Machine Learning

LABORATORY: Gradient Descent Homework

NAME:	STUDENT ID#:

Objectives:

- Understand and implement Mini-batch SGD (Algorithm 7.2).
- Extend the algorithm to support Momentum (Algorithm 7.3) and Adam optimization (Algorithm 7.4).
- Apply each optimizer to a binary classification task using the MNIST dataset.
- Evaluate and compare model behavior through accuracy and misclassified samples.
- Practice implementing mathematical update rules directly from textbook equations using NumPy.

Part 1. Instruction

- In this assignment, you will implement **Mini-batch Stochastic Gradient Descent (SGD)** and its extensions using **Algorithms 7.2, 7.3, and 7.4**.
- Your task is to build a **binary classifier** to determine whether an MNIST image matches a specific digit or not (e.g., "Is this a 4 or not?").
- You will implement **three** different methods: **Mini-batch SGD** (Algorithm 7.2), **SGD with Momentum** (Algorithm 7.3) and **Adam Optimizer** (Algorithm 7.4)
- You may write all algorithms in **one file with selectable modes**, or in **three separate files**.
- The code must be implemented **entirely with NumPy**. Do not use external machine learning libraries (e.g., scikit-learn, PyTorch).
- The model should output:
  - Final **accuracy** on the test set.
  - At least **five misclassified test samples**, with true and predicted labels shown.
- Use the **last digit of your student ID** as the `TARGET_DIGIT` for binary classification (e.g., ID ending in 7 → `TARGET_DIGIT = 7`).

Lecture: Prof. Hsien-I Lin
TA: Satrio Sanjaya and Muhammad Ahsan

Part 2. Arithmetic Instructions.

Algorithm 7.2: Mini-batch stochastic gradient descent

Input: Training set of data points indexed by $n \in \{1, \ldots, N\}$
Batch size B
Error function per mini-batch $E_{n:n+B-1}(\mathbf{w})$
Learning rate parameter η
Initial weight vector $\mathbf{w}$
Output: Final weight vector $\mathbf{w}$

$n \leftarrow 1$
repeat
  $\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla E_{n:n+B-1}(\mathbf{w})$ `// weight vector update`
  $n \leftarrow n + B$
  if $n > N$ **then**
    shuffle data
    $n \leftarrow 1$
  end if
until convergence
return $\mathbf{w}$

Algorithm 7.3: Stochastic gradient descent with momentum

Input: Training set of data points indexed by $n \in \{1, \ldots, N\}$
Batch size B
Error function per mini-batch $E_{n:n+B-1}(\mathbf{w})$
Learning rate parameter η
Momentum parameter μ
Initial weight vector $\mathbf{w}$
Output: Final weight vector $\mathbf{w}$

$n \leftarrow 1$
$\Delta\mathbf{w} \leftarrow \mathbf{0}$
repeat
  $\Delta\mathbf{w} \leftarrow -\eta \nabla E_{n:n+B-1}(\mathbf{w}) + \mu \Delta\mathbf{w}$ `// calculate update term`
  $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$ `// weight vector update`
  $n \leftarrow n + B$
  if $n > N$ **then**
    shuffle data
    $n \leftarrow 1$
  end if
until convergence
return $\mathbf{w}$

Algorithm 7.4: Adam optimization

Input: Training set of data points indexed by $n \in \{1, \ldots, N\}$
Batch size B
Error function per mini-batch $E_{n:n+B-1}(\mathbf{w})$
Learning rate parameter η
Decay parameters β_1 and β_2
Stabilization parameter δ
Output: Final weight vector $\mathbf{w}$

$n \leftarrow 1$
$\mathbf{s} \leftarrow \mathbf{0}$
$\mathbf{r} \leftarrow \mathbf{0}$
repeat
  Choose a mini-batch at random from $\mathcal{D}$
  $\mathbf{g} = -\nabla E_{n:n+B-1}(\mathbf{w})$ // evaluate gradient vector
  $\mathbf{s} \leftarrow \beta_1 \mathbf{s} + (1 - \beta_1)\mathbf{g}$
  $\mathbf{r} \leftarrow \beta_2 \mathbf{r} + (1 - \beta_2)\mathbf{g} \odot \mathbf{g}$ // element-wise multiply
  $\hat{\mathbf{s}} \leftarrow \mathbf{s}/(1 - \beta_1^\tau)$ // bias correction
  $\hat{\mathbf{r}} \leftarrow \mathbf{r}/(1 - \beta_2^\tau)$ // bias correction
  $\Delta\mathbf{w} \leftarrow -\eta \dfrac{\hat{\mathbf{s}}}{\sqrt{\hat{\mathbf{r}}} + \delta}$ // element-wise operations
  $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$ // weight vector update
  $n \leftarrow n + B$
  if $n + B > N$ **then**
    shuffle data
    $n \leftarrow 1$
  end if
until convergence
return $\mathbf{w}$

Part 3. Code Template.

Step	Procedure
1	#Load Dataset

```
import struct
import numpy as np
import matplotlib.pyplot as plt

# ==========Load IDX Files ==========
def load_images(filename):
    with open(filename, 'rb') as f:
        _, num, rows, cols = struct.unpack(">IIII",
f.read(16))
        images=np.frombuffer(f.read(),
dtype=np.uint8)
        images = images[:(len(images)//(rows * cols))
* rows * cols]
        return images.reshape(-1, rows *
cols).astype(np.float32) / 255.0

def load_labels(filename):
    with open(filename, 'rb') as f:
        _, num = struct.unpack(">II", f.read(8))
        labels = np.frombuffer(f.read(),
```


	dtype=np.uint8) return labels[:num]

2	# ========== 1. Sigmoid Function ==========
	def sigmoid(z):
	    # TODO: Implement sigmoid function
	    pass
	
	# ====== 2. Mini Batch SGD: Algorithm 7.2 ======
	def sgd_minibatch(X, y, eta=0.01, max_iters=10000,
	batch_size=64):
	
	    pass
	
	# ===== 3. Mini Batch SGD with Momentum: Algorithm 7.3 =====
	def sgd_minibatch_momentum(X, y, eta=0.01, max_iters=10000,
	batch_size=64, momentum=0.9):
	
	    pass
	
	# ====== 4. Adam Optimizer: Algorithm 7.4 ======
	def sgd_Adam(X, y, eta=0.001, max_iters=10000, batch_size=64,
	beta1=0.9, beta2=0.999, delta=1e-8):
	
	    pass

3	# ==========Show Misclassified Samples ==========
	def show_misclassified(X, true_labels, pred_labels,
	max_show=10):
	    mis_idx = np.where(true_labels !=
	pred_labels)[0][:max_show]
	    plt.figure(figsize=(10, 2))
	    for i, idx in enumerate(mis_idx):
	        plt.subplot(1, len(mis_idx), i + 1)
	        plt.imshow(X[idx, 1:].reshape(28, 28), cmap='gray')
	        plt.axis('off')
	        plt.title(f"T:{true_labels[idx]}
	P:{pred_labels[idx]}")
	    plt.suptitle("Misclassified Samples")
	    plt.show()

4	# ========== 3. Main ==========
	if __name__ == "__main__":
	    # === Load Data ===
	    X_train = load_images("train-images.idx3-ubyte__")
	    y_train = load_labels("train-labels.idx1-ubyte__")
	    X_test = load_images("t10k-images.idx3-ubyte__")
	    y_test = load_labels("t10k-labels.idx1-ubyte__")
	
	    # === Choose binary classification target digit ===
	    TARGET_DIGIT = 0 # TODO: Fill in (0 to 9)
	
	    y_train_bin = np.where(y_train == TARGET_DIGIT, 1, 0)
	    y_test_bin = np.where(y_test == TARGET_DIGIT, 1, 0)

```
    # === Add bias term ===
    X_train   =   np.hstack([np.ones((X_train.shape[0],   1)),
X_train])
    X_test    =    np.hstack([np.ones((X_test.shape[0],    1)),
X_test])

    # === Set parameters ===

    # === Train ===

    # === Predict ===

    # === Evaluate ===

    # === Show Misclassified Samples ===
```

Grading Assignment & Submission (70% Max)

Implementation(50%):

1. **Correctly** implemented, runs, shows accuracy and sample misclassification of:
   a. **(15%) Mini-batch SGD (Algorithm 7.2)**
   b. **(10%) SGD with momentum (Algorithm 7.3)**
   c. **(5%) SGD with Nesterov momentum (Eq. 7.34)**
   d. **(15%) Adam Optimizer (Algorithm 7.4)**
2. **(5%)** Compare the accuracy and test sample for each algorithm.

Question(20%):

1. (7%) Which optimizer gave you the best test accuracy? Why do you think it performed better than the others?
2. (8%) What is the differences in learning stability, convergence speed, or misclassification types across all algorithm? Please explain with examples or observation from your results.
3. (7%) How did your choice of learning rate, batch size, or momentum affect each optimizer? What values worked best in your experiments?

Submission :

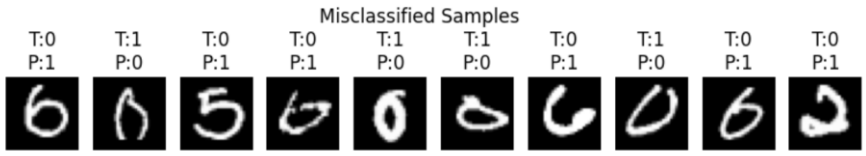
1. Report: Answer all conceptual questions. Include screenshots of your results in the last pages of this PDF File.
2. Code: Submit your complete Python script in either .py or .ipynb format.
3. Upload both your report and code to the E3 system (**Labs4_Homework_Assignment**). Name your files correctly:
   a. Report: StudentID_Lab4_Homework.pdf
   b. Code: StudentID_Lab4_ Homework.py or StudentID_Lab4_Homeworkipynb
4. Deadline: Sunday, 21:00 PM
5. Plagiarism is **strictly prohibited**. Submitting copied work from other students will result in penalties.

Example Output (Just for reference):

```
[INFO] Header: 60000 images, 28x28
[INFO] Loading 60000 images based on file size
[INFO] Loading 60000 labels based on file size
[INFO] Header: 10000 images, 28x28
[INFO] Loading 10000 images based on file size
[INFO] Loading 10000 labels based on file size

[INFO] Binary classification: '0' vs not-0

Test Accuracy (is 0 or not): 0.9927
```



Code Results and Answer: