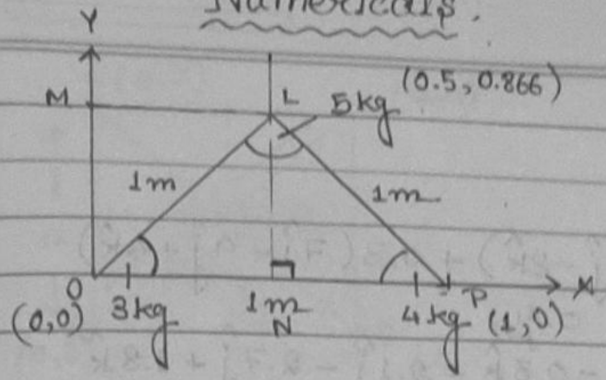
Numericals.

1)



$ON = \frac{1}{2} OP = 0.5\,m$

$OM = \sqrt{OL^2 - ON^2}$

$= \sqrt{1^2 - 0.5^2}$

$= \sqrt{0.75}$

$= 0.866$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}$$

$$= \frac{3\times 0 + 4\times 1 + 5\times 0.5}{3+4+5} = \frac{6.5}{12} = 0.5416\ m$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3}$$

$$= \frac{3\times 0 + 4\times 0 + 5\times 0.866}{12}$$

$$= 0.36\ m$$

Ans) (0.5416 m, 0.3608 m) → position of COM.

2) $m_1 = 100g$ $\quad \vec{r_1} = 2\hat{i} + 5\hat{j} + 13\hat{k}$ $\quad \vec{v_1} = 10\hat{i} - 7\hat{j} - 3\hat{k}$

$m_2 = 300g$ $\quad \vec{r_2} = -6\hat{i} + 4\hat{j} - 2\hat{k}$ $\quad \vec{v_2} = 7\hat{i} - 9\hat{j} + 6\hat{k}$

$$\vec{P}_{cm} = \frac{\Sigma m\vec{r}}{M}$$

$$= \frac{0.1(2\hat{i} + 5\hat{j} + 13\hat{k}) + 0.3(-6\hat{i} + 4\hat{j} - 2\hat{k})}{0.1 + 0.3}$$

$$= \frac{(0.2 - 1.8)\hat{i} + (0.5 + 1.2)\hat{j} + (1.3 - 0.6)\hat{k}}{0.4}$$

$$= \frac{-1.6\hat{i} + 1.7\hat{j} + 0.7\hat{k}}{0.4}$$

$$= -4\hat{i} + 4.25\hat{j} + 1.75\hat{k}$$

or $\dfrac{-16\hat{i} + 17\hat{j} + 7\hat{k}}{4}$ (Ans).

$$\vec{v}_{cm} = \frac{\sum m\vec{v}}{M}$$

$$= \frac{0.1(10\hat{i} - 7\hat{j} - 3\hat{k}) + 0.3(7\hat{i} - 9\hat{j} + 6\hat{k})}{0.4}$$

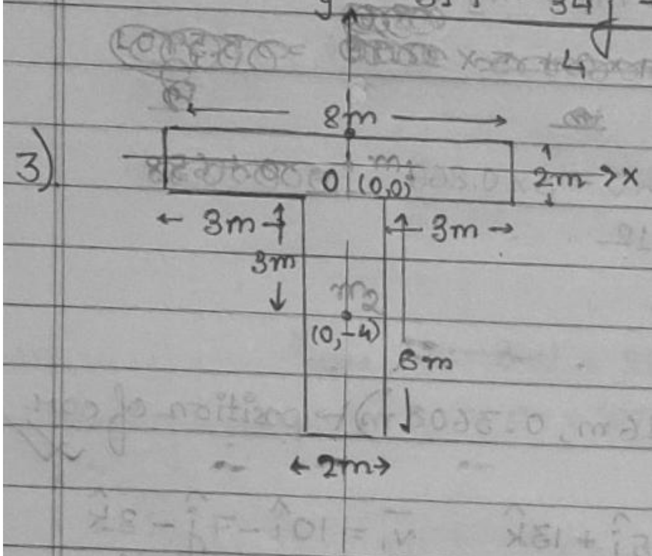
$$= \frac{1\hat{i} - 0.7\hat{j} - 0.3\hat{k} + 2.1\hat{i} - 2.7\hat{j} + 1.8\hat{k}}{0.4}$$

$$= \frac{(1+2.1)\hat{i} + (-0.7-2.7)\hat{j} + (-0.3+1.8)\hat{k}}{0.4}$$

$$= \frac{3.1\hat{i} + (-3.4)\hat{j} + 1.5\hat{k}}{0.4}$$

$$= \frac{31\hat{i} - 34\hat{j} + 15\hat{k}}{4} \quad \text{(Ans)}$$

3)



mass per unit area = σ

m_1 = area × σ

$= 8 \times 2 \times \sigma = 16\sigma$

m_2 = area × σ

$= 6 \times 2 \times \sigma = 12\sigma$

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{16\sigma \times 0 + 12\sigma \times 0}{(16+12)\sigma} = 0$$

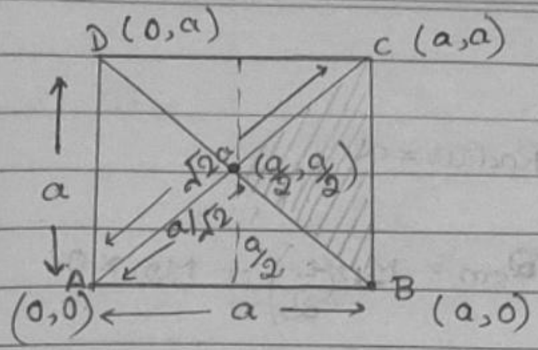
$$y_{com} = \frac{m_1y_1 + m_2y_2}{m_1+m_2} = \frac{16\sigma \times 0 + 12\sigma \times (-4)}{28\sigma}$$

$$= \frac{-48\cancel{\sigma}}{28\cancel{\sigma}} = \frac{-12}{7} = -1.7$$

COM = $(0, -1.7)$ — Ans

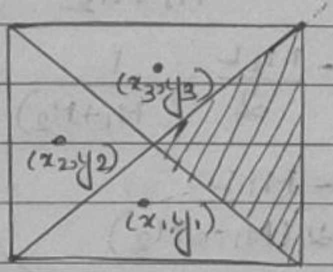
centroid of Angle = $\left(\frac{x_1+x_2+x_3}{3} \; ; \; \frac{y_1+y_2+y_3}{3}\right)$

4)



Let m be mass of each triangle

$$x_1 = \frac{0+a+\frac{a}{2}}{3} = \frac{2a+a}{2} \times \frac{1}{3} = \frac{a}{2}.$$



$$y_1 = \frac{0+\frac{a}{2}+0}{3} = \frac{a}{6}.$$

$$x_2 = \frac{0+0+\frac{a}{2}}{3} = \frac{a}{6}. \qquad y_2 = \frac{a+0+a/2}{3} = \frac{a}{2}.$$

$$x_3 = \frac{0+a+a/2}{3} = \frac{a}{2}. \qquad y_3 = \frac{a+a+a/2}{3} = \frac{5a}{2} \times \frac{1}{3} = \frac{5a}{6}.$$

$$x_{cm} = \frac{\frac{a}{2} \times m + \frac{a}{6} m + \frac{a}{2} m}{3m} = \frac{\frac{3am+am+3am}{6}}{3m}$$

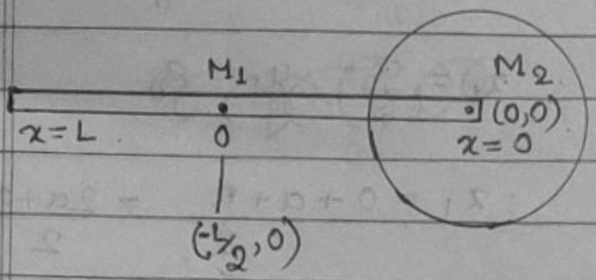
$$= \frac{7a}{18}.$$

$$y_{cm} = \frac{\frac{a}{6} m + \frac{a}{2} m + \frac{5am}{6}}{3m} = \frac{am+3am+5am}{6 \times 3m}.$$

$$= \frac{9a}{18} = \frac{a}{2}.$$

Ans) $\left(\frac{7a}{18}, \frac{a}{2}\right)$

5)



Radius = a

$$x_{cm} = \frac{M_1 \times \left(-\frac{L}{2}\right) + M_2 \times 0}{M_1 + M_2}$$

$y_{com} = 0$ by symmetry

$$= -\frac{M_1 L}{2} \times \frac{1}{(M_1 + M_2)}$$

$$= -\frac{M_1 L}{2(M_1 + M_2)}$$

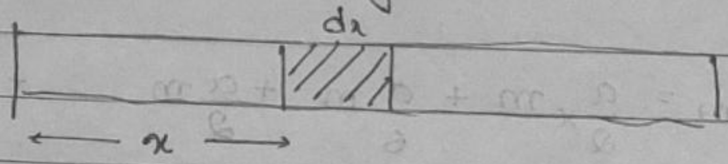
6) $\lambda = \frac{\delta_0 x^2}{L}$

$\delta \to$ mass per unit length

$\delta_0 \to$ constant

$x \to$ distance

$L \to$ length

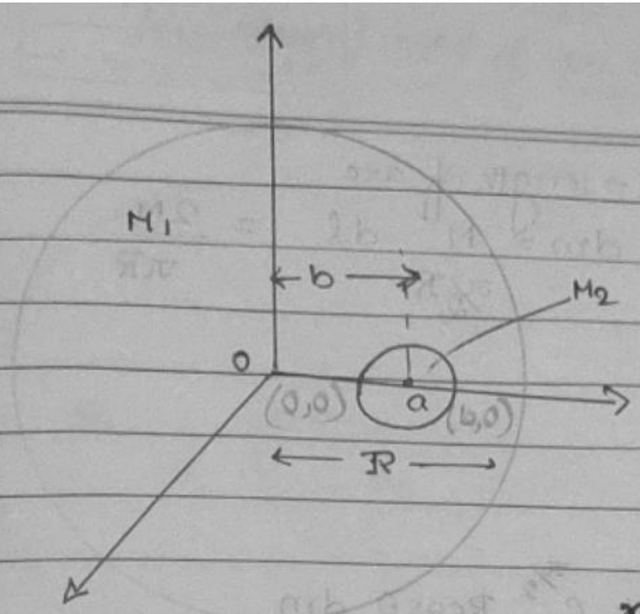


$$x_{cm} = \frac{\int_0^L \frac{\delta_0 x^2 dx}{L} \cdot x}{\int_0^L \frac{\delta_0 x^2}{L} dx} = \frac{\int_0^L \frac{\delta_0 x^3}{L} dx}{\int_0^L \frac{\delta_0 x^2}{L} dx}$$

$$= \frac{\cancel{\frac{\delta_0}{L}} \left[\frac{x^4}{4}\right]_0^L}{\cancel{\frac{\delta_0}{L}} \left[\frac{x^3}{3}\right]_0^L} = \frac{\frac{L^4}{4}}{\frac{L^3}{3}} = \frac{L^4}{4} \times \frac{3}{\cancel{L^3}} = \frac{3L}{4}$$

Ans) $\frac{3L}{4}$.

7)



density ρ

$$\Rightarrow M_1 = \left(\rho \times \frac{4}{3}\pi R^3\right)$$

$$M_2 = \rho \times \frac{4}{3}\pi a^3$$

$$\cancel{x_{com} = \frac{m_1x_1 + m_2x_2}{m_1+m_2}}$$

$M' =$ mass of the remaining portion.

$$= M_1 - M_2$$

$$= \frac{4}{3}\pi\rho(R^3 - a^3)$$

$$x_{com} = \frac{M' \times x - M_2(b)}{M' - M_2}$$

$$= \frac{\frac{4}{3}\pi\rho(R^3 - a^3)x - \frac{4}{3}\pi a^3\rho(b)}{\frac{4}{3}\pi\rho(R^3-a^3) - \rho\frac{4}{3}\pi a^3}$$

$$= \frac{x(R^3 - a^3) - a^3 b}{(R^3 - a^3) - a^3} = 0$$

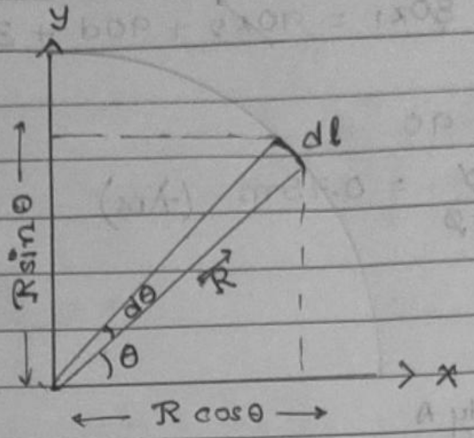
$$\therefore x(R^3 - a^3) - a^3 b = 0$$

$$\Rightarrow x = \frac{a^3 b}{R^3 - a^3}$$

$$\therefore x = \frac{-a^3 b}{R^3 - a^3}$$ (shifted in left of origin)

8)

$dl = R\,d\theta$



$$\lambda = \frac{M}{L} = \frac{M}{\frac{2\pi R}{4}} = \frac{M \times 2}{\pi R}$$

$$dm = \lambda\, dl = \frac{2M}{\pi R} R\,d\theta$$

$$x_{com} = \frac{1}{M}\int_0^{\pi/2} x\, dm$$

$$= \frac{1}{M}\int_0^{\pi/2} R\cos\theta \cdot \frac{2M}{\pi \cancel{R}} \cancel{R}\,d\theta$$

$$= \frac{2R\cancel{M}}{\cancel{M}\pi}\int_0^{\pi/2} \cos\theta\, d\theta$$

$$= \frac{2R}{\pi}\left[\sin\theta\right]_0^{\pi/2}$$

$$= \frac{2R}{\pi} \times 1$$

$$= \frac{2R}{\pi}$$

$$y_{com} = \frac{1}{M}\int_0^{\pi/2} y\, dm = \frac{1}{M}\int_0^{\pi/2} R\sin\theta \cdot \frac{2M}{\pi}\, d\theta$$

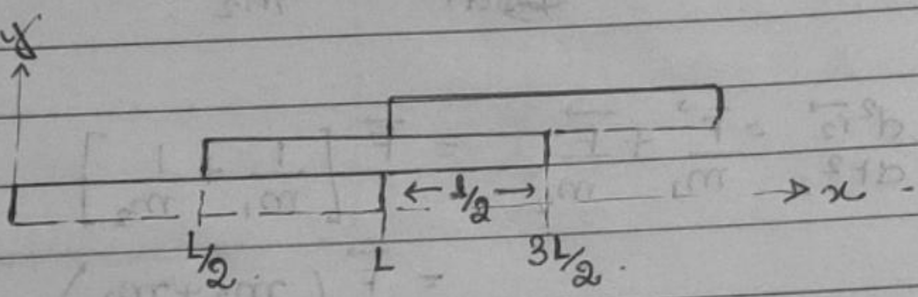
$$= \frac{R \times 2\cancel{M}}{\cancel{M} \times \pi}\int_0^{\pi/2} \sin\theta \cdot d\theta$$

$$= \frac{2R}{\pi}\left[-\cos\theta\right]_0^{\pi/2}$$

$$= \frac{2R}{\pi}(0+1) = \frac{2R}{\pi}$$

Ans) $\left(\frac{2R}{\pi}, \frac{2R}{\pi}\right)$

9).



$$\frac{M \times L + M \times 3L/2}{3M} = \frac{6L}{2} \times \frac{1}{3} = L.$$

$$\frac{M \times L + M \times 3L/2}{2M} = \frac{5L}{4}.$$

$$\frac{5L}{4} - \frac{L}{2} = \frac{5L - 2L}{4} = \frac{3L}{4}$$