LAWS OF MOTION

Force: Force may be defined as an external cause which changes or tends to change the state of rest or of uniform motion or the direction of motion of a body.

Inertia and its different types:
The inherent property of a body by virtue of which it cannot change, by itself, its state of rest or of uniform motion in a straight line is called inertia.

Inertia of rest: The tendency of a body to remain in its position of rest is called inertia of rest.

Inertia of motion: The tendency of a body to remain in its state of uniform motion in a straight line is called inertia of motion.

Example of inertia of rest: A person standing in a bus falls backward when the bus suddenly starts moving forward.
Example of inertia of motion: When a moving bus suddenly stops, a person sitting in it falls forward.

Newton's First Law or The Law of Inertia:
Everybody in this universe continues to be in a state of rest or of uniform motion in a straight line until and unless it is compelled by an external unbalanced force to change that state.

Linear Momentum:
Momentum of a body determines the dynamical state of a body. It is measured by the product of mass and velocity of the body.

$$\text{Momentum } (p) = \text{Mass } (m) \times \text{Velocity } (v)$$

$$\vec{p} = m\vec{v}$$

It is a vector quantity. Its direction is along the direction of the velocity. Its SI unit is $kg\,ms^{-1}$ or Ns.

Newton's second Law or Law of Momentum:
The rate of change of linear momentum is directly proportional to the applied force and the momentum change takes place in the direction of applied force.

As we know $\quad \vec{p} = m\vec{v}$

Differentiating both side w.r.t time t we get

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

If m remains constant during the application of force, then $\frac{dm}{dt} = 0$ $\quad \therefore \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$

$\therefore$ According to Second's Law $\quad \vec{F} \propto \frac{d\vec{p}}{dt} \quad \therefore \vec{F} \propto m\vec{a}$

$\therefore \quad \vec{F} = Km\vec{a}$

In S.I system $K = 1$ $\quad \therefore \vec{F} = m\vec{a}$ or $F = ma$.

Unit of force

Absolute Unit :

newton (N) : One newton (N) is defined as that much force which produces an acceleration of $1\,ms^{-1}$ in a body of mass 1 kg.

$$1N = 1kg \times 1ms^{-2} = 1kgms^{-2}$$

Gravitational unit:

kilogram weight (kg-wt / kgf) : It is defined as that much force by which earth pulls an object of mass 1 kg towards its centre.

$$1\,kg\text{-}wt = 1\,kg\text{-}f = 9.8\,N.$$

Impulse: A large force acting for a short duration of time to produce a finite change in momentum is called an impulsive force. The product of force and time is known as impulse.

Example:
- Force exerted by a bat while hitting a ball.
- Blow of a hammer on a nail.

Impulse = Force × time interval = Change in momentum.

$$F = \frac{\Delta p}{\Delta t} \quad \therefore \quad J = \Delta p = F \cdot \Delta t \,. \quad \therefore \vec{J} = \vec{F} \cdot \Delta t$$

Impulse is a vector quantity.

Impulse momentum theorem

According to Newton's Second law of motion

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F}dt = d\vec{p}$$

If in time 0 to t, the momentum of the body changes from $\vec{p_1}$ to $\vec{p_2}$, then integrating the above equation within these limits, we get

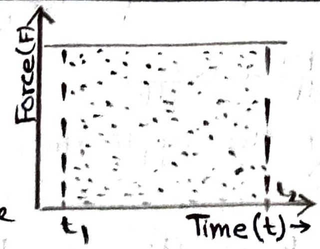
$$\int_0^t \vec{F} \cdot dt = \int_{\vec{p_1}}^{\vec{p_2}} d\vec{p} = [\vec{p}]_{\vec{p_1}}^{\vec{p_2}} = \vec{p_2} - \vec{p_1}$$

But $\int_0^t \vec{F}dt = \text{Impulse } (\vec{J}) \quad \therefore \vec{J} = \vec{p_2} - \vec{p_2}$

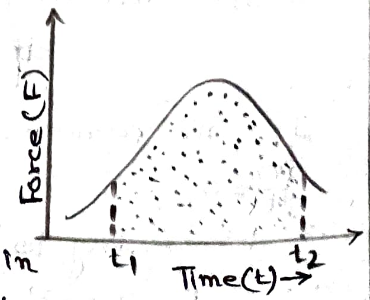
Thus the impulse of a force is equal to the total change in momentum produced by the force.

Impulse for Constant and variable force

Area under the graph
$= F(t_2 - t_1) = F \cdot \Delta t$
= Magnitude of impulse of force F in the Δt time.



Impulse $(\vec{J}) = \int_{t_1}^{t_2} F \cdot dt$ = Area under the force-time curve within time t_1 to t_2.



Practical applications of impulse

If two forces $\vec{F_1}$ and $\vec{F_2}$ acts on a body to produce the same impulse (or change in momentum), then their time durations t_1 and t_2 should be such that

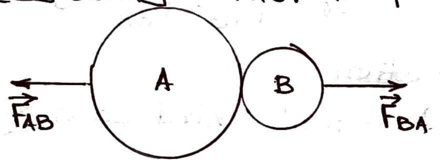
$$\vec{F_1} t_1 = \vec{F_2} t_2$$

Clearly, if the time duration of an impulse is large, the force exerted will be small. The following examples will make this concept clear

- A cricket player lowers his hands while catching a ball.
- A person falling from a certain height receives more injuries when he falls on a cemented floor than when he falls on a heep of sand.
- Automobiles are provided with shockers.
- Buffers are provided between the boggies of a train.
- Chinawares are packed in straw paper before packing.

Newton's Third Law:

To every action, there is always an equal and opposite reaction. The action and reaction act on two different bodies. The forces always exist in pairs.



$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Horse and cart problem:

$F \rightarrow$ Force by the horse on the ground

$H \rightarrow$ Horizontal Component of R

$V \rightarrow$ Vertical Component of R

$R \rightarrow$ Reaction of the ground

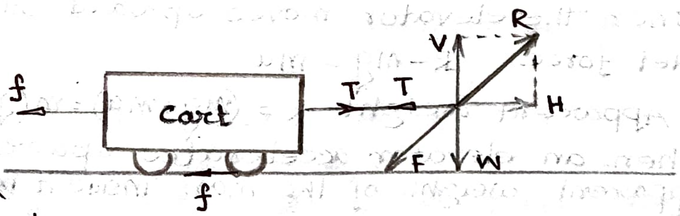
$W \rightarrow$ Weight of the horse

$f \rightarrow$ frictional force

$m \rightarrow$ mass of horse

$a \rightarrow$ Acceleration of the horse-cart system.

$M \rightarrow$ Mass of the cart.



For horse

$$H - T = ma \quad \text{----(i)}$$

For cart

$$T - f = Ma \quad \text{----(ii)}$$

By adding eqn (i) & (ii)

$$H - f = (M+m)a$$

$$\therefore a = \frac{H-f}{M+m}$$

$\therefore$ a is positive if $H > f$, thus the system moves if $H > f$.

Second Law is the real law of motion:

- Derivation of first Law from second Law

From Second Law $F = ma$

In absense of any external force $F=0$ or $ma=0$

as $m \neq 0$, therefore, $a=0$

Thus there is no acceleration when no force is applied. That is in the absence of any external force a body at rest will remain at rest and a body in uniform motion will continue to move uniformly along the same straight path. This is nothing but first law of motion.

- Derivation of third Law from second Law

Consider an isolated system of two bodies A and B.
Let $\vec{F}_{AB}$ force on A by body B and $\vec{F}_{BA}$ force on B by body A.

$\therefore$ From Second Law $\vec{F}_{AB} = \frac{d\vec{P}_A}{dt}$ and $\vec{F}_{BA} = \frac{d\vec{P}_B}{dt}$

$$\therefore \quad \vec{F}_{AB} + \vec{F}_{BA} = \frac{d\vec{P}_A}{dt} + \frac{d\vec{P}_B}{dt} = \frac{d}{dt}(\vec{P}_A + \vec{P}_B)$$

In absence of any external force the total momentum remains constant

$$\therefore \quad \frac{d}{dt}(\vec{P}_A + \vec{P}_B) = 0 \quad \text{or} \quad \vec{F}_{AB} + \vec{F}_{BA} = 0 \quad \therefore \vec{F}_{AB} = -\vec{F}_{BA}$$

or Action = $-$ Reaction.

This is nothing but Newton's third Law of motion.

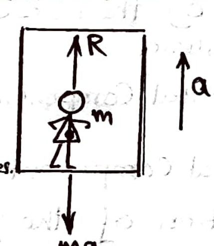
Apparent weight of a person in an Elevator:

(a) When the elevator moves upward with acceleration 'a'

Net force $R - mg = ma$

$\therefore$ Apparent weight $R = (mg + ma) = m(g+a)$

So when an elevator accelerates upwards, the apparent weight of the man inside it increases.

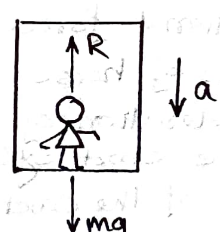


(b) When the elevator moves downward with acceleration 'a'.

Net force $mg - R = ma$

$\therefore$ Apparent weight $R = (mg - ma) = m(g-a)$

So when a lift accelerates downwards, the apparent weight of a man inside it decreases.



(c) When the elevator is at rest or moving with uniform velocity v downward or upward.

Net force $R - mg = 0$ $\therefore R = mg$.

$\therefore$ Apparent weight = Actual weight.

(d) When the life falls freely

Then acceleration $a = g$

$\therefore$ Net downward force $R = m(g-g) = 0$

So the apparent weight of the man becomes zero.

Conservation of Linear Momentum

When no external force acts on a system of several interacting particles, the total linear momentum of the system is conserved. The total linear momentum is the vector sum of the linear momenta of all the particles of the system.

Derivation of the Law of Conservation of linear momentum

Consider an isolated system (the system on which no external force acts) of n particles. Suppose the n particles have masses $m_1, m_2, m_3, \ldots m_n$ and are moving with velocities $\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots \vec{v}_n$ respectively. Then total linear momentum of the system is

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n$$
$$= \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots + \vec{P}_n$$

If $\vec{F}$ is the external force acting on the system then from Newton's second law,

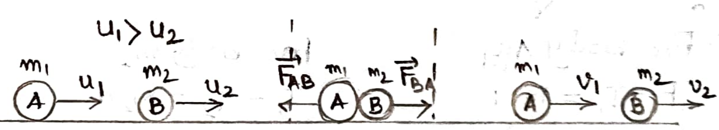
$$\vec{F} = \frac{d\vec{P}}{dt}$$

For an isolated system $\vec{F} = 0$ or $\frac{d\vec{P}}{dt} = 0$ $\therefore \vec{P} = \text{Const.}$

or $\quad \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots + \vec{P}_n = \text{Const.}$

Thus in the absense of any external force, the total linear momentum of the system is constant.

Alternative Method :



- m_1 and m_2 are the masses of the two bodies respectively
- u_1 and u_2 are the initial velocity of mass m_1 and m_2 respectively
- v_1 and v_2 are the final velocity of mass m_1 and m_2 respectively
- $\vec{F}_{AB}$ is the interacting force on body A by the body B;
- $\vec{F}_{BA}$ is the interacting force on body B by the body A.

From Newton's third Law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

$\therefore$ Impulse of $\vec{F}_{AB} = \vec{F}_{AB} \cdot \Delta t$ = change in momentum of A.

$$= m_1\vec{v}_1 - m_1\vec{u}_1$$

Impulse of $\vec{F}_{BA} = \vec{F}_{BA} \cdot \Delta t$ = change in momentum of B

$$= m_2\vec{v}_2 - m_2\vec{u}_2$$

But $\quad \vec{F}_{AB} \cdot \Delta t = -\vec{F}_{BA} \cdot \Delta t$

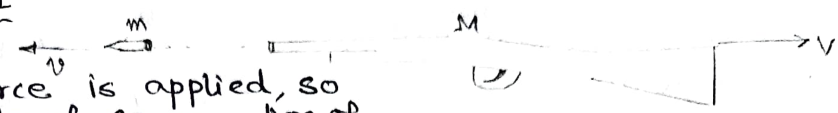
$\therefore \quad m_1\vec{v}_1 - m_1\vec{u}_1 = -(m_2\vec{v}_2 - m_2\vec{u}_2)$

or $\quad m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2$

$\therefore$ Total linear momentum after collision = Total linear momentum before collision.

Practical Application of The Law of Conservation of Motion

(i) Recoil of a gun



As no external force is applied, so according to principle of Conservation of momentum,

Total momentum before firing = Total momentum after firing

$$0 = m\vec{v} + M\vec{V}$$

or $$MV = -m\vec{v}$$

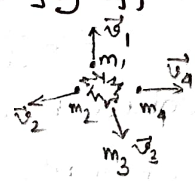
or $$V = -\frac{m}{M}\vec{v}$$

The negative sign indicates $\vec{V}$ and $\vec{v}$ are in opposite direction. Further, as $M >> m$, so $V << v$ i.e the recoil velocity of the gun is much smaller than the forward velocity of the bullet.

(ii) Explosion of a bomb

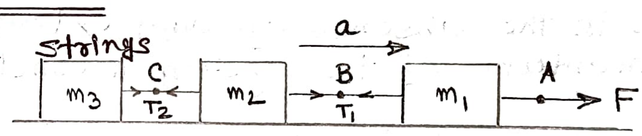
Suppose the bomb is at rest before the explosion. Its total momentum is zero. As it explodes, it breaks up into many parts of masses m_1, m_2, m_3, etc which fly off in different directions with velocities $\vec{v_1}$, $\vec{v_2}$, $\vec{v_3}$ etc.

$$\therefore \quad m_1v_1 + m_2v_2 + m_3v_3 + \cdots + m_nv_n = 0$$



Motion of Connected Bodies

- Bodies connected with strings



Let a be the acceleration of the system.

$\therefore$ For body m_1

$$F - T_1 = m_1a$$

$$\therefore T_1 = F - m_1a$$

For body m_2

$$T_1 - T_2 = m_2a$$

For body m_3

$$T_2 = m_3a$$

$$\therefore \quad F - m_1a - m_3a = m_2a$$

$$\therefore \quad a = \frac{F}{(m_1+m_2+m_3)} \quad \text{---- (i)}$$

$$\therefore T_1 = F - m_1\frac{F}{M} \quad (\text{where } M \text{ is } (m_1+m_2+m_3))$$

$$T_1 = F\left(\frac{m_2+m_3}{M}\right) \quad \text{--- (ii)}$$

$$T_2 = F\left(\frac{m_3}{M}\right) \quad \text{--- (iii)}$$

- Bodies connected through a fixed pulley.

For body m_1

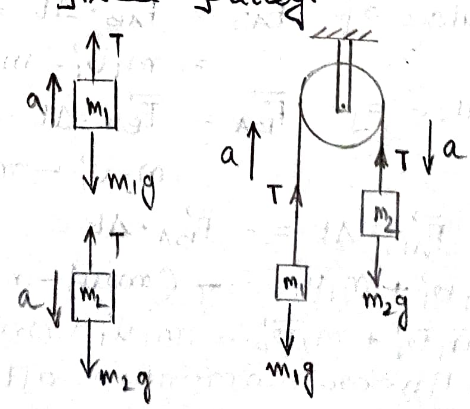
$$T - m_1g = m_1a \quad \text{--- (i)}$$

For body m_2

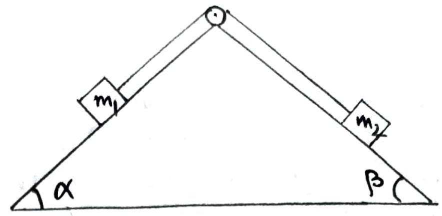
$$m_2g - T = m_2a \quad \text{--- (ii)}$$

on solving $$a = \frac{m_2 - m_1}{m_1 + m_2}g$$

$$T = \frac{2m_1m_2}{m_1+m_2}g.$$



• Two blocks of masses m_1 and m_2 connected by a cord passing over a small frictionless pulley rests on frictionless plane inclined at α and β.



For body m_1

$T - m_1 g \sin\alpha = m_1 a$

For body m_2

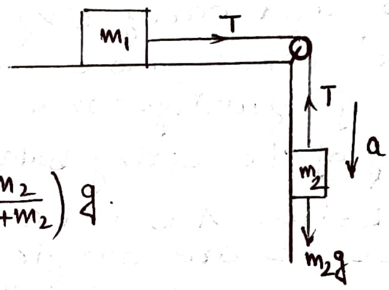
$m_2 g \sin\beta - T = m_2 a$.

Adding and solving for a

$$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha) g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2}{m_1 + m_2} g (\sin\alpha + \sin\beta)$$

• A block of mass m_1 on a smooth horizontal table is pulled by a string which is attached to a mass m_2 hanging over a pulley



For body m_1

$T = m_1 a$

For body m_2

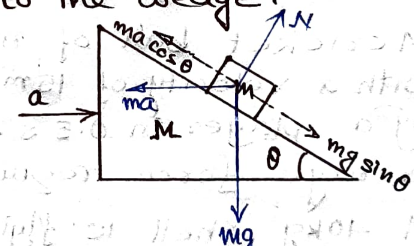
$m_2 g - T = m_2 a$

Adding and solving

$$a = \left(\frac{m_2}{m_1 + m_2}\right) g \quad , \quad T = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g.$$

• <u>Non-Inertial Frames</u>

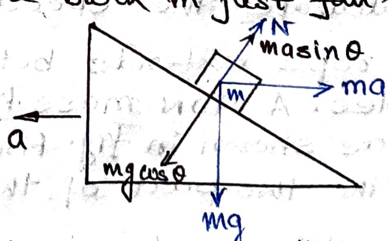
• A wedge of mass M is moving horizontally with acceleration 'a' towards right on a smooth surface and a block of mass m stays freely at rest with respect to the wedge.



For equilibrium $mg \sin\theta - ma \cos\theta = 0$

$a = g \tan\theta$.

• A block of mass m is resting on a wedge. The wedge is given an acceleration a, so that the block m just fall freely.



At equilibrium $N + ma \sin\theta - mg \cos\theta = 0$

For just to fall $N = 0$

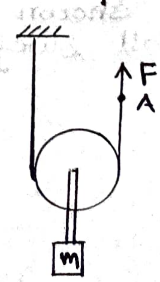
$\therefore \quad ma \sin\theta - mg \cos\theta = 0$

$a = g \cot\theta$.

• Motion of a body attached with single movable pulley.

Displacement of point A is twice of the mass m

$\therefore$ Acceleration of point A is half of the acceleration of mass m.

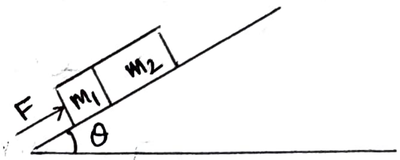


Newtons Laws of Motion

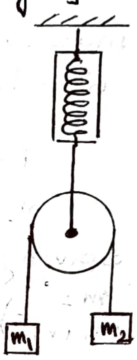
WORKSHEET (DOSE-I)

1. A train is moving along a straight horizontal track. A pendulum suspended from the roof of a carriage of the train is inclined at 45° to the vertical. Calculate the acceleration of the train. ($g = 10 m/s^2$).

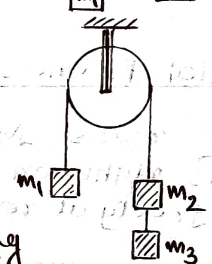
2. Two blocks of masses $m_1 = 3 kg$ and $m_2 = 2 kg$ in contact are pushed up by a force F on an inclined plane making an angle 30° with the horizontal. If the applied force F = 30N and acts parallel to the plane, find (i) the acceleration (ii) the force between the blocks.



3. Figure shows a massless pulley hanging from a spring balance. The free ends of the weight less cord passing over the pulley and carry 5 kg and 1kg weights which move under the action of gravity. What is the reading of the spring balance?



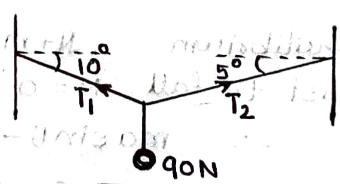
4. Three bodies A, B and C each of mass m are hanging on a string over a fixed pulley, as shown in figure. What are the tension in the strings connecting bodies A to B and B to C?



5. A cricket ball of mass 200 g moving with a velocity of 15 m/s is brought to rest by a player in 0.5 s. Calculate the impulse of the ball and average force required by the player.

6. A 40kg shell is flying at a speed of 72 km/h. It explodes into two pieces. One of the pieces of mass 15kg, stops. Calculate the speed of the other.

7. A rope extends between two poles. A 90N mass hangs from it as shown in fig. Find the tension in the two part of the rope.



8. Find the acceleration of the cart and the mass shown in fig. The pulleys are light and all surfaces are frictionless.