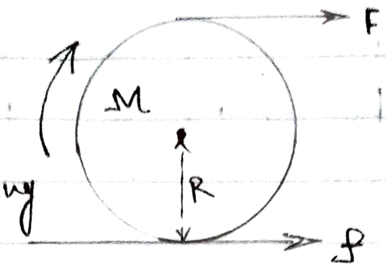
Accelerated Pure Rolling :

Linear velocity = v
Angular velocity = ω
Condition for pure rolling
$v = R\omega$



$\frac{dv}{dt} = R\frac{d\omega}{dt}$ or $a = R\alpha$

Linear Acceleration = R× angular acceleration.

- If $a = R\alpha$, then frictional force $f = 0$.
- If $a < R\alpha$, f will act in forward direction
- If $a > R\alpha$, f will act in the backward direction.

From figure

$$a = \frac{F_{net}}{M} = \frac{F+f}{M}$$

$$\alpha = \frac{\tau_c}{I} = \frac{(F-f)R}{I}$$

For pure rolling $a = R\alpha$.

$$\frac{F+f}{M} = \frac{(F-f)R^2}{I}$$

$$\Rightarrow IF + If = MR^2F - MR^2f$$

$$\Rightarrow f(I + MR^2) = (MR^2 - I)F$$

$$\boxed{f = \left(\frac{MR^2 - I}{MR^2 + I}\right)F}$$

(i) $I = MR^2$ then $f = 0$

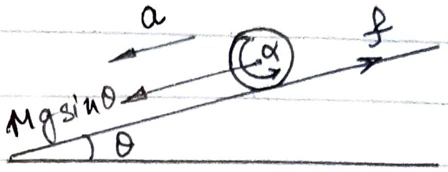
(ii) If $I < MR^2$ $\therefore f$ is positive, f is in forward direction.

(iii) If $I > MR^2$ f is in backward direction.

Although $I > MR^2$ not possible so the force of friction is either in the forward direction or zero.

Rolling on rough inclined plane

$$a = \frac{Mg\sin\theta - f}{M}$$



$$\alpha = \frac{\tau}{I} = \frac{fR}{I}$$

For pure rolling

$$a = R\alpha$$

$$\frac{Mg\sin\theta - f}{M} = \frac{fR^2}{I}$$

$$\Rightarrow \quad IMg\sin\theta - If = MfR^2$$

$$\Rightarrow \quad f(I + MR^2) = IMg\sin\theta$$

$$f = \frac{IMg\sin\theta}{(I+MR^2)} = \frac{Mg\sin\theta}{\left(1 + \frac{MR^2}{I}\right)}$$

$$a = \frac{Mg\sin\theta - f}{M} = g\sin\theta - \frac{f}{M}$$

$$a = g\sin\theta - \frac{Mg\sin\theta}{\left(1+\frac{MR^2}{I}\right)M}$$

$$= g\sin\theta\left[1 - \frac{1}{1+\frac{MR^2}{I}}\right]$$

$$= g\sin\theta\left[1 - \frac{I}{I+MR^2}\right]$$

$$= g\sin\theta\left[\frac{MR^2}{I+MR^2}\right]$$

$$\boxed{a = \frac{g\sin\theta}{\left[1 + \frac{I}{MR^2}\right]}}$$

Solid sphere, hollow sphere, cylinder.

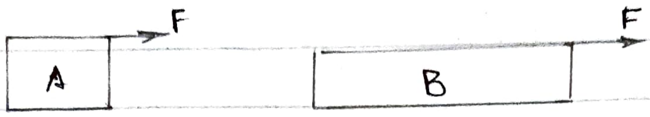
$I_{sphere} = \frac{2}{5}MR^2$ $\quad I_{hollow} = \frac{2MR^2}{3}$ $\quad I_{cyl} = \frac{1}{2}MR^2$

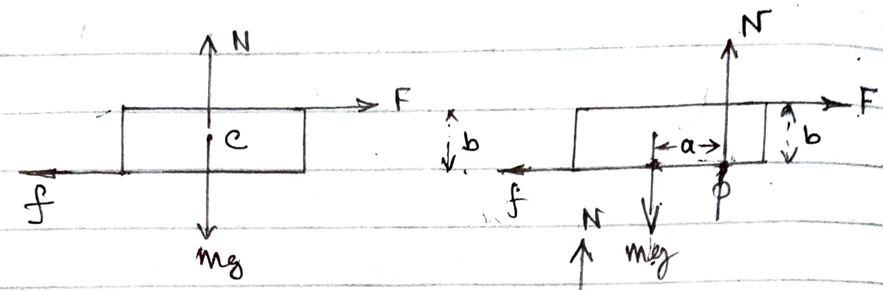
Angular Impulse :

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \therefore \quad \vec{\tau}\cdot dt = d\vec{L}$$

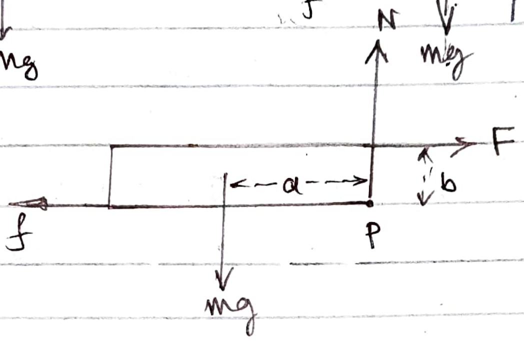
$$\int_{t_1}^{t_2} \vec{\tau}\,dt = \text{Angular impulse} = d\vec{L} = \vec{L_2} - \vec{L_1}$$

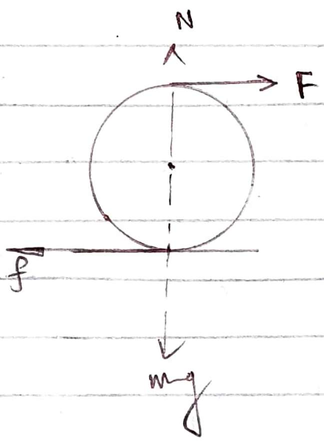
Toppling of a body





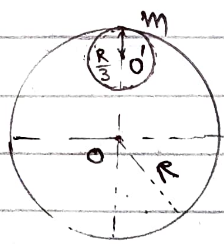
$$Fb = (mg)\,a$$





Numericals

1. Find the M.I of the remaining disc about an axis perpendicular to the plane of the disc passing through O.



Let $\sigma \rightarrow$ Mass per unit area.

$$m = \pi\left(\frac{R}{3}\right)^2 \sigma \qquad \frac{m}{M} = \frac{1}{9} \qquad \therefore M = 9m$$

$$M = \pi R^2 \sigma$$

M.I of the small disc about $O' = \frac{1}{2} m \left(\frac{R}{3}\right)^2$

$$= \frac{1}{2} m \frac{R^2}{9}$$

M.I of the small disc about $O = \frac{1}{2} m \frac{R^2}{9} + m\left(\frac{2R}{3}\right)^2$

$$= \frac{1}{9}\cdot\frac{1}{2} MR^2 + \frac{1}{9}\cdot\frac{4MR^2}{9}$$

$$\therefore \frac{1}{2}\,\frac{M}{9}\,R^2 = \frac{1}{18} MR^2$$

$$= \frac{1}{18} MR^2 + \frac{8}{18} MR^2$$

$$= \frac{1}{2} MR^2$$

$\therefore$ M.I of the full disc about O $= \frac{1}{2}MR^2$

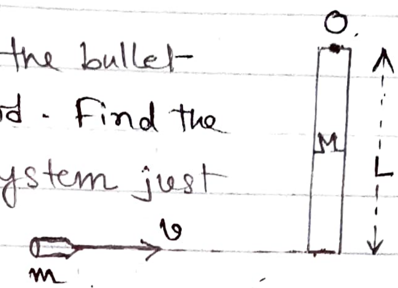
$\therefore$ M.I of the remaing portion $= \left(\frac{1}{2}MR^2 - \frac{1}{18}MR^2\right)$

$$= \frac{MR^2}{2}\left[1 - \frac{1}{9}\right]$$

$$= \frac{4MR^2}{9}.$$

Problem 2

After the strike, the bullet gets embedded in the rod. Find the angular velocity of the system just after the impact.



Before impact initial angular momentum

$L_i = mvL$

After impact final angular momentum.

$L_f = I\omega$

But $I = I_{bullet} + I_{rod}$

$= mL^2 + \frac{1}{3}ML^2$

$= \frac{M+3m}{3}L^2$

$\therefore$ By Conservation of angular momentum

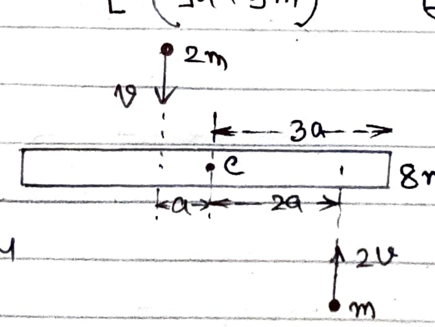
$L_i = L_f$

$mvL = \left(\frac{M+3m}{3}L^2\right)\omega$

$\omega = \dfrac{3mvL}{L^2(M+3m)} = \dfrac{3mv}{(M+3m)L}$

Problem 3

After collision m and 2m stick to the rod.



Determine (i) velocity of COM

(ii) ω about COM

(iii) Total Kinetic energy just after the collision

By Conservation of linear momentum

$2mv - 2mv + 8m \times 0 = (8m + m + 2m) v_c$

$v_c = 0$

Initial angular momentum $L_i = 2mv(a) + 2mv(2a)$

$= 6mva.$

Final angular momentum $L_f = I_{total}\,\omega$

$$I_{total} = 2ma^2 + m(2a)^2 + 8m\frac{(6a)^2}{12}$$

$$= 2ma^2 + 4ma^2 + 24ma^2$$

$$= 30ma^2$$

$\therefore$ $L_f = 30ma^2\omega$

$\therefore$ By conservation of angular momentum

$$L_i = L_f$$

$$6mva = 3ma^2\omega$$

$$\omega = \frac{6mva}{30ma^2} = \frac{v}{5a}$$

Total Kinetic energy $E_K = \frac{1}{2} I_{total}\,\omega^2$

$$= \frac{1}{2}(30ma^2)\,\omega^2$$

$$= 15ma^2 \cdot \frac{v^2}{25a^2}$$

$$E_K = \frac{3}{5}mv^2.$$