Four Cycles

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Notations

In general the variables used in the question and their definitions are:

- P is the Pressure of the Gas
- V is the Volume of the Gas
- T is the Temperature of the Gas
- C_v is the specific heat of the gas at constant volume
- C_p is the specific heat of the gas at constant pressure
- γ is the adiabatic index which is the ratio of C_p and C_v
- S is the entropt of the gas
- Q is the heat absorbed or released by the gas
- W is Work
- η is the thermodynamic efficiency of the cycle

Two important areas of application for thermodynamics are power generation and refrigeration. Both are usually accomplished by systems that operate on a thermodynamic cycle. Thermodynamic cycles can be divided into two general categories: power cycles and refrigeration cycles. Thermodynamic cycles can also be categorized as gas cycles and vapor cycles, depending on the phase of the working fluid. In gas cycles, the working fluid remains in the gaseous phase throughout the entire cycle, whereas in vapor cycles the working fluid exists in the vapor phase during one part of the cycle and in the liquid phase during another part.

The devices or systems used to produce a net power output are often called engines, and the thermodynamic cycles they operate on are called power cycles. In this question we will be dealing with the workings and applications of 4 gas power cycles.

Use the following assumptions in answering the questions:

- The cycle does not involve any friction. Therefore, the working fluid does not experience any pressure drop as it flows in pipes or devices such as heat exchangers.
- All expansion and compression processes take place in a quasiequilibrium manner.
- The pipes connecting the various components of a system are well insulated, and heat transfer through them is negligible.
- We are dealing with n moles of the gas in all the subparts

The efficiency of a cycle can be determined by the formula

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

where :

- η is the efficiency of the cycle
- Q_{out} is the heat released in the cycle
- and Q_{in} is the heat absorbed in the cycle

A. Carnot Cycle

The Carnot cycle is composed of four totally reversible processes:

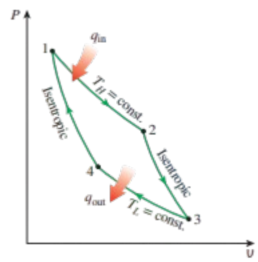
1-2: Isothermal expansion at a temperature T_H

2-3: Isentropic (Adiabatic) expansion to a temperature T_L

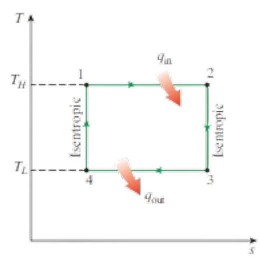
3-4: Isothermal expansion at a temperature T_L

4-1: Isentropic expansion back to temperature T_H

The pressure (P) versus Volume (V) curve:



The Temperature (T) versus Entropy curve(S):



Using the following information answer the questions given below

1. Express Q_{in} and Q_{out} in terms of entropies at processes 4-1 and 2-3 (s_1 and s_2), sink temperature (T_L) and source temperature (T_H).

2. Evaluate the work done (W) in the cycle.

3. Evaluate the efficiency of the cycle (η) in terms of T_H and T_L.

4. Compare the value of efficiency obtained in A.3 to the ratio of W and Q_{in}.

The actual gas power cycles are rather complex. To reduce the analysis to a manageable level, we utilize the following approximations, commonly known as the cold air-standard assumptions:

- The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.

- All the processes that make up the cycle are internally reversible.

- The combustion process is replaced by a heat-addition process from an external source
- The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.
- Air has constant specific heats whose values are determined at room temperature

The reciprocating engine (basically a piston–cylinder device) is one of the rare inventions that has proved to be very versatile and to have a wide range of applications.

The piston reciprocates in the cylinder between two fixed positions called the top dead center (TDC)—the position of the piston when it forms the smallest volume in the cylinder—and the bottom dead center (BDC)—the position of the piston when it forms the largest volume in the cylinder in a reciprocating engine.

In the following questions we will be understanding the working of two kinds of reciprocating engines and their ideal cycles.

B. Otto Cycle

The Otto cycle is the ideal cycle for spark-ignition reciprocating engines. Its working is described below:

Initially, both the intake and the exhaust valves are closed, and the piston is at its lowest position (BDC). During the compression stroke, the piston moves upward, compressing the air–fuel mixture. Shortly before the piston reaches its highest position (TDC), the spark plug fires and the mixture ignites, increasing the pressure and temperature of the system. The high-pressure gases force the piston down, which in turn forces the crankshaft to rotate, producing a useful work output during the expansion or power stroke. Toward the end of expansion stroke, the exhaust valve opens and the combustion gases that are above the atmospheric pressure rush out of the cylinder through the open exhaust valve. This process is called exhaust blowdown, and most combustion gases leave the cylinder by the time the piston reaches BDC. The cylinder is still filled by the exhaust gases at a lower pressure at BDC. Now the piston moves upward one more time, purging the exhaust gases through the exhaust valve (the exhaust stroke), and down a second time, drawing in fresh air–fuel mixture through the intake valve (the intake stroke)

The thermodynamic processes in an ideal Otto Cycle is given below:

1-2: Isentropic compression to TDC

2-3: Isochoric heat addition

3-4: Isentropic expansion to BDC

4-1: Isochoric heat rejection

Answer the questions below:

1. We define a compression ratio r which is ratio of maximum volume occupied by the gas to the minimum volume occupied by the gas. Find a relation between r, volume occupied in BDC (V_{BDC}) and volume occupied in TDC (V_{TDC}).

2. Sketch the P-V and T-S curve for the Otto Cycle.

3. Evaluate Q_{in}, Q_{out} and η in terms of specific heat at constant volume C_v of air and the temperatures. Assume that all the temperatures are known and the temperature at a point i is T_i for i = 1,2,3,4.

4. Find a relation between $\frac{T_1}{T_2}$, $\frac{T_4}{T_3}$, $r = \frac{V_1}{V_2}$ and adiabatic index γ using laws of adiabatic and isochoric expansion.

5. Hence express η_{otto} in terms of r and adiabatic index γ.

6. You are given three gases to use in an Otto cycle. The first gas A is monoatomic, the second gas B is diatomic and the third gas C is triatomic. Which gas would you use to maximize the power output. Support your answer with a reason.

C. Diesel Cycle

The Diesel cycle is the ideal cycle for Compression Ignition reciprocating engines. The working is given below:

In CI engines (also known as diesel engines), the air is compressed to a temperature that is above the autoignition temperature of the fuel, and combustion starts on contact as the fuel is injected into this hot air. Therefore, the spark plug is replaced by a fuel injector in diesel engines. In gasoline engines, a mixture of air and fuel is compressed during the compression stroke, and the compression ratios are limited by the onset of autoignition or engine knock. In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition. Therefore, diesel engines can be designed to operate at much higher compression ratios, typically between 12 and 24. Not having to deal with the problem of autoignition has another benefit: many of the stringent requirements placed on the gasoline can now be removed, and fuels that are less refined (thus less expensive) can be used in diesel engines. The fuel injection process in diesel engines starts when the piston approaches TDC and continues during the first part of the power stroke. Therefore, the combustion process in these engines takes place over a longer interval. Because of this longer duration, the combustion process in the ideal Diesel cycle is approximated as a constant-pressure heat-addition process. In fact, this is the only process where the Otto and the Diesel cycles differ. The remaining three processes are the same for both ideal cycles.

The thermodynamic processes in an ideal Diesel Cycle is given below:

1-2: Isentropic compression

2-3: Isobaric pressure heat addition

3-4: Isentropic expansion

4-1: Isochoric heat rejection

1. We define another ratio called the cutoff ratio r_c which is defined as the ratio of cylinder volumes after and before the combustion. If the volumes for each of the points i are known and are equal to V_i for i=1,2,3,4, find the expression for r_c in terms of the appropriate volumes.

2. Sketch the P-V and T-S curve for the Diesel Cycle.

3. The pressure, volume and temperature at the point 1 are respectively P_1, V_1 and T_1. The compression ratio r and cutoff ratio r_c are known. The adiabatic index of the gas used is γ. Evaluate the pressure, volume and temperature at the points 2, 3 and 4 using the variables given in this subpart.

4. Proceeding from C.3 evaluate Q_{in}, Q_{out} and W using specific heats of the gas (C_p and C_v), adiabtic index γ, T_1, r and r_c.

5. Hence evaluate η_{diesel} in terms of adiabatic index γ, r and r_c.

6. Find:

$$\lim_{r_c \to 1} \eta_{diesel}\left(r, r_c\right).$$

What is the physical significance of this result in relation with the Otto Cycle?

7. Compare the efficiencies of Otto Cycle and Diesel under similar conditions with reasonable approximations.

D. Dual Cycles

In modern high-speed compression ignition engines, fuel is injected into the combustion chamber much sooner than in the early diesel engines. Fuel starts to ignite late in the compression stroke, and consequently part of the combustion occurs almost at constant volume. Fuel injection continues until the piston reaches the TDC, and combustion of the fuel keeps the pressure high well into the expansion stroke. Thus, the entire combustion process can be better modeled as the combination of constant-volume and constant-pressure processes. The ideal cycle based on this concept is called the dual cycle. The relative amounts of heat transferred during each process can be adjusted to approximate the actual cycle more closely. Dual cycle is a more realistic model than Diesel cycle for representing modern, high-speed compression ignition engines.

The processes in a cold-air standard dual cycle is given as:

1-2: Isentropic compression with a compression ratio, $r = \frac{V_1}{V_2}$

2-3: Isochoric heat addition with a pressure ratio, $r_p = \frac{P_3}{P_2}$

3-4: Isobaric heat addition with a volume ratio, $r_c = \frac{V_4}{V_3}$

4-5: Isentropic expansion while work is done until $V_5 = V_1$

5-1: Isochoric heat rejection to the initial state

1. Sketch the P-V and T-S curves for this cycle.

2. Obtain an expression for the cycle thermal efficiency $\eta\left(\gamma, r, r_c, r_p\right)$ as a function of adiabatic index γ, r, r_c, and r_p.

3. Evaluate:

$$\lim_{r_c \to 1} \eta\left(\gamma, r, r_c, r_p\right)$$

Compare the answer with the Otto cycle efficiency.

4. Evaluate:

$$\lim_{r_p \to 1} \eta\left(\gamma, r, r_c, r_p\right)$$

Compare the answer with the Diesel cycle efficiency.

ANSWERS

A.

1. $Q_{in} = T_H(s_2 - s_1)$ $Q_{out} = T_L(s_2 - s_1)$
2. $(T_H - T_L) \cdot (s_2 - s_1)$
3. 1 - $\frac{T_L}{T_H}$
4. The ratio is equal to the efficiency obtained in A.3

B.

1. r = $\frac{V_{BDC}}{V_{TDC}}$
2.
3. $Q_{in} = nC_v(T_3 - T_2)$ $Q_{out} = nC_v(T_4 - T_1)$
η = 1 - $\frac{T_4 - T_1}{T_3 - T_2}$
4. $\frac{T_1}{T_2} = \frac{T_4}{T_3} = \frac{1}{r^{\gamma-1}}$
5. $\eta_{otto} = 1 - \frac{1}{r^{\gamma-1}}$

C.

1. $r_c = \frac{V_3}{V_2}$
2.
3. Let the Pressure, Volume and Temperature be indicated by (P, V, T)
At Point 2 $(P_1 r^\gamma,\ \frac{V_1}{r},\ T_1 r^{\gamma-1})$
At Point 3 $(P_1 r^\gamma,\ \frac{r_c}{r}V,\ T_1 r_c r^{\gamma-1})$
At Point 4 $(P_1 {r_c}^\gamma,\ V_1,\ T_1 r_c^\gamma)$
4. $Q_{in} = nC_p T_1 r^{\gamma-1}(r_c - 1)$ $Q_{out} = nC_v T_1(r_c^\gamma - 1)$
$W = nC_p T_1 r^{\gamma-1}(r_c - 1) - nC_v T_1(r_c^\gamma - 1)$
5. $\eta_{diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^\gamma - 1}{\gamma(r_c - 1)}\right]$
6. The limit tends to the value $1 - \frac{1}{r^{\gamma-1}}$ which is the efficiency of an Otto cycle with a compression ratio r
7. Generally for values of $r_c > 1$ $\eta_{otto} > \eta_{diesel}$

D.

1.

2. $\eta_{dual} \;=\; 1 \;-\; \left[\frac{r_p r_c^{\gamma}-1}{r^{\gamma-1}\;[\;(r_P-1\;)+\gamma{\cdot}r_p{\cdot}(r_c-1)\;]}\right]$

3. The limit approaches $1 \;-\; \frac{1}{r^{\gamma-1}}$ which is the efficiency of an Otto cycle with a compression ratio r

3. The limit approaches $\eta_{diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma}-1}{\gamma(r_c-1)}\right]$ which is the efficiency of an Diesel cycle with a compression ratio r and cutoff ratio r_c

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This question was made by me with the help of the book Engineering Thermodynamics by Cengel, Boles and Kanoglu Ninth Edition. For the P-V and T-S diagram please refer to the book mentioned.