Moment of Inertia of rigid body cut from the whole Mass

$M' = M \cdot \left(\frac{\theta}{2\pi}\right)$

If the surface mass density is uniform

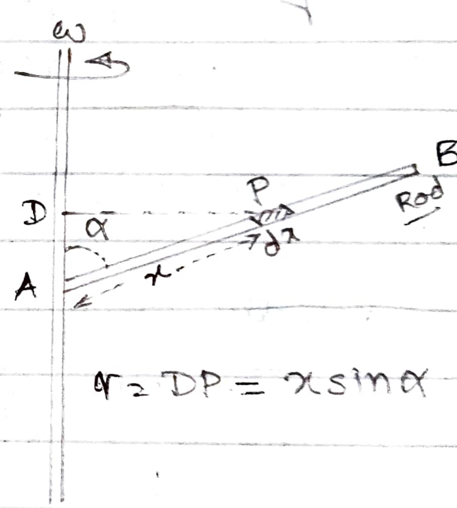
$I_{section} = \frac{1}{n} I_{Total}$

Ex: For disc

$I_{sec} = \frac{1}{n} I_{disc} = \frac{1}{n}\left(\frac{1}{2} MR^2\right)$

$= \frac{1}{2}\left(\frac{M}{n}\right) R^2$

Moment of inertia of a rotating rod making an angle α with the verticle.



Mass per unit length $= \left(\frac{M}{l}\right)$

Mass of the element

$dm = \left(\frac{M}{l}\right) dx$

$dI = (dm) r^2 = \frac{M}{l} dx \cdot x^2 \sin^2\alpha$

$r = DP = x\sin\alpha$

$= \frac{M}{l} \sin^2\alpha \quad x^2 dx$

$I = \int dI = \frac{M}{l} \sin^2\alpha \int_0^l x^2 dx$

$I = \frac{M}{l} \sin^2\alpha \left[\frac{x^3}{3}\right]_0^l = \frac{M}{3l} \sin^2\alpha \cdot l^3$

$I = \frac{Ml^2 \sin 2\alpha}{3}$

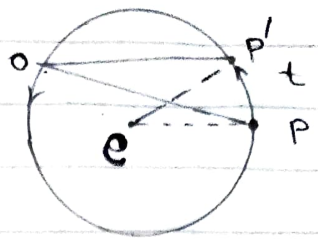
$I = \frac{Ml^2}{3} \sin^2\alpha$

If $\alpha = \frac{\pi}{2}$ then $I = \frac{Ml^2}{3}$

Angular velocity:

$\omega_c \rightarrow$ Angular velocity about C

$\omega_o \rightarrow$ Angular velocity about O



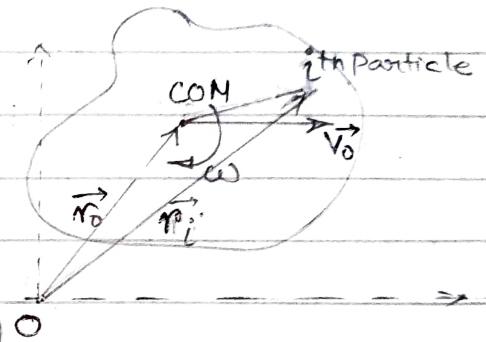
$$\omega_c = \frac{\angle P'CP}{t} \qquad \omega_o = \frac{\angle P'OP}{t}$$

AS $\angle P'CP = 2\angle P'OP$

$\therefore \omega_c = 2\omega_o.$

Angular momentum of a rigid body in Combination of rotation and translation:

$\vec{L}$ = Angular momentum.



$$\vec{L} = \sum(\vec{r_i} \times \vec{p_i})$$

$$= \sum m_i(\vec{r_i} \times \vec{v_i})$$

$$= \sum_i m_i(\vec{r}_{icm} + \vec{r_0}) \times (\vec{V}_{icm} + \vec{V_0})$$

$$\vec{L} = \sum m_i(\vec{r}_{icm} \times \vec{V}_{icm}) + \left\{\sum m_i \vec{r}_{icm}\right\} \times \vec{V_0} + \vec{r_0} \times \left\{\sum m_i \vec{V}_{icm}\right\} + \left\{\sum m_i\right\} \vec{r_0} \times \vec{V_0}$$

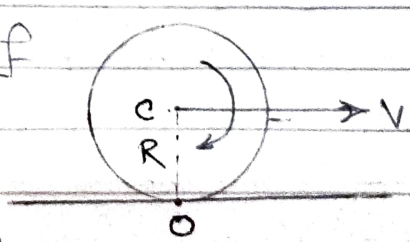
Now, $\sum m_i \vec{r}_{i\,cm} = M\vec{R}_{cm} = 0$

$$\sum_i m_i V_{icm} = 0$$

$$L = \sum_{i=1}^{n} m_i(\vec{r}_{icm} \times \vec{V}_{icm}) + M\vec{r_0} \times \vec{V_0}$$

$$\boxed{\vec{L} = \vec{L}_{cm} + M\vec{r_0} \times \vec{V_0}}$$

Find the angular momentum of the disc about O.



$L = L_{cm} + M(\vec{r_0} \times \vec{V_0})$

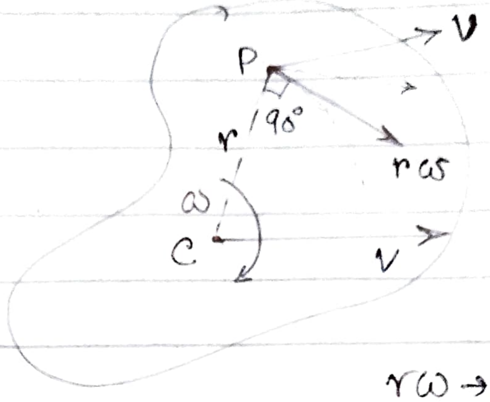
Here $L_{cm} = I\omega = \left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right) = \frac{1}{2}MvR$

$M(\vec{r_0} \times \vec{V_0}) = m(\vec{R} \times \vec{V_c}) = mvR$

$\therefore L = \left(mvR + \frac{1}{2}mvR\right) = \frac{3}{2}mvR$

Combined Translational & Rotational Motion of a Rigid body :

$\vec{V}_P = \vec{V}_{COM} + \vec{V}_{P,COM}$



$|\vec{V}_{COM}| = v$

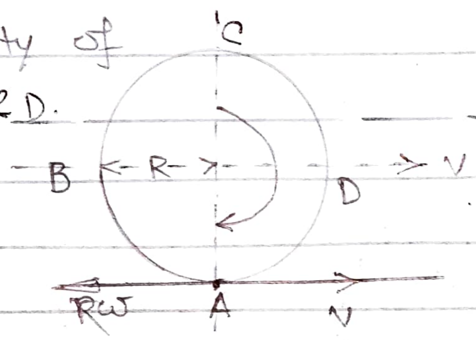
$|\vec{V}_{P,COM}| = r\omega$

$\vec{V}_P = \vec{v} + \vec{r\omega}$

$r\omega \rightarrow$ Linear velocity of point P about C

v translational velocity of point.

Find the velocity of point A, B, C & D.



At Point A

$V_A = 0$

$V_A = \vec{V} + r\omega = 0$

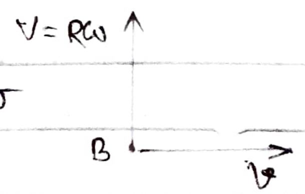
$V = -r\omega$

$\therefore V = r\omega$ in magnitude

At Point B

$V_B = \sqrt{v^2 + v^2}$

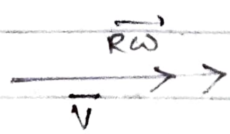
$= \sqrt{2}\,V = \sqrt{2}\,R\omega$



At point C

$V_C = V + V = 2V$

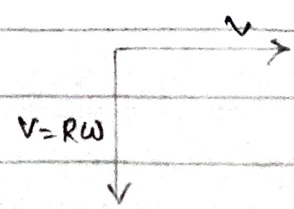
$= 2R\omega.$



At point D

$V_D = \sqrt{V^2 + V^2} = \sqrt{2}\,V$

$= \sqrt{2}\,R\omega.$

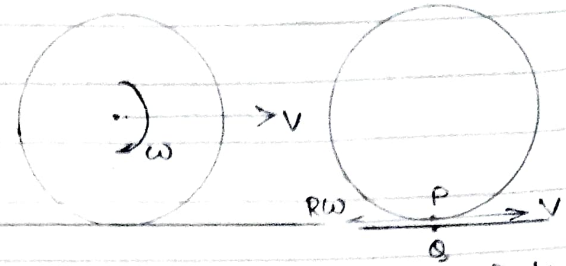


Uniform pure Rolling

Condition of pure rolling

$V_P = V_Q$

or $V - R\omega = 0$ or $V = R\omega$



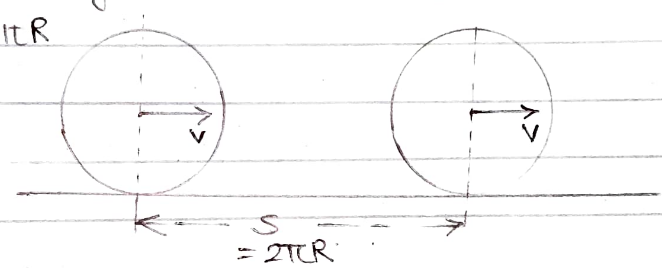
P → Bottom most point of the disc

Q → Point on the surface in contact with the disc.

If $V_P > V_Q$ or $V > R\omega$, this motion is said to be forward slipping

If $V_P < V_Q$ or $V < R\omega$, the motion is said to be backward slipping.

For pure rolling, dist. moved by the COM in one complete rotation is $2\pi R$



$$S = V \cdot T = R\omega \cdot \left(\frac{2\pi}{\omega}\right) = 2\pi R.$$

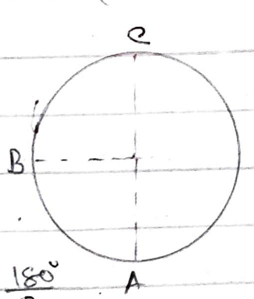
In forward slipping $S > 2\pi R$

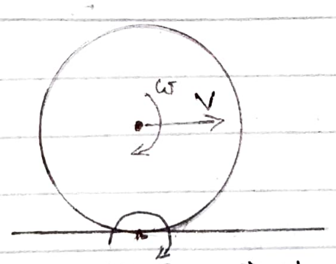
In backward slipping $S < 2\pi R$

$V_A = 0$

$V_B = 2V \cdot \sin\frac{90°}{2} = \sqrt{2}V$

$V_C = 2V\sin\frac{180°}{2} = 2V$





About COM

$\therefore E_K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

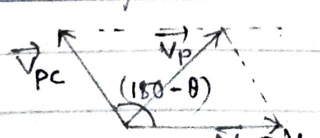
About IAOR

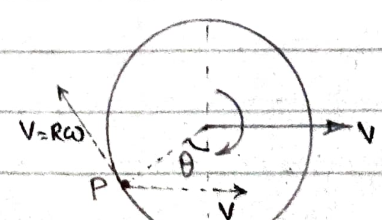
$E_K = \frac{1}{2}I'\omega^2.$

IAOR → Inst. Axis of rotation. (velocity is zero)

Only rotation motion about IAOR = Translation + Rotation about the axis passing through COM.

The speed of a point on the circumference of the body at any instance





$$\vec{V}_P = \vec{V}_C + \vec{V}_{PC}$$

$$|\vec{V}_P| = \sqrt{V^2 + V^2 + 2V \cdot V \cdot \cos(180 - \theta)} = \sqrt{2V^2(1 - \cos\theta)} = 2V\sin\frac{\theta}{2}$$