A Gas Compression Plant

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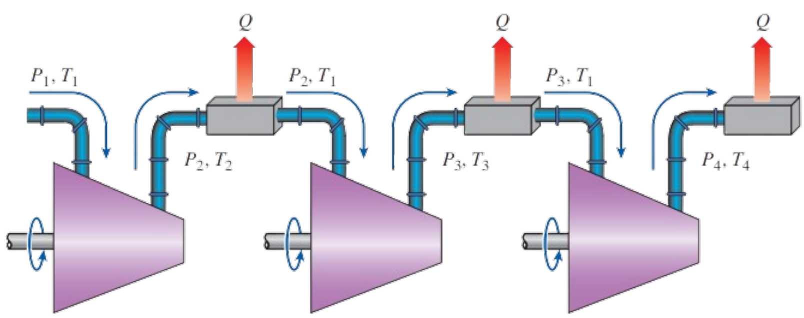
A. The Physics behind the plant

In large gas-compression stations (for example, on a natural gas pipeline), the compression is done in several stages as illustrated in the figure. At the end of each stage, the compressed gas is cooled at constant pressure back to the temperature at the inlet of the compressor. Consider a compression station that is to compress n moles of methane from P_1 to P_{N+1} in N stages, where each stage has an isentropic compressor coupled to a reversible, isobaric cooling unit. The initial temperature of the gas is T_1. We define a ratio r_p to be the ratio of final pressure of the gas to initial pressure for simplicity.



Use these values where needed:

- Universal gas constant R = 8.314 $J\ mol^{-1}\ K^{-1}$
- Adiabatic index of methane $\gamma = 1.31$

Answer in terms of variables unless mentioned otherwise

1. If the compression is carried out with a single isentropic compressor, what is the magnitude of the work done?

As an engineer, you are tasked with initializing the intermediate N-1 pressures in such a way that the work done is minimized in the N-fold process. With this target in mind answer the following questions.

2. What is the magnitude of the work done in terms of n, γ, R, T_1 and the pressure P_i where $i = 1, 2, \ldots, N+1$

3. Find out the value(s) of the consecutive pressure ratios $\frac{P_1}{P_2}$, $\frac{P_2}{P_3}$, ... , $\frac{P_N}{P_{N+1}}$ and how they differ from each other in the optimized scenario.

4. Evaluate the intermediate temperatures T_i for i =2, 3, ..., N

5. What would be the magnitude of work done $W_{min}(N)$ if the intermediate pressures are optimized to yield the lowest absolute value of work done.

6. The function $W_{min}(N)$ is generalized to $W_{min}(x)$ where x can take any value. Plot $W_{min}(x)$ in first quadrant of the cartesian plane.

7. Write the equation of the asymptote of $W_{min}(x)$ in the first quadrant. What is the physical significance of this asymptote?

8. What is the approximate percentage decrease in the work done $\alpha(N)$ when N coupled units are used instead of one coupled unit.

9. What is the global maximum value of $\alpha(N)$?

B. The Economics behind the plant

10. Complete the following table with numerical values for a gas compression plant which compresses 10 moles of methane at 300K from a pressure of 1 atm to $\frac{1}{40}$ atm.

N	$W_{min}(N)$ (in kJ)	$\alpha(N)$
1		
2		
5		
10		
20		
40		
100		

11. As an engineer, you must also take care of not overspending and getting the best bang for the buck. Assume it costs your company \$$a$ per coupled unit and \$$b$ for every joule of work done. Write the cost function $C(N)$ for N coupled units in the plant.

12. Using the previous cost function set-up an equation to find out the optimum value of N in terms of relevant parameters.

Answers

A.

1. $n\frac{\gamma R}{\gamma-1}T_1\left(r_p^{\frac{\gamma-1}{\gamma}}-1\right)$

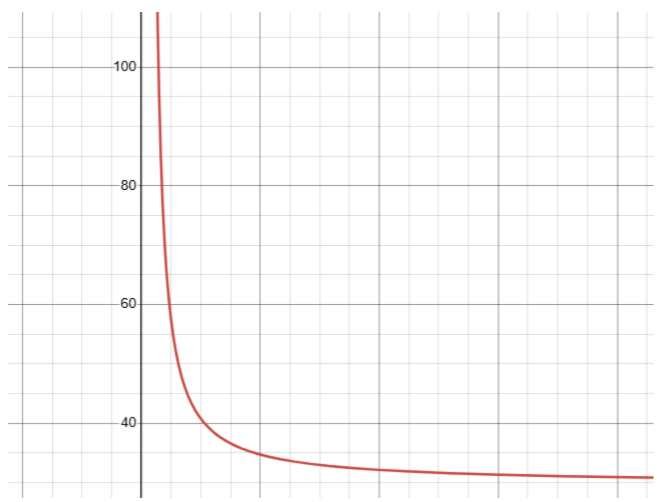
2. $n\frac{\gamma R}{\gamma-1}T_1\left[\left(\sum_{i=1}^{N}\left(\frac{P_{i+1}}{P_i}\right)^{\frac{\gamma-1}{\gamma}}\right)-N\right]$

3. In the optimized scenario, the consecutive pressure ratios are equal to one another i.e. they form a geometric progression and the common ratio $r = r_p^{\frac{\gamma-1}{N\gamma}}$

4. $T_i = T_1 r_p^{\frac{i-1}{N}\cdot\left(\frac{\gamma-1}{\gamma}\right)^2}$

5. $W_{min}(N) = nN\frac{\gamma R}{\gamma-1}T_1\left(r_p^{\frac{\gamma-1}{N\gamma}}-1\right)$

6.



7. $x = nRT_1\ln\left(r_p\right)$. The asymptotic value is the minimum work done when the number of coupled units tends to infinity. It is also equal to the work done if the process was carried out in a single isothermal process.

8. $\alpha(N) = 100\cdot\left(1-N\frac{\left(r_p^{\frac{\gamma-1}{N\gamma}}-1\right)}{\left(r_p^{\frac{\gamma-1}{\gamma}}-1\right)}\right)$

9. $\alpha_{max} = 100 \cdot \left(1 - \frac{\gamma-1}{\gamma} \frac{\ln(r_p)}{\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)}\right)$

B.

10.

N	$W_{min}(N)$ (in kJ)	$\alpha(N)$
1	146.92	0
2	115.36	21.48
5	100.53	31.57
10	96.14	34.56
20	94.05	35.98
40	93.02	36.69
100	92.41	37.10

11. $C(N) = aN + bnN\frac{\gamma R}{\gamma-1}T_1\left(r_p^{\frac{\gamma-1}{N\gamma}} - 1\right)$

12. Suppose $\alpha > 0$ satisfies

$$a + bn\alpha\frac{\gamma R}{\gamma - 1}T_1\left(r_p^{\frac{\gamma-1}{\alpha\gamma}}\left(1 - \frac{\gamma - 1}{\gamma}\frac{\ln(r_p)}{\alpha}\right) - 1\right) = 0$$

Then N could be $\lfloor\alpha\rfloor$ or $\lceil\alpha\rceil$.