II.

a	ь	a.b	a'	(a.b).a'
0	0	0	1	0
0	1	0	1	0
1	0	0	0	0
1	1	1	0	0

Hence, (a.b).a' is a contradiction.

Contingency: When we construct a truth table, the values applied in the columns may be 0's and 1's. If the final result of the truth table consists of the combination of 0's and 1's, is termed as 'Contingency'.

a	b	a+b	a.(a+b)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Solved examples

Example 1: Show that the expression (p.(p+q)) is equal to p.

p	q	p+q	p.(p+q)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Thus,
$$p = p.(p+q)$$

Example 2: Show that the expression $(p \Longrightarrow q) = (\sim q \Longrightarrow \sim p)$.

p	q	~p	~q	$\sim q \Longrightarrow \sim p$	$p \Longrightarrow q$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	. 0	0	1	1

Example 3. The propositions are given as:

p = Today is Sunday.

q = My father is at home.

r = We will go to a Mega Shop.

Express each of the following statement in words:

p + q	Today is Sunday OR my father is at home.
$(\sim p) + (\sim q)$	Today is not Sunday OR my father is not at home.
(~ p).q	Today is not Sunday AND my father is at home.
$\sim (p + q)$	It is not true that today is Sunday OR my father is at home.
~ (p.q)	It is not true that today is Sunday AND my father is at home
6. p+q.r Today is Sunday OR my father is at home Al Mega Shop.	
~(p + q).r	It is not true that today is Sunday OR my father is at home we will go to a Mega Shop.
	(~ p) + (~ q) (~ p).q ~ (p + q) ~ (p.q) p+q.r

Exercise

I. Write short answers:

- 1. What do you understand by 'Logic' and 'Proposition Logic'?
- 2. What is a proposition?
- 3. How will you distinguish a proposition from a compound proposition?
- 4. Define the following terms with a suitable example for each of them:
 - (a) Conjunction
- (b) Disjunctions
- (c) Negation

- (d) Converse
- (e) Inverse
- (f) Contradiction

- (g) Contrapositive
- (h) Tautology
- (i) Contingency

II. Answer the following:

- 1. The statements are given as:
 - p = Jitendra Kumar is a computer teacher.
 - q = Rahul is a student.

Write these statements in symbolic form:

- (a) Jitendra Kumar is a computer teacher and Rahul is a student.
- (b) Jitendra Kumar is a computer teacher and Rahul is not a student.
- (c) Jitendra Kumar is not a computer teacher and Rahul is a student.
- (d) Neither Jitendra Kumar is a computer teacher nor Rahul is a student.
- (e) Either Jitendra Kumar is a computer teacher or Rahul is a student.
- 2. Construct the 'Truth Table' for the following:

(a)
$$p + (\sim q)$$

(b) (
$$\sim$$
 p).q

$$(d) \sim (p.q)$$

(e)
$$(\sim p) + (\sim q)$$

- **3.** The statements are given as:
 - p = Today is a holiday.
 - q = I will go to attend a birth day party.

Express each of the following statement in words:

(a)
$$p \sim q$$

$$(b) \sim q$$

$$(c) \sim p$$

(d)
$$(\sim p) + (\sim q)$$

$$(f) \sim (p + q)$$

$$(g) P + q$$

$$(h) \sim (p.q)$$

- (i) \sim [(\sim p).q) (j) (\sim p).) \sim q) 4. Consider the statements:
 - p = You will work hard.
 - q = You will succeed in your life.

Translate each of these symbolic expressions in meaningful sentences.

(a)
$$p \Longrightarrow q$$

(b)
$$q \Longrightarrow p$$

(c)
$$(\sim p) \Longrightarrow (\sim q)$$
 (d) $(\sim q) \Longrightarrow (\sim p)$

5. Construct the truth tables for the following:

(a)
$$(p \Longrightarrow q)^{\wedge} (q \Longrightarrow p)$$

(b)
$$q = [(\sim p)v \ q]$$

- 6. Write converse, inverse and contrapositive of each statement:
 - (a) If it rains, then you will not play.
 - (b) If you work hard, then you will pass.
 - (c) If I run fast, then I will win the race.
- 7. Complete the following tables:

p	q	~p	$p \Longrightarrow q$	~pvq
0	0			
0	1			
1	0			
1	1			

(b)

p	q	p^q	pvq	$(p^q) \Longrightarrow (pvq)$
0	0			
0	1			1.00
1	0 .	1144		
1	1	p p l	4	

(c)

р	q	~p	~q	~pvq	p^(~q)
0	0				
0	1	emage Ties			
1	0				
1	1				

- 8. Show that the given statements are Tautologies.
 - (a) $[(p \Longrightarrow q) \land (q \Longrightarrow r)] \Longrightarrow (p \Longrightarrow r)$
 - (b) $[(p \Longrightarrow q)^{\land} p] \Longrightarrow q$

LOGIC GATES

INTRODUCTION

HISTORICAL DEVELOPMENT OF BOOLEAN ALGEBRA

Mathematicians and scientists have spent centuries to find the relation between Mathematics and Logic. The dream came true, when **George Boole** discovered a link between Mathematics and Logic in the year 1854. He developed 'Symbolic Logic', which became useful to solve logical problems in Mathematics and is known as 'Boolean Algebra'. Boolean algebra deals with either of two values 'True' or 'False' (0's or 1's).

In the year 1938, Claude Shannon developed a practical approach of Boolean algebra by applying in signal diversion through an electronic telephone. He used relays in the circuit to show the application of two states switching ('On and Off' process of relays). This is the reason, why Boolean algebra is also called as 'Switching Algebra'.