# A HIGH SCHOOLER'S COLLECTION OF INTEGRALS A-Z

D. Majumder

March 18, 2025

## 1 Questions

A.

$$\int_0^1 e^{-\frac{1}{2}\lfloor \frac{1}{x} \rfloor} \prod_{p=1}^{\lfloor \frac{1}{x} \rfloor} e^{px}\, dx$$

B.

$$\int_1^\infty \frac{dx}{\left(\sum_{p=1}^{\infty} (-1)^p \;\; max\{0, x-p\}\right)^2}$$

C.

$$\sum_{n=1}^{\infty} \int_0^1 \left(\prod_{p=1}^{n} (x_p)^p\right) dx_1\, dx_2 \, ...dx_n$$

D.

$$\forall\, n \in W \;,\; T_{n+1} \; = 2xT_n \; - T_{n-1} \; with \; T_0 = 1 \; and \; T_1 = x.$$

$$Compute : \; \sum_{n=2}^{\infty} \int_0^1 T_n \; dx$$

E.

$$\lim_{n\to\infty} \int_0^1 \int_0^1 ... \int_0^1 \sin^2\left(\frac{\pi}{2n}(x_1 + x_2 + ... + x_n)\right) \; dx_1 \; dx_2 \; ... \; dx_n$$

F.

$$\sum_{p=1}^{\infty} e^{\int_1^0 \frac{x^p+x^{p+1}-2}{ln(x)}dx}$$

G.

$$\sum_{n=0}^{\infty}\left(\int_0^{\pi}\sin^{2n}(x)\,dx\cdot\int_0^{\pi}\sin^{2n+1}(x)\,dx\right)^2$$

H.

$\forall\, n \in Z^+$ *let* $p_n(x)$ *denote the unique polynomial with integer coefficients suchthat*

$$p_n(\ln(2)) \;=\; \int_1^2 (\ln(x))^n$$

$$Compute \sum_{n=0}^{\infty}\frac{1}{p_n(0)}$$

I.

$$Compute: \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(1/4)^{m+n}}{(2n+1)(m+n+1)}$$

J.

*Let* $A_n$ *be the* $n\times n$ *matrix whose element in the* $i^{th}$ *row and* $j^{th}$ *column ,* $a_{i,j}$ *is given by*

$$a_{i,j}=\frac{1}{min(i,j)}$$

*Let* $B_n(x)$ *be the* $(n-1)\times(n-1)$ *matrix whose element in the* $i^{th}$ *row and* $j^{th}$ *column ,* $b_{i,j}$ *is given by*

$$b_{i,j} \;=\begin{cases} x & i\neq j\\ x\cdot(i+2) & i=j\end{cases}$$

$$Evaluate: f(\pi)\ if\ f(x)=\lim_{n\to\infty}\sum_{k=2}^{n}\left(|\det(A_k)|\cdot\int_0^x\left(\frac{\det(B_k(t))}{\sum_{p=1}^{k}\frac{1}{p}}\right)dt\right)$$

K.

$$The\ Chebyshev\ Polynomials\ are\ defined\ as:$$

$$P_n(x) \;=\; \cos\left(n \arccos(x)\right)\ \forall\, n \in W$$

$$Compute: \sum_{n=0}^{\infty} \int_{-1}^{1} P_n(x)\, dx$$

L.

$$Define\ f_0(x) = 1\ and\ \forall\, n \in\ N\ f_n(x) = x^{f_{n-1}(x)}.\ Compute$$

$$\lim_{n\to\infty} \sum_{p=1}^{\infty} \left( \int_{e}^{e^{2p}} \frac{f_n^{'}(x)}{f_n(x) f_{n-1}(x) \ln(x)} \; - \; \frac{f_{n-1}^{'}(x)}{f_{n-1}(x)}\, dx \right.$$

$$- \int_{e}^{e^{2p-1}} \frac{f_{n-1}^{'}(x)}{f_{n-1}(x) f_{n-2}(x) \ln(x)} - \frac{f_{n-2}^{'}(x)}{f_{n-2}(x)}\, dx$$

$$+ \int_{e}^{e^{2p}} \frac{f_{\lfloor \pi \rfloor}^{'}(x)}{f_{\lfloor \pi \rfloor}(x) f_{\lfloor e \rfloor}(x) \ln(x)} \; - \; \frac{f_{\lfloor e \rfloor}^{'}(x)}{f_{\lfloor e \rfloor}(x)}\, dx$$

$$\left. - \int_{e}^{e^{2p+1}} \frac{f_{n-3}^{'}(x)}{f_{n-3}(x) f_{n-4}(x) \ln(x)} \; - \; \frac{f_{n-4}^{'}(x)}{f_{n-4}(x)}\, dx \right)$$

$$\left( Use: \prod_{n=1}^{\infty} \frac{4n^2}{4n^2 - 1} = \frac{\pi}{2} \right)$$

M.

$$Let\ D_0(x) \;=\; \frac{\sin(x)}{x}\ and\ D_{n+1}(x) = e^{-x} D_n(x)\ \forall\, n \in\ W.\ If\ \int_0^{\infty} D_0(x)\, dx \;=\; \frac{\pi}{2}\ .$$

$$Compute\ the\ value\ of: \sum_{n=0}^{\infty} \int_0^{\infty} D_{2n^2}(x)\, dx$$

N.

$$\textit{Compute}: \lim_{n\to\infty} n^{-\frac{1}{2}\left(1+\frac{1}{n}\right)} \left(1^1 \cdot 2^2 \cdot 3^3 ... n^n\right)^{\frac{1}{n^2}}$$

O.

$$\textit{Let } P_0(x) = x^3 - \frac{3}{45}x^2 + \frac{4x}{(45)^2} - \frac{2}{(45)^3} + \frac{1}{45} \textit{ and } P_{n+1}(x) = P_n(P(x)).$$

$$\textit{Compute}: \int_0^{\frac{2}{45}} P_{2025}(x)\, dx$$

P.

*Let* $T(s,\ \alpha)$ *denote the coefficient of* $x^s$ *in the expansion about* $x = 0$ *of* $(1+x)^\alpha$

$$\textit{Evaluate}: \sum_{n=1}^{\infty} \frac{1}{\int_0^1 \left(T^2(n,\ -y\ -1) \sum_{k=0}^{n} \frac{1}{y+k}\right) dy}$$

Q.

$$\textit{Evaluate} \int_{-100}^{-10} \left(\frac{x^2 - x}{x^3 - 3x + 1}\right)^2 dx + \int_{\frac{1}{101}}^{\frac{1}{11}} \left(\frac{x^2 - x}{x^3 - 3x + 1}\right)^2 dx + \int_{\frac{101}{100}}^{\frac{11}{10}} \left(\frac{x^2 - x}{x^3 - 3x + 1}\right)^2 dx$$

*Answer in the form of* $\frac{a}{b} - \frac{c}{d}$ *where a and b are coprime and c and d are coprime*

## 2 Answers

$$A.\ 2(e^{\frac{1}{2}} - 1)$$

$$B.\ 1 + \frac{\pi^2}{6}$$

$$C.\ e - 2$$

$$D.\ -1$$

$$E. \frac{1}{2}$$

$F.\ 1$

$G. \frac{\pi^4}{2}$

$H.\ \frac{1}{e}$

$I.\ \ln^2(3)$

$J. e^{\pi} - \pi - 1$

$K.\ 1$

$L.\ \ln\left(\frac{\pi}{2}\right)$

$M. \frac{3\pi}{4}$

$N.\ e^{-\frac{1}{4}}$

$O. \frac{2}{2025}$

$P.\ \frac{3}{2}$

$Q.\ \frac{110}{969} - \frac{10100}{999699}$