Leg Diagram

Inverse Kinematics derivations

→ $h = h_A + h_B$

→ $h_A = \tan^{-1}\left(\frac{x}{z}\right)$

→ Use Law of cosines to find h_B

→ $L_2^2 = L_1^2 + Z_2^2 - L_1 Z_2 \cos(h_B)$

$L_1 Z_2 \cos(h_B) = L_1^2 + Z_2^2 - L_2^2$

$$\cos(h_B) = \frac{L_1^2 + Z_2^2 - L_2^2}{L_1 Z_2}$$

$$h_B = \cos^{-1}\left(\frac{L_1^2 + Z_2^2 - L_2^2}{L_1 Z_2}\right)$$

→ $\boxed{h = \tan^{-1}\left(\frac{x}{z}\right) + \cos^{-1}\left(\frac{L_1^2 + Z_2^2 - L_2^2}{L_1 Z_2}\right)}$

→ $K + K' = 180°$

→ $K = 180° - K'$

→ Use law of cosines to find K'

→ $Z_2^2 = L_1^2 + h_2^2 - L_1 h_2 \cos(K')$

→ $L_1 h_2 \cos(K') = L_1^2 + h_2^2 - Z_2^2$

→ $$\cos(K') = \frac{L_1^2 + L_2^2 - Z_2^2}{L_1 h_2}$$

→ $\boxed{K' = \cos^{-1}\left(\frac{L_1^2 + h_2^2 - Z_2^2}{L_1 h_2}\right)}$

→ $a + a_a + a_b = 180°$

→. $a = 180° - a_a - a_b$

→ Use Law of cosines to find a_b

$$L_1^2 = z_2^2 + L_2^2 - z_2 L_2 \cos(a_b)$$

→ $z_2 L_2 \cos(a_b) = z_2^2 + L_2^2 - L_1^2$

→ $\cos(a_b) = \dfrac{z_2^2 + L_2^2 - L_1^2}{L_2 z_2}$

→ $a_b = \cos^{-1}\left(\dfrac{z_2^2 + L_2^2 - L_1^2}{L_2 z_2}\right)$

→ $a_a = \tan^{-1}\left(\dfrac{z}{x}\right)$

→ $\boxed{a = 180° - \tan^{-1}\left(\dfrac{z}{x}\right) - \cos^{-1}\left(\dfrac{z_2^2 + L_2^2 - L_1^2}{L_2 z_2}\right)}$