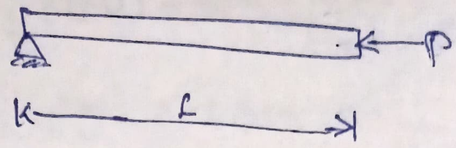
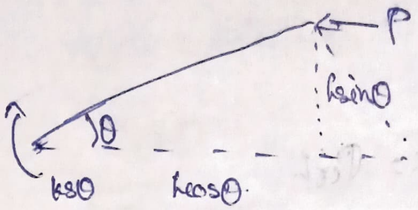
Tutorial - 1

2. a)





$\Sigma M_A = 0$

$-k\theta + Pl\sin\theta = 0$

$P = \dfrac{k\theta}{l\sin\theta}$ For small deflection

$P_{cr} = \dfrac{k\theta}{l\theta} = \dfrac{k}{l}$

$P = \dfrac{k\theta}{l\sin\theta}$

Energy Approach

$\Pi = U + V$

$U = \frac{1}{2}(k\theta)\theta = \frac{1}{2}k\theta^2$

$V = Pl(1-\cos\theta)$

$\Pi = \frac{1}{2}k\theta^2 - Pl(1-\cos\theta)$

For eqm TPE

$\dfrac{d\Pi}{d\theta} = 0$

$k\theta - Pl\sin\theta = 0$

$P_{cr} = \dfrac{k\theta}{l\sin\theta}$

To get Post buckling nature of Equilibrium -

$$\frac{d^2\Pi}{d\theta^2} = k - PL\cos\theta$$

In small defflection

$$\frac{d^2\Pi}{d\theta^2} = k - PL$$

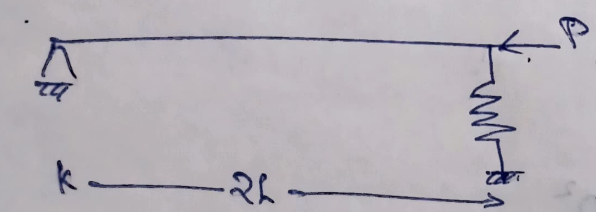
w.k.t $P_{cr} = k/L$ (or) $k = P_{cr}L$

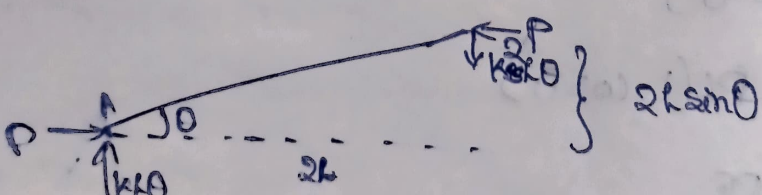
If $P < P_{cr} \Rightarrow \frac{d^2\Pi}{d\theta^2} \Rightarrow (+) \rightarrow$ stable.

$P > P_{cr} \Rightarrow \frac{d^2\Pi}{d\theta^2} \Rightarrow (-) \rightarrow$ unstable.

$P = P_{cr} \Rightarrow \frac{d^2\Pi}{d\theta^2} = 0 \rightarrow$ No conclusion.

b)





$\Sigma M_A = 0$

$$-k2L\theta \times 2L + P \times 2L\sin\theta = 0$$

$$P 2L\sin\theta = 2kL^2\theta$$

$$P = 2kL'$$

$$P\sin\theta = kL\theta$$

$$P = 2kL\theta.$$

$\boxed{P_{cr} = \frac{2}{?}kL}$ — bifurcation approach

$$\Pi = \frac{1}{2}k(2L\theta)^2 - 2PL(1-\cos\theta)$$

$$\frac{d\Pi}{d\theta} = 4kL^2\theta - 2PL\sin\theta = 0$$

$$4kL\theta = 2P\sin\theta$$

$$2kL\theta = P\theta$$

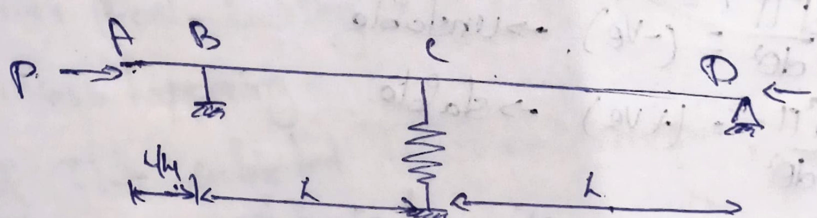
$$\boxed{P_{cr} = 2kL}$$ —

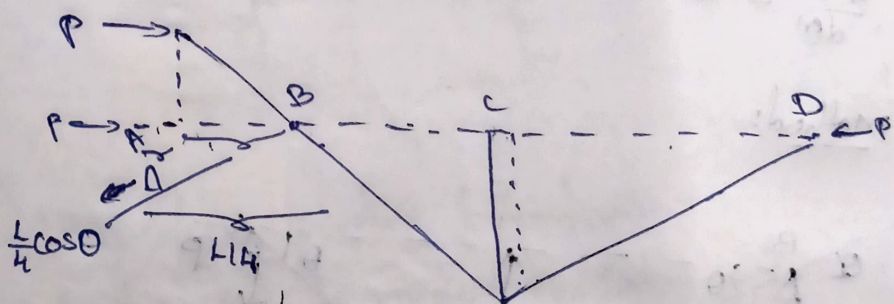
If $P < P_{cr}$, $d^2\Pi/d\theta^2 \rightarrow$ (+ve) stable

$P > P_{cr}$, $\frac{d^2\Pi}{d\theta^2} \rightarrow$ (−ve) unstable

$P = P_{cr}$, $\frac{d^2\Pi}{d\theta^2} = 0 \rightarrow$ No conclusion

4.





AB = $P\frac{L}{4}(1-\cos\theta)$

BC = $PL(1-\cos\theta)$

CD = $PL(1-\cos\theta$

$$U = \frac{1}{2}k(L\theta)^2$$

$$V = -\frac{PL}{4}(1-\cos\theta) - PL(1-\cos\theta) - PL(1-\cos\theta)$$

$$\Pi = \frac{1}{2}(k)(L\theta)^2 - \frac{PL}{4}(1-\cos\theta) - 2PL(1-\cos\theta)$$

$$\frac{d\Pi}{d\theta} = kL^2\theta - \frac{PL}{4}\sin\theta - 2PL\sin\theta = 0$$

For small deflection.

$$kL^2 = \frac{PL}{4} + 2PL$$

$$KL = \frac{9P}{4}$$

$$\frac{4PL}{9} = P$$

$$\boxed{P_{cr} = \frac{4kL}{9}}$$

$$\frac{d^2\Pi}{d\theta^2} = kL^2 - \frac{PL}{4}\cos\theta - 2PL\cos\theta$$

$$\frac{d^2\Pi}{d\theta^2} = kL^2 - \frac{9LP}{4}$$

$P > P_{cr}$ $\frac{d^2\Pi}{d\theta^2} = (-ve) \rightarrow$ unstable

$P < P_{cr}$ $\frac{d^2\Pi}{d\theta^2} = (+ve) \rightarrow$ stable.

$P = P_{cr}$ $\frac{d^2\Pi}{d\theta^2} = 0 \rightarrow$ No conclusion.

Bifurcation method:-

$$\Sigma M_B = 0$$

$$P\left(\frac{L\theta}{4}\right) - (kL\theta)L + R_D(2L) = 0$$

$$\frac{P\theta}{4} - kL\theta = -2R_D$$

$$\frac{kL\theta}{2} - \frac{P\theta}{8} = R_D$$

$$R_D = \frac{4kL\theta - P\theta}{8}$$

$\Sigma M_C = 0$

$R_D L - PL\theta = 0$

$R_D = P\theta$

$4kL\theta - P\theta = 8P\theta$

$4kL\theta = 9P\theta$

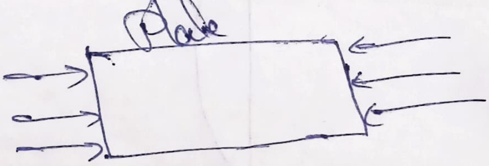
$$\boxed{P = \frac{4kL}{9}}$$

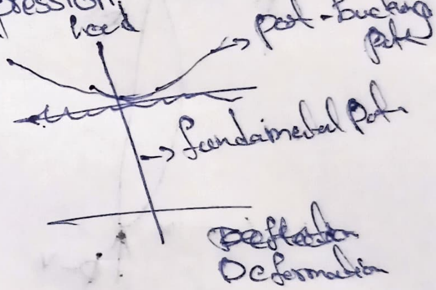
1.) Types of bifurcation instability:-

a) Stable symmetric bifurcation:-

Load required to maintain equilibrium increases after post-buckling (after P_{cr}) implies there is a hardening process happening.

Eg:- Plate subjected to uniaxial compression.

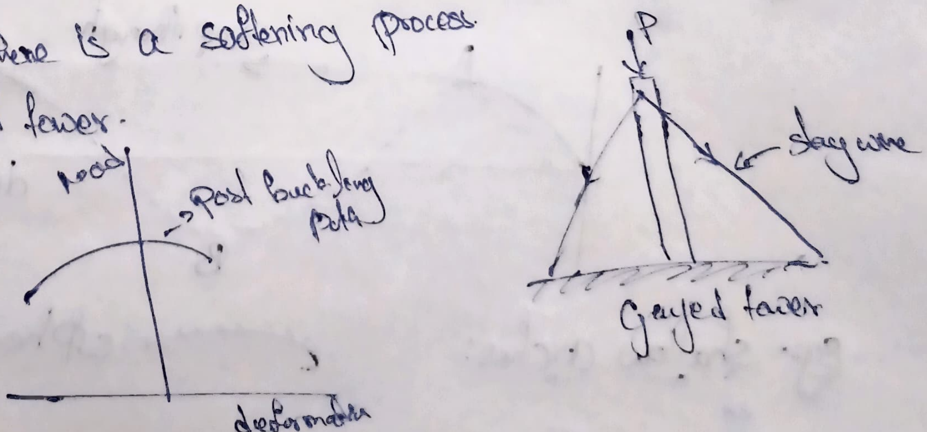




b) Unstable symmetric bifurcation:-

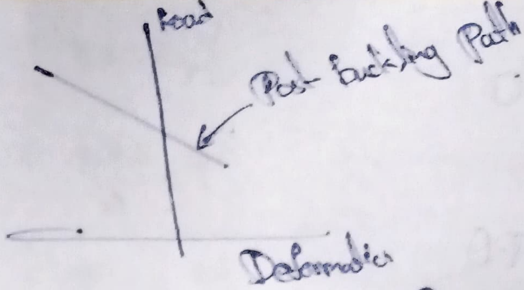
* Load required to maintain equilibrium reduces post-buckling.
* Implies there is a softening process.

Eg: guyed tower.

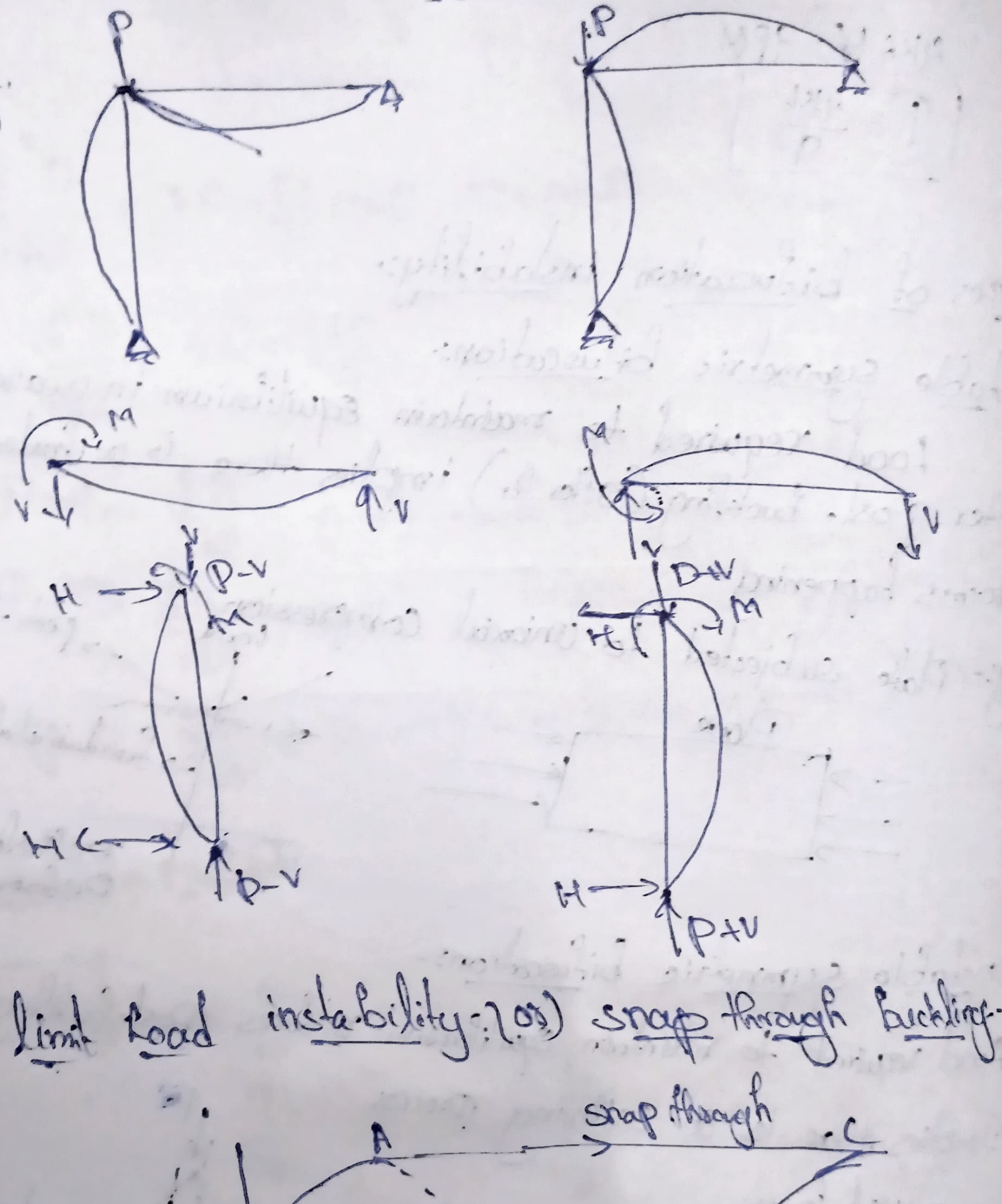


c) Assymmetric bifurcation:-

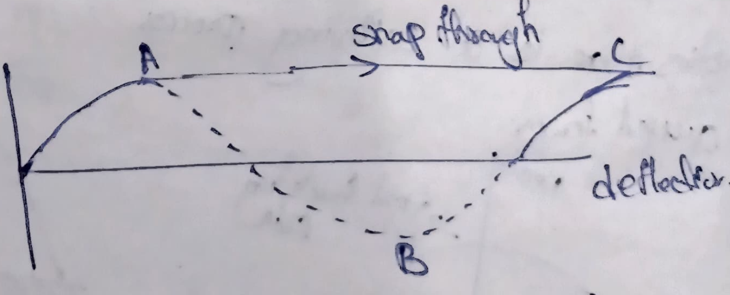
buckling may increses (or) decrease the load required to maintain equilibrium post-buckling.



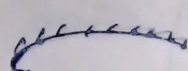
Eg:-



d) limit load instability :- (or) snap through buckling.

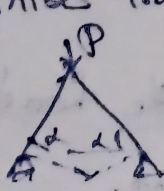


Eg:- shallow arches:



spherical caps

Van mises Truss:



* When load increases, there should exist a horizontal forces for this member.
* There should be a case where $\alpha = 0$ i.e, member become horizontal then, the member came to another side opposite (-ve side) called snap through buckling.

e) Shell Buckling (or) Finite disturbance buckling:-

(This has feature of Bifurcation and snap through.