Elasticity

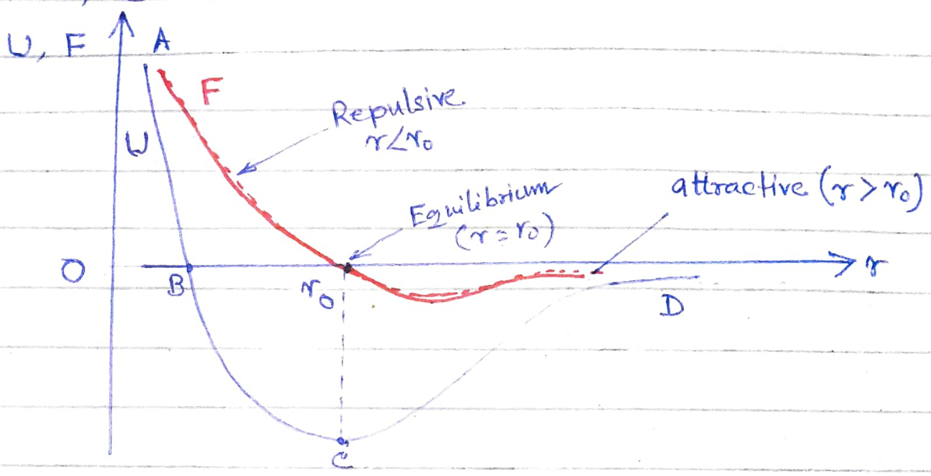
Interatomic and Intermolecular Forces.

Potential Energy $U = \frac{a}{r^p} - \frac{b}{r^q}$

Also $F = -\frac{dU}{dr}$

$$F = -\frac{d}{dr}\left(\frac{a}{r^p} - \frac{b}{r^q}\right) = \frac{pa}{r^{p+1}} - \frac{qb}{r^{q+1}}$$

where a, b are const.



For solid state $r = r_0$

For Liquid state $r > r_0$

For Gaseous state $r >> r_0$.

Elasticity : The property of a material by virtue of which it regains its original configuration on the removal of the deforming force is called elasticity.

Stress :

$$\text{Stress} = \frac{\text{Force }(F)}{\text{Area }(A)}$$

Types of Stress:

i) Tensile stress (Longitudinal stress)

ii) Compressive Stress.

iii) Hydrostatic stress

iv) Tangential stress or shearing stress

Strain : $\frac{\text{change in dimension}}{\text{original dimension}}$

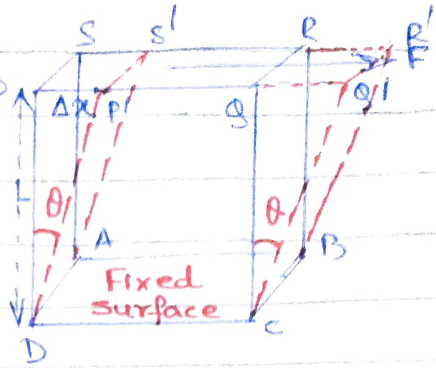
Types of Strain

i) Longitudinal strain $= \frac{\text{change in length}}{\text{original Length}} = \frac{\Delta l}{l}$

ii) Volumetric strain $= \frac{\text{change in vol.}}{\text{original vol.}} = \frac{\Delta V}{V}$

Shear strain

$PP' = QQ' = SS' = RR' = \Delta x$



Shear strain = $\frac{\text{Lateral disp}}{\text{perpendicular dist}}$

Shear strain = $\frac{\Delta x}{L}$

$\therefore \tan\theta = \frac{\Delta x}{L}$

For θ very small $\tan\theta \simeq \theta$

$\theta = \frac{\Delta x}{L}$

Hooke's Law

Within elastic limit, stress and strain are proportional to each other.

Stress $\propto$ Strain

Stress = $E \times$ Strain

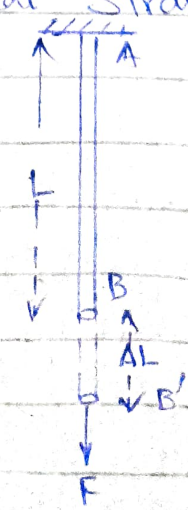
$E \rightarrow$ Modulus of elasticity or coefficient of elasticity.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Elastic Moduli

(a) Young's Modulus:

Ratio of longitudinal stress (tensile or compressive) to the longitudinal strain is called Young's Modulus

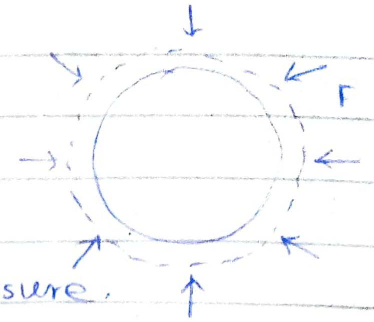


$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\,\Delta L}$$ unit Nm^{-2}

$A \rightarrow$ Cross-sectional area.

Bulk Modulus: The ratio of volume stress to volume Strain is called bulk modulus



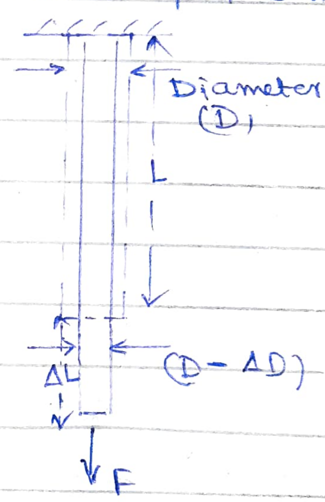
$$\beta = \frac{\text{Volume stress}}{\text{Volume Strain}} = -\frac{P}{\Delta V/V}$$

−Ve sign indicates a decrease in volume with increase in pressure.

Modulus of rigidity (η): The ratio of shear stress to the shere strain is called the shear modulus or modulus of rigidity.

$$\eta = \frac{\text{Shear stress}}{\text{Shear Strain}} = \frac{F/A}{\Delta x/L} = \frac{F/A}{\theta}.$$

Poisson's Ratio (σ): The ratio of the lateral strain to the Longitudinal Strain is called poisson's ratio.



$$\sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = -\frac{\Delta D/D}{\Delta L/L}$$

−Ve sign indicates as L increases D decreases.

Show that $\sigma = 0.5$ for a perfectly incompressible Substance ($\Delta V = 0$).

$$V = \frac{\pi D^2 L}{4}$$

On differentiating both side

$$dV = \frac{\pi}{4}\left[D^2\, dL + L\, 2D\, dD\right]$$

For zero volume change $\Delta V = 0$

$$D^2\, dL + L\, 2D\, dD = 0$$

$$D\, dL = -2L\, dD$$

$$\Rightarrow -\frac{dD/D}{dL/L} = \frac{1}{2} = 0.5$$

$$\sigma = 0.5$$

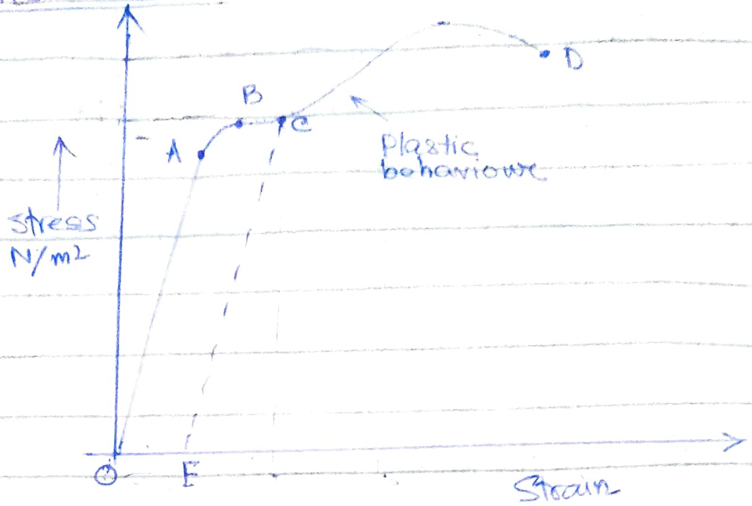
Stress vs Strain curve

A → Proportional limit

B → Yield point / Elastic limit

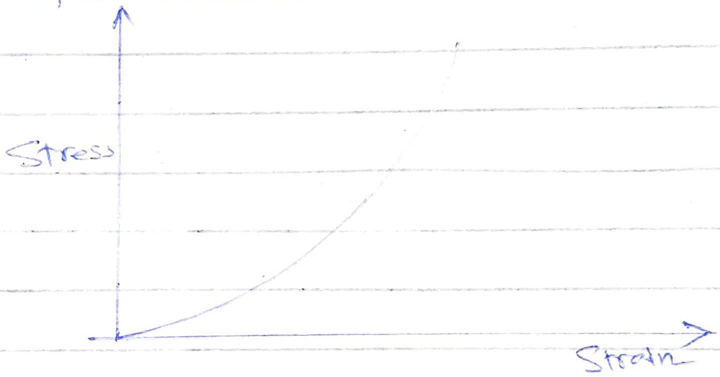
D → Fracture point / Breaking point

OE → Permanent set



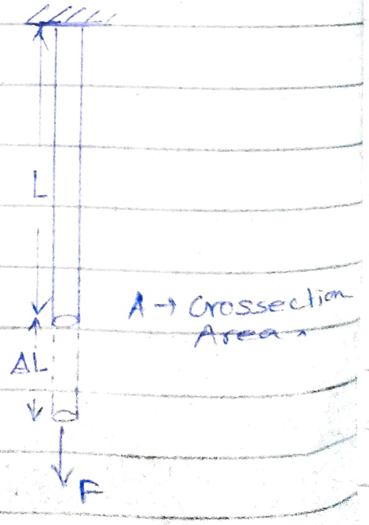
Stress vs strain graph for Elastomers

The material which can be elastically stretched to large values of strain are called elastomers. Example rubber.



Stress - Strain and Elastic potential energy

As we know $Y = \frac{FL}{A l} \Rightarrow F = \frac{YAl}{L}$ ---(i)



Workdone for dL extension

$dw = F \cdot dl = \frac{YAL}{L} dl$

Total work done for increase in length $l = 0$ to $l = l$.

$$W = \int dw = \int_0^l \frac{YAl}{L} dl$$

$$= \frac{YA}{L} \int_0^l l\, dl = \frac{YA}{L} \left[\frac{l^2}{2}\right]_0^l$$

$$W = \frac{YAl^2}{2L} = \frac{1}{2}\left(\frac{YAl}{L}\right) l = \frac{1}{2} F l$$

$$W = \frac{1}{2} F \times \Delta L$$

This workdone is stored as elastic potential energy

$\therefore U = W = \frac{1}{2} \times F \times \Delta l$

$$U = \frac{1}{2} \times \left(\frac{F}{A}\right) \times \left(\frac{\Delta l}{l}\right) \times A l$$

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$u = \frac{U}{V} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$\therefore$ Potential Energy stored per unit volume $(u) = \frac{1}{2} \times$ Stress $\times$ Strain.

Why steel is more elastic than rubber band.

For steel

$$Y_s = \frac{FL}{A \Delta L_s}$$

For Rubber

$$Y_r = \frac{FL}{A \Delta L_r}$$

$$\therefore \frac{Y_s}{Y_r} = \frac{\Delta L_r}{\Delta L_s}$$

As $\Delta L_r >> \Delta L_s \quad \therefore Y_s >> Y_r$

$\therefore$ Young's modulus for steel is much greater than rubber band, so steel is more elastic than rubber band.