MOTION IN A STRAIGHT LINE

Frame of Reference :

The body relative to which a given mechanical motion is being considered is called a reference body.

The combination of a reference body and a co-ordinate system is called a reference frame or the frame of reference.

There are two types of frame of reference

(a) Inertial (or Newtonian or Galilean) frame of reference.
A frame of reference in which Newton's Laws of motion and other laws of Newtonian mechanics hold good.

(b) Non-Inertial frame of reference
A frame of reference which has an acceleration.

Rest and Motion :

- An object is said to be at rest if it does not change its position with respect to its immidiate surroundings with time. Like a book lying on a table.
- An object is said to be in motion if it changes its position with respect to its surroundings with the passage of time. Like a train is moving on a rail.
- Absolute rest and motion are unknown.

Motion in one, two and three dimensions :

- The motion of an object is said to be one dimensional if only one of the three co-ordinates specifying the position of the object changes with time. Here the object is moving along a straight line. Like: motion of a freely falling body.
- The motion of an object is said to be two dimensional if only two of the three co-ordinate specifying the position of the object changes with time. Here the object is moving in a plane. Like: Motion of a planet around the Sun.
- The motion of an object is said to be three dimensional if all the three co-ordinates specifying its position of the object changes with time. Here the object is moving in space.

Distance and Displacement :

Distance is the length of the actual path traversed by a body between its initial and final positions. It is a scalar quantity.

Displacement is the shortest (or the straight line) path measured in the direction from initial point to the final point. It is a vector quantity.

Speed :

The rate of change of position with time in any direction is known as speed.

$$\text{speed} = \frac{\text{Distance}}{\text{time}}$$

It is a scalar quantity.

Different types of speed:

(i) Uniform speed: An object is said to be in uniform speed if it covers equal distance in equal interval of time.

(ii) Variable speed: An object is said to be in variable speed if it covers unequal distance in equal interval of time.

(iii) Average speed: Average speed = $\frac{\text{Total distance}}{\text{Total time taken}}$.

(iv) Instantaneous speed: The speed of an object at any particular instant of time.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Velocity: The rate of change of position of an object with time in a given direction is called its velocity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{time}}$$

It is a vector quantity.

Different types of velocities:

(i) Uniform velocity: An object is said to be moving with uniform velocity if it covers equal displacement in equal interval of time.

(ii) Variable velocity: An object is said to be moving with variable velocity if ~~either~~ its speed changes or direction of motion changes or both changes. with time.

(iii) Average velocity: Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}}$

(iv) Instantaneous velocity: The velocity of an object at a particular instant of time.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}.$$

Uniform and Non-uniform motion:

The motion of an object is said to be uniform if it covers equal displacement in equal interval of time, howsoever small these intervals may be.

Important points related to uniform motion

- Velocity is independent of the choice of origin.
- Velocity in independent of the choice of time interval.
- Magnitude of distance and displacement is equal for motion along a straight line
- No external force is required. Therefore net force on it is zero.
- Instantaneous and average velocity are same.

The motion of the object is said to be non-uniform if it covers unequal distance in equal interval of time.

Equation of motion by Calculus method

$v = u + at$: From definition of acceleration

$$a = \frac{dv}{dt} \quad \therefore \quad dv = a\,dt$$

Integrating both side with limit $v = u$ to $v = v$ for time $t = 0$ to $t = t$.

$$\int_u^v dv = \int_0^t a\,dt$$

$$[v]_u^v = a[t]_0^t$$

$$v - u = at$$

$$\therefore \boxed{v = u + at}$$

$s = ut + \frac{1}{2}at^2$

As we know $v = \frac{ds}{dt} \quad \therefore ds = v\,dt = (u + at)\,dt$

Now integrating both side with limit $t = 0$ to $t = 0$ and $s = 0$ to $s = s$

$$\int_0^s ds = \int_0^t (u + at)\,dt = u\int_0^t dt + a\int_0^t t\,dt$$

$$[s]_0^s = u[t]_0^t + \frac{a}{2}[t^2]_0^t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

$v^2 = u^2 + 2as$

As we know $a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \cdot v$.

$$\therefore a\,ds = v\,dv.$$

Now integrating both side with the limit $s = 0$ to $s = s$ and $v = u$ to $v = v$,

$$\int_0^s a\,ds = \int_u^v v\,dv$$

$$\therefore a[s]_0^s = \left[\frac{v^2}{2}\right]_u^v = \frac{v^2}{2} - \frac{u^2}{2}$$

$$\therefore 2as = v^2 - u^2$$

$$\therefore \boxed{v^2 = u^2 + 2as}$$

$S_{nth} = u + \frac{a}{2}(2n - 1)$

As we know $v = \frac{ds}{dt} \quad \therefore ds = v\,dt = (u + at)\,dt$

Integrating on both side with limit $t = (n-1)$ to $t = n$ when $s = S_{n-1}$ to $s = S_n$.

$$\int_{S_{n-1}}^{S_n} ds = \int_{(n-1)}^{n} (u + at)\,dt = \int_{n-1}^{n} u\,dt + a\int_{n-1}^{n} t$$

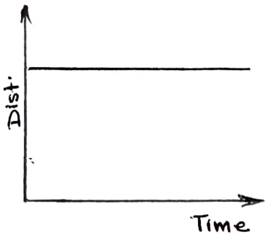
$$[s]_{S_{n-1}}^{n} = u[t]_{n-1}^{n} = \frac{a}{2}[t^2]_{n-1}^{n}$$

$$\therefore S_n - S_{n-1} = u[n - (n-1)] + \frac{a}{2}[n^2 - (n-1)^2]$$

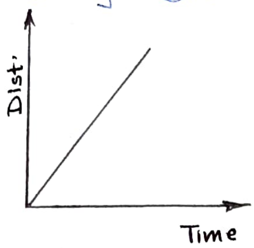
$$\boxed{S_{nth} = u + \frac{a}{2}(2n - 1)}$$

Analysing Nature of Motion From various Graphs :

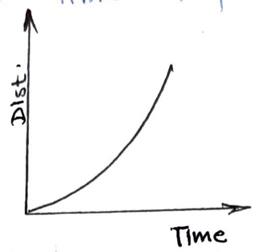
- Different types of Distance - Time Graph :



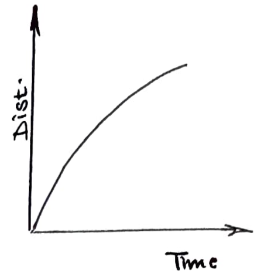
- Body at rest



- Body moves with uniform speed

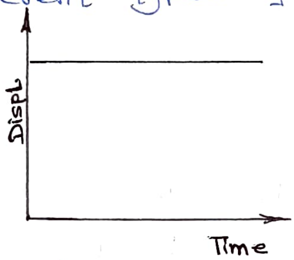


- Accelerated Motion

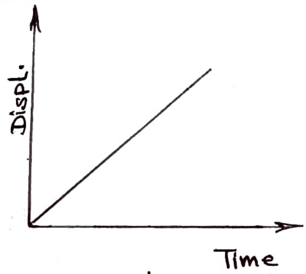


- Decelerated motion

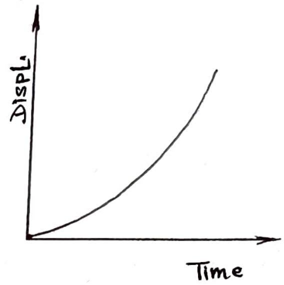
- Different types of displacement time graph:



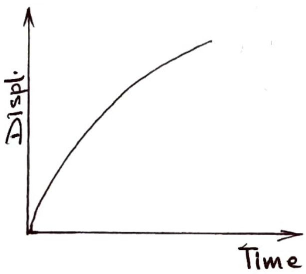
- Body at rest



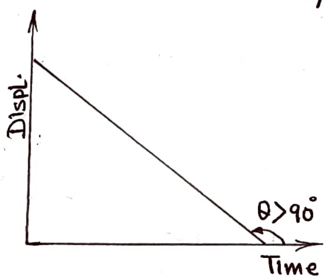
- Constant velocity



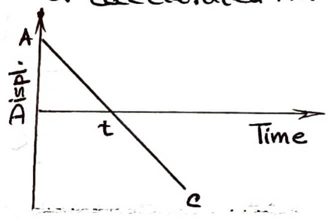
- Increasing velocity or accelerated motion



- Decreasing velocity or deceleration.

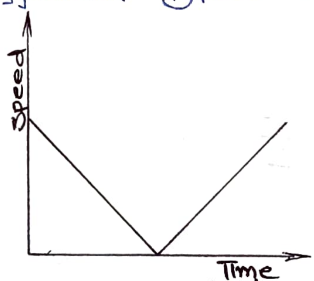


- Uniform negative velocity. The body is returning back to its original position with uniform velocity.

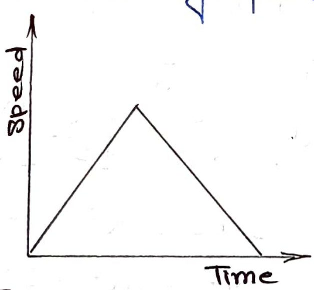


- Body returning from position A, moves past the original position B and then moves towards c with uniform velocity.

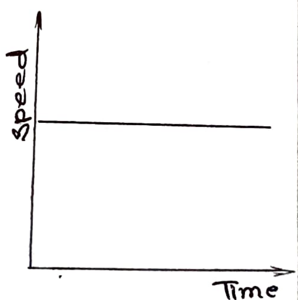
- Different types of Speed - Time graph:



- A body projected upward

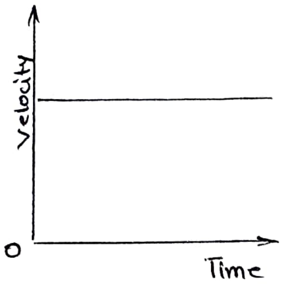


- For a ball dropped on the ground from a certain height and bounces back without any energy loss.

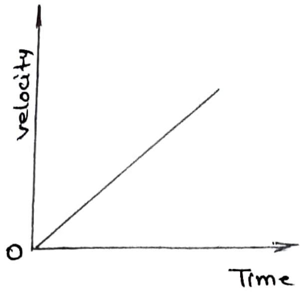


- Uniform speed.

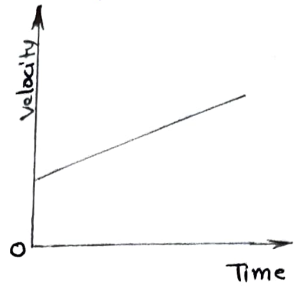
• Different types of Velocity - Time graph :



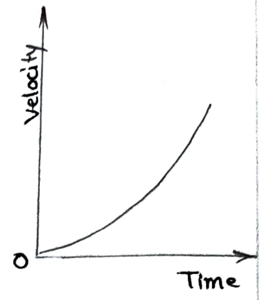
• Uniform velocity or zero acceleration



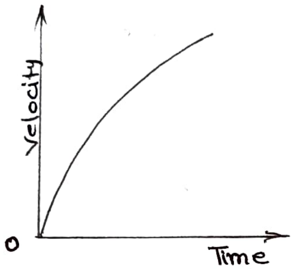
• Uniform Acceleration



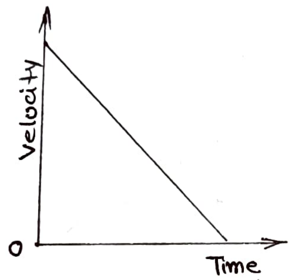
• Uniform acceleration with an initial velocity



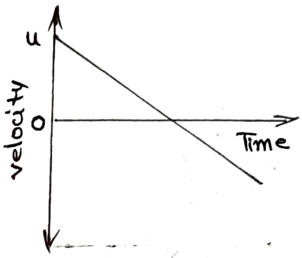
• Increasing acceleration.



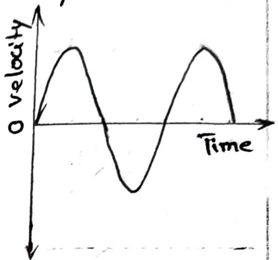
• Decreasing Acceleration



• Velocity decreases uniformely. Negative acceleration.

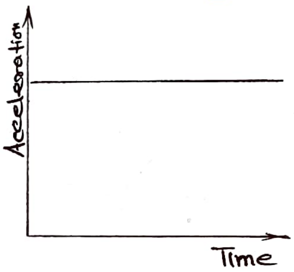


• Body projected upwards with initial velocity u.

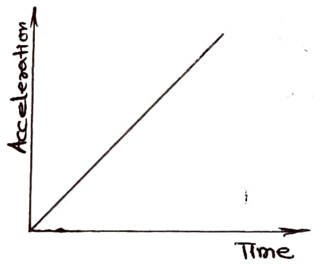


• Velocity varies sinusoidally with time

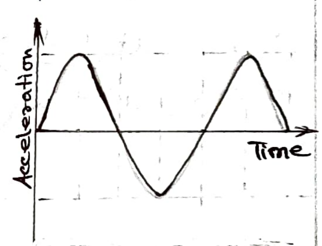
• Different Types of Acceleration - Time graph :



• Body moves with const acceleration

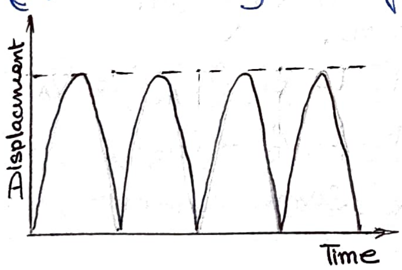


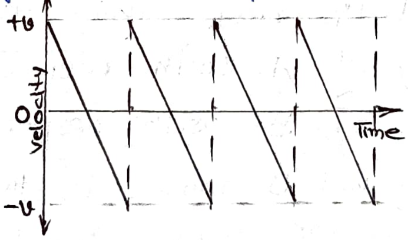
• Body moves with uniformely increase in acceleration.

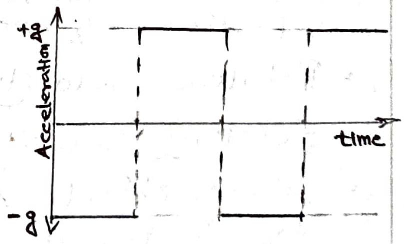


• Acceleration varies sinusoidally with time

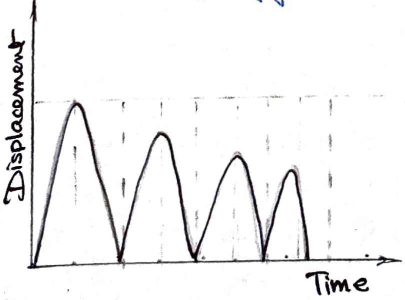
• Graph of a body thrown upward to a height and rebounding (without any energy loss during the impact)

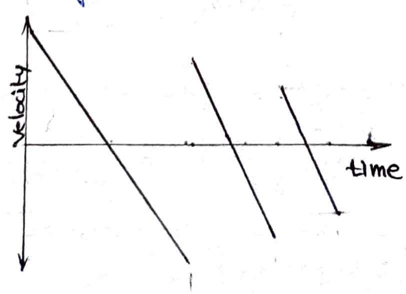


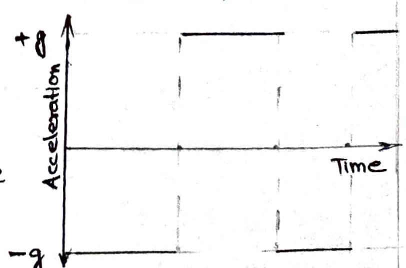




• Graph of a body thrown upward to a height and rebounding (with energy loss during the impact on the ground)







Motion under Gravity :

Sign conventions :

- The initial position of the body is taken to be the origin 'O'.
- For a freely falling body, the vertically downward direction is taken as positive. $(a = +g)$
- For a body projected vertically upwards, the vertically upward direction is taken as positive. $(a = -g)$

Equations of motion :

A body falls from rest from a height h.

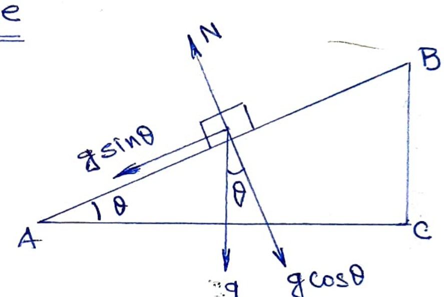
- $v = gt$ or $t = \frac{v}{g}$
- $h = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2h}{g}}$
- $v^2 = 2gh$ or $v = \sqrt{2gh}$.
- $h_{nth} = (2n-1)\frac{g}{2}$

A body is projected vertically up with initial velocity v_0.

- $v_0 = gt$ or $t = \frac{v_0}{g}$
- $h = v_0t - \frac{1}{2}gt^2$
- $v_0^2 = 2gh$ or $v_0 = \sqrt{2gh}$

Motion along a smooth inclined plane

- $v = u + (g\sin\theta)t$
- $S = ut + \frac{1}{2}(g\sin\theta)t^2$
- $v^2 = u^2 + 2(g\sin\theta)S$



Relative Velocity :

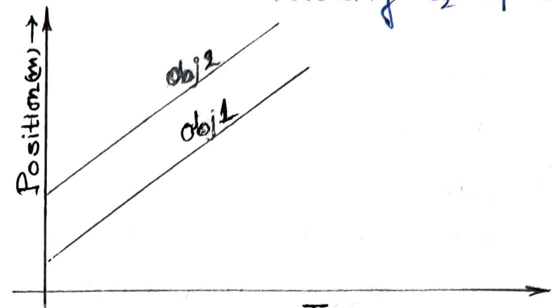
The relative velocity of a body A w.r.t another body B is the rate at which body A changes its position w.r.t body B.

Let us consider two bodies A and B moving with uniform velocities v_A and v_B respectively, then,

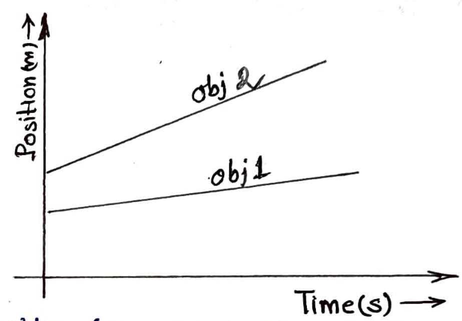
- Relative velocity of A with respect to B $(\vec{v}_{AB}) = \vec{v}_A - \vec{v}_B$
- Relative velocity of B with respect to A $(\vec{v}_{BA}) = \vec{v}_B - \vec{v}_A$
- When object A and B moves in the same direction, then $v_{AB} = v_A - v_B$ or $v_{BA} = v_B - v_A$
- When the object B moves in the opposite direction of A, then $v_{AB} = v_A - (-v_B) = v_A + v_B$.

Relative Velocity interms of Position - time graph :

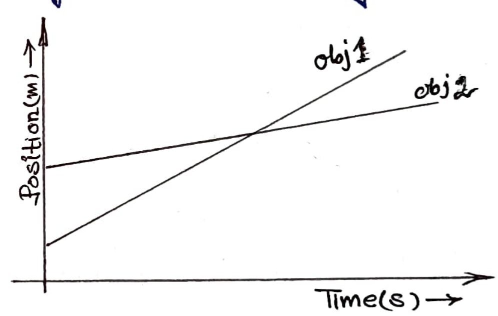
(i) When the two objects move with same velocity in the same direction. That is $v_1 = v_2$ and the relative velocity $v_2 - v_1 = 0$



(ii) When $v_2 > v_1$ or relative velocity $(v_2 - v_1)$ is positive



(iii) When $v_2 < v_1$ or relative velocity $(v_2 - v_1)$ is negative.



Determination of relative velocity :

If $\vec{V}_{AB}$ = Relative velocity of A. w.r.t B then

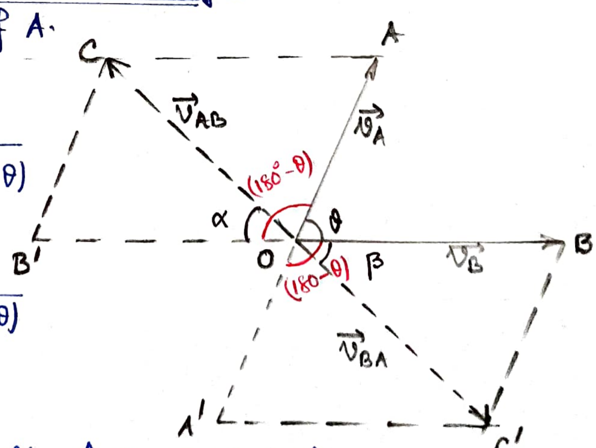
$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\therefore \; v_{AB} = \sqrt{v_A^2 + v_B^2 + 2 v_A v_B \cos(180^\circ - \theta)}$

$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos\theta}$

Again $\tan\alpha = \dfrac{v_A \sin(180 - \theta)}{v_B + v_A \cos(180 - \theta)}$

$\therefore \quad \tan\alpha = \dfrac{v_A \sin\theta}{v_B - v_A \cos\theta}$.



Similarly $\vec{v}_{BA}$ = Relative velocity of B w.r.t A then

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

$\therefore \; v_{BA} = \sqrt{v_A^2 + v_B^2 + 2 v_A v_B \cos(180^\circ - \theta)} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos\theta}$

Again $\tan\beta = \dfrac{v_A \sin(180^\circ - \theta)}{v_B + v_A \cos(180^\circ - \theta)} = \dfrac{v_A \sin\theta}{v_B - v_A \cos\theta}$.

$\therefore \quad v_{AB} = v_{BA} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos\theta}$ and

$\tan\alpha = \tan\beta$ or $\beta = \alpha$. $\therefore$ Magnitude is same but the direction is opposite to each other.

Special cases:

(i) When both the bodies are moving along parallel straight line in the same direction ($\theta = 0°$)

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos 0°} = (v_A - v_B)$$

Thus the relative velocity of A w.r.t B is equal to the difference between the magnitudes of their velocities.

(ii) When the two bodies are moving along parallel straight lines in the opposite directions. ($\theta = 180°$)

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos 180°} = v_A + v_B$$

Thus the relative velocity of body A w.r.t body B is equal to the sum of the magnitudes of their velocities.