Computational Neurodynamics

Topic 11
Synchronous Oscillation Analysis

Murray Shanahan

Overview

- Measuring synchrony
- Idealised oscillators

The Analysis Pipeline

- Synchronisation can be assessed formally in terms of the *synchronisation index* ϕ, which we'll now define
- But this requires the extraction of the phase of the data

Firings ⇨ Moving average ⇨ *Mean firing rates* ⇨ Fourier analysis ⇨ *Power spectrum*
(Is the data periodic?)

Mean firing rates ⇩ Hilbert transform ⇨ *Phase* ⇨ Sync. analysis ⇨ *Sync. index* ϕ

- If we know the data is periodic, we can extract its phase using the *Hilbert transform*

Extracting Phase 1

- A time series $X(t)$ that is periodic about the origin (zero) can be mapped into the complex plane as follows

$$\zeta(t) = X(t) + iX_{\mathrm{H}}(t) = A(t)e^{i\theta(t)}$$

where $A(t)$ is the instantaneous amplitude of X at time t, $\theta(t)$ is the instantaneous phase of X at t, and X_{H} is the Hilbert transform of X at t

- So, given $X_{\mathrm{H}}(t)$, we have

$$\theta(t) = \tan^{-1}\left(\frac{X_{\mathrm{H}}(t)}{\mathrm{X(t)}}\right)$$

Extracting Phase 2

- The Hilbert transform of $X(t)$ is defined as

$$X_H(t) = \frac{1}{\pi}\,\mathrm{P.V.}\int_{-\infty}^{\infty} \frac{X(\tau)}{t-\tau}\,d\tau$$

where "P.V." indicates that we are taking the Cauchy principal value of the integral

- Fortunately, the Matlab signal processing toolbox provides a function to compute this directly
- The Hilbert transform of a time series represented as a vector `X` is given by `hilbert(X)`, and the instantaneous phase is given by `angle(hilbert(X))`

Quantifying Synchrony 1

- The instantaneous level of synchrony ϕ_c in a community c of oscillators can be quantified by

$$\phi_c(t) = \left| \left\langle e^{i\theta_k(t)} \right\rangle_{k \in c} \right|$$

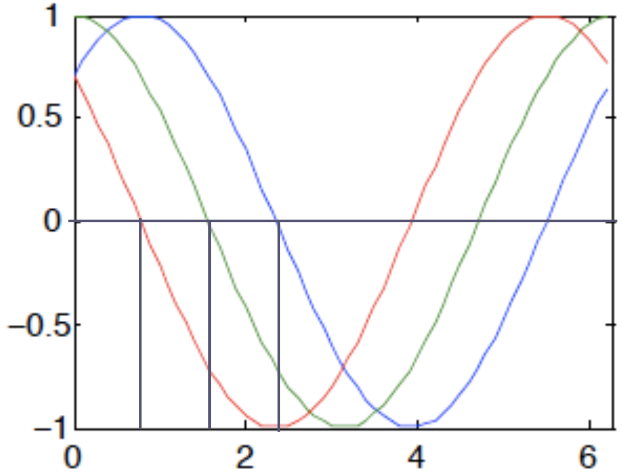
where $\theta_k(t)$ is the instantaneous phase of oscillator k, $\langle\ \rangle$ denotes the average and $|\ |$ denotes the absolute value

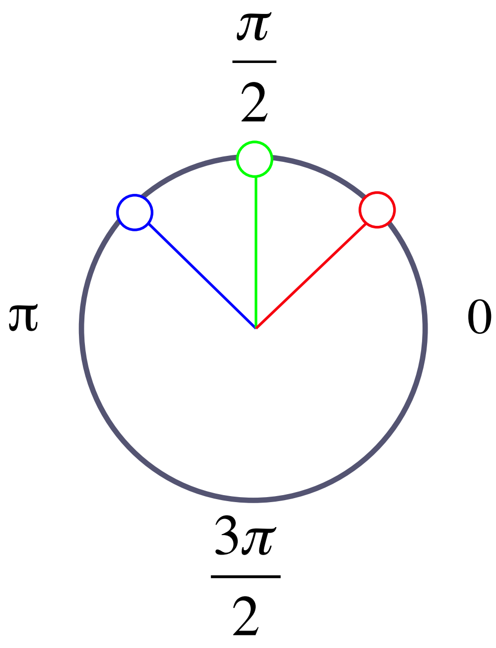
- Note that (according to the Euler formula)

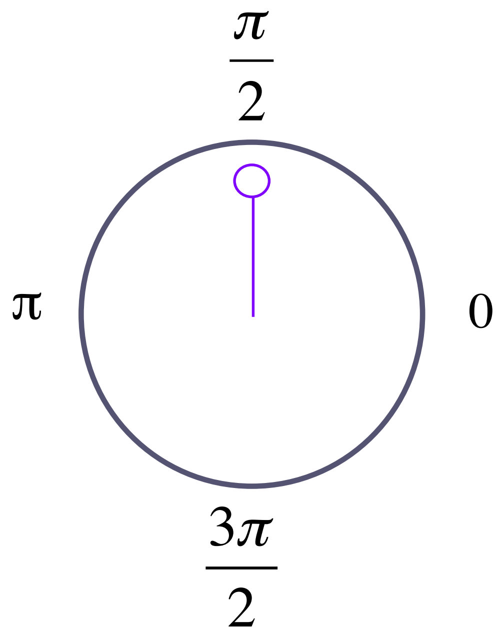
$$e^{i\theta} = \cos\theta + i\sin\theta$$

Quantifying Synchrony 2

Example







$$\theta_1 = 0.7854$$

$$\theta_2 = 1.5708$$

$$\theta_3 = 2.3562$$

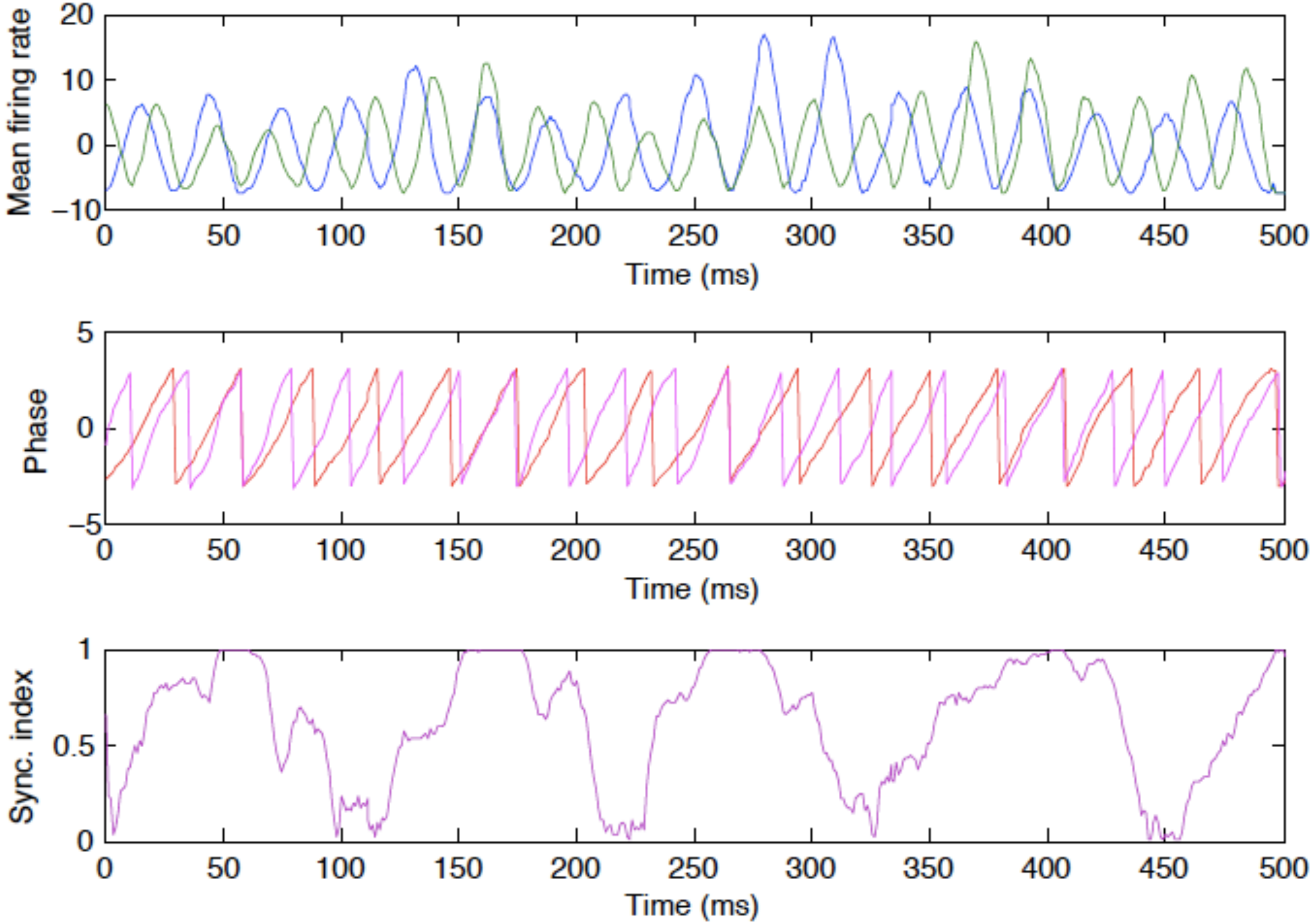
$$\left\langle e^{i\theta_k} \right\rangle_{k \in \{1,2,3\}} = 0 + 0.8047i \qquad |0 + 0.8047i| = 0.8047$$

$$\phi = 0.8047$$

Preprocessing the Data

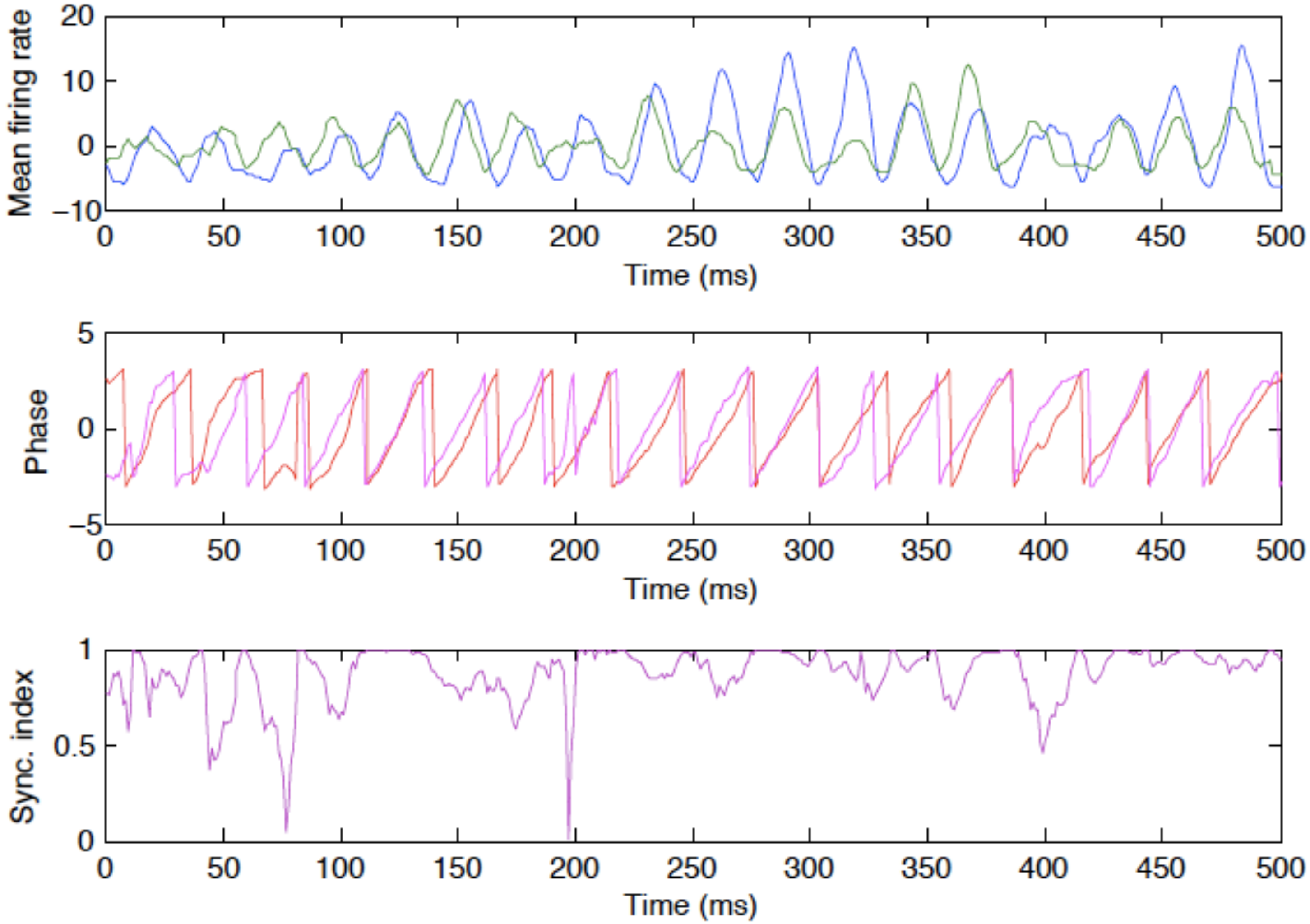
- Although not shown on our analysis pipeline, it may be necessary to filter the data before applying the Hilbert transform
- For example, if the data has high gamma power, and we are interested in gamma-band synchronisation, we can apply a *band-pass filter* that removes frequencies above and below the gamma band
- Also, the data has to be *periodic about the origin*. This means it has to oscillate around zero
- However, there's no such thing as a negative firing rate, so our data is all positive
- To render it suitable for the Hilbert transform, we can subtract the mean of the time series from all the data

ϕ in Decoupled Oscillators



- Let's return to the two populations generating gamma oscillations
- This is the uncoupled case, where the oscillators run free
- It's clear that ϕ tracks instantaneous synchrony

ϕ in Coupled Oscillators



- This is the coupled case, where the excitatory neurons of one population are connected to inhibitory neurons of the other
- Again we csn see that ϕ tracks synchrony, which is much higher

Averaging ϕ

- We can see that the synchronisation index ϕ is effective at tracking the instantaneous synchrony between the two populations
- In the case of two populations, we get lots of synchronisation even in the decoupled case, because the oscillators spend equal time in every phase relationship with each other
- In fact, if we take the mean ϕ over time, we get $\phi = 0.8697$ for the coupled case, which is high as we might expect. But we get $\phi = 0.6221$ even for the decoupled case
- However, this example includes just two oscillators, and of course, the measure applies to more than two oscillators. As the number of oscillators goes up, the average ϕ becomes an increasingly accurate indicator of synchronisation

Idealised Oscillators

- In order to model synchronisation phenomena at a higher level of abstraction, we can consider communities of idealised oscillators, ignoring the details of neuron firing
- There are a number of applicable oscillator models. We will use the *Kuramoto model*, which captures phase dynamics and ignores amplitude
- We can use the Kuramoto model to study various phenomena of relevance to brain dynamics
  - Synchronisation in complex networks (modular and small-world)
  - Chimera states (in which synchronisation and desynchronisation co-exist)
  - Metastability (highly synchronised states arise but are temporary)

Kuramoto Oscillators

- In a system of N Kuramoto oscillators, the phase θ_i of oscillator i is given by

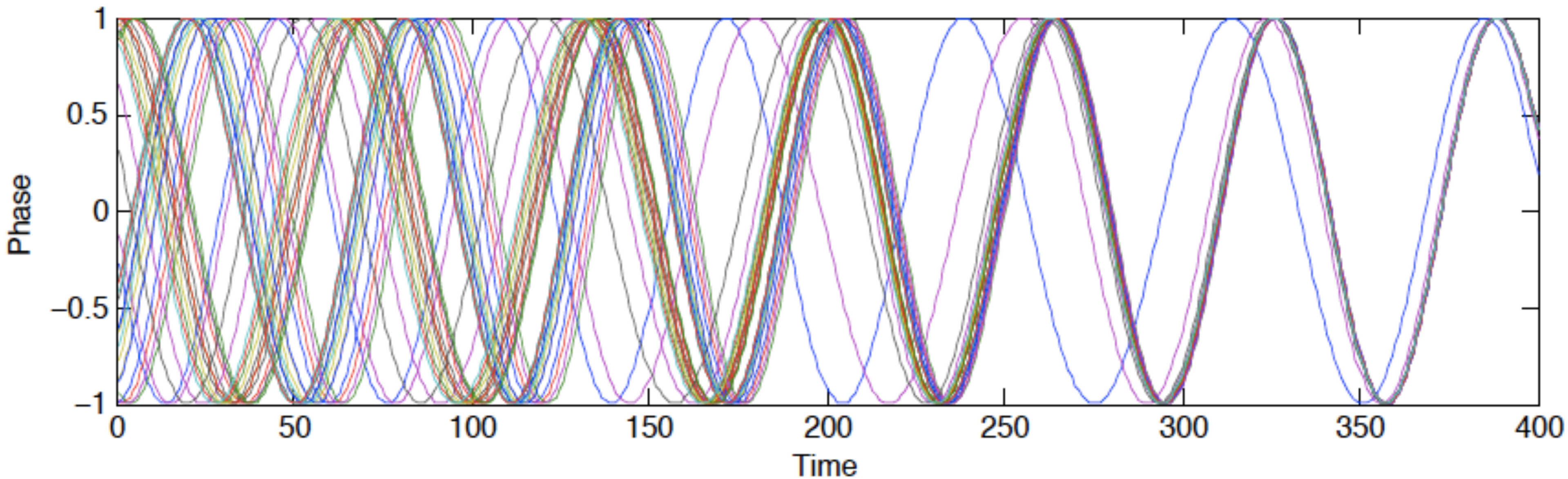
$$\frac{d\theta_i}{dt} = \omega_i + \frac{1}{N}\sum_{j=1}^{N} K_{i,j}\sin(\theta_j - \theta_i - \alpha)$$

where

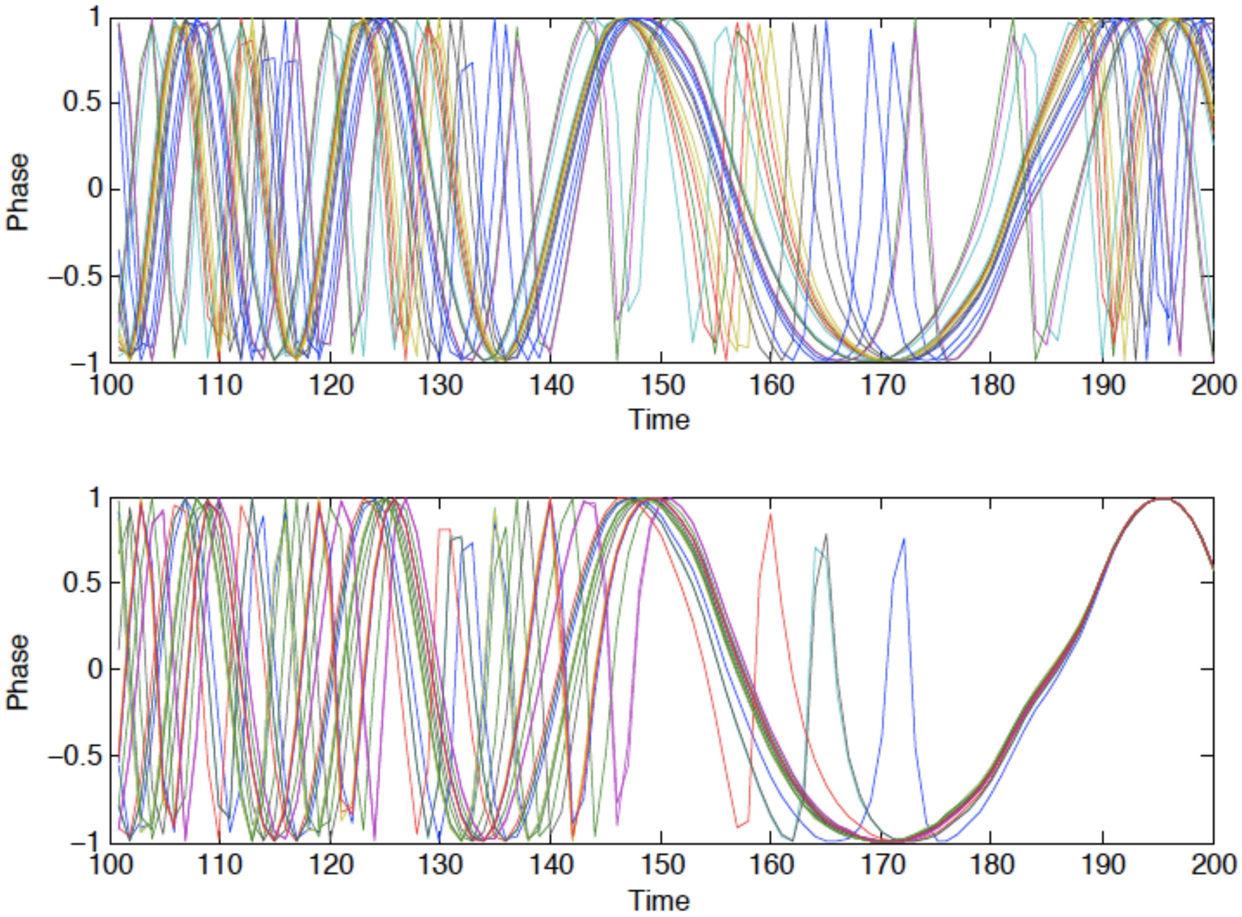
- ω_i is the natural frequency of the oscillator,
- α is a constant phase lag, and
- $K_{i,j}$ is the coupling strength from oscillator j to oscillator i

Idealised Synchronisation

- A system of weakly-coupled Kuramoto oscillators with similar natural frequencies will synchronise
- Here we have a system of 32 oscillators with zero phase lag, identical natural frequencies (ω_i = 0.1), and all-to-all coupling ($K_{i,j}$ = 0.02)

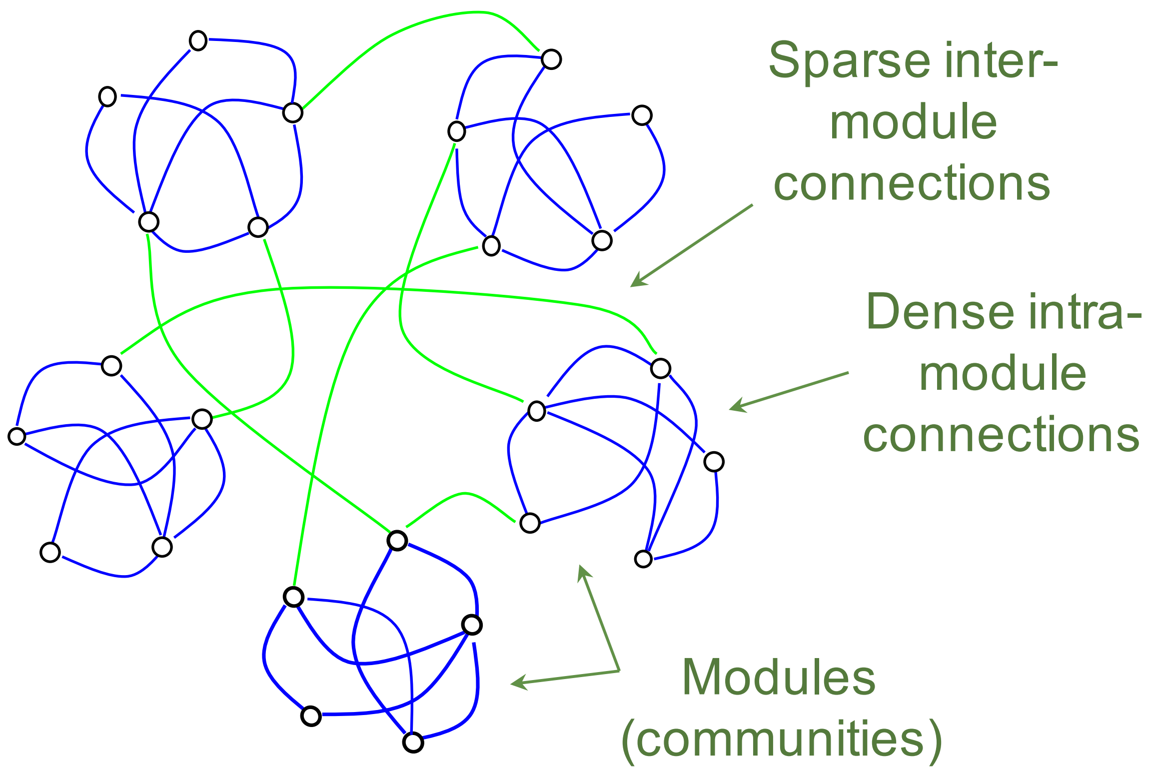


Chimera States



- If the topology of connections is right then, under certain circumstances, a system of identical oscillators spontaneously partitions into synchronised and desynchronised subsets
- This phenomenon of *chimera states* was only discovered in 2002

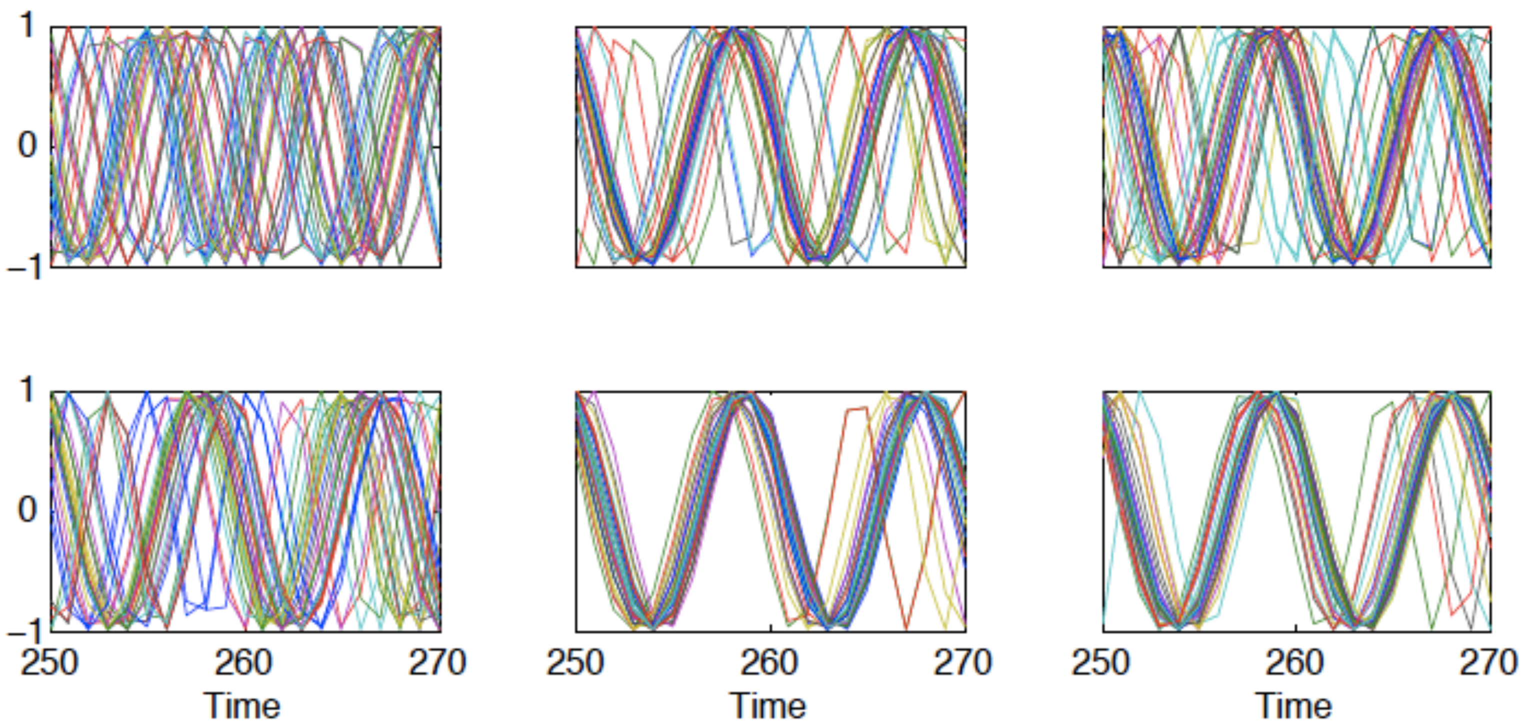
Community Structure



- We'll consider a system of eight communities, each comprising 32 identical phase-lagged Kuramoto oscillators
- All-to-all connectivity within each community
- 32 inter-community (slightly weaker) symmetrical connections per oscillator, randomly chosen
- This yields a modular small-world network similar to those we've seen before

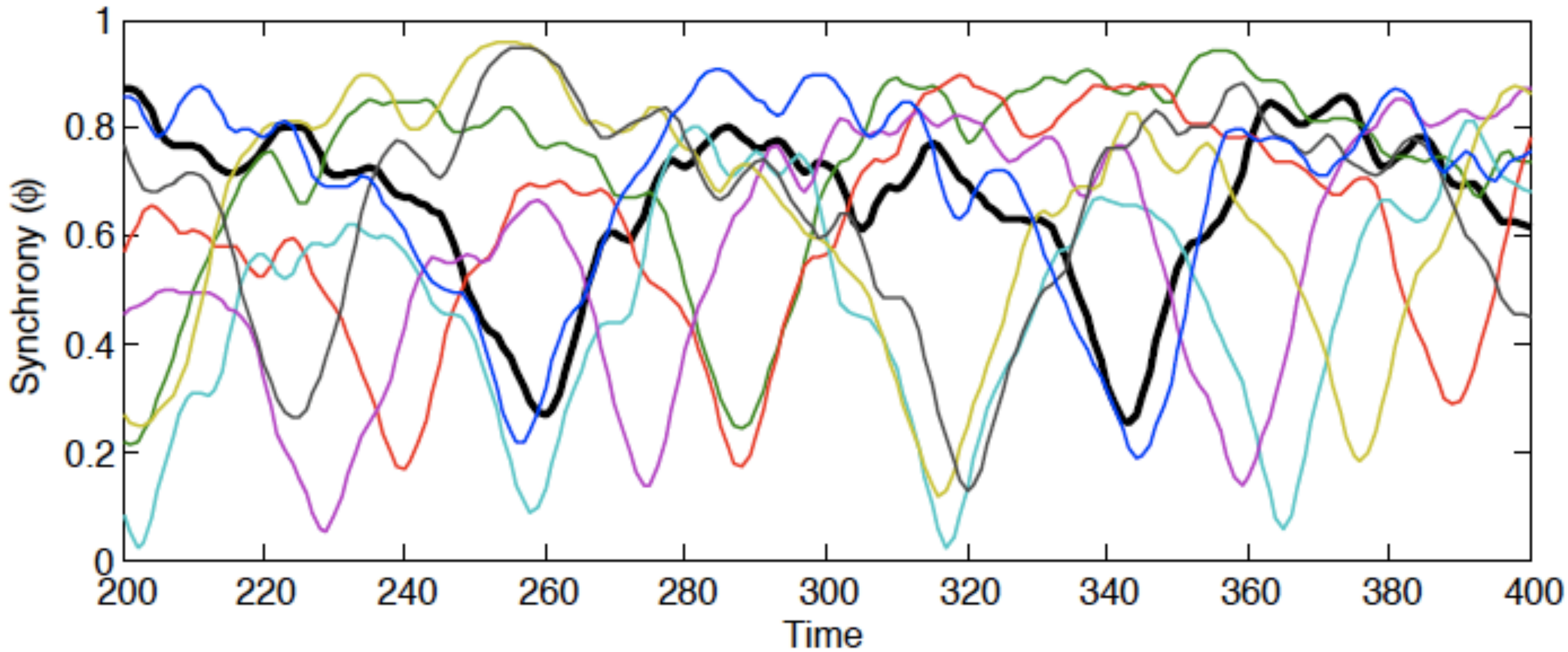
Chimera States and Community Structure

- If the phase lag α is close to but slightly smaller than $\pi/2$, then chimera-like states arise



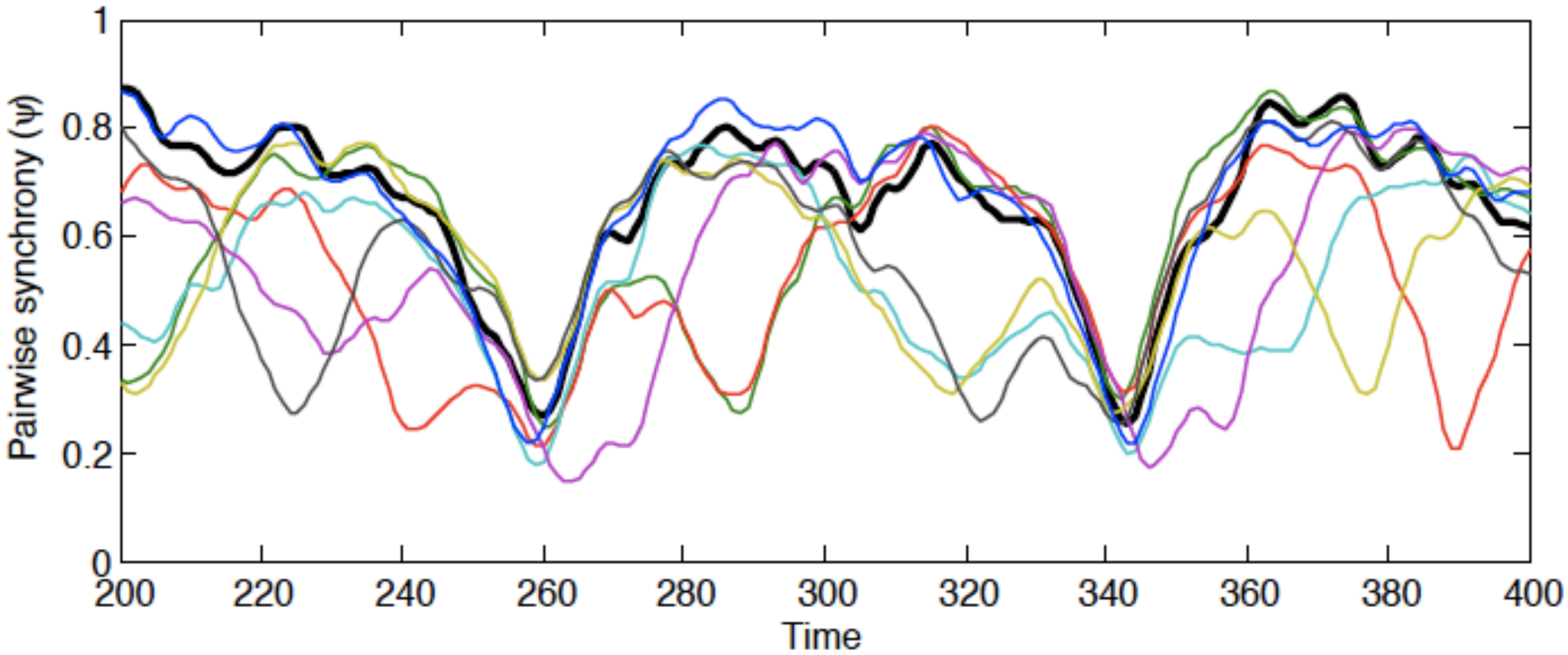
Metastability

- When we examine the time-course of synchronisation, we might expect the chimera state to form, then to stabilise, or to fluctuate regularly
- But this is not what we see. Rather, the chimera-like states come and go in irregular patterns



Inter-community Synchrony

- When two communities are highly internally synchronised they tend to be highly synchronised with each other
- To see this we can track pairwise synchrony wrt a selected community



Useful Measures

$$\sigma_{\text{chi}}(t) = \frac{1}{M-1}\sum_{c \in C}\left(\phi_c(t) - \langle \phi(t) \rangle_C\right)^2$$

$$\sigma_{\text{met}}(c) = \frac{1}{T-1}\sum_{t \le T}\left(\phi_c(t) - \langle \phi_c \rangle_T\right)^2$$

$$\chi = \langle \sigma_{\text{chi}} \rangle_T$$

$$\lambda = \langle \sigma_{\text{met}} \rangle_C$$

…

Spatial variation (σ_{chi}) in ϕ = chimera state

Temporal variation (σ_{met}) in ϕ = metastability

- Metastability can be quantified as the mean of the temporal variance of synchrony
- The mean of the spatial variance of synchrony says how chimera-like its states are

Measuring Metastability

- More precisely, if we have a set c of oscillators (a community), then the synchrony within c at time t is $\phi_c(t)$ as defined earlier
- The metastability within c is the variance of the synchrony over time, given by

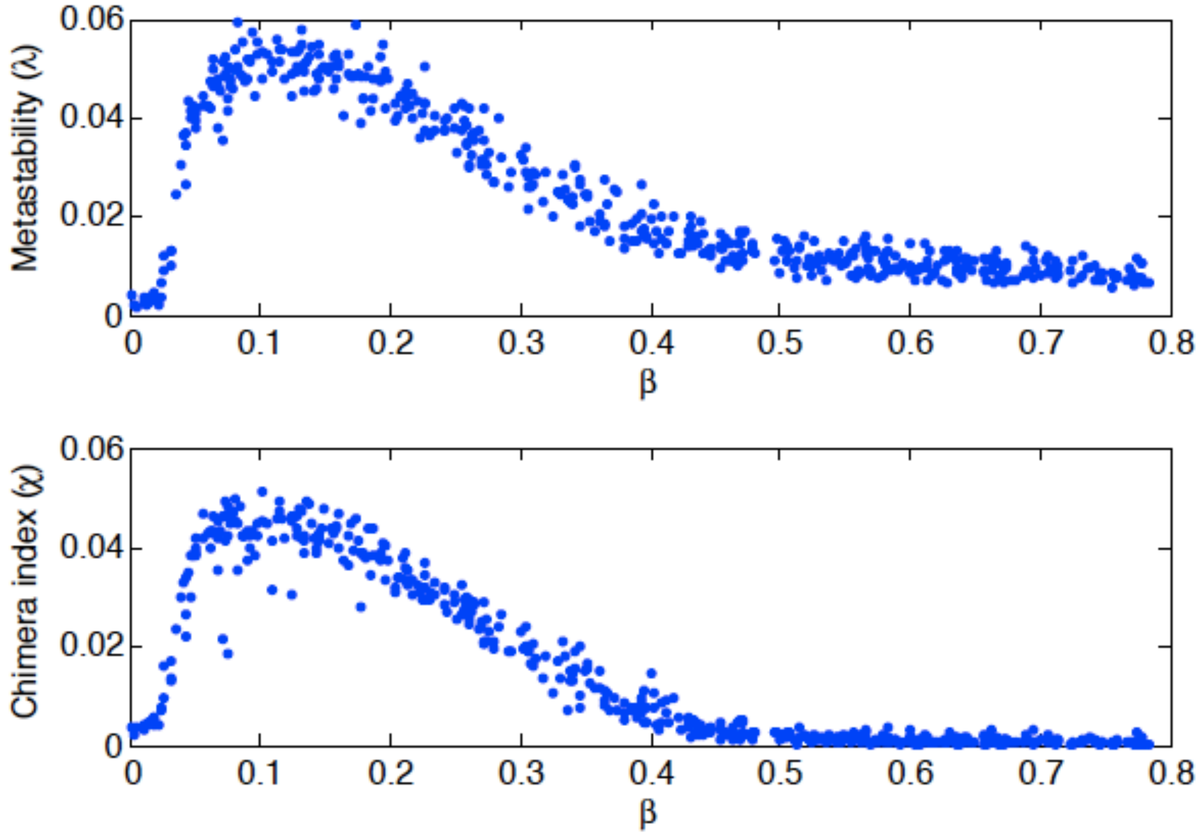
$$\sigma_{\text{met}}(c) = \frac{1}{T-1} \sum_{t \leq T} (\phi_c(t) - \langle \phi_c \rangle_T)^2$$

where T is the number of time points and $\langle \phi_c \rangle_T$ is the mean synchrony over time

- We take the mean over all communities C, to give the *metastability index* λ

$$\lambda = \langle \sigma_{\text{met}} \rangle_C$$

Peak Complexity

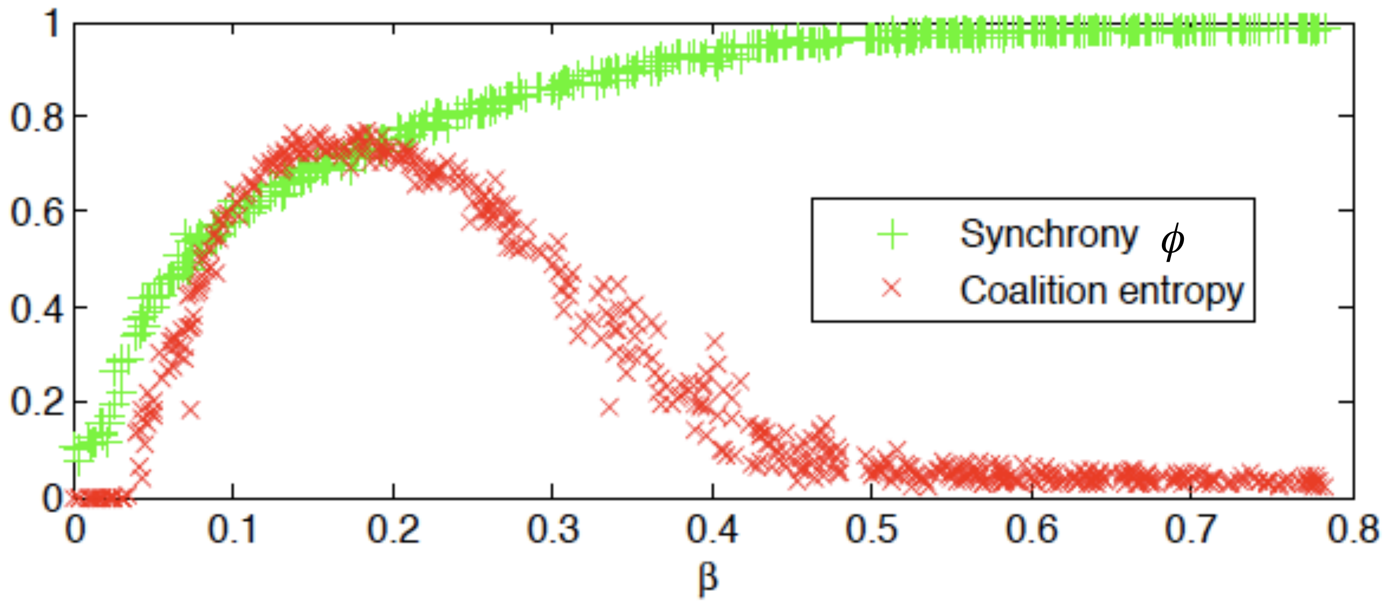


$$\beta = \pi/2 - \alpha$$

- Metastable chimera states only arise in a narrow region of parameter space, when the phase lag α is close to $\pi/2$
- We also get a peak of coalition entropy at this point

Criticality and Complexity

- Metastable chimera states are a *criticality* phenomenon, nestled between order and disorder.
  - If α is too high we get total desynchronisation (disorder)
  - If α is too low we get total synchronisation (order)



ϕ is synchrony averaged over time and communities

Related Reading

Pikovsky, A., Rosenblum, A. & Kurths, J. (2001). *Synchronization: A Universal Concept in Nonlinear Systems*. Cambridge University Press.

Shanahan, M.P. (2010). Metastable Chimera States in Community-structured Oscillator Networks. *Chaos* 20, 013108.

Cabral, J., Hugues, E., Sporns, O. & Deco, G. (2011). Role of Local Network Oscillations in Resting-state Functional Connectivity. *NeuroImage* 57, 130–139.

Hellyer, P., Shanahan, M., Scott, G., Wise, R.J.S., Sharp, D.J. & Leech, R. (2014). The Control of Global Brain Dynamics: Opposing Actions of Frontoparietal Control and Default Mode Networks on Attention. *Journal of Neuroscience* 34 (2), 451–461.