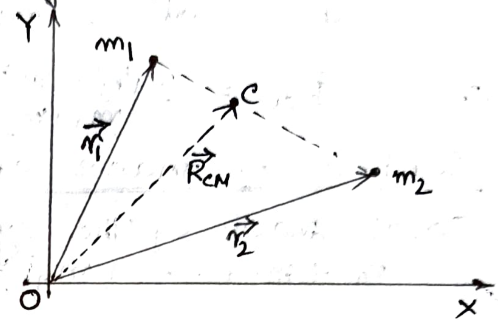
System of Particles

Centre of Mass : Centre of mass of a system of particles is that single point which moves in the same way in which a single particle having the total mass of the system and acted upon by the same external force.

Centre of mass of a two particle - System :



Consider a system of two particles P_1 and P_2 of masses m_1 and m_2. Let $\vec{r_1}$ and $\vec{r_2}$ be their position vectors with respect the origin O, as shown in fig.

The position vector $\vec{R}_{CM}$ of the centre of mass C of the two-particle system is given by

$$\vec{R}_{CM} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2}.$$

$$\therefore \quad (m_1 + m_2)\vec{R}_{CM} = m_1\vec{r_1} + m_2\vec{r_2}$$

If $m_1 = m_2 = m$ (say)

$$\vec{R}_{CM} = \frac{\vec{r_1} + \vec{r_2}}{2}$$

If (x_1, y_1) and (x_2, y_2) are the co-ordinates of the locations of the two particles, the coordinates of their centre of mass are given by

$$\vec{x}_{CM} = \frac{m_1\vec{x_1} + m_2\vec{x_2}}{m_1 + m_2}$$

and

$$\vec{y}_{CM} = \frac{m_1\vec{y_1} + m_2\vec{y_2}}{m_1 + m_2}.$$

The velocity and acceleration vectors of the two particles

$$\vec{v_1} = \frac{d\vec{r_1}}{dt} \quad \text{and} \quad \vec{a_1} = \frac{d\vec{v_1}}{dt} = \frac{d^2\vec{r_1}}{dt^2}$$

$$\vec{v_2} = \frac{d\vec{r_2}}{dt} \quad \text{and} \quad \vec{a_2} = \frac{d\vec{v_2}}{dt} = \frac{d^2\vec{r_2}}{dt^2}$$

$\therefore$ Total force on P_1 and P_2

$$\vec{F_1} = \vec{F_{12}} + \vec{F_1}^{ext} \quad \text{and} \quad \vec{F_2} = \vec{F_{21}} + \vec{F_2}^{ext}$$

But

$$m_1\vec{a_1} = \vec{F_1} = \vec{F_{12}} + \vec{F_1}^{ext}$$

$$m_2\vec{a_2} = \vec{F_2} = \vec{F_{21}} + \vec{F_2}^{ext}$$

$$\therefore \quad m_1\vec{a_1} + m_2\vec{a_2} = \vec{F_{12}} + \vec{F_{21}} + \vec{F_1}^{ext} + \vec{F_2}^{ext}$$

According to Newton's third Law, the internal forces mutually exerted by the two particles are equal and opposite. i.e

$$\vec{F_{12}} = -\vec{F_{21}} \quad \text{and} \quad \vec{F_{12}} + \vec{F_{21}} = 0$$

$$m_1\vec{a_1} + m_2\vec{a_2} = \vec{F}$$

Suppose the total mass of the system is $M = m_1 + m_2$

Let $\vec{F}$ be the total force acting on the system.

$$M\vec{a}_{cm} = \vec{F}.$$

$$\therefore \quad M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 = m_1\frac{d^2\vec{r}_1}{dt^2} + m_2\frac{d^2\vec{r}_2}{dt^2}$$

$$= \frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2)$$

$$\therefore \quad \vec{a}_{cm} = \frac{1}{M}\frac{d^2}{dt^2}(m_1\vec{r}_1 + m_2\vec{r}_2)$$

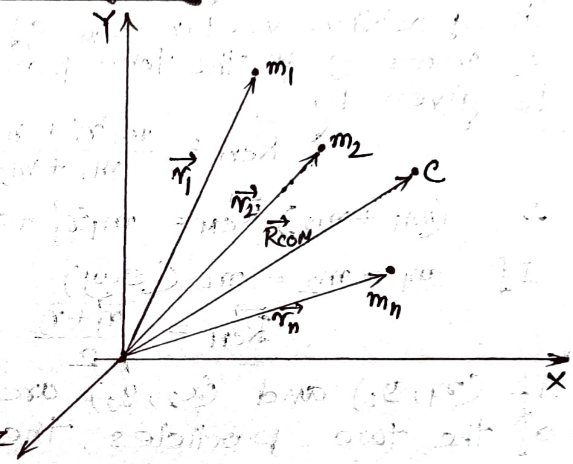
$$\frac{d^2\vec{R}_{cM}}{dt^2} = \frac{d^2}{dt^2}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}\right)$$

$$\therefore \quad \vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}.$$

Centre of mass of n-particle system

Total Mass of the system

$$m_1 + m_2 + \cdots + m_n = M$$



Position of COM

$$\vec{R}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1+m_2+\cdots+m_n}$$

$$\vec{R}_{COM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$

If x_{COM}, y_{COM} and z_{COM} are the cartesian coordinates of the COM of the n-particles system, then

$$\vec{x}_{COM} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1+m_2+\cdots+m_n} = \frac{\sum_{i=1}^{n} m_ix_i}{M}$$

$$\vec{y}_{COM} = \frac{m_1\vec{y}_1 + m_2\vec{y}_2 + \cdots + m_n\vec{y}_n}{m_1+m_2+\cdots+m_n} = \frac{\sum_{i=1}^{n} m_i\vec{y}_i}{M}$$

$$\vec{z}_{COM} = \frac{m_1\vec{z}_1 + m_2\vec{z}_2 + \cdots + m_nz_n}{m_1+m_2+\cdots+m_n} = \frac{\sum_{i=1}^{n} m_iz_i}{M}$$

Position of COM of Continuous Bodies

$$x_{COM} = \frac{\int x\,dm}{\int dm} = \frac{\int x\,dm}{M}$$

$$y_{COM} = \frac{\int y\,dm}{\int dm} = \frac{\int y\,dm}{M}$$

$$z_{COM} = \frac{\int z\,dm}{\int dm} = \frac{\int z\,dm}{M}$$

Motion of the COM

Centre of mass for n-particle system.

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + \cdots + m_n} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{M}$$

$$\therefore \quad M\vec{r}_{COM} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n$$

Differentiating this expression with respect to time t.

$$M\frac{d\vec{r}_{COM}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt}.$$

Since $\frac{d\vec{r}}{dt}$ = velocity

$$\therefore \quad M\vec{V}_{COM} = m_1\vec{V}_1 + m_2\vec{V}_2 + \cdots + m_n\vec{V}_n \quad \text{------ (i)}$$

$$\vec{V}_{COM} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + \cdots + m_n\vec{V}_n}{M}$$

$$\vec{V}_{COM} = \frac{\sum_{i=1}^{n} m_i\vec{V}_i}{M}$$

Furthure, $m\vec{V}$ = momentum of a particle $\vec{P}$

$$\therefore \quad \vec{P}_{COM} = \vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n$$

$$\vec{P}_{COM} = \sum_{i=1}^{n} \vec{P}_i$$

Differentiating Eq (i) w.r.t time t, we get

$$M\frac{d\vec{V}_{COM}}{dt} = m_1\frac{d\vec{V}_1}{dt} + m_2\frac{d\vec{V}_2}{dt} + \cdots + m_n\frac{d\vec{V}_n}{dt}.$$

$$M\vec{a}_{COM} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_n$$

$$\vec{a}_{COM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_n}{M}$$

$$\vec{a}_{COM} = \frac{\sum_{i=1}^{n} m_i\vec{a}_i}{M}$$

$\therefore$ Net force

$$\vec{F}_{COM} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

$$\vec{F}_{COM} = \sum_{i=1}^{n} \vec{F}_i$$

Momentum conservation and Centre of mass motion.

Suppose $\vec{F}_1, \vec{F}_2, \cdots \vec{F}_n$ forces are exerted on the masses $m_1, m_2 \cdots m_n$ respectively and produces an acceleration $\vec{a}_1, \vec{a}_2, \cdots \vec{a}_n$ respectively.

$$\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_n$$

$$\therefore \quad \vec{F}_{ext} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + \cdots + m_n\frac{d\vec{v}_n}{dt}$$

$$= \frac{d}{dt}(m_1\vec{v}_1) + \frac{d}{dt}(m_2\vec{v}_2) + \cdots + \frac{d}{dt}(m_n\vec{v}_n)$$

$$= \frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n)$$

If no external force is applied then $\vec{F}_{ext} = 0$

$$\therefore \frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n\right) = 0$$

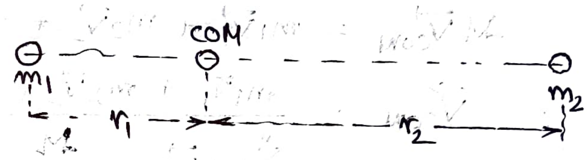
$$\frac{d}{dt}\left(\vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n\right) = 0$$

or, $$\vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n = \text{Const.}$$

$\therefore$ $\vec{P} = \vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n$ = Const if no external force is applied on the system

Position of Centre of mass of two particle System

Centre of mass of two particles of mass m_1 and m_2 separated by a distance of d lies in between two particles. The distance of COM of mass from any of the particle (r) is inversely proportional to the mass of the particle (m)



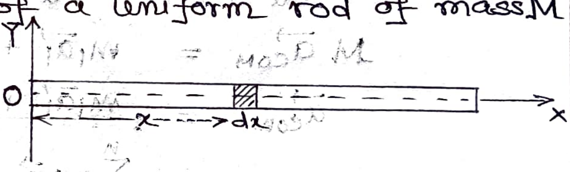
$$r \propto \frac{1}{m}$$

$$\therefore \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$\therefore m_1 r_1 = m_2 r_2$$

$$\therefore r_1 = \left(\frac{m_2}{m_1+m_2}\right)d \quad \text{and} \quad r_2 = \left(\frac{m_1}{m_1+m_2}\right)d$$

- Determination Centre of mass of a uniform rod of mass M and Length L.

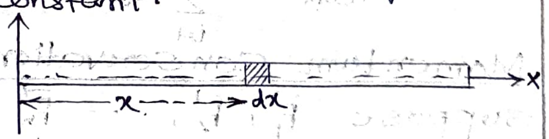


Mass of the small element $dm = \frac{M}{L}dx$

$$\therefore x_{COM} = \frac{1}{M}\int_0^L x\,dm$$

$$= \frac{1}{M}\int_0^L x\cdot\frac{M}{L}dx = \frac{1}{L}\int_0^L x\,dx = \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{L}{2}.$$

- Determination of COM of a rod with linear density $\rho = a + bx$ where a and b are constant.



$dm = \rho\,dx = (a+bx)\,dx$

The coordinates of COM $(x, 0, 0)$

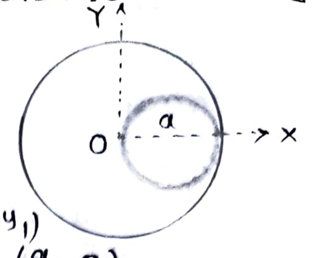
$$\therefore x_{COM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L (x)(a+bx)\,dx}{\int_0^L (a+bx)\,dx}$$

$$= \frac{\left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^L}{\left[ax + \frac{bx^2}{2}\right]_0^L}$$

$$\therefore x_{COM} = \frac{3aL + 2bL^2}{6a + 3bL}$$

$\therefore$ Hence position of COM $\left[\frac{3aL+2bL^2}{6a+3bL}, 0, 0\right]$

• Find the position of COM of a uniform lamina shown in fig.



Method 1 (Negative Mass)

A_1 = Area of the Complete circle $= \pi a^2$

$A_2 = \pi (a/2)^2 = \frac{\pi a^2}{4}$ = Area of the cut portion.

Position of COM of the full lamina $(0,0) = (x_1, y_1)$

Position of COM of the small lamina $(x_2, y_2) = (\frac{a}{2}, 0)$

$$\therefore \quad x_{COM} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$x_{COM} = \frac{0 - \frac{\pi a^2}{4}(a/2)}{\pi a^2 - \frac{\pi a^2}{4}} = \frac{-(1/8)}{(3/4)} a = -\frac{a}{6}$$

$\therefore \quad y_{COM} = 0$ as y_1 and y_2 both are zero.

$\therefore$ Co-ordinates of COM of the lamina shown in figure are $(-\frac{a}{6}, 0)$.

Method 2 (Moment of mass)

Mass of the Complete Lamina $M = \pi a^2 \sigma$ [$\sigma \rightarrow$ Linear mass density]

Mass of the cut portion $m = \pi (a/2)^2 \sigma = \frac{\pi a^2 \sigma}{4}$

Mass of the remaining portion $M' = (M - m) = \pi a^2 \sigma - \frac{\pi a^2 \sigma}{4}$

$$M' = \frac{3}{4} \pi a^2 \sigma.$$

Let the COM is located x distance apart from the Centre of the lamina.

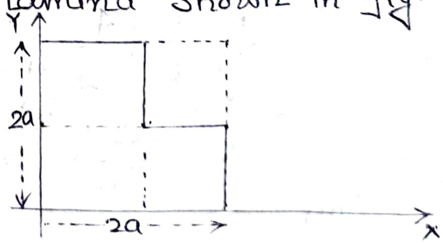
$$\therefore \quad M' x = m \times \frac{a}{2}$$

$$x = \frac{m}{M'} \times \frac{a}{2} = \frac{\frac{\pi a^2 \sigma}{4}}{3/4 \, \pi a^2 \sigma} = \frac{a}{6} \text{ (Left of O)}$$

$\therefore$ By Symmetry $y = 0$

$\therefore$ Position of COM $(-\frac{a}{6}, 0)$

• Find the position of COM of a uniform lamina shown in fig.



As we know
$$X_{COM} = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2}$$

$$Y_{COM} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2}$$

In this case

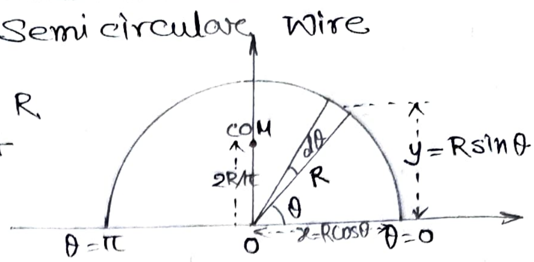
$$X_{COM} = \frac{4a^2 \times a - a^2 (3a/2)}{4a^2 - a^2} = \frac{4a - 3a/2}{3}$$

$$= \frac{5a}{6}$$

$$Y_{COM} = \frac{(4a^2) a - a^2 (3a/2)}{4a^2 - a^2}$$

$$= \frac{5a}{6}$$

• Center of mass of a Uniform Semi circular Wire



If M is the mass of the wire and R is its radius, then mass per unit length of the wire, $\lambda = \frac{M}{\pi R}$

Let us consider an element of length dl of the wire

$\therefore$ Mass of the element, $dm = \lambda dl = \left(\frac{M}{\pi R}\right)(R d\theta) = \frac{M}{\pi} d\theta$.

$\therefore$ Co-ordinate of COM

$$x_{COM} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^{\pi}(R\cos\theta)\,\frac{M}{\pi}\,d\theta$$

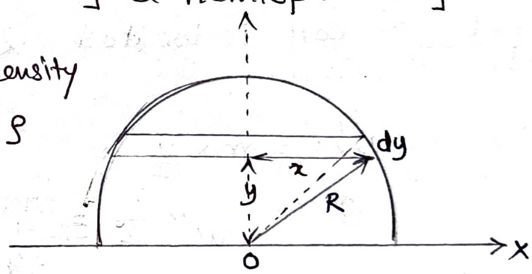
$$= \frac{R}{\pi}\int_0^{\pi}\cos\theta\,d\theta = \frac{R}{\pi}\left[\sin\theta\right]_0^{\pi} = 0$$

$$y_{COM} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{R}(R\sin\theta)\,\frac{M}{\pi}\,d\theta$$

$$= \frac{R}{\pi}\int_0^{\pi}\sin\theta\,d\theta = -\frac{R}{\pi}\left[\cos\theta\right]_0^{\pi} = \frac{2R}{\pi}$$

$\therefore$ Co-ordinate of COM $\left(0, \frac{2R}{\pi}\right)$

• Determine the position of COM of a hemisphere of radius R



Mass of elementary disc = vol × density

$$dm = \pi x^2 dy \times \rho = \pi(R^2 - y^2)\,dy\,\rho$$

$$\therefore \quad X_{COM} = \frac{1}{M}\int x\,dM = 0$$

$$Y_{COM} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{R} y\pi(R^2 - y^2)\rho\,dy$$

$$= \frac{\pi\rho}{M}\int_0^{R}(R^2 y - y^3)\,dy = \frac{\pi\rho}{M}\left[R^2\frac{y^2}{2} - \frac{y^4}{4}\right]_0^{R}$$

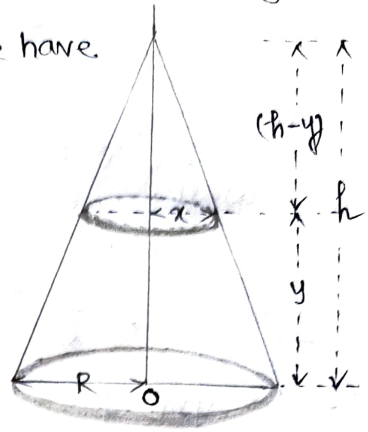
$$= \frac{\pi\rho}{M}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = \frac{\pi\rho}{M}\left[\frac{R^4}{4}\right]$$

$$= \frac{\pi\rho}{2/3\,\pi R^3\rho}\left(\frac{R^4}{4}\right) = \frac{3}{8}R.$$

$$Z_{COM} = 0$$

Hence the co-ordinate of the COM of the hemisphere are $(0, 3/8 R, 0)$

• Determin the co-ordinates of the Centre of mass of a right circular cone of base radius R and height h.



In similar triangles AOD and ABC, we have

$$\frac{h}{h-y} = \frac{R}{x} \quad \text{or} \quad x = \frac{R(h-y)}{h}$$

Mass of elementary disc is

$dm =$ volum $\times$ density

$$= \pi x^2 dy \times \rho = \pi\rho \frac{R(h-y)^2}{h^2} dy$$

∴ The Co-ordinates of the COM can be determined as follows:

$$x_{COM} = \frac{1}{M}\int x\,dm = 0$$

$$y_{COM} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^h y\,\pi\rho\left(\frac{R^2(h-y)^2}{h^2}\right)dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\int_0^h y(h-y)^2 dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\int_0^h y(h^2+y^2-2hy)\,dy$$

$$= \frac{\pi\rho R^2}{Mh^2}\left[h^2\left[\frac{y^2}{2}\right]_0^h + \left[\frac{y^4}{4}\right]_0^h - 2h\left[\frac{y^3}{3}\right]_0^h\right]$$

$$= \frac{\pi\rho R^2}{Mh^2}\left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3}\right] = \frac{\pi\rho R^2}{Mh^2}\left[\frac{h^4}{12}\right]$$

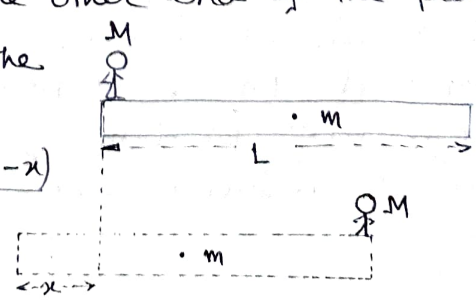
$$= \frac{\pi\rho R^2}{\frac{1}{3}\pi R^2 h\rho}\cdot\frac{h^2}{12} = \frac{h}{4} \qquad \left[\because M = \tfrac{1}{3}\pi r^2 h\rho\right]$$

Similarly,

$$z_{COM} = \frac{1}{M}\int z\,dm = 0$$

Hence the co-ordinates of the COM are $(0, \frac{h}{4}, 0)$

• A wooden plank of mass m kg is resting on a smooth horizontal floor. A man of mass M kg starts moving from one end of the plank to the other end. The length of the plank is L. Find the displacement of the plank over the floor when the man reaches the other end of the plank.



As, no external force is applied, the position of COM remains same.

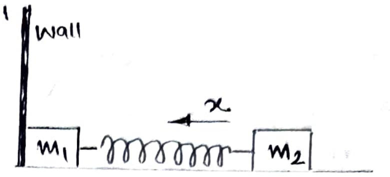
$$\frac{M\times 0 + m\left(\frac{L}{2}\right)}{M+m} = \frac{M(L-x) + m\left(\frac{L}{2}-x\right)}{M+m}$$

$$\therefore \frac{mL}{2} = ML - Mx + \frac{mL}{2} - mx$$

$$\therefore x(M+m) = ML$$

$$\therefore x = \left(\frac{M}{M+m}\right)L.$$

• Two blocks 1 and 2 of masses m_1 and m_2 connected by a weightless spring of spring constant K rest on a smooth horizontal plane as shown in figure. Block 2 is shifted a small distance x to the left and then released. Find the velocity of the COM after block-1 break off the wall.



By the conservation of energy

$$\frac{1}{2}m_2v_2^2 = \frac{1}{2}Kx^2$$

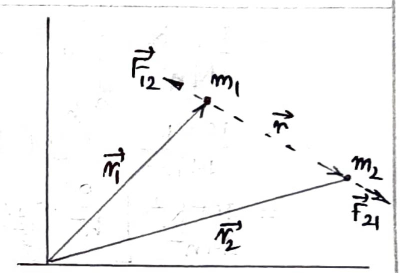
$$\therefore \quad v_2 = \sqrt{\frac{K}{m_2}}\,x$$

As we know, the velocity of COM

$$V_{COM} = \frac{m_1V_1 + m_2V_2}{m_1+m_2} = \frac{m_1\times 0 + m_2\sqrt{\frac{K}{m_2}}\,x}{m_1+m_2} \qquad [\because v_1 = 0]$$

$$V_{COM} = \frac{x\sqrt{Km_2}}{(m_1+m_2)}$$

Concept of Reduced Mass:



From figure $\vec{r} = \vec{r_1} - \vec{r_2}$

Now, $\vec{F_{12}} = m_1\frac{d^2\vec{r_1}}{dt^2}$ and $\vec{F_{21}} = m_2\frac{d^2\vec{r_2}}{dt^2}$

By Newton's third law

$$|\vec{F_{12}}| = -|\vec{F_{21}}| = \vec{F} \text{ (say)}$$

$$\therefore \quad \frac{d^2\vec{r_1}}{dt^2} = \frac{\vec{F}}{m_1} \quad \text{and} \quad \frac{d^2\vec{r_2}}{dt^2} = -\frac{\vec{F}}{m_2}$$

$$\therefore \quad \frac{d^2\vec{r_1}}{dt^2} - \frac{d^2\vec{r_2}}{dt^2} = \frac{\vec{F}}{m_1} - \frac{\vec{F}}{m_2} = \vec{F}\left[\frac{1}{m_1} - \frac{1}{m_2}\right]$$

$$\therefore \quad \frac{d^2}{dt^2}\left[\vec{r_1} - \vec{r_2}\right] = \vec{F}\left[\frac{1}{m_1} - \frac{1}{m_2}\right]$$

But $\vec{r_1} - \vec{r_2} = \vec{r}$. $\quad \therefore \frac{d^2\vec{r}}{dt^2} = \frac{F}{\mu}$

or $\quad \vec{F} = \mu\frac{d^2\vec{r}}{dt^2}$ where $\frac{1}{\mu} = \left[\frac{1}{m_1} + \frac{1}{m_2}\right]$

$$\mu = \frac{m_1m_2}{m_1+m_2}$$

This equation represents the eqn. of motion of a single particle of mass μ at a vector distance $\vec{r}$ from the fixed centre which exerts on it a central force $\vec{F}$. Thus the original two body problem has been reduced to a single body problem of mass $\mu = \frac{m_1m_2}{m_1+m_2}$, which is known as reduced mass of two two body system.

System of Particle (Centre of Mass)

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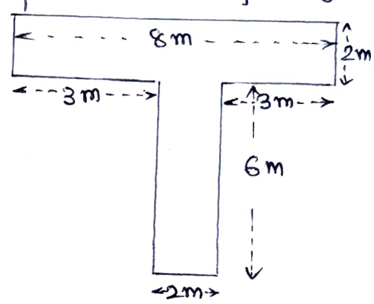
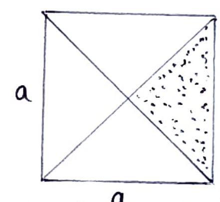
1. Three masses 3, 4 and 5 kg are located at the corners of an equilateral triangle of side 1 m, Locate the centre of mass of the system. Ans: (0.54 m, 0.36 m)

2. Two particles of masses 100 g and 300 g at a given time have positions $2\hat{i}+5\hat{j}+13\hat{k}$ and $-6\hat{i}+4\hat{j}-2\hat{k}$ m respectively and velocities $10\hat{i}-7\hat{j}-3\hat{k}$ and $7\hat{i}-9\hat{j}+6\hat{k}$ ms^{-1} respectively. Determine the instantaneous position and velocity of COM.
Ans. $R_{COM} = \frac{-16\hat{i}+17\hat{j}+7\hat{k}}{4}$, $V_{COM} = \frac{31\hat{i}-34\hat{j}+15\hat{k}}{4}$

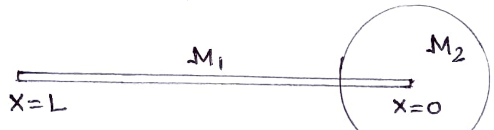
3. Find the position of COM of the 'T' shaped plate from O.



4. From a square sheet of uniform density, a portion is removed shown shaded in figure. Find the COM of the remaining portion if the side of the square is a.



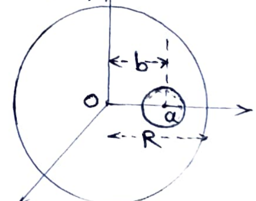
5. A thin uniform rod of length L and mass M_1 has a uniform disc of radius a and mass M_2 fastened to the rod



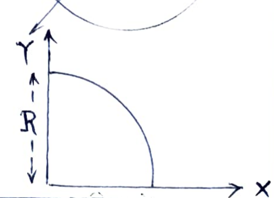
so that the disc and the rod are coplanar as shown in figure. The centre of the disc is at the rod's end. Find the distance of the disc's centre from the centre of mass of the combination.
Ans: $LM_1/2(M_1+M_2)$

6. Calculate the centre of mass of a non-uniform rod whose mass per unit length λ varies as $\lambda = \lambda_0 x^2/L$ where λ_0 is a constant, L is the length of the rod and x is the dist. of any point on the rod measured from one end. Ans: (3/4) L

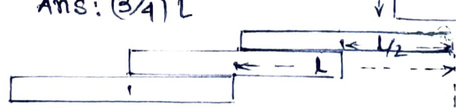
7. The uniform solid sphere shown in fig. has a spherical hole in it. Find the position of centre of mass.
$x_{com} = -\frac{a^3 b}{(R^3 - a^3)}$



8. Find the centre of mass of a uniform quarter-circular wire. Ans: $\left(\frac{2R}{\pi}, \frac{2R}{\pi}\right)$



9. Three identical bricks of length L are placed on the top of each other, so that a part of each overhangs the one below. Find the maximum value of total overhang distance in terms of L. Ans: (3/4) L



10. A gun of mass M fires a bullet of mass m with a speed v_r relative to barrel of the gun which in inclined at an angle 60° with horizontal. The gun is placed over a smooth horizontal surface. Find the recoil speed of gun. Ans: $\frac{mv_r}{2(M+m)}$