Degrees of Freedom

Boyle's Law:

$$p = \frac{1}{3}\rho v_{rms}^2$$

$M \rightarrow$ mass of the gas $\qquad V \rightarrow$ volume $\quad \rho = \frac{M}{V}$

$$p = \frac{1}{3}\frac{M}{V} v_{rms}^2$$

$$pV = \frac{1}{3} M v_{rms}^2$$

At const T $\quad v_{rms}$ const $\quad$ and if M is const

$$pV = \text{const}$$

$$p \propto \frac{1}{V}$$

Charle's Law & Gay-Lussac's Law

$$pV \propto v_{rms}^2$$

But $\quad v_{rms}^2 \propto T$

$pV \propto T. \qquad \frac{V}{T} = \text{const}$ at p const

$\frac{p}{T} = \text{const}$ at V const

$m_1, m_2, m_3 \ldots$ $\qquad$ Molecular mass

$n_1, n_2, n_3 \ldots$ $\qquad$ no. of molecules. per unit vol.

$$p = \frac{1}{3} m_1 n_1 v_{1rms}^2 + \frac{1}{3} m_2 n_2 v_{2rms}^2 + \frac{1}{3} m_3 n_3 v_{3rms}^2 + \cdots$$

$\frac{1}{3} m_1 n_1 v_{rms}^2 = p_1$ $\qquad$ Partial pressure for gas 1

$\frac{1}{3} m_2 n_2 v_{2rms}^2 = p_2$ $\qquad$ " " " " 2

$$p = p_1 + p_2 + \cdots + p_n = \sum p_i$$

$\boxed{\text{gas 1}} \qquad \boxed{\text{gas 2}}$

$$\frac{1}{3} m_1 \frac{n_1}{V} v_{1rms}^2 = \frac{1}{3} m_2 \frac{n_2}{V} v_{2rms}^2$$

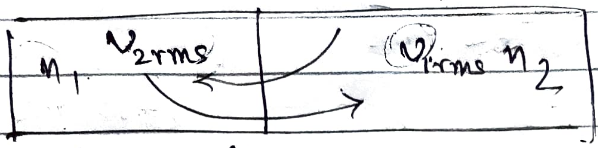
$$\frac{1}{3} m_1 n_1 v_{1rms}^2 = \frac{1}{3} m_2 n_2 v_{2rms}^2$$

At the same temp.

$$\frac{1}{2} m_1 v_{1rms}^2 = \frac{1}{2} m_2 v_{2rms}^2$$

$$n_1 = n_2$$

Graham's Law of diffusion.



$$\frac{n_1}{n_2} = \frac{v_{1rms}}{v_{2rms}}$$

$$\frac{1}{3}\rho_1 v_{1rms}^2 = \frac{1}{3}\rho_2 v_{2rms}^2$$

$$\frac{v_{1rms}}{v_{2rms}} = \sqrt{\frac{\rho_2}{\rho_1}} = \frac{n_1}{n_2}$$

$$\frac{n_1}{n_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad n \propto \frac{1}{\sqrt{\rho}}.$$

Clapeyron's Eqn.

$$p = \frac{1}{3} m n v_{rms}^2 = \frac{1}{3}\frac{n}{N_A} m N_A v_{rms}^2$$

$$= \frac{1}{3}\frac{n}{N_A} M v_{rms}^2 = \frac{n}{N_A} pV$$

$$\left(p = \frac{1}{3}\frac{M}{V} v_{rms}^2\right)$$

$$p = \frac{n}{N_A} pV = \frac{n}{N_A} RT$$

$$\boxed{p = n K_B T.}$$

Average Speed, R.M.S Speed, Most probable speed

Average Speed: $\bar{v} = \frac{v_1 + v_2 + v_3 + \cdots + v_n}{n}$

$$\bar{v} = \sqrt{\frac{8K_BT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8PV}{\pi M}}$$

R.M.S Speed:

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}}$$

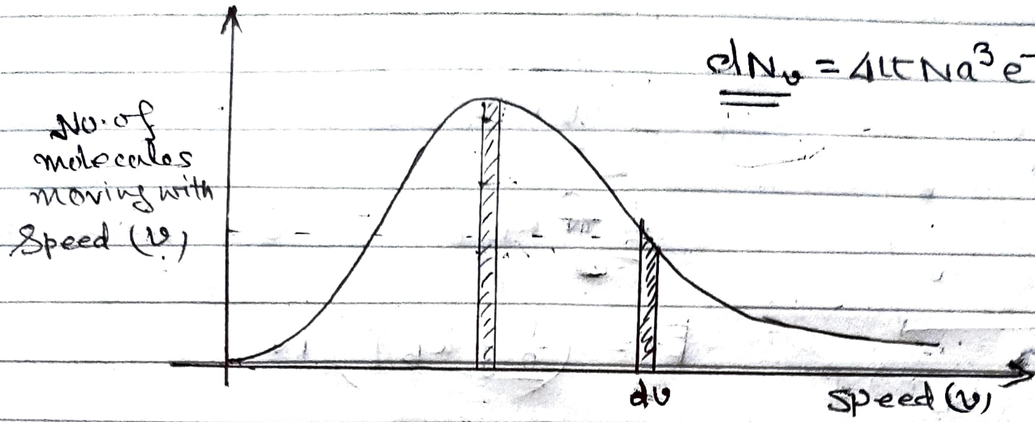
$$v_{rms} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

Most probable velocity:

$$v_{mp} = \sqrt{\frac{2K_BT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

Maxwell's Speed dist.

$$dN_v = 4\pi N a^3 e^{-bv^2} v^2 dv$$



$\boxed{v \rightarrow v+dv}$

$N \rightarrow$ No. of particle

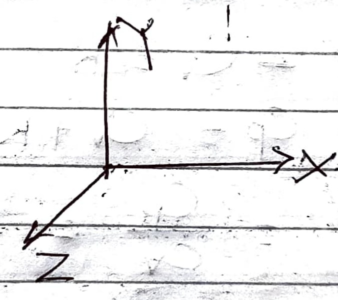
$K \rightarrow$ No. of independent relations between the particles.

3 Translation

$$\boxed{f = 3 \times N - K.}$$

$E_K = \frac{1}{2} I \omega^2$

mr^2



Mono atomic

$N = 1 \qquad K = 0$

$f = 3 \qquad$ (translation).

Diatomic

$N = 2 \qquad K = 1 \qquad f = 3 \times 2 - 1 = 5$

$\left.\begin{matrix} T - 3 \\ R - 2 \end{matrix}\right\} 5$

$V - 2$

$f = 5 + 2$ (Including rotation)

→ Including vibration

Triatomic

~~N~~on Linear

$N = 3 \qquad K = 3$

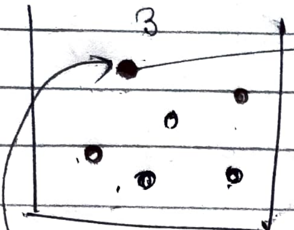
$f = 3 \times 3 - 3 = 6$

Linear

$N = 3 \qquad K = 2$

$f = 3 \times 3 - 2 = 7$. ✓

Law of equipartition of energy



$\frac{1}{2} K_B T$ →

$\frac{1}{2} f K_B T$ → One molecules

$E = \left(\frac{1}{2} f k_B T\right) N$

$\left(\frac{1}{2} K_B T\right) f$

$\frac{3}{2} k_B T$

Monoatomic gas

Energy per molecules $= \frac{3}{2} k_B T$

$U = \frac{3}{2} k_B T \times N_A = \frac{3}{2} RT$ for One mole

$C_v = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{3}{2} RT\right) = \frac{3}{2} R$

$C_p - C_v = R \Rightarrow \cancel{C_p - R}$

$C_p = C_v + R = \frac{3}{2} + R = \frac{5}{2} R$

$\gamma = \frac{C_p}{C_v} = \frac{5/2\,R}{3/2\,R} = \frac{5}{3} = 1.67$

p

$\frac{nR}{\gamma - 1}(T - T_0)$

Diatomic

$U = \frac{5}{2} k_B T \times N_A = \frac{5}{2} RT$

$C_v = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{5}{2} RT\right) = \frac{5}{2} R$

$P \Delta V$

$PV = nRT$

$PdV = nRd$

$C_p = 7/2$

$\gamma = 7/5$

$dQ = -1200\,J$

$dQ = dU + dW$

$W_{AB} + W$

Triatomic (linear)

$U = \frac{7}{2} k_B T N_A = \frac{7}{2} RT$

$nR\Delta T$

$+ W_{BC} + 0$

$C_v = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{7}{2} RT\right) = \frac{7}{2} R$

$C_p = C_v + R = 9/2$

$\gamma = 9/7$