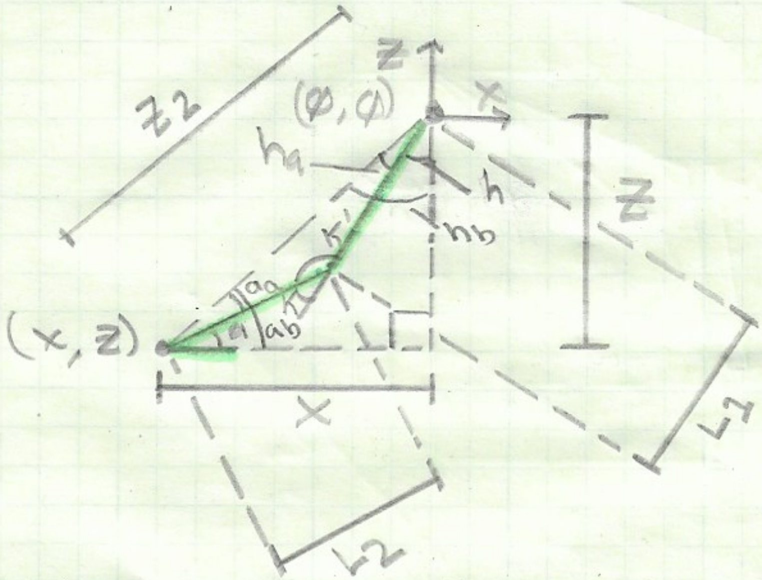
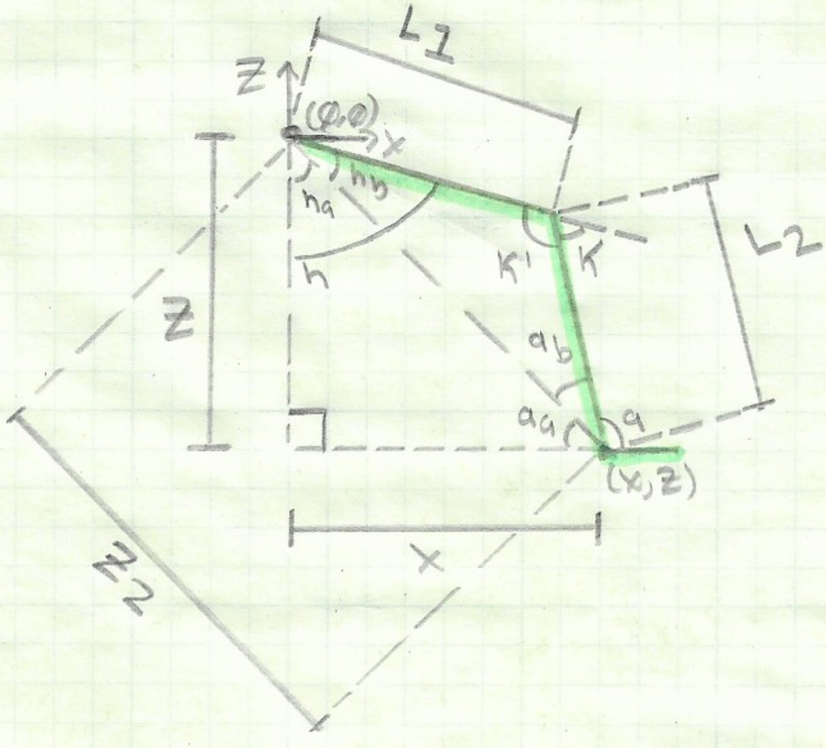
Given; a) Back leg to thrust robot forward:

→



b) Front leg to take a step:



Solution: a) Find angle K in terms of x, z, L_1, and L_2.

→ $K + K' = 180°$

→ $K = 180° - K'$

→ Use Law of cosines to find K':

- $Z_2^2 = L_1^2 + L_2^2 - 2L_1L_2\cos(K')$

→ $2L_1L_2\cos(K') = L_1^2 + L_2^2 - Z_2^2$

→ $$\cos(K') = \frac{L_1^2 + L_2^2 - Z_2^2}{2L_1L_2}$$

→ Use the pythagorean theorem to find Z_2 and z^2:

- $Z_2^2 = x^2 + z^2$
- $Z_2 = (x^2 + z^2)^{1/2}$

→ $$K' = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - (x^2 + z^2)}{2L_1L_2}\right)$$

→ $$\boxed{K = 180° - \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - z^2}{2L_1L_2}\right)}$$

Solution (cont.): aii) Find angle a in terms of x, Z, L_1, and L_2.

→ Use the Law of Cosines to find angle a_a:

- $L_1^2 = L_2^2 + Z_2^2 - 2L_2Z_2\cos(a_a)$

→ $2L_2Z_2\cos(a_a) = L_2^2 + Z_2^2 - L_1^2$

→ $$a_a = \cos^{-1}\left(\frac{L_2^2 + Z_2^2 - L_1^2}{2L_2Z_2}\right)$$

→ Recall that:

- $Z_2^2 = x^2 + z^2$
- $Z_2 = (x^2 + z^2)^{1/2}$

→ $$a_a = \cos^{-1}\left(\frac{L_2^2 + x^2 + z^2 - L_1^2}{2L_2(x^2+z^2)^{1/2}}\right)$$

→ Recall:

- $\tan(a_b) = (\text{opposite})(\text{adjacent})^{-1} = \frac{z}{x}$

→ $a_b = \tan^{-1}\left(\frac{z}{x}\right)$

→ $a_b = a + a_a = \tan^{-1}\left(\frac{z}{x}\right)$

→ $$\boxed{a = \tan^{-1}\left(\frac{z}{x}\right) - \cos^{-1}\left(\frac{L_2^2 + x^2 + z^2 - L_1^2}{2L_2(x^2+z^2)^{1/2}}\right)}$$

Solution (cont.): a iii) Find angle h in terms of x, z, L_1, and L_2.

→ Use Law of cosines to find ha:

• $L_2^2 = L_1^2 + Z_2^2 - 2L_1 Z_2 \cos(h_a)$

→ $2L_1 Z_2 \cos(h_a) = L_1^2 + Z_2^2 - L_2^2$

→ $$h_a = \cos^{-1}\left(\frac{L_1^2 + Z_2^2 - L_2^2}{2L_1 Z_2}\right)$$

→ $$h_a = \cos^{-1}\left(\frac{L_1^2 + x^2 + z^2 - L_2^2}{2L_1(x^2+z^2)^{1/2}}\right)$$

→ $180^\circ = 90^\circ + a_b + h + h_a$

→ $h = 90^\circ - a_b - h_a$

→ $$\boxed{h = 90^\circ - \tan^{-1}\left(\frac{z}{x}\right) - \cos^{-1}\left(\frac{L_1^2 + x^2 + z^2 - L_2^2}{2L_1(x^2+z^2)^{1/2}}\right)}$$

Solution (cont.): b) Find angle K in terms of x, z, L_1, and L_2.

→ $180° = K + K'$

→ $K = 180° - K'$

→ Use the Law of cosines to find K':

- $Z_2^2 = L_1^2 + L_2^2 - 2L_1L_2\cos(K')$

→ $2L_1L_2\cos(K') = L_1^2 + L_2^2 - Z_2^2$

→ $$\cos(K') = \frac{L_1^2 + L_2^2 - Z_2^2}{2L_1L_2}$$

→ $$K' = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - Z_2^2}{2L_1L_2}\right)$$

→ Use the pythagorean theorem to find Z_2^2 and Z_2:

- $Z_2^2 = x^2 + z^2$
- $Z_2 = (x^2 + z^2)^{1/2}$

→ $$K' = \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - z^2}{2L_1L_2}\right)$$

→ $$\boxed{K = 180° - \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - z^2}{2L_1L_2}\right)}$$

Solution (cont.): b ii) Find angle a in terms of x, Z, L_1, and L_2.

$\rightarrow$ $180° = a_a + a_b + a$

$\rightarrow$ Recall that:

- $\tan(a_a) = \frac{\text{opposite}}{\text{Adjacent}} = \frac{Z}{x}$

$\rightarrow$ $a_a = \tan^{-1}\left(\frac{Z}{x}\right)$

$\rightarrow$ Use Law of cosines to find a_b:

- $L_1^2 = Z_2^2 + L_2^2 - 2L_2Z_2\cos(a_b)$

$\rightarrow$ $2L_2Z_2\cos(a_b) = Z_2^2 + L_2^2 - L_1^2$

$\rightarrow$ $\cos(a_b) = \dfrac{Z_2^2 + L_2^2 - L_1^2}{2L_2Z_2}$

$\rightarrow$ $a_b = \cos^{-1}\left(\dfrac{x^2+Z^2+L_2^2-L_1^2}{2L_2(x^2+Z^2)^{1/2}}\right)$

$\rightarrow$ $\boxed{a = 180° - \cos^{-1}\left(\dfrac{x^2+Z^2+L_2^2-L_1^2}{2L_2(x^2+Z^2)^{1/2}}\right) - \tan^{-1}\left(\dfrac{Z}{x}\right)}$

Solution (cont.): biii) Find angle h in terms of x, y, L1, and L2.

→ $h = h_a + h_b$

→ $h_b + k' + a_b = 180°$

→ $h_b = 180° - k' - a_b$

$$= 180° - \cos^{-1}\left(\frac{x^2+z^2+L_2^2-L_1^2}{2L_2(x^2+z^2)^{1/2}}\right) - \cos^{-1}\left(\frac{L_1^2+L_2^2-x^2 z^2}{2L_1L_2}\right)$$

→ Recall that:

- $\tan(h_a) = \frac{x}{z}$
- $h_a = \tan^{-1}\left(\frac{x}{z}\right)$

→ $$\boxed{h = 180° - \cos^{-1}\left(\frac{x^2+z^2+L_2^2-L_1^2}{2L_2(x^2+z^2)^{1/2}}\right) - \cos^{-1}\left(\frac{L_1^2+L_2^2-x^2-z^2}{2L_1L_2}\right) + \tan^{-1}\left(\frac{x}{z}\right)}$$

Summary of Equations:

a) Equations for the back leg to thrust robot forward

$$\rightarrow a = \tan^{-1}\left(\frac{z}{x}\right) - \cos^{-1}\left(\frac{L_2^2 + x^2 + z^2 - L_1^2}{2L_2(x^2+z^2)^{1/2}}\right)$$

$$h = 90^\circ - \tan^{-1}\left(\frac{z}{x}\right) - \cos^{-1}\left(\frac{L_1^2 + x^2 + z^2 - L_2^2}{2L_1(x^2+z^2)^{1/2}}\right)$$

$$K = 180^\circ - \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - z^2}{2L_1L_2}\right)$$

b) Equations for the front leg to take a step

$$\rightarrow a = 180^\circ - \cos^{-1}\left(\frac{x^2 + z^2 + L_2^2 - L_1^2}{2L_2(x^2+z^2)^{1/2}}\right) - \tan^{-1}\left(\frac{z}{x}\right)$$

$$h = 180^\circ - \cos^{-1}\left(\frac{x^2 + z^2 + L_2^2 - L_1^2}{2L_2(x^2+z^2)^{1/2}}\right) - \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - z^2}{2L_1L_2}\right) + \tan^{-1}\left(\frac{x}{z}\right)$$

$$K = 180^\circ - \cos^{-1}\left(\frac{L_1^2 + L_2^2 - x^2 - z^2}{2L_1L_2}\right)$$