When Air Resistance isn't Ignored

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June 2025

In physics, drag forces are forces acting opposite to the direction of motion of any object moving with respect to a surrounding fluid. They can exist between two fluid layers, two solid surfaces, or between a fluid and a solid surface. Drag forces tend to decrease fluid velocity relative to the solid object in the fluid's path. In this question we will be going over various everyday examples involving drag forces.

A. Low-velocity drag

Consider the example of a sphere falling in a vat of viscous liquid with a small velocity. When the velocity is small the resistive force acting on the sphere is linearly proportional to the velocity. Assume that the proportionality constant in this case is k. For parts 1-10, the liquid density can be considered negligible.

1. Write the relation between instantaneous drag force F_{drag}, k and the instantaneous velocity v.

2. A sphere of mass m is dropped in the viscous liquid. What is the net instantaneous force (F) acting on the sphere in terms of m, k, instantaneous velocity (v) and acceleration due to gravity (g)?

3. Set up a differential equation to calculate the velocity $v(t)$ as a function of time t. Use $F = \frac{dv}{dt}$ to achieve this.

4. Integrate the expression obtained in A.3 to obtain $v(t)$. Assume that the ball had an initial velocity v_0 when it entered the liquid.

5. Using the expression obtained in A.4 find the distance travelled by the sphere in the liquid as a function of time $x(t)$.

6. With the equations obtained in A.5 and A.4 express velocity as a function of distance travelled $v(x)$.

When such a sphere is dropped in such a vat of viscous liquid with no initial speed the constant k is a function of the viscosity of the liquid η and the diameter (D) of the sphere.

7. Using dimensional analysis, find out how k varies with η and D. Give the exact expression of k taking the proportionality constant to be 3π.

8. What is the maximum velocity attained (v_{max}) by the sphere?

9. Find the time taken in seconds (t_α) to reach a velocity αv_{max} (where $\alpha \leq 1$).

10. Discuss the variation of this time t_α with change in η and D for a fixed α.

In reality, the density of the liquid is not negligible. For the parts assume that the liquid density is ρ_L and that the density of the sphere is ρ_S

11. What is the buoyant force acting on the sphere?

12. Calculate the terminal velocity attained by the sphere

13. Find the new expression for $v(t)$

B. High-velocity drag

On the other hand, the drag force experienced by a body with a constant drag coefficient, moving through the fluid at a relatively large velocity, i.e. high Reynolds number is linearly proportional to the square of the velocity with the proportionality constant being the drag coefficient C.

Suppose the same sphere is dropped into the vat with a high initial velocity such that the instantaneous power dissipated due to resistive forces is linearly dependent on the cube of its instantaneous velocity. Assume that the fluid density is negligible.

Answer the questions given below:

1. Establish a numeric relation between instantaneous drag force F_{drag}, C and instantaneous velocity v.

2. Express velocity as a function of distance travelled $v(x)$ if the sphere entered with a velocity v_B.

3. What is the terminal velocity of the sphere?

C. Theoretical v/s Practical

In kinematics, we often assume that the surrounding medium offers no resistance when we throw a ball in projectile motion. In reality, there is a resistive force on the ball modelled by the low-speed drag equation. However, since projectile motion is a two dimensional, there are two components of the drag force. These components individually alter the time of flight, range and maximum height of the ball by affecting the ball's overall velocity. To tackle the dynamics of this model, we only need to extend the one dimensional drag equation and use it two times - along the x-axis and the y-axis to get the modified parameters of the flight.

In this question we assume a ball of mass m has been thrown with an initial velocity u_0 in air at an angle θ with the horizontal. Assume that the horizontal is x-axis and the vertical is the y-axis. The drag coefficient of air is k and the acceleration due to gravity is g.

1. Write the vector form of the drag equation pertinent to this scenario.

2. Evaluate the time of ascent τ_{ascent}.

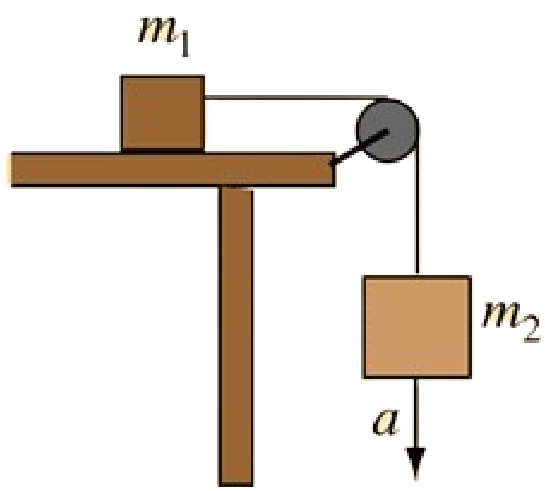
3. What is the maximum height reached by the ball?

4. Find the relation between time of descent $\tau_{descent}$ and maximum height.

5. What is the range of the ball?

D. Oiled Atwood

Consider a modified horizontal atwood machine with blocks X and Y of masses m_1 and m_2 respectively. The table is coated with an oil of viscosity η and thickness d. The surface area of X in contact with the table is A. Using Newton's shear-stress viscosity relation determine the velocity of m_1 as a function of time. Also indicate the terminal velocity in this case.



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Answers

A.

1. $F_{drag} = -kv$

2. $F = mg - kv$

3. $m\frac{dv}{dt} = mg - kv$

4. $v(t) = \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right) \ + \ v_0 e^{-\frac{kt}{m}}$

5. $x(t) = \frac{mg}{k}t \ + \ \frac{m^2 g}{k^2}\left(e^{-\frac{kt}{m}} - 1\right) \ + \ \frac{v_0 m}{k}\left(1 - e^{-\frac{kt}{m}}\right)$

6. $x(v) = \frac{m}{k}(v_0 - v) \ + \ \frac{m^2 g}{k^2} \cdot \ln\left(\frac{mg - kv_0}{mg - kv}\right)$

7. $k = 3\pi\eta D$

8. $v_{max} = \frac{mg}{3\pi\eta D}$

9. $t_\alpha = -\frac{m}{3\pi\eta D} \ \ln(1-\alpha)$

10. Directly proportional to η and inversely proportional to D

11. $mg\frac{\rho_L}{\rho_S}$

12. $\frac{mg}{3\pi\eta D}\left(1 - \frac{\rho_L}{\rho_S}\right)$

13. $v(t) = \frac{mg}{3\pi\eta D}\left(1 - \frac{\rho_L}{\rho_S}\right)\left(1 - e^{-\frac{kt}{m}}\right)$

B.

1. $F_{drag} = -Cv^2$

2. $v(x) = \sqrt{\frac{mg - \left(mg - cv_B^2\right)e^{-\frac{2cx}{m}}}{c}}$

3. $\sqrt{\frac{mg}{c}}$

C.

1. $\vec{F}_{drag} = -k\vec{v}$

2. $\tau_{ascent} = \frac{m}{k}\ln\left(1 \ + \ \frac{kv_0\sin(\theta)}{mg}\right)$

3. $H_{max} = \frac{mv_0\sin(\theta)}{k} \ + \ \frac{m^2 g}{k^2}\ln\left(1 \ + \ \frac{kv_0\sin(\theta)}{mg}\right)$

4. $\frac{mg\tau_{descent}}{k} \ + \ \frac{m^2 g}{k^2}\left[e^{-\frac{k\tau_{descent}}{m}} - 1\right] \ = \ H_{max}$

5. $R \ = \ \frac{mv_0}{k}\left[1 - e^{-\frac{k(\tau_{ascent}+\tau_{descent})}{m}}\right]$

D. $v(t) = \frac{m_2 \cdot g \cdot d}{\eta A}\left(1 \ - \ e^{-\frac{\eta A t}{d(m_1+m_2)}}\right)$