Venus Dirigible Project
Complete Mathematical Formulation

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1 Introduction

This document provides the complete mathematical formulation for the Venus airship atmospheric entry simulation program.

2 Basic Notation

Symbol	Description
h	Altitude above surface [m]
v	Velocity [m/s]
m	Mass [kg]
T	Temperature [K]
ρ	Density [kg/m^3]
g	Gravitational acceleration [m/s^2]
C_D	Drag coefficient
A	Cross-sectional area [m^2]
t	Time [s]
q	Heat flux [W/m^2]
σ	Stress [Pa]

3 Venus Atmospheric Models

3.1 Gravitational Acceleration

Gravitational acceleration at altitude h:

$$g(h) = g_0 \left(\frac{R_V}{R_V + h} \right)^2$$

where:

- $g_0 = 8.87$ m/s^2 - gravitational acceleration at Venus surface

- $R_V = 6.0518 \times 10^6$ m - Venus radius

3.2 Atmospheric Density

Tabular data interpolation:

$$\log_{10} \rho(h) = a_0 + a_1 h + a_2 h^2 + a_3 h^3$$

$$\rho(h) = 10^{\log_{10} \rho(h)}$$

3.3 Atmospheric Temperature

$$T(h) = b_0 + b_1 h + b_2 h^2 + b_3 h^3$$

4 Aerodynamics

4.1 Drag Coefficient

Velocity-dependent empirical coefficient: Variable exponent $n(v)$ with resonance near 340 m/s

4.2 Drag Force

$$F_D = \frac{1}{2} C_D A \rho(h) |v|^{n(v)}$$

Components in Cartesian system:

$$F_{Dx} = -F_D \frac{v_x}{v}, \quad F_{Dy} = -F_D \frac{v_y}{v}$$

where $v = \sqrt{v_x^2 + v_y^2}$

5 Equations of Motion

5.1 System of Differential Equations

$$\begin{cases} \dfrac{dv_x}{dt} = \dfrac{F_{Dx}}{m} \\ \dfrac{dv_y}{dt} = -g(h) + \dfrac{F_{Dy}}{m} \\ \dfrac{dh}{dt} = v_y \end{cases}$$

5.2 Numerical Integration

4th order Runge-Kutta method:

$$k_1 = h f(t_n, y_n)$$

$$k_2 = h f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

6 Thermal Loads

6.1 Heat Flux

$$q = K\rho v^3$$

where $K = 1.0 \times 10^{-5}$ - empirical coefficient.

6.2 Accumulated Thermal Energy

$$Q = \int_0^t q(t')A_{\text{heat}}dt'$$

Trapezoidal integration:

$$Q_{\text{accum}} = \sum_{i=1}^{N} \frac{q_i + q_{i-1}}{2}\Delta t_i A_{\text{heat}}$$

6.3 Heat Shield Heating

Without melting:

$$\Delta T = \frac{Q}{m_{\text{heat}}c_s}, \quad T_{\text{final}} = T_{\text{init}} + \Delta T$$

where $m_{\text{heat}} = \rho_{\text{heat}}t_{\text{heat}}A_{\text{heat}}$

With melting:

$$Q_{\text{met}} = m_{\text{heat}}c_s(T_{\text{met}} - T_{\text{init}})$$

$$Q_{\text{full met}} = m_{\text{heat}}L$$

7 Airship Parameter Calculation

7.1 Buoyancy Equation

$$V(\rho_2 - \rho_1) = m_{\text{envelope}} + m_{\text{payload}} + m_{\text{heat}}$$

7.2 Spherical Airship Geometry

$$V = \frac{4}{3}\pi R^3, \quad S = 4\pi R^2$$

$$m_{\text{envelope}} = \rho_{\text{envelope}}S = 4\pi R^2 \rho_{\text{envelope}}$$

7.3 Cubic Equation for Radius

$$\frac{4}{3}\pi Z R^3 = 4\pi D R^2 + P$$

where:

- $Z = \rho_2 - \rho_1$ - buoyancy force [kg/m³]

- $D = \rho_{\text{envelope}}$ - envelope areal density [kg/m²]

- $P = m_{\text{payload}} + m_{\text{heat}}$ - payload mass [kg]

Reduced form:

$$R^3 + aR^2 + c = 0$$

$$a = -\frac{3D}{Z}, \quad c = -\frac{3P}{4\pi Z}$$

7.4 Solution of Cubic Equation

Reduction:

$$y^3 + py + q = 0$$

$$p = -\frac{a^2}{3}, \quad q = \frac{2a^3}{27} + c$$

Discriminant:

$$\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$$

Case $\Delta \geq 0$:

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}$$

Case $\Delta < 0$:

$$\phi = \frac{1}{3}\arccos\left(-\frac{q}{2}\sqrt{-\frac{27}{p^3}}\right)$$

$$y = 2\sqrt{-\frac{p}{3}}\cos\phi$$

$$R = y - \frac{a}{3}$$

8 Structural Calculations

8.1 Frame Load

$$F_{\text{total}} = m_{\text{total}} a_{\text{max}}$$

where $a_{\text{max}} = n_{\text{max}} g_0$

8.2 Equivalent Pressure

$$p = \frac{F_{\text{total}}}{\pi D L}$$

8.3 Wall Thickness (Strength)

$$t_{\mathrm{min}} = \frac{pD}{2\sigma_{\mathrm{allow}}}$$

where $\sigma_{\mathrm{allow}} = \sigma_y / SF$

8.4 Stability (Buckling)

Moment of inertia:

$$I = \frac{\pi D^3 t}{8}$$

Critical buckling load:

$$P_{\mathrm{cr}} = \frac{\pi^2 EI}{(kL)^2}$$

where $k = 0.7$

8.5 Frame Mass

$$m_{\mathrm{frame}} = \pi DLt\rho_{\mathrm{mat}}$$

9 Thermodynamic Analysis of Gas Systems

9.1 Basic Notation and Assumptions

Symbol	Description
ρ_1 [kg/m³]	Internal gas mixture density
ρ_2 [kg/m³]	External gas density
P_1 [Pa]	Internal gas mixture pressure
P_2 [Pa]	External gas pressure
T_1 [K]	Internal gas mixture temperature
T_2 [K]	External gas temperature
M_1 [kg/mol]	Gas component 1 molar mass
M_2 [kg/mol]	Gas component 2 molar mass
M_{avg} [kg/mol]	Mixture average molar mass
ϕ_1	Volume fraction of gas 1 in mixture
F	Pressure ratio $F = P_1/P_2$
Z [kg/m³]	Useful load per cubic meter
R [J/(mol·K)]	Universal gas constant (8.314)
c_p [J/(kg·K)]	Specific heat at constant pressure
c_v [J/(kg·K)]	Specific heat at constant volume
$\gamma = c_p/c_v$	Adiabatic index

9.2 Basic Definitions

Average molar mass of gas mixture:

$$M_{\text{avg}} = M_2 + \phi_1(M_1 - M_2)$$

Internal gas density (ideal gas law):

$$\rho_1 = \frac{P_1 M_{\text{avg}}}{RT_1} = \frac{F P_2 M_{\text{avg}}}{RT_1}$$

External gas density:

$$\rho_2 = \frac{P_2 M_2}{RT_2}$$

9.3 Fundamental Buoyancy Equation

$$\rho_2 - \rho_1 = Z$$

or equivalently:

$$\frac{P_2 M_2}{RT_2} - \frac{P_1 M_{\text{avg}}}{RT_1} = Z$$

9.4 Derivation of T_1 for Isothermal Case $(T_2 = T_1)$

From buoyancy equation with $T_2 = T_1$:

$$\frac{P_2 M_2}{RT_1} - \frac{F P_2 M_{\text{avg}}}{RT_1} = Z$$

Solving for T_1:

$$T_1 = \frac{P_2(M_2 - F M_{\text{avg}})}{ZR}$$

9.5 Derivation of T_1 from Density Difference

From $\rho_2 - \rho_1 = Z$:

$$\rho_2 - Z = \rho_1 = \frac{F P_2 M_{\text{avg}}}{RT_1}$$

$$T_1 = \frac{F P_2 M_{\text{avg}}}{R(\rho_2 - Z)}$$

Substituting M_{avg}:

$$T_1 = \frac{F P_2[M_2 + \phi_1(M_1 - M_2)]}{R(\rho_2 - Z)}$$

9.6 Derivation of T_1 from Pressure-Temperature Relation

From fundamental equation:

$$\frac{P_2 M_2}{RT_2} - \frac{F P_2 M_{\text{avg}}}{RT_1} = Z$$

Rearranging:

$$\frac{FP_2M_{\text{avg}}}{RT_1} = \frac{P_2M_2}{RT_2} - Z$$

$$\frac{FP_2M_{\text{avg}}}{R} = T_1\left(\frac{P_2M_2}{RT_2} - Z\right)$$

$$T_1 = \frac{FP_2M_{\text{avg}}}{R\left(\frac{P_2M_2}{RT_2} - Z\right)}$$

$$T_1 = \frac{FP_2[M_2 + \phi_1(M_1 - M_2)]}{\frac{P_2M_2}{T_2} - ZR}$$

9.7 Volume Calculation for Buoyancy

Basic buoyancy condition:

$$V(\rho_2 - \rho_1) = m_{\text{payload}}$$

where:

- V [m^3] - Gas envelope volume

- m_{payload} [kg] - Payload mass

Volume formula:

$$V = \frac{m_{\text{payload}}}{Z}$$

where $Z = \rho_2 - \rho_1$

Expanded form:

$$V = \frac{m_{\text{payload}}}{\frac{P_2M_2}{RT_2} - \frac{FP_2[M_2 + \phi_1(M_1 - M_2)]}{RT_1}}$$

Isothermal case ($T_1 = T_2 = T$):

$$V = \frac{m_{\text{payload}}RT}{P_2[M_2 - F(M_2 + \phi_1(M_1 - M_2))]}$$

10 Appendix: Physical Constants

Parameter	Value	Units
g_0 (Venus)	8.87	m/s^2
R_V (Venus)	6.0518×10^6	m
R (gas constant)	8.314	J/(mol·K)