# Fluids

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Question 1

A milling machine is used in factories and metalworking shops to cut slots and grooves in metal workpieces (Figure A). The material to be machined is held and moved beneath a rapidly rotating cutting tool. The workpiece and its holder slide over smooth guideways that are lubricated with oil having a viscosity of **240 cP**. The two guideways are each of length **40cm** and width **8cm** (Figure B). While setting up for a particular cut, a machinist disengages the drive mechanism, applies **90N** of force to the table holding the workpiece, and is able to push it **15cm** in **1 s**. Calculate the thickness of the oil film between the table and the guideways.

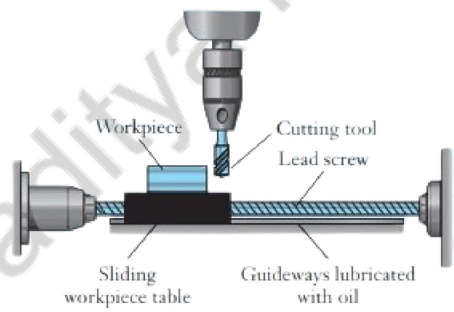



*Figure A*

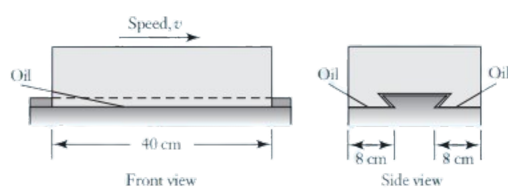



*Figure B*

Question 2

At any time, approximately 20 volcanoes are actively erupting on the Earth, and 50–70 volcanoes erupt each year. Over the past 100 years, an average of 850 people have died each year from volcano eruptions. As scientists and engineers study the mechanics of lava flow, accurately predicting the flow rate (velocity) of the lava is critical to saving lives after an eruption. Jeffrey's equation captures the relationship between flow rate and viscosity as:

$$V = \frac{\rho g t^2 \sin(\alpha)}{3\mu}$$

where $\rho$ is the density of the lava, g is gravity, t is the flow thickness, $\alpha$ is the slope, and $\mu$ is the lava viscosity. Typical values for the viscosity and density of lava are $\mathbf{4.5} \times \mathbf{10}^3 kg/(m\cdot\ \mathbf{s})$ and $\mathbf{2.5}\ g/cm^3$, respectively. Find the velocity of the flow in Figure C in cm/s and mph.

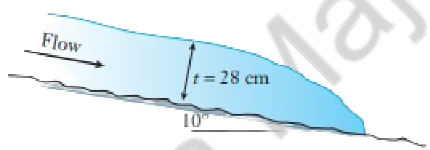



*Figure C* ($\alpha = 10°$)

Using the volcanic flow model and flow parameters given, prepare two charts:

(a) On one chart, plot the flow velocity in mph as a function of the slope, varying the slope from 0○ to 90○.

(b) On the other chart, plot the flow velocity in mph as a function of the flow thickness, varying the thickness from 0 to 300 cm.

(c) Discuss and compare the influence of the slope and the thickness on the flow velocity.

Question 3

Compressible fluids are substances, typically gases, whose density changes significantly under varying pressure and temperature conditions, unlike incompressible fluids where density remains nearly constant. A model of a compressible fluid is provided below.

Consider a long narrow cylinder (Figure D) of cross section A filled with compressible liquid upto a height h whose density $\rho$ is a function of the pressure $P(z)$ as

$$\rho(z) = \frac{\rho_0}{2}\left(1 + \frac{P(z)}{P_0}\right)$$

Where $P_0$ and $\rho_0$ are constants. The depth z is measured from the free surface of the liquid where the pressure is equal to the atmospheric pressure ($P_{atm}$).

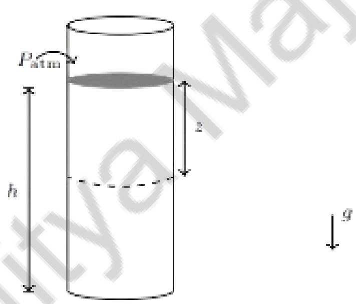



*Figure D*

(a) Obtain $P(z)$ as a function of $z$

(b) Obtain the mass M of the liquid in the cylinder

(c) Let $P_i(z)$ be the pressure at z if the liquid were incompressible with a density $\frac{\rho_0}{2}$. Assuming $P_0 >> \rho_0 gh$ obtain an approximated expression for

$$\Delta P = P(z) - P_i(z)$$

Question 4

As shown in Figure E, we consider two stationary, spherical water drops on the surface of a superhydrophobic material, i.e., very strong repulsive force exists between material and water. Initially two identical neighboring spherical water drops are placed on the surface; then these two drops are merged after touching each other to form a larger spherical water drop, which suddenly jumps up.

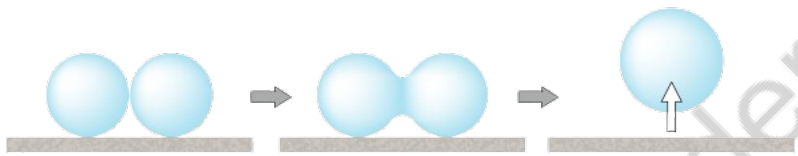


*Figure E*

A portion k of the difference of surface energy before and after the merger $\Delta E$ is transformed into the kinetic energy of the jumping water drop

The radius $a$ of the water drops before the merger is 100 $\mu m$. The density $\rho$ of water is $1.0 \times 10^3$ $kg/m^3$. The surface tension $\gamma$ is $7.27 \times 10^{-2}$ $J/m^2$.

Assuming:

- k=0.06
- The volume of the water before and after the merger is conserved

Determine:

(a) The velocity of the combined water drop immediately after the merger

(b) The maximum height the water drop rises assuming the entire kinetic energy is converted to potential energy

Question 5

In this question you will analyse a simplified models of blood flow in capillaries using Poiseuille's law.

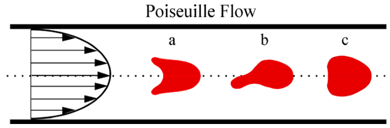



*Figure F*

Let us regard blood as an incompressible viscous fluid with mass density $\mu$ similar to that of water and dynamic viscosity $\eta = 4.5\ g\ m^{-1}\ s^{-1}$. We model blood vessels as circular straight pipes with radius r and length L and describe the blood flow by Poiseuille's law,

$$\Delta p = RD$$

the Fluid Dynamics analog of Ohm's law in Electricity. Here $\Delta p$ is the pressure difference between the entrance and the exit of the blood vessel, D = S$\nu$ is the volume flow through the cross-sectional area S of the blood vessel and $\nu$ is the blood velocity. The hydraulic resistance R is given by

$$R = \frac{8\eta L}{\pi r^4}$$

For the systemic blood circulation (the one flowing from the left ventricle to the right auricle of the heart), the blood flow is D $\approx$ 100cm$^3$ $s^1$ for a man at rest. Answer the following questions under the assumption that all capillary vessels are connected in parallel and that each of them has radius r = 4 $\mu$m and length L = 1 mm and operates under a pressure difference $\Delta p$ = 1 kPa.

(a) How many capillary vessels are in the human body?

(b) How large is the velocity with which blood is flowing through a capillary vessel?

Question 6

Fluids in which shear stress is not directly proportional to deformation rate are non-Newtonian. Two familiar examples are toothpaste and paint. Numerous empirical equations have been proposed to model the observed relations between shear stress $\tau_{yx}$ and $\frac{du}{dy}$ for time-independent fluids. They may be adequately represented for many engineering applications by the power law model, which for one-dimensional flow becomes

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n ...(A)$$

where the exponent, n, is called the flow behavior index and the coefficient, k, the consistency index. This equation reduces to Newton's law of viscosity for n= 1 with k $=\mu$.

To ensure that $\tau_{yx}$ has the same sign as $\frac{du}{dy}$, Eq. A is rewritten in the form

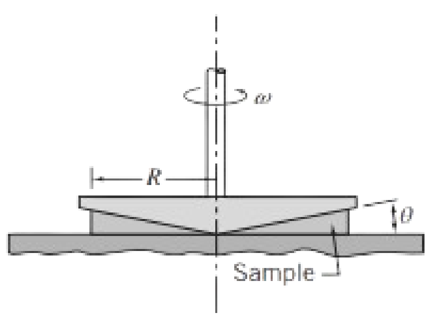
$$\tau_{yx} = k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} = \eta\frac{du}{dy}$$

The term $\eta = k\left|\frac{du}{dy}\right|^{n-1}$ is termed as the apparent viscosity

With the following information answer the question below

The cone and plate viscometer shown is an instrument used frequently to characterize non-Newtonian fluids. It consists of a flat plate and a rotating cone with a very obtuse angle (typically $\theta$ is less than 0.5 degrees). The apex of the cone just touches the plate surface and the liquid to be tested fills the narrow gap formed by the cone and plate. The viscometer is used to measure the apparent viscosity of a fluid. The data below are obtained.

| Speed (rpm) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|
| $\mu$ ($N \cdot m/s^2$) | 0.121 | 0.139 | 0.153 | 0.159 | 0.172 | 0.172 | 0.183 | 0.185 |



(a) Find the values of k and n used in defining the apparent viscosity of the fluid. (Assume $\theta$ is 0.5 degrees.)

(b)Predict the viscosity at 90 and 100 rpm.

Answers

1. 25.6 $\mu$m
2. 2.47 cm/s
3. (a) $P(z) = \left[(P_0 + P_{atm})e^{\frac{\rho_0 g z}{2P_0}} - P_0\right]$
   (b) $M = \frac{A}{g}(P_0 + P_{atm})\left(e^{\frac{\rho_0 g h}{2P_0}} - 1\right)$
   (c) $\Delta P = \frac{(\rho_0 g z)^2}{8P_0} + \frac{P_{atm}}{P_0}\frac{\rho_0 g z}{2} + \frac{P_{atm}}{2}\left(\frac{\rho_0 g z}{2P_0}\right)^2$ which is correct upto second order in $\frac{\rho_0 g z}{2P_0}$
4. (a)0.23 m/s (b)2.699 mm
5. (a) 4.5 x$10^9$ (b) 0.44 mm/s
6. (a) k = 0.0449 n = 1.2 (b) At 90 rpm: 0.190; 100 rpm: 0.194