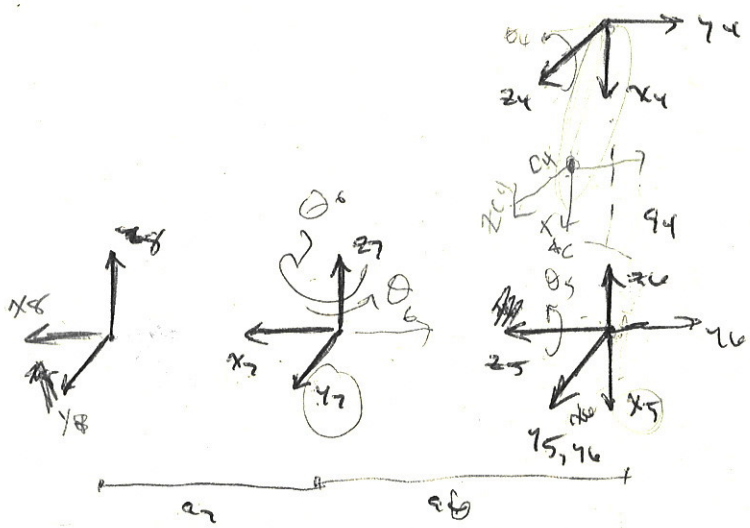
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

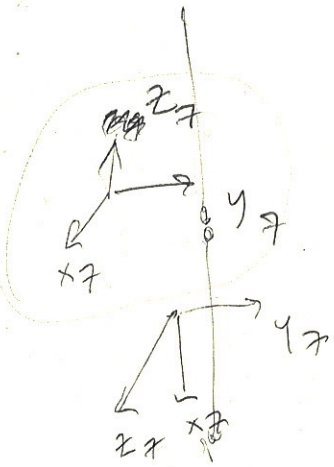
$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & a_6 C_6 \\ S_6 & C_6 & 0 & a_6 S_6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

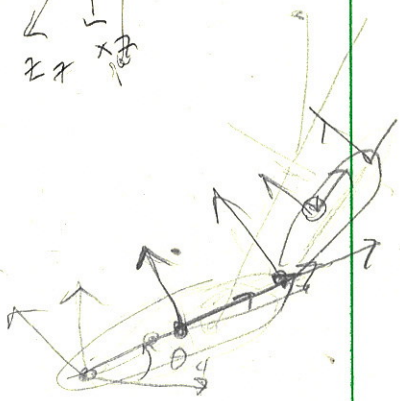
Forward Kinematics #1



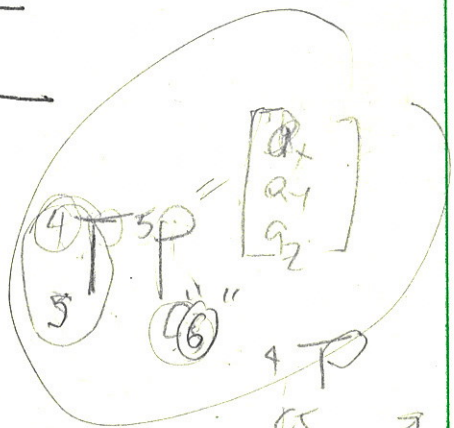


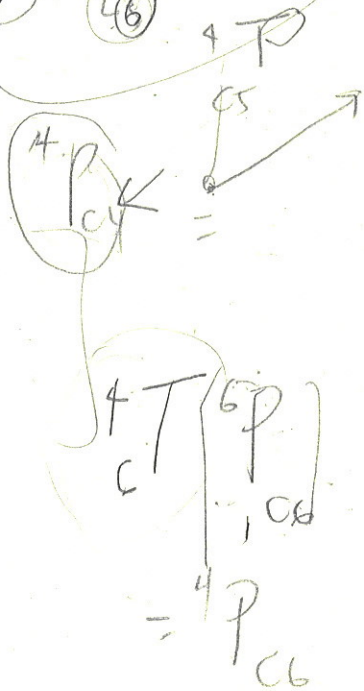
D-H

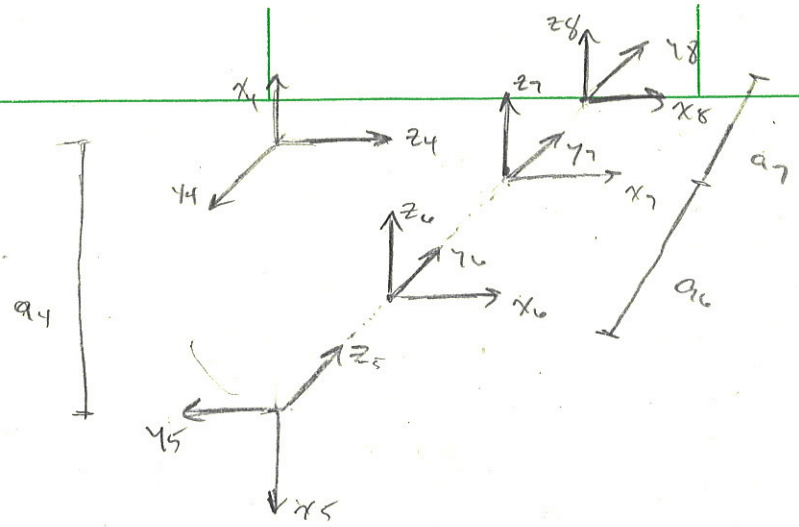
	d	θ	a	α
4-5	0	θ_4	a_4	$+90^\circ$
5-6	0	$\theta_5 + 90$	0	-90
6c	0	$\theta_6 + 90$	a_4	0
7		θ_7	a_7	



$$A_4 = \begin{bmatrix} c_4 & 0 & s_4 & a_4 c_4 \\ s_4 & 0 & -c_4 & a_4 s_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$







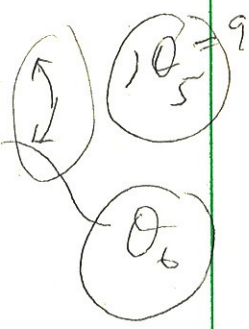
z_4 = axis for passive flexion/extension

z_5 = axis for actuated pronation/supination

z_6 = axis for passive radial/ulnar deviation

z_7 = axis for actuated flexion/extension

z_8 = axis for hand position



DH

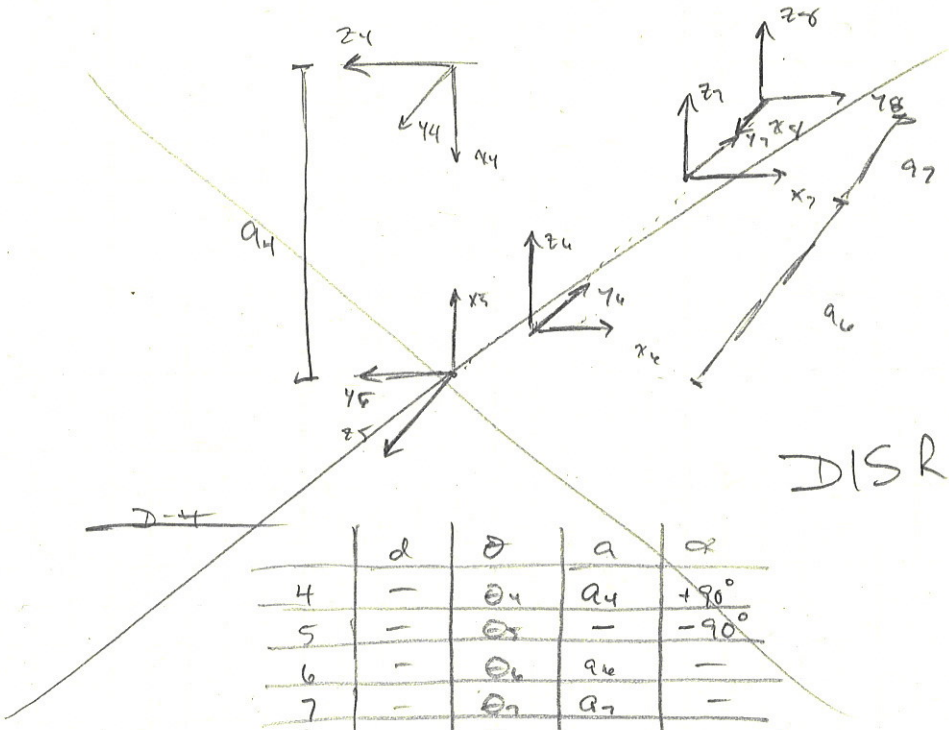
	d	θ	a	α
4-5	–	θ_4	q_4	$+90°$
5-6	–	θ_5	–	$-90°$
6-7	–	θ_6	q_6	–
7-8	–	θ_7	q_7	–
8	–	θ_8		

P

q_4 = distance from connection of orthosis and robot arm to trough

q_6 = distance from gimbal to anthropometric wrist joint

q_7 = distance from wrist joint to metacarpophalangeal joints



DISREGARD

	d	θ	a	α
4	–	θ_4	a_4	+90°
5	–	θ_5	–	-90°
6	–	θ_6	a_6	–
7	–	θ_7	a_7	–
8	–	θ_8	–	–

$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & a_4 C_4 \\ S_4 & 0 & -C_4 & a_4 S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & a_6 C_6 \\ S_6 & C_6 & 0 & a_6 S_6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_7 = \begin{bmatrix} C_7 & -S_7 & 0 & a_7 C_7 \\ S_7 & C_7 & 0 & a_7 S_7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_8 = \begin{bmatrix} C_8 & -S_8 & 0 & 0 \\ S_8 & C_8 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 A_5 = \begin{bmatrix} c_4c_5 & -s_4 & -c_4s_5 & q_4c_4 \\ s_4c_5 & c_4 & -s_4s_5 & q_4s_4 \\ s_5 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{45}A_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & q_6(c_4c_5c_6 - s_4s_6) + q_4c_4 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & -s_4s_5 & q_6(s_4c_5c_6 + c_4s_6) + q_4s_4 \\ s_5c_6 & -s_5s_6 & c_5 & q_6s_5c_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{4567}A_7 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} \qquad A_{4567}A_8 = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}$$

$$r_{11} = c_7(c_4c_5c_6 - s_4s_6) + s_7(-c_4c_5s_6 - s_4c_6)$$
$$r_{21} = c_7(s_4c_5c_6 + c_4s_6) + s_7(-s_4c_5s_6 + c_4c_6)$$
$$r_{31} = s_5c_6c_7 - s_5s_6s_7$$
$$r_{41} = 0$$
$$r_{12} = -s_7(c_4c_5c_6 - s_4s_6) + c_7(-c_4c_5s_6 - s_4c_6)$$
$$r_{22} = -s_7(s_4c_5c_6 + c_4s_6) + c_7(-s_4c_5s_6 + c_4c_6)$$
$$r_{32} = -s_5c_6s_7 - s_5s_6c_7$$
$$r_{42} = 0$$
$$r_{13} = -c_4s_5$$
$$r_{23} = -s_4s_5$$
$$r_{33} = c_5$$
$$r_{43} = 0$$
$$r_{14} = q_7c_7(c_4c_5c_6 - s_4s_6) + q_7s_7(-c_4c_5s_6 - s_4c_6) + q_6(c_4c_5c_6 - s_4s_6) + q_4c_4$$
$$r_{24} = q_7c_7(s_4c_5c_6 + c_4s_6) + q_7s_7(-s_4c_5s_6 + c_4c_6) + q_6(s_4c_5c_6 + c_4s_6) + q_4s_4$$
$$r_{34} = q_7s_5c_6c_7 - q_7s_5s_6s_7 + q_6s_5c_6$$
$$r_{44} = 1$$

$$\begin{bmatrix} x = q_6(c_4c_5c_6 - s_4s_6) + q_4c_4 \\ y = q_6(s_4c_5c_6 + c_4s_6) + q_4s_4 \\ z = q_6s_5c_6 \end{bmatrix}$$

$$\begin{bmatrix} x = q_6 + q_4 \\ y = 0 \\ z = 0 \end{bmatrix}$$

$\theta_4 = 0$, $\theta_5 = q_5$, $\theta_6 = q_6$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$[0, 0, 0]$ $\qquad$ $[0, q_5, -q_6]$

roll, pitch, yaw

$$S_{11} = C_8\left[C_7(C_4C_5C_6 - S_4S_6) + S_7(-C_4C_5S_6 - S_4C_6)\right] + S_8\left[-S_7(C_4C_5C_6 - S_4S_6) + C_7(-C_4C_5S_6 - S_4C_6)\right]$$

$$S_{21} = C_8\left[C_7(S_4C_5C_6 + C_4S_6) + S_7(-S_4C_5S_6 + C_4C_6)\right] + S_8\left[-S_7(S_4C_5C_6 + C_4S_6) + C_7(-S_4C_5S_6 + C_4C_6)\right]$$

$$S_{31} = C_8(S_5C_6C_7 - S_5S_6S_7) + S_8(-S_5C_6S_7 - S_5S_6C_7)$$

$$S_{41} = 0$$

$$S_{12} = -S_8\left[C_7(C_4C_5C_6 - S_4S_6) + S_7(-C_4C_5S_6 - S_4C_6)\right] + C_8\left[-S_7(C_4C_5C_6 - S_4S_6) + C_7(-C_4C_5S_6 - S_4C_6)\right]$$

$$S_{22} = -S_8\left[C_7(S_4C_5C_6 + C_4S_6) + S_7(-S_4C_5S_6 + C_4C_6)\right] + C_8\left[-S_7(S_4C_5C_6 + C_4S_6) + C_7(-C_4C_5S_6 - S_4C_6)\right]$$

$$S_{32} = -S_8(S_5C_6C_7 - S_5S_6S_7) + C_8(-S_5C_6S_7 - S_5S_6C_7)$$

$$S_{42} = 0$$

$$S_{13} = -C_4S_5$$

$$S_{23} = -S_4S_5$$

$$S_{33} = C_5$$

$$S_{43} = 0$$

$$S_{14} = q_7C_7(C_4C_5C_6 - S_4S_6) + q_7S_7(-C_4C_5S_6 - S_4C_6) + q_6(C_4C_5C_6 - S_4S_6) + q_4C_4$$

$$S_{24} = q_7C_7(S_4C_5C_6 + C_4S_6) + q_7S_7(-S_4C_5S_6 + C_4C_6) + q_6(S_4C_5C_6 + C_4S_6) + q_4S_4$$

$$S_{34} = q_7S_5C_6C_7 - q_7S_5S_6S_7 + q_6S_5C_6$$

$$S_{44} = 1$$